THE CONCEPT OF ASYMPTOTIC COEFFICIENTS OF VARIATION OF STRUCTURAL RESPONSE APPLIED FOR FOUNDATION DESIGN

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In this paper, firstly basic concepts of the structural reliability will be summarized in terms of two basic variables, i.e. structural response (R) and load effect (S). The uncertainty in structural response could be statistically characterized by mean and coefficient of variation (Ω_R). Based on these formulations, there must be an upper limit of Ω_R for the pre-specified acceptable level of reliability (p_f). The increment of coefficient of variation of load effect (Ω_S) shows minor influence on the central factor of safety (FS) and its effect diminishes rapidly where Ω_R approaches the upper limit. Below this limit, the structural system could be used safely for a pre-specified target reliability. For lower value of Ω_R, the target FS could be determined from the quadratic relationship between Ω_R and Ω_S.

The structural response for foundations is typically a function of soil properties, sections and dimensions. It is not uncommon that uncertainties in soil properties could be normal or non-normal probability distribution and the relationship among basic variables in forming the structural response could be either non-linear or so complicated that results could be obtained from finite element analyses only. Fortunately, the randomness of structural response could be obtained by Monte Carlo simulation technique. Then the fitted distribution of outcome experiments could be specified by Goodness-of-Fit tests. The applicability of proposed concepts could be demonstrated in numerical examples, e.g. driven pile, spread footing and bored pile. For the conventional design approach, soil parameters are considered to be constant. The solution is simplified thorough the use of deterministic safety factor. In reality, soil is neither isotropic nor homogeneous such that their uncertainties could not be ignored. References to the calculated failure probability evidence that deterministic safety factor could not guarantee enough safety. In some cases, an FS of 3 or more is not considered too conservative to apply for the structural response.

Key words : reliability, structural response, central factor of safety, Monte Carlo simulation

1. Introduction

In recent study [1], the reliability analyses revealed that there were asymptotic coefficients of variation for structural responses (Ω_R) against corresponding target values of structural reliability. The asymptote for Ω_R is independent of variation of load effect (Ω_S). Below this upper limit, structures could resist safely with the acceptable level of risk or failure probability. The central factor of safety (*FS*) for normal variate could be determined from the quadratic relationship between Ω_R and Ω_S. Extended studies [2] confirmed that the concept of asymptotic coefficients of variation could be generalized for non-normal variates

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and any kind of system. Therefore, it is the aim of this paper to apply the concepts of limit of Ω_R and influence of uncertainties in soil parameters on *FS* for the foundation design.

2. Limit state function

For time invariant reliability analyses, the structural reliability (p_s) may be defined as the probability that the structural response (R) will not be exceeded by load effect (S) within the whole service life as shown in Eq. (1). In this case, the limit state function of a structural system may be defined as Eq. (2).

$$p_s = \Pr(R - S > 0) \; , \tag{1}$$

$$g(\mathbf{X}) = R - S \; , \tag{2}$$

where $\mathbf{X}$ is a vector of basic random variables. $g(x) > 0$ defines safe state of structural system and defines failure state, otherwise, as shown in Fig. 1.
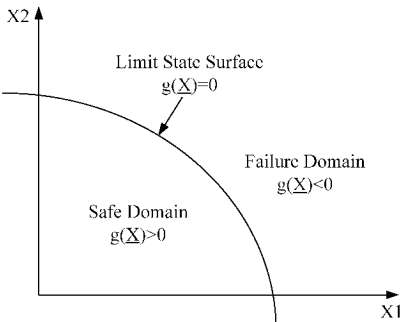

Fig.1: Failure and safe domain for two random variables

The limit state can be classified into three types as followed: 1) an ultimate limit state for structural safety which may correspond to the loss of equilibrium, rupture and fatigue; 2) a serviceability limit state means an excessive deformation such as crack, obvious oscillation or vibration, loss of durability or visual damage and 3) a limit state in other forms.

3. Failure probability, safety index and safety factor

For the case of two random variables, the limit state function for a structural system may be simply defined as in Eq. (2). For the limit state, where $g(\mathbf{x}) = 0$, the failure probability can be obtained directly from the z-score as:

$$p_f = \Phi\left(\frac{0 - \mu_g}{\sigma_g}\right) = \Phi(-\beta) = \Phi(-z) \; , \tag{3}$$

$$\beta = \frac{\mu_g}{\sigma_g} \; , \tag{4}$$

where $\Phi(\cdot)$ is the distribution function (*cdf*) of the standard normal variable. The term β is well-known as the safety or reliability index. Relationships between the values of failure probability and the safety indices are summarized in Table 1.

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-6}	10^{-8}	10^{-10}
β	1.370	2.326	3.090	3.719	4.753	5.612	6.361

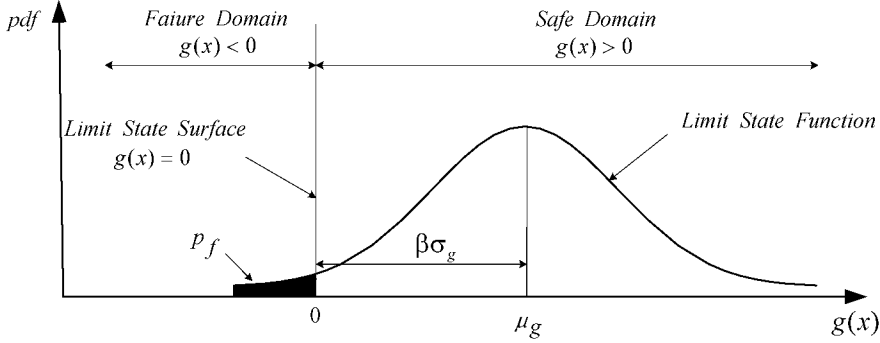*Tab.1: Relationship between safety index and failure probability*

Fig.2: Relationship between safety index and failure probability

Fig. 2 interprets the probability of failure depending on the ratio of the mean value of the limit state function to its standard deviation. The higher value of β implies lower value of failure probability. In the conventional design approach, the central factor of safety (FS) is defined as the ratio of the mean values of the structural response and load effect.

$$FS = \frac{\mu_R}{\mu_S} \; . \tag{5}$$

4. Limit Ω_R for normal variate

Let R and S be normally distributed. The safety index may be rewritten as in Eq. (6).

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \; . \tag{6}$$

A standard measure for the dispersion about mean is the coefficient of variation (Ω). By defining $\Omega_R = \sigma_R/\mu_R$ and $FS = \mu_R/\mu_S$, then Eq. (6) can be further rewritten as:

$$FS - 1 = \beta \sqrt{FS^2 \, \Omega_R^2 + \Omega_S^2} \; . \tag{7}$$

The relationships between FS and Ω_R for given values of Ω_S can be shown in Fig. 3.

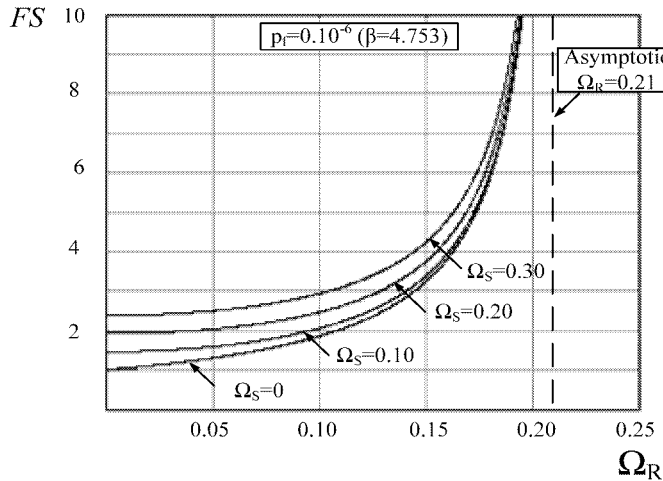Fig.3: Relationship between Ω_R and FS for $p_f = 10^{-6}$

Fig. 3 shows that the asymptote for Ω_R is independent of variation of Ω_S. Furthermore, the value of FS tends to increase rapidly with respect to Ω_R. Whereas, the increment of Ω_S shows minor influence on FS and its effect diminishes rapidly where Ω_R approaches the upper limit. However, lower values of FS and Ω_R are practical interests. For lower value of Ω_R, the structural system could be used safely for a pre-specified target reliability with lower bound of factor of safety as in the case of $\Omega_S = 0$. In fact, material properties are random in nature and Ω_R could be higher than the limit of Ω_R. For these cases, the structural system might not be safe for the expected level of reliability. The limit values of Ω_R with respect to corresponding values of p_f and β are summarized in Table 2.

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-6}	10^{-8}	10^{-10}
β	1.370	2.326	3.090	3.719	4.753	5.612	6.361
Ω_R	0.730	0.430	0.324	0.269	0.210	0.178	0.157

Tab.2: Limits of Ω_R against p_f and β for normal variate

5. Limit of Ω_R for non-normal structural response

In fact, the relationship among basic variables in the structural response could be either linear or non-linear. It is not uncommon that the randomness of the structural response could be non-normal distribution. If the structural response is not normally distributed the limit of Ω_R may be obtained using the equivalent normal concepts. The distribution function (*cdf*) and the probability density function (*pdf*) of the non-normal distribution should be equal to those of the corresponding equivalent normal distribution at the checking point (x^*) [3]. The concept of equivalent normal variable for determining limit Ω_R can be interpreted in Fig. 4. If a lognormal is need for equivalent, the equations corresponding to those can be interpreted as in Eq. (8) and Eq. (9).

$$\Phi(-\beta) = \Phi\left(\frac{\ln x^* - \mu}{\zeta}\right) \ , \tag{8}$$

$$\frac{1}{\sigma}\,\phi(Z) = \frac{1}{\zeta\,x^*}\,\phi\left(\frac{\ln x^* - \lambda}{\zeta}\right) \ , \tag{9}$$

where λ and ζ are parameter of a lognormal distribution. These parameters could be determined from corresponding mean and standard deviation as shown in Eq. (10) and Eq. (11).

$$\lambda = \ln \mu - \frac{1}{2}\,\zeta^2 \ , \tag{10}$$

$$\zeta = \sqrt{\ln[\Omega^2 + 1]} \ . \tag{11}$$

Solving for Eq. (8) and Eq. (9), leads the relationship between β and Ω for the lognormal distribution in the following form:

$$\beta = \left[\ln x^* - \ln \mu + \ln(\Omega^2 + 1)\right] \frac{x^*}{\sigma} \ . \tag{12}$$

Employing procedures similar to preceding concepts, the limits of Ω_R for other types of commonly used distribution for structural response are summarized in Table 3.
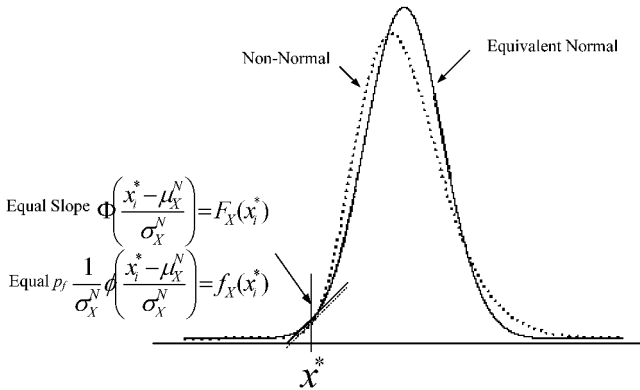
Fig.4: Schematic sketch of equivalent normal concepts

	Ω_{R}				
Type	$p_{\mathrm{f}} = 10^{-2}$ ($\beta = 2.326$)	$p_{\mathrm{f}} = 10^{-3}$ ($\beta = 3.090$)	$p_{\mathrm{f}} = 10^{-4}$ ($\beta = 3.719$)	$p_{\mathrm{f}} = 10^{-6}$ ($\beta = 4.753$)	$p_{\mathrm{f}} = 10^{-10}$ ($\beta = 6.361$)
Lognormal	0.253	0.187	0.155	0.120	0.090
Gamma	0.370	0.301	0.255	0.204	0.154
Normal	0.430	0.324	0.269	0.210	0.157
Weibull	0.402	0.350	0.324	0.295	0.269

Tab.3: Limits of Ω_{R} for structural response distribution

6. Numerical example

Example 1 – prestress concrete pile : A driven pre-cast concrete pile in length of 26.0 m with 60 cm in diameter is selected to carry on a service load of 100 ton. Following conventional design approach, the pile driving criterion for ultimate load (Q_{ult}) can be obtained from Danish's Formula [4] with $FS = 2.5$ as shown :

$$Q_{\mathrm{ult}} = \frac{E\,W\,H}{S + \sqrt{\dfrac{E\,W\,H\,L}{2\,A\,E_{\mathrm{p}}}}} \;,\tag{13}$$

where W is the weight of a steel hammer (7 ton), E_{p} is the modulus of elasticity of pile material (340 ton/cm^2), H is the drop height (70 cm), A is the area of pile cross section (1 571 cm^2) and L is the pile length (2 600 cm). The hammer efficiency of rig (E) evaluated by dynamic test is found 0.80 . Substituting all deterministic values and $Q_{\mathrm{ult}} = 250$ ton into Eq. (13), the pile settlement S is found 0.59 cm/blow for driving control. The statistics of 50 pilot piles from a new airport building at the southern part of Bangkok are summarized in Table 4. Determine whether driven piles could be used safely for the ultimate limit state ($p_{\mathrm{f}} = 10^{-6}$).

Variable	μ	Ω	Distribution
L	2410 cm	0.22	Normal
E_{p}	328 ton/cm^2	0.18	Normal
S	0.57 cm/blow	0.18	Lognormal

Tab.4: Statistical properties of driven pile parameters variable

In this case, the limit of service load which should not exceed the ultimate capacity is considered for reliability analyses. Recall Eq. (2), the limit state function could be further rewritten as Eq. (14).

$$g(x) = \frac{E\,W\,H}{S + \sqrt{\dfrac{E\,W\,H\,L}{2\,A\,E_{\mathrm{p}}}}} - 100 \; . \tag{14}$$

The first and second term in the right hand side of Eq. (14) are represented for the structural response (R) and load effect (S), respectively. Note that the relationship among basic variables in forming R is nonlinear. Therefore, the direct integration technique could not be used to simply this solution easily. Substituting $E = 0.8$, $W = 7$ ton, $H = 70$ cm and $A = 1\,571$ cm^2 into Eq. (14), the failure probability may be obtained using the Monte Carlo simulation technique through Eq. (15).

$$g(x_i) = \frac{0.8 \times 7 \times 70}{S_i + \sqrt{\dfrac{0.8 \times 7 \times 70 \times L_i}{2 \times 1571 \times E_{\mathrm{p}i}}}} - 100 \; . \tag{15}$$

The simulation numbers of 2^{10} is used to generate for random numbers of L, E_{p} and S based on its *pdf*. Substituting 3 sets of random numbers into Eq. (15), then numbers of failure could be counted. The application of random experiments for failure probability calculation can be conceptualized as in Fig. 5.
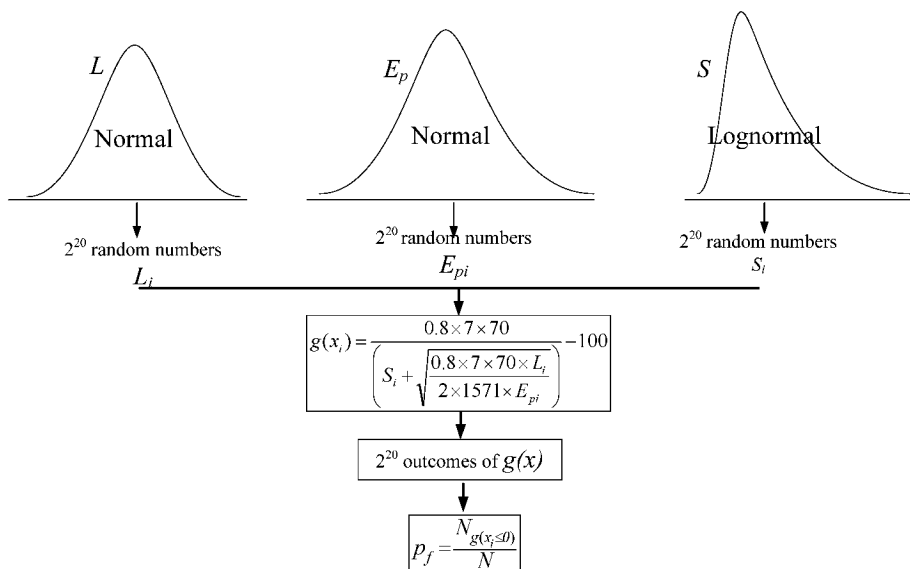

Fig.5: The scheme of failure probability calculation

For simulation numbers of 2^{10}, the failure probability using Monte Carlo simulation technique and variance reduction technique, e.g. importance sampling, is found to be 3.81×10^{-6}. Since the calculated failure probability is higher than the target failure probability, $3.81 \times 10^{-6} \geq 1 \times 10^{-6}$, piles could not be used safety for the ultimate limit state. On the other hand, the reliability of structural system may be examined using the limit of Ω_{R}.

Since the ultimate capacity of pile reflects the structural response, Eq. (16) can be pursued for obtaining randomness.

$$R_i = \frac{0.8 \times 7 \times 70}{S_i + \sqrt{\dfrac{0.8 \times 7 \times 70 \times L_i}{2 \times 1571 \times E_{\mathrm{p}i}}}} \ . \tag{16}$$

The statistical characteristics of R can be generated from statistical data, as shown in Table 4, using the Monte Carlo Simulation technique with 2^{10} simulations. The randomness or uncertainty of R could be characterized by mean value $\mu_{\mathrm{R}} = 264\,\mathrm{ton}$ and coefficient of variation $R = 0.214$. The Goodness-of-Fit test for simulation outcomes may be performed by using CESTTEST software [5]. Where the confidence interval of 99 %, the lognormal distribution could be accepted by both Chi-Square and K-S Test as interpreted in Fig. 6.
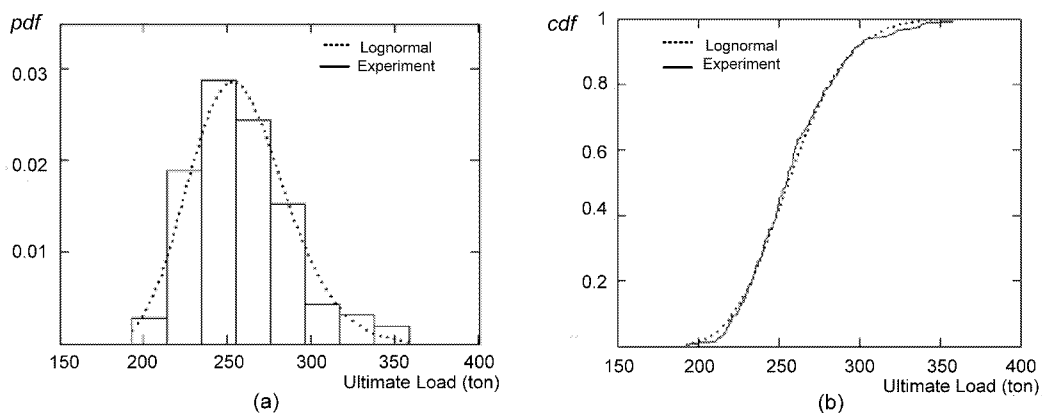

Fig.6: Goodness of fit tests for lognormal : (a) chi-square, (b) K-S

If the target failure probability is 10^{-6} ($\beta = 4.753$) the corresponding limit of Ω_{R} using equivalent normal distribution, presented in Table 3, is 0.120. It is confirmed that if Ω_{R} of the structural system is higher than the limit of Ω_{R}, $0.214 > 0.120$, the structure could not be used safely with the target failure probability.

Reference to the calculated failure probability evidences if coefficient of variation of the structural response is higher than the limit, the structure could not be used safely with the corresponding acceptable level of risk. In views of conventional design approach, a higher level of deterministic safety factor should be reviewed for piling criterion. Since the structural response is non-normal, Eq. (7) is not valid to review the target *FS* corresponding to $p_{\mathrm{f}} = 10^{-6}$ for further driving. In this case, the direct integration approach may be applied for obtaining the value of μ_{R}. The concept for finding value of μ_{R} could be interpreted in Fig. 7.

The parameter λ and ζ could be obtained from the relationship in Eq. (10) and Eq. (11), respectively. For $p_{\mathrm{f}} = 1 \times 10^{-6}$, 100 ton and $\Omega_{\mathrm{R}} = 0.214$, then μ_{R} is found to be 281 ton and *FS* for the new design criteria becomes $(281/100) = 2.81$. By substituting deterministic value $E = 0.8$, $L = 2\,410\,cm$, $W = 7\,\mathrm{ton}$, $A = 1\,571\,\mathrm{cm}^2$, $H = 70\,\mathrm{cm}$, $Q_{\mathrm{ult}} = 281\,\mathrm{ton}$ and $E_{\mathrm{p}} = 328\,\mathrm{ton/cm}^2$ into Eq. (13), the pile settlement for further driving becomes $0.44\,\mathrm{cm/blow}$. For lower values of pile settlement, a pile is needed to be driven into soil layers for more embedded length.
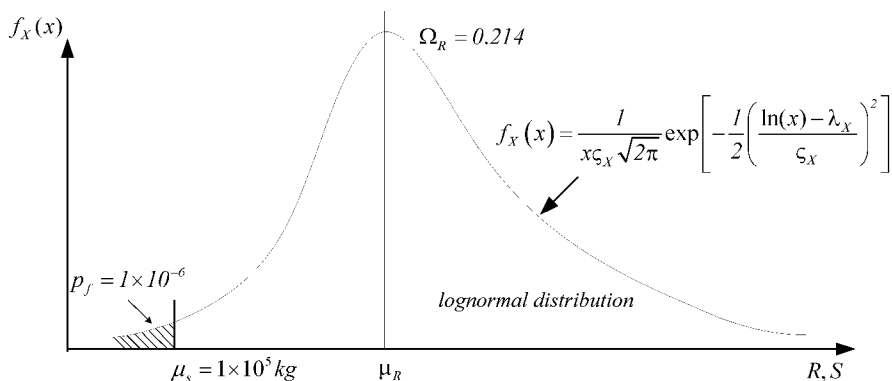
Fig.7: Concepts of integration

This example shows that design criterion was not accounted on how uncertainties of basic parameters influence the safety factor. The solution is simplified by considering parameters to be constant and performing for structural response thorough the use of deterministic $FS = 2.5$. If uncertain parameters are considered, $FS = 2.81$ or more would meet the pile diving criteria.

Example 2 – spread footing: A square footing, subjected to gross load P, has to be constructed in the soil condition as shown in Fig. 8. Using $FS = 3$, determine the size of the footing.
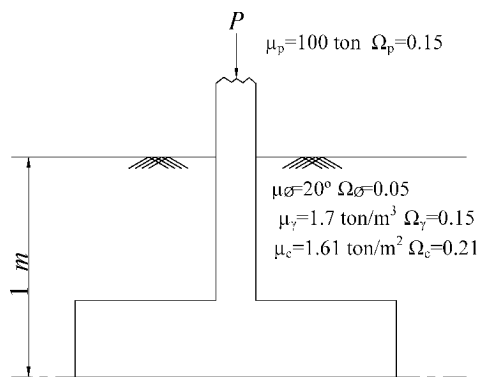
Fig.8: Square footing subjected to load P

In the case of general shear failure, Terzaghi [6] suggested the equation for ultimate load Q_u as:

$$Q_u = (1.3\, c\, N_c + q\, N_q + 0.4\, \gamma\, B\, N_\gamma)\, B^2 \ . \tag{17}$$

In fact, ϕ is less sensitive, coefficient of variation < 0.1, it can assume to be constant. For $\phi = 20°$ the corresponding Terzaghi's bearing factors are found to be $N_c = 17.69$, $N_q = 7.44$ and $N_\gamma = 4.97$. Since the limit of ultimate load is considered for reliability analyses the limit state function and structural response, in terms of Q_u, can be written as in Eq. (18) and Eq. (19), respectively.

$$g(x) = (23\, c + 7.44\, \gamma + 1.99\, \gamma\, B)\, B^2 - P \ , \tag{18}$$

$$R = (23\, c + 7.44\, \gamma + 1.99\, \gamma\, B)\, B^2 \ . \tag{19}$$

Employing a procedure similar to that used in the preceding numerical example, it is possible to develop expressions in Eq. (19) relating Monte Carlo simulation technique to the structural response for various footing sizes. Based on 210 simulations and probabilistic values of c and γ, the randomness corresponding to the base size could be expressed in Table 5.

$B \times B$	Deterministic	Probabilistic		
(m×m)	R (ton)	μ_R (ton)	Ω_R	Distribution
2.3 × 2.3	304	303	0.145	Normal
2.4 × 2.4	333	332	0.145	Normal
2.5 × 2.5	363	362	0.145	Normal

Tab.5: Randomness of ultimate capacity

Since structural response is normally distributed, Eq. (7) is valid for obtaining FS. For the target failure probability is 10^{-6}, the safety index β for normal distribution becomes 4.753. Substituting $\beta = 4.753$ and $\Omega_R = 0.146$ into Eq. (7), leads typical cases of FS as shown in Table 6 and 7.

$B \times B$	$\Omega_R = 0, \Omega_S = 0$		
(m×m)	R (ton)	S (ton)	FS
2.3 × 2.3	**304**	**100**	**3.04**
2.4 × 2.4	333	100	3.33
2.5 × 2.5	363	100	3.6

Tab.6: FS based on deterministic approach

$B \times B$	$\Omega_R = 0.146, \Omega_S = 0$			$\Omega_R = 0.146, \Omega_S = 0.15$		
(m×m)	μ_R (ton)	S (ton)	FS	μ_R (ton)	S (ton)	FS
2.3 × 2.3	303	93.2	3.25	303	85	3.56
2.4 × 2.4	**332**	**102.2**	**3.25**	332	93.3	3.56
2.5 × 2.5	362	111.4	3.25	**362**	**101.7**	**3.56**

Tab.7: FS based on probabilistic approach

Observe from Table 6 and 7, as might be expected, uncertainties of R and S are significantly affected to the values of FS. If uncertainties are fully involved in basic parameters, probabilistic R and S, the footing size of 2.5×2.5 m or larger would meet the design criterion and could be used safely.

It should be emphasized that conventional approaches simplify the solution by considering the uncertain parameters to be constant and defining for structure resistance thorough the use of deterministic safety factor. In reality, soil is neither isotropic nor homogeneous such that their uncertainties could not be ignored. Following the conventional design approach, an FS of 3 or more is not considered too conservative to apply for the allowable bearing capacity.

Example 3 – bored pile : A bored pile $\emptyset$ 800 mm in diameter is embedded in the soil profile shown in Fig. 9. The column load is of value 300 ton and $FS = 2.5$. The ultimate load, based on frictional resistance (Q_f) and end bearing (Q_b) in clay could be obtained

from following equations:

$$Q_{\text{ult}} = Q_{\text{f}} + Q_{\text{b}} \ ,$$
$$Q_{\text{ult}} = \sum (\alpha \, L \, \phi_{\text{p}} \, S \, u) = 9 \, A_{\text{p}} \, Su \ , \tag{20}$$

where α is the reduction coefficient, Su is the undrained shear strength which is referred to the SPT-N value, ϕ_{p} is the pile circumference $(2.5\,\text{m})$, L is the thickness of soil layer and A_{p} is the base area of pile $(0.5\,\text{m}^2)$. Based on conventional approaches, the procedures to determine for the desirably embedded length are illustrated in Table 8.
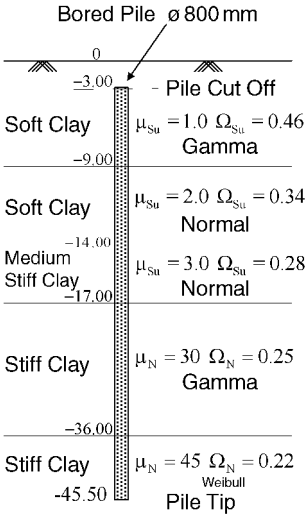

Fig.9: Soil Profiles

Soil Layers	From (m)	To (m)	L (m)	N (blow/ft)	Su (ton/m²)	α	Q_{f} (ton)	Q_{b} (ton)
1	3	9	6	—	1	1	15	—
2	9	14	5	—	2	0.97	24	—
3	14	17	3	—	3	0.87	20	—
4	17	36	19	30	20.4	0.4	390	—
5	36	45.5	9.5	45	25	0.34	203	113

Tab.8: Procedures to determine for desirably embedded lengths

The ultimate load is found to be $652 + 113 = 765$ ton and allowable load corresponding to $FS = 2.5$ would be 306 ton.

For probabilistic approaches, the limit of ultimate load is considered for reliability analyses. Recall Eq. (2), the limit state function and the structural response can be written as in Eq. (21).

$$q(x) = \alpha_1 \, L_1 \, \phi_{\text{p}1} \, Su_1 + \alpha_2 \, L_2 \, \phi_{\text{p}2} \, Su_2 + \alpha_3 \, L_3 \, \phi_{\text{p}3} \, Su_3 +$$
$$+ \alpha_4 \, L_4 \, \phi_{\text{p}4} \, Su_4 + \alpha_5 \, L_5 \, \phi_{\text{p}5} \, Su_5 + 9 \, A_{\text{p}} \, Su_5 - 300 \ . \tag{21}$$

Therefore, the structural response could be represented by the first term in the right hand side of Eq. (21). For the sake of simplicity, the parameter α, L, ϕ_{p} and A_{p} are assumed to

be constant. Substituting deterministic values in Eq. (22), then the performance function could be shown as:

$$R_i = (1 \times 6 \times 2.5)\, Su_{1i} + (0.97 \times 5 \times 2.5)\, Su_{2i} + (0.87 \times 3 \times 2.5)\, Su_{3i} +$$
$$+ (0.4 \times 19 \times 2.5)\, Su_{4i} + (0.34 \times 9.5 \times 2.5)\, Su_{5i} + (9 \times 0.5)\, Su_{5i}\,. \tag{22}$$

Based on these data, the randomness of structural response may be characterized by Monte Carlo simulation technique through Eq. (22). The number of simulations is 1024 (2^{10}). For a confidence interval of 99 % the gamma distribution could be accepted by both the Chi-Square and K-S test for the Goodness-of-Fit tests. Then the randomness of structural response, in terms of Q_{ult}, could be characterized by $\mu_R = 769\,\mathrm{ton}$ and $\Omega_R = 0.107$. For the fitted distribution of structural responses is gamma, the limit of Ω_R using the concept of equivalent normal distribution becomes 0.204 ($p_f = 10^{-6}$). Where Ω_R of the structural system is lower than the limit of Ω_R, $0.107 < 0.204$, the structure could be used safely with the target failure probability.

7. Conclusions

1. The classical reliability analyses in terms of normal structural response (R) and load effect (S) show that there should be an asymptotic coefficient of variation of structural response (Ω_R) for the corresponding target reliability (p_f).

2. The asymptotic limit value of coefficient of variation is inversely proportional to the safety index. For instance, the asymptotic Ω_R of 0.157, 0.210 and 0.269 would correspond to p_f of 10^{-10}, 10^{-6} and 10^{-4}, respectively.

3. If Ω_R is below this limit the structure could resist safely with the corresponding acceptable level of p_f. Otherwise, the expected structural reliability could not be achieved. The central safety factor (FS) for a particular value of pf could be determined from the quadratic relationship between Ω_R and Ω_S.

4. The concepts of asymptotic coefficient of variation could be extended for non-normal variates using the concepts of equivalent normal distribution. If the target p_f is 10^{-6}, the corresponding asymptotic Ω_R is 0.120, 0.204, 0.210 and 0.295 when the randomness of structural response is characterized by lognormal, gamma, normal and the Weibull distribution, respectively.

5. The applicability of the proposed concept for non-normal variates is shown in several numerical examples, i.e. deep and shallow foundation. It could be observed from these examples that the relationship among basic variables for structural response could be either non-linear.

6. For the case of a non-linear or complex relationship statistics of the structural response could be obtained by Monte Carlo simulation technique. Then the fitted distribution of structural responses could be confirmed by the Goodness-of-Fit tests, i.e. Chi-Square and K-S.

7. For non-normal structural response, fitted distributions could be obtained easily with CESTTEST Software. Then the limit of structural response corresponding to failure probability could be determined by using the concept of equivalent normal variates.

8. The conventional approach simplifies the solution by considering uncertain parameters to be constant and performing for structural response thorough the use of deterministic FS.

The design criterions would not account of information on how soil parameters influence the safety factor.

9. In fact, soil is neither isotropic nor homogeneous such that their uncertainties could not be ignored. The value of structural response is depended mainly on the variation of soil parameters and type of distributions. Therefore, the deterministic *FS* could not guarantee enough safety for structural system. In some cases, an *FS* of 3 or more is not considered too conservative to apply for the allowable capacity of the structural usage.

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