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THE CONCEPT OF QUANTUM STATE: NEW VIEWS ON OLD PHENOMENA

> "I think (…) that a theory cannot be produced from results of observations, but only from an invention."
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> "I am not ashamed to put the concept of «real state of a physical system» ["existing objectively, independently of any observation or measure, and that can in principle be described through the means of expression of physics"] at the very centre of my meditation."
> Albert Einstein[1]

> "(…) It is interesting to speculate on the possibility that a future theory will not be *intrinsically* ambiguous and approximate. Such a theory could not be fundamentally about «measurements», for that would again imply incompleteness of the system and unanalyzed interventions from outside. Rather, it should again become possible to say of a system not that such and such may be *observed* to be so, but that such and such *be* so. The theory would not be about *observ*ables, but about «*be*ables»."
> John S. Bell (Bell 1973; 1987a)

Abstract. Recent developments in the area of quantum systems have led to accept statements, which originally appeared to be mere interpretations, as representing physical facts that appeared formerly to be more related to interpretation with free options. Of such a nature are the statements relating to quantum behavior of individual particles (diffraction, etc.), neutrinos oscillations, distant quantum correlations (local non-separability), Bose-Einstein condensation, cooling isolation of atoms and, recently, decoherence of quantum superposition states interacting with environment measurement apparatus, that allows a better understanding of the transition from the quantum domain to the classical-macroscopic one. The debate on the interpretation of quantum mechanics has imperceptibly changed its nature through these developments, giving higher weight to a "physical interpretation" more clearly distinct from the philosophical one than in the old days of quantum mechanics. In particular, the concept of quantum state has undoubtedly acquired a direct physical meaning, in terms of *properties of a physical system* that is fully represented by a linear superposition of eigenstates, and able to propagate as such in space and time. The price for this new situation is an extension of meaning of the concepts of *physical magnitude* and *physical state* towards ones that do not correspond directly with numerical values.

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1. INTRODUCTION:
STATE FUNCTION AND “DIRECT REPRESENTATION” OF A QUANTUM SYSTEM OR STATE

Quantum physics aims at understanding the deep structure of matter in general, from bodies of our environment and molecular associations of atoms to atomic nuclei and to elementary particles actually or “virtually” contained in the latter, including even cosmic objects as well as the primordial phases of cosmology. In our understanding it underlies the unity of matter in the variety of its organization patterns. The means of the theoretical understanding of this domain of physics is constituted by *quantum mechanics*. This is, in turn, applied to particular (atomic, nuclear) theoretical models, and enlarged, from a more fundamental point of view, to *quantum field theory* ranging from quantum electrodynamics (QED)[2] to electroweak and chromodynamic gauge field theories.[3] These recent theories have been established within the conceptual frame of quantum mechanics and, as an effect, have confirmed its heuristic power and permitted at the same time physicists to get used to work with this tool-for-thought that is indispensable to explore quantum phenomena.

Quantum mechanics, as a theoretical scheme, is *practiced* successfully. Today physicists, while strictly applying the rules that govern the use of quantum magnitudes in the working process of their physical thought on the phenomena being studied, no longer worry very much about the “difficulties of the interpretation” that had heavily preoccupied the founding fathers and their immediate successors. As for interpretation, if they had to propose any, this would be for most of them the following: “what is important is that it works.” And indeed, this might be a mark of unconcern or the expression of an immediately pragmatic philosophy that would remain blind to deeper reasons. This attitude comprises, in all events, a part of truth, of the kind—walking before knowing how: they have the theoretical (and even conceptual) tool and know how to handle it before knowing its exact nature and worrying about it.

However, as soon as physicists ask themselves questions concerning the intelligibility of physical phenomena in the quantum area, they again find the terms of the old debate. But, in contrast to their elders, they meet these “from outside,” so to speak, in the sense that these questions appear to them as posed only to a “second order” of the understanding: i.e. when they question themselves *not about this understanding*, that is itself provided by theory, a theory they know so well that it has become “second nature” to their thought, but *about the reasons for it*.

To understand “to the first degree,” that is at the level of their work in physics itself, is achieved by handling the concepts and magnitudes that represent, reproduce or create the phenomena of interest. When physicists today speak of an “elementary particle” (for example, a proton), they mean, indeed, that it is described by “quantum numbers” or quantities that are “eigenvalues” of operators representing the adequate physical magnitudes. They have abandoned the classical image of a directly visible corpuscle, that no longer belongs to their referential background. *Quarks* themselves are quantum particles considered in this sense. Such *entities* or *physical “systems”* or *quantons*,[4] conceived in the specific way of quantum physics, are implicitly supposed to constitute in one way or another, according to the modalities of their description, objective elements of the real world that manifest themselves to human understanding. Symmetries of quantum particles and fields allow us to *understand* in that way their properties and their arrangements.

Philosophical and epistemological difficulties arise only when one intends to understand *the nature of this understanding*: this is what we mean by "intelligibility to the second degree." This difference of degree with the founding debates comes from the fact that the theoretical tool, "formalism," in the usual expression, is now already justified as a representation of its success. Physics builds its tools in an abstract manner and contents itself with these being well conceived, without trying to naturalize their origin; admittedly they are abstract, symbolic, mathematical, and elaborated by thought from necessities enforced by phenomena (physics has indeed established rules for that purpose such as, for example, statements in the form of principles).

From this one sees clearly and undoubtedly better than in the past when it was necessary to construct these tools, that the problem of interpretation is twofold but separate: physical and philosophical. The physical interpretation deals essentially, as was traditionally the case since the birth of modern physics (in the seventeenth century), with the relationships between mathematically expressed magnitudes and the corresponding physical contents. The difference, from the physical point of view, between theoretical and conceptual elaborations related to quantum phenomena and systems, and those dealing with classical ones, is that the quantum phenomena are farther than the classical ones removed from the processes of observation by which these phenomena reach our senses.

Niels Bohr rightly emphasized this difference of nature between the quantum and the classical. But he formulated it in a manner that abruptly changed a simple physical state of things into a philosophically problematic state of knowledge. There was, according to him, a barrier between the quantum and the classical worlds, a barrier that was due to measurement. It resulted from this state of affairs, in his view, that the knowledge of quantum phenomena cannot grasp these directly, but has always to refer to classical representations.

The disjunction between the representation of quantum phenomena and that of classical ones can be pointed at in a philosophically more neutral manner: while classical physical phenomena and systems are *homogeneous* to the processes of their observation, quantum phenomena and systems are not, since observations and measurements relative to them belong ultimately to the domain of classical physics. But that does not entail any impossibility to represent "directly," that is to say in terms of *properties* and *objects*, quantum phenomena and systems, at least if one refers *intelligibility* not to *perception* but to *understanding*, as it seems logical. If one can conceive objects of a quantum area or "world" in this sense, the questions of *physical interpretation* will be therefore largely independent from those bearing on more general considerations on knowledge, i.e. *philosophical interpretations*. One would then have shifted, so to speak, from a concern for *interpretation in general* to a more precise interest for the *physical meaning of quantum magnitudes* provided by quantum theory itself.

The philosophical aspects of interpretation would then present themselves at a different level, granting a large autonomy to quantum physical thought, sensibly the same as for the other areas of physics: the former would no longer have to sacrifice to a so-called "foundational" need of being based on a peculiar philosophical interpretation, as in the early period of quantum mechanics.[5] And one would have then satisfied, up to some point, the realist demand of the theoretical physicist asking, with John Bell, for a theory that is not fundamentally about measurements, that considers physical systems in their inner completeness, and of which one could tell "not that such and such *may be observed* to be so, but that such and such *be* so." In other words, a theory that "would not be on *observ*ables" [magnitudes able to be *observed*], "but about *be*ables" [magnitudes able to *be*].[6] This is what will be achieved in the following without modifying in any way the form of standard quantum theory, and only by understanding it differently (to an intermediate degree of understanding, involving some kind of *physical* interpretation).

It remains to see how to "interpret physically" the theoretical, conceptual or factual states of things that were problematic in a physico-philosophical mode for the "orthodox" or the "complementarity" "interpretation." We shall restrict ourselves, in what follows, to revisiting some characteristic and relatively simple quantum phenomena, renewed by recent results from high precision experiments, in the light of the proposed perspective on the physical interpretation of magnitudes and of theoretical formalism referred to the description of a world of properly quantum objects or systems. We shall see that they invite us directly to conceive quantum magnitudes in this manner, which entails the need to expand the meaning generally given to the concept of physical magnitude, and especially to the concept of the state function representative of a physical system.

Since the beginnings of quantum mechanics, these phenomena usually served to illustrate the problems of interpretation. By a fair reversal of things, it is today possible to extract directly from them the physical interpretation they are calling for. These phenomena are, first, *local non-separability*, whose epistemological status has undergone changes from a formal feature with optional physical meaning to an established physical fact, corroborated by experiments with distant correlated systems. Then, *diffraction of quantum particles*, no longer performed with many particle beams for statistical results, but with individual quantum systems for probabilities of individual events. Also, *indistinguishability of identical particles*, initially postulated or conceived as a formal property, and thereafter demonstrated by direct physical effects such as Bose-Einstein condensation, where a great number of identical atoms are accumulated in the same fundamental state up to a quasi-macroscopic level. Finally, recent *experiments of "decoherence"* have permitted to visualize superpositions of states in relation with mesoscopic systems when measured by a classical device, in a tiny time interval before the dissipative loss of information occurred from interaction with the environment.

All these results converge towards a specification of the physical meaning of quantum concepts and magnitudes implied by the corresponding phenomena, obliging us to associate factual evidence and physical contents conceived in terms of *properties of systems*, with "formal" properties whose interpretation remained until then optional or problematical. We will analyze some aspects of this new situation, trying to make out in which way they may contribute to deepening, modifying, or finding a foundation for our theoretical comprehension of quantum features, by reducing the latitude of arbitrary choice in the interpretation and by adapting the norms of our intelligibility.

2. LOCAL NON-SEPARABILITY AS A FACT AND AS A PRINCIPLE

The objection opposed by Einstein in 1935 to the claim that quantum mechanics is a fundamental theory that will serve as the *basis* for any further progress in physics raised several questions that overlap with interpretation.[7] It became known as the "EPR argument," and was later reformulated and refined by its main author. The problem was to know whether the theory (quantum mechanics) describes, or not, *real individual physical systems*, and if it describes them *completely*, that is to say adequately to all aspects rightfully attached to their *individuality* and in a one-to-one manner. The orthodox interpretation (in the philosophical sense) challenged the legitimacy of speaking of elements of reality independently of their conditions of observation, and Bohr's reply to Einstein's argument was exactly founded upon this position (Bohr 1935). It had no chance to be listened to by Einstein, who could not accept its principle. Any progress in the debate on this question supposes from then on that one tries to leave aside the philosophical *diktat* of Bohr's reply, and adheres to physical theory and the content of its concepts.

The question, as contemplated by Einstein, was to know whether the theory is complete in the indicated sense (it was to him the minimal requirement for a theory to be considered fundamental).[8] If one can completely characterize an individual quantum system (be its theoretical representation probabilistic or not), its state function has to represent it as such. If that is impossible because of some feature of the theory, then this theory can be only a statistical description (such was, indeed, the conclusion of the "EPR" argument). We will see further that this is also the point at stake with the *interference* of distinctive *individual* quantum systems when put actually in evidence. The EPR argument suppressed in principle the possibility to elude such a question in the name of an operationalist philosophical interpretation, and indeed the construction of rarefied beams later suppressed it effectively. Individualization for a system was indeed usually prevented by the alleged necessity to detect and measure it, if one wanted to know something of it (with a particle counter to know whether there came only one), and this act would destroy it immediately as a quantum system (it would project it on a classical particle state), forbidding all further knowledge of its quantum state (through the manifestation of wave properties).

In the EPR case, the system under study (U) was conceived in correlation to another one (V) while it did not maintain any dynamical interaction with it.[9] The correlation, expressed by the conservation of a magnitude (A) used in the description of these systems, and known for the initial state formed by the two subsystems,[10] allowed the determination of the state of the first without perturbing it by a measurement, by deducing it from the state of the second, measured (supposedly) independently of it.[11] Measuring magnitude (A) of the second destroyed its state at the very moment of its determination, forbidding any meaning to the consideration of an alternative measurement for another magnitude (B) incompatible with the first: the initial system being no longer available, no effective comparison can be made. But it would have nevertheless been logically possible, as a matter of principle, to perform the second measurement instead of the first, and it would have provided another state function for the second system; from it, the first system would have been deduced, *a priori* different from the preceding result.[12] One could therefore have two different state functions to describe one and the same physical system: it would obviously be a theoretical weakness.

But this reasoning was dependent on a statement that did not belong to quantum formalism and that was at this time considered optional: the separability of two far distant systems, that is to say their mutual independence in their respective locations. Einstein gave a precise definition of this *principle of separability,*[13] although recognizing that he added it to quantum theory. Without this principle, he believed, however, one could not characterize separately localized individual systems, unless one admitted a non-physical interaction (instantaneous action at a distance) between them. He concluded from this that quantum mechanics does not describe individual physical systems, but only statistical ensembles of systems, for which the objection does not hold.[14]

Further progress, both theoretical, with the establishment of John Bell's theorem (1964), and experimental, with experiments of correlation from a distance, has essentially consisted in analyzing local separability, a concept identified by Einstein, and in testing it for quantum systems. Bell's theorem on non-locality demonstrated the existence of a contradiction between *local separability* and some predictions of *quantum mechanics* for systems of two correlated particles (*strong correlation* relationships for quantum systems expressed by *equalities* between averages for magnitudes were opposed to *weaker correlations* in the form of inequalities for the local separation case). This theorem provided the sensitive relationships able to discriminate the local separability hypothesis and quantum theory.[15] From then on, experiments have decided in favor of quantum mechanics in a hardly disputable manner, especially that of Freedman and Clauser, realized in 1972, and that of higher precision performed in 1981 by Alain Aspect.[16] *Local non-separability* was henceforth established as a physical *fact,* a *general property* of quantum systems having been put in correlation, well identified from the phenomenal point of view.[17]

This property corresponds to a characteristic feature of the state function in quantum mechanics: the state functions of subsystems that have been once correlated are not factorizable (i.e. independent of each other, i.e. separable). Having been linked together to form, even momentarily, one single system, two quantum (sub-) systems cannot be dissociated: this *"entanglement"*[18] is a fundamental property of quantum formalism, and possesses therefore a direct counterpart in the phenomena it describes, *local non-separability.* This can equally be considered as an aspect of *non-locality* of quantum systems. The general and fundamental character of this property, and its inscription in the formulations that define in the theory the state of a system, suggest at the same time a fact *of experience* and a *principle* for quantum physics.

An important conceptual aspect of local non-separability is its place in the economy of quantum theory. From the point of view of the conceptual and theoretical consistency requirement adopted here (which may also be called of "critical realism"), one can analyze it in the following manner. As it is directly linked to the definition of quantum systems and of the magnitudes by which we represent their states, local non-separability relies only on these and does not have to refer to other magnitudes that would be defined outside this theory. It bears on systems that, spatially speaking, are "extended systems," and for which the space variables have not, as an effect, a direct part in their definition; in this sense, it is not concerned by special relativity. It does not contradict it and has nothing to do with (instantaneous) actions at a distance.[19]

It is fair to say, however, that many physicists and philosophers of science would still disagree with this conclusion, which seems compelling from the point of view adopted here. It is, indeed, difficult to think physically without the help of spatial intuition, and this is probably the main reason for their dissatisfaction with "pure quantum reasoning." But who can say what kind of intuition is adequate for the quantum domain? It seems to me that quantum physicists have developed over the years an adequate intuition in this respect that is basically founded on the quantum formalism as a practiced intellectual tool for exploring and understanding quantum phenomena (the epistemological implications of which we are exploring here, taking a point of view of general consistency). John Bell, who was reluctant to accept the above argument, which he viewed as too formal and even as a "verbal" solution,[20] admitted nevertheless non-separability as a fundamental fact and eventually as a physical principle.[21] But he would have preferred to have a dynamical interpretation of it. It seems to me, on the contrary, that as a principle it definitely does not need an explanation, but stands as one of the primary conceptual references toward which the other quantum concepts must be consistently obliged (in a way similar to the principle of special relativity ruling the transformation laws of the concepts related to the motion of bodies).

As a fundamental quantum fact, one should perhaps consider that local non-separability is to quantum physics, for example, what the principle of equivalence (of inertial and gravitational masses) is to the general relativity theory of gravitation. One can see it as a true *principle*, both a synthetic proposition based on experimental facts and a theoretical statement of a central, and perhaps foundational, importance; it could serve to formulate quantum theory in a less formal manner than the usual presentation, which would make it come closer to the other physical theories from which until now it parted in this respect.

Local non-separability can be seen as an even wider theme of reflection, rejoining a cosmological perspective. One can, indeed, make rapprochements with other features of "disindividualization"[22] or of "desingularization" or, better, of *indifferentiation* such as *indistinguishability* (that partakes as well of the superposition principle), and perhaps as symmetries of matter, that are important features of primordial cosmology (Paty 1999b). One can also see in it, with David Bohm, the mark of a more general indivisible wholeness of material reality (Bohm 1980). With regard to this point, one must nevertheless observe that to grasp an underlying order, thought separations in such a wholeness are needed as a necessary approximation, without which the concept of wholeness would lose all utilizable physical content. Extended in an absolute manner to the whole Universe, the principle of non-separability would present the same kind of disadvantage as the one pointed out by Poincaré[23] regarding the principle of relativity of space if we were to formulate it with respect to all bodies of the Universe: being tautological it would not give us a hold on phenomena. But, at any rate, it might give us some hint on cosmological conditions, of the kind Einstein obtained for a closed and unlimited Universe[24] (for example, in quantum cosmology, some coherence condition for having finite time inside Planck's limit in the primordial Universe).

3. INDIVIDUAL SYSTEMS AND THE TRANSFORMATION OF PHYSICAL PROBABILITY

The phenomenon that is the simplest in its principle to characterize quantum properties is that of *interference*, which confirmed the wave-particle duality of matter and inspired Max Born's idea of the probabilist interpretation of the state function. This archetypal phenomenon illustrates some fundamental aspects of the description of quantum systems and helps, from the physical point of view, to make explicit the interpretation problems that had been raised.

The "orthodox" interpretation of complementarity and observationalism sees in it the necessity of wave-particle duality and the impossibility to go beyond it. The interference pattern (concentric rings, alternately obscure and bright), similar to those of classical waves, is due to the wave property of quantum systems; whilst, on the other hand, the materialization of these varied intensity rings on the screen covered with a sensitized film comes from the corpuscular property of these systems (through their interaction with the grains of photographic emulsion, producing an image). The dual properties, contradictory if they are considered for individual "particles" or systems, can be reconciled as soon as one ceases to be concerned with causality of individual events, and shifts the focus to the statistical aspect of the experiment. If one wanted to examine, in this experiment, the behavior of an individual quantum system, a meaning could not be derived according to the complementarity interpretation, in the name of the very definition of the systems. As a matter of fact, if one wanted to characterize a quantum system as individual, it would be necessary to submit it to a counting experiment, that would indicate which of the slits the quantum system has gone through; by being localized in that way, the system would suffer a perturbation and therefore lose its quantum aspect and its capacity to produce interferences.

Yet, in 1930, Paul Dirac, in his book *The Principles of Quantum Mechanics* (Dirac 1930), already indicated that, according to this theory, one photon interferes with itself and that this is the reason for the interference phenomenon in the case of a single quantum system. This is also the case for any quantum system (particle, atom, etc.). The meaning of it would be that interference is a property of any individual quantum system, and that quantum physics is the theoretical description of such individual systems. The probabilistic turn of this description would not *a priori* be a hindrance for this purpose (after all, statistical mechanics does the same). However, the "complementarity explanation," to which we just referred, blunts and dissolves the force of this statement by making it a mere feature of the formalism in claiming the impossibility by principle to observe it in experiments.

As for the ensemble interpretation of quantum mechanics, according to which the theory is only a statistical one (incomplete for Einstein, complete for others), it only knows averages that have no physical meaning except for an ensemble of systems, and can not pronounce on the significance of individual quantum events.

However, for approximately two decades, experiments have been realized and continuously improved thanks to technical advances with *individual quantum systems* (photons, electrons, neutrons, atoms) that are known to be such without needing to be counted by detection on their path, and therefore without destruction of their quantum state. It has actually been possible to produce beams of such “particles” or quantum systems, extremely rarefied and with a high time definition (better than 0.1 ns), in such a manner that particles get to the interferometer one by one, spread in time, each having got across the experimental arrangement within an interval of time sufficiently small to ensure that the following one has not yet entered.[25] One can then be fairly confident that only one particle at a time has crossed the interference apparatus (and interfered with itself). The detection of impacts on the screen seems in the beginning to be at random. When many single “particles” have gone through the interferometer, the distribution of impacts is seen to obey a law: one obtains, in the end, the same interference pattern as in the traditional experiment with a beam of N identical particles crossing simultaneously the interferometer.

These results require that a physical meaning be attributed to an individual event in an interference experiment. Clearly, the final interference pattern with individual particles can only be obtained statistically, by the realization of a great number of successive one-particle or individual quantum system experiments. The result of N such experiments with single quantum systems gives the same result as a single experiment performed, in the same interferometer, with a beam of N identical systems. But the theoretical inference that one is allowed to make in the two cases is very different. The second case, consisting of the traditional experiment with a great number of simultaneous systems, satisfies a frequentist and purely statistical interpretation of the probability given by the state function.[26]

But the effective occurrence of the first case, N experiments with an individual system identical each time, and represented by the same state function, assures us that each individual phenomenon, independently from the others, contributes to the final interference pattern. One is therefore led to conclude that *it is the individual quantum systems that make the phenomenon* and therefore that, in a way that remains to be specified, each individual phenomenon occurring with each (independent) system potentially constitutes the overall interference phenomenon revealed by the final pattern, obtained statistically. In other words, each phenomenon relative to an individual system is a *quantum phenomenon*, collected on the screen through a *classical measurement* process (the “photon” or quantum particle impact on a grain of silver bromide of the photographic emulsion). One is then inclined to consider that, just before interception on the screen, each of the individual systems having interfered with themselves is in a quantum superposition state. And so, as nothing distinguishes them from each other, all these individual systems in interference are strictly identical. From then on, the only remaining problem would be the measurement process: identical quantum systems provide, after detection, different results, but endowed with probabilities corresponding to the amplitude of probability of their state of superposition.[27]

As a result of what precedes, the ψ state function must be considered as the theoretical representation of an individual particle, which entails the following important consequence of its physical meaning: the *physical probability*, given by the ψ state function[28] (the latter being often named "probability amplitude," in a sense that can only be physical, since nothing of the kind exists in mathematical probabilities), is not liable to be reduced to statistics for ensembles of systems. It has a *theoretical function* from the physical point of view, as it is deduced from a magnitude having a direct physical meaning, the probability amplitude (i.e. the state function itself). One can therefore consider this probability as a *physical magnitude*, which makes it differ from probability in a merely mathematical sense, as well as from probability conceived physically as expressing a frequency.[29]

4. INDISTINGUISHABILITY AND STATE FUNCTION

In quantum physics, the state function that represents a quantum system allows the complete description of all the properties attributed to this system, in such a way that systems represented by the same state function are effectively in the same state and are absolutely indistinguishable. That means that, external to the theory, no other possibility exists to distinguish them. In others words, *a quantum "particle" has no other characteristics than those of its state*, differently from physical systems as described by other theories such as classical mechanics, thermodynamics or relativity theory. These theories describe *what happens* to physical objects that are in other respects defined outside of them. For example, the three-body problem of classical astronomy is about the mechanical processes occurring to celestial objects that are supposedly given. The theory bears not on these objects, but on their interaction properties. The Moon, the Sun and the Earth, for example, possess an identity—and an opacity—defined prior to the laws and equations under study in mechanics and astronomy.

The only theory, except quantum physics, for which the eventuality has been considered that it could be able by itself to describe its object, instead of obtaining it from outside, is the general theory of relativity, at least in a further more elaborated formulation foreseen as a distant purpose (by A. Einstein and J. A. Wheeler notably), where it would be possible to describe in the same system of equations both a field and its source. Such was the "strong" meaning Einstein attached to the notion of theoretical *completeness*.[30] To him, quantum mechanics was not a "complete theory," in this sense obviously, as its status of a framework theory rather than a dynamics suffices to show. But there was another weaker meaning of the same notion, which he considered crucial for the fundamental nature of quantum physics, as we have seen earlier. A theory would be "complete" in a minimal sense if it were able to describe fully its *object*, that is all the properties than can be physically considered about it. It was not the case for Einstein, with quantum mechanics because of EPR type correlations that, invalidating the principle of separability, excluded the description of individual systems.[31]

We do not any more consider this argument in this form, such correlations having proved to be factual and to concern individual correlated systems. On the contrary, actually, completeness at least in the weaker sense would characterize, in principle, the description of *quantum systems* on the background of the physical interpretation envisaged here. The main obstacle to this requirement seems today to remain the "quantum measurement problem." If one sets aside the latter for a moment, one can rightfully be struck by the purpose of quantum mechanics to achieve an *exact covering* of the *described system* by its *state function*, going therefore even, in a way, beyond the restricted completeness requirement.

The most remarkable expression of this covering appears, finally, to be the property of *indistinguishability of identical quantum systems*. But is it a mere feature of the formalism, or a property of physical systems? Both aspects, as always with quantum mechanics interpretation problems, seemed closely connected and not easily disentangled. This property was identified on the eve of the constitution of wave and quantum mechanics, by Satyendra N. Bose and Albert Einstein for quantum systems of null or integer spin (photons and atoms named afterwards "bosons," obeying "Bose-Einstein statistics"), and by Enrico Fermi and Paul Dirac for quantum systems of half-integer spin (electrons, protons, and other "fermions," obeying "Fermi-Dirac statistics"). Indistinguishability of identical bosons (in the case of photons) appeared to be the real underlying reason of the quantification procedure for radiation energy exchanges in black body as performed by Planck in 1900;[32] and indistinguishability of fermions (here, electrons) gave the explanation of the Pauli exclusion principle that accounted for the constitution of atom levels in terms of state occupations by electrons.

This property, corresponding to two types of statistical (or probabilist) processing of quantum systems (the admission of several particles in the same state in the first case or, on the contrary, their mutual exclusion in the second one), opposed to the classical statistical processing *à la* Boltzmann of particles always distinguishable even when occupying the same state (for they possess a proper identity). Indistinguishability therefore limits drastically the possible state occupations. It indicates, actually, that quantum systems *do not occupy* states, but that *they are themselves* states, and are identified with their states.[33] Indistinguishable quantum systems have no other element of identity than those furnished by the theoretical description of their state. The notion of state is identified with that of "particle": a quantum "particle" (or system) *is its state*: it is not "in its state," as a classical system. This situation corresponds to a closer determination of the physical system by the theory. Contrary to the idea that prevailed for quantum physics of a looser determination and a limitation of knowledge because of "indeterminacy" relations.

This formal property, indirectly dictated by factual reasons and finding expression in the principle of superposition,[34] has proven to correspond to fundamental physical properties of quantum systems that could be directly tested and that have implications to the macroscopic level itself.

Supraconductivity and *superfluidity* are such properties directly connected to indistinguishability. Bose-Einstein condensation, already predicted in 1925 by Einstein from the indistinguishability of the identical for some kinds of atoms (it was, actually, the first theoretical description of a phase transition), was long considered as being very far from possibilities of verification. Yet it has recently been experimentally proven thanks to the high technical realization of extreme colds and atoms trapped by laser rays.[35] Tens of thousands of atoms are thus condensed in the lowest energy state (called "of the zero point") with nothing distinguishing them from each other: the superatom they then form corresponds to a fluid in absolute superfluidity state, without viscosity, that can show itself at the macroscopic level (by an effect of *visible non- locality*, the fluid occupying quasi-instantaneously all the space offered to it, rising on the container walls). At this stage, restrictions claimed by the orthodox complementarity interpretation about the direct physical character of the state function appear rather ridiculous, and as an exercise of twisted rhetoric serving only to hide evidence.

One may invoke for the exclusion principle—and therefore for indistinguishability of identical fermions—direct consequences at a highly macroscopic level, concerning cosmic objects corresponding to particular phases of the evolution of stars. "White dwarfs" are compact stars in a state of equilibrium between the gravitational tendency to collapse and the pressure of degeneracy of electrons that cannot fall in the same fundamental state because of the Pauli exclusion principle.[36] "Neutron stars" resist in the same way, they collapse in on themselves because of gravitation due to the degeneracy pressure of the neutrons into which all atomic nuclear constituents have been transformed.

By its directly physical consequences, indistinguishability of identical quantum systems is indeed a physical property of these systems, and not only a feature of the theoretical formalism. It is described precisely by quantum theory in terms of *state function* (submitted to the principle of superposition). There is therefore, as we suggested earlier, a liaison of the property indicated by *indistinguishability* (equivalence of particles of similar characteristics, occupying the same state within a system, that one can count but that nothing distinguishes)[37] and the *theoretical description* by the state function of quantum mechanics (or, at a further stage, of quantum field theory). All this encourages us to see indistinguishability not as a "lack," as would suggest the common intuition of the notion of "particle," taken from the immediate experience of bodies in our environment as well as from the habit of classical physics, but rather as a characteristic and determining physical *property.* For nothing authorizes us, when dealing with such objects, to think of properties that are not pointed out by the theory.

5. REAL PHYSICAL STATE AND SUPERPOSITION, MEASURED STATE AND PROJECTION

The state of a quantum system, as we have tried to characterize it physically, is not identified with that obtained directly by one measurement alone. This last, indeed, is a reduction or at least a projection of the state physically defined by one of its components, according to the choice of the preparation of the system (by a complete set of compatible magnitudes).[38] A measurement device in the usual sense can only measure a classical magnitude. With respect to the state of superposition that represents a system before the operation of measurement, it can only provide one of the components (one of the "eigenstates" of the measured set of magnitudes). One should not be surprised by this as such is its function and its only ability.

The measurement apparatus is, as a matter of fact, by definition a projection device (in the geometrical sense) of the various components of the state of the system. One has claimed that quantum measurement is a non-causal interaction, but this means to pronounce oneself *a priori* on the nature of the interaction between the quantum system and the macroscopic device. If one speaks rightly of a *rule of projection*, or eventually *of reduction*, this rule does not, up to now, mean any directly physical process and nothing allows it to be raised to the status of a physical principle. In the absence of a theory, in the proper sense, of quantum measurement, that would be a general theory of the interaction between quantum system and macroscopic measurement apparatus; one must regard this as merely a practical rule.

Each measurement provides a numerical value for the measured magnitude, one of its possible (classical) values (among the eigenvalues) with some frequency, given by the corresponding probability amplitude (eigenfunction). An experiment with a great number of identical systems, or a great number of independent experiments performed on such systems taken individually, provide the whole spectrum of values of the magnitude with probabilities for each (corresponding to the amplitudes in the superposition). From these results in terms of classical magnitudes, one infers the quantum superposition state that has been submitted to measurement, and of which one can reasonably think that it represents *the quantum system before measurement*, in one of the possible bases; the one chosen by preparation. The state function reconstituted in that way is not a simple catalogue of data, since the system that it represents has the capacity, a clearly physical one, to propagate, to evolve in the course of time, to make interferences or to possibly oscillate between different physical states (on which we shall give more details below). Measurement to determine the state will intervene only after these transformations, which owe nothing to man's hand or thought but everything to nature.

In summary, we propose to consider that the physical quantum states are the states expressed as superpositions themselves, which one can determine from the determination of their components. This reduces to magnitudes endowed with numerical values by classical measurement devices. Actually, this is nothing more than taking von Neumann and Dirac's geometrical vector representations as meaning it: state vectors in Hilbert space are the physical ones, represented by their various possible bases (determined from the preparations according to their possible sets of commuting magnitudes). As a vector, the system state is a basis-free geometrical representation of a physical state, and is more fundamental, because of its invariance, than its "contextual" components.

As an effect, physicists, familiarized by their experience of quantum systems, consider them in this manner: what is important is the representation of these systems' quantum states, i.e. the overall final reconstitution and not the contingent and particular (classical) values obtained by measurement. These values are intermediate entities given by experiment, whose deep physical meaning is obtained only from their immediate translation in quantum terms, necessary in returning to the description of the physical quantum system under study.

6. PHYSICAL PHENOMENA LINKED TO PROPAGATION OF SUPERPOSITION STATES

A physical state, as considered by physicists in their representation of quantum phenomena, and how they *think* about it in their *theoretical work*, is given in an invariant form with respect to its "vector projections," while being generally presented at the same time as a state of superposition on one basis or the other. This is more general than being restricted to the consideration of measurement alone, which after all is nothing else than one of the moments of verification or of experimental test, and is not a purely formal property: *this form rules the physical properties* of quantum systems. We have seen this for the phenomena evoked above, but one can also evoke a number of others of a different nature that show to what extent this is indeed the universal form of the description of all quantum systems. Two examples, both borrowed from elementary particle physics, will show this in a clear and striking manner, all the more as they have no classical analogous: these are the *"mixtures"* of particles states and the *"oscillations"* from one state to another, these mixings and oscillations being expressed directly in terms of state superpositions that propagate.[39]

The neutral "strange pseudoscalar" meson K^0 and its antiparticle, $\overline{K^0}$, are eigenstates of their "mass matrix" (M) and of the *strong interaction* Hamiltonian (H_s) production process (they are physical states in *associated production* conserving the "strangeness" magnitude, $S = +1$ for K^0, $S = -1$ for $\overline{K^0}$[40] or for any other associated strange particle in the production interaction, for instance the "strange baryon" Λ^0). They behave differently in their *decay through weak interaction*, with the strangeness of non-conserving Hamiltonian H_F.[41] The eigenstates for such processes are the mesons as observed from their decays, characterized by proper lifetimes (τ) and decay modes, the short-lived K_S^0 $(\tau = 10^{-10}\text{ s})$ and the long-lived K_L^0 $(\tau = 10^{-8}\text{ s})$. The initial states K^0 (resp. $\overline{K^0}$) are expressed as linear superpositions of K_S^0 and K_L^0 states, which progressively transform according to the law of exponential decrease in time. If one considers a K^0 meson initially produced (actually, a beam of such mesons, appropriately selected), and one worries about its state at a time *t*, the superposition initially containing the states K_S^0 and K_L^0 in equal parts impoverishes in K_S^0, whose time decay is faster, and enriches in K_L^0, that will in the end completely dominate. The then nearly pure beam of K_L^0 states can be written as a superposition of the states K^0 and $\overline{K^0}$ in equal proportions. One therefore obtains, in the beam of K^0 mesons, a "regeneration" of $\overline{K^0}$ mesons that were absent in the initial beam. These can be detected through a strong interaction process with respect to which they are well defined, i.e. of which they are eigenstates.

Let us note, incidentally, that the qualification of eigenstate concerns definite states of a Hamiltonian and other physical magnitudes that are not, here, of a classical nature. At this level, the identification of quantum systems in given states does not call for measurement in the classical sense. The latter is needed only at the end of the chain of experimental processes of the detection of "particles" typical of the considered interactions. In a general fashion, an eigenstate given for a set of compatible magnitudes can be projected (in the vector sense) onto another (preparation) basis relative to another set of magnitudes incompatible (non-commuting operators) with the first. This eigenstate of the first set of magnitudes will therefore be written as a superposition of eigenstates of the second set. In others words, the “preparation” of a quantum system concerns proper quantum magnitudes as well as magnitudes submitted to a classical determination by measurement. “Preparation” for measurement is only a particular case of “preparation” in general, that means the choice of a set of physical magnitudes corresponding to a set of eigenstates taken as referential (or as vector basis in the Hilbert space of their eigenfunctions).

One can also consider the behavior of these neutral K particles under the transformation by the CP operator[42] as a product of charge conjugation (C, that changes a particle into its antiparticle) and parity (P, or space symmetry) or, equivalently, by the time reversal (T) operator, the equivalence ($CP = T$) being due to the conservation of the product CPT, following a theorem of the quantum theory of fields.[43] If one represents the eigenstates of the CP magnitude by K_1^0 (with a corresponding eigenvalue $CP = +1$), and K_2^0 ($CP = -1$), and if the operator CP does not commute with the weak interaction Hamiltonian (H_F),[44] the K_L^0 and K_S^0 states are different from the K_1^0 and K_2^0 states and can be considered as linear superpositions of these states. The coefficients in the superpositions are functions of the parameters of CP violation in these weak interaction processes.

Such physical systems propagate with time between the moment of their production and that of their detection. The state that is attributed to them during this course is that given by the state vector (invariant with respect to the basis), that is, for the chosen basis, the linear superposition, whose coefficients vary with time (let the function $\psi_K(t)$ be the representation of this state). That is to say that the *superposition* here is the *physical state*, without any circumlocution that would bring physical existence only to the state detected after observation or measurement. The quantum system under study (represented by the $\psi_K(t)$ state function) is analyzed by a detector placed on its line of flight that projects it (in the geometrical sense of vector projection) at time *t onto* one of its components chosen by fixing the detection conditions (“preparation”). From the frequencies for each detected state that are a measure of their probabilities, one obtains the coefficients of the superposition or probability amplitudes (probabilities are the absolute squares of the coefficients), as in the usual case. One observes statistically, for K_L^0, a given number of states in the $CP = +1$ mode (for example, $K_L^0 \to 2\pi$) and another one in the $CP = -1$ mode ($K_L^0 \to 3\pi$).

What is interesting for physicists, from a physical point of view, is not so much the final state observed at the detection, which choice is, as a matter of fact, purely contingent, as the indication it provides about the physical state of the K^0 meson *at a time t before its detection*, given by the basis-free or invariant state vector. This state vector is given, for each group of (compatible) magnitudes corresponding to a physical content (either M and H_S, or H_F, or CP), as a superposition of their eigenstates. Conversely, the knowledge of this state permits the characterization of the properties of these magnitudes (for instance, the degree of CP violation in the weak interaction process with a Hamiltonian H_F).[45]

The so-called *“oscillation”* phenomena between quantum particles states are described and thought of in a similar way.[46] Consider *neutrinos*, electrically neutral (fermion) “leptonic” particles existing under the form of three different species,

ν_e, ν_μ, ν_τ, each one endowed with a distinct conservative magnitude, the leptonic, *electronic*, *muonic*, *tauic* charges or quantum numbers, shared with the electrically charged corresponding particles, electron, muon, tauon[47] (respectively e^-, μ^-, τ^-), together with which they constitute the three families of *leptons* (the most elementary "particles" of matter with *quarks*). Their mass is very small, possibly null.

If the mass of neutrinos is not exactly zero, one can distinguish three states of mass, ν_1, ν_2 and ν_3, distinct from the states that represent the ("leptonic") neutrinos observed through their "weak interactions" (ν_e, ν_μ and ν_τ and the corresponding antineutrinos). The latter can be described as linear superpositions of the mass states.[48] Neutrinos emitted in nuclear reactions (in β decays of nuclei) are of the type ν_e (or $\overline{\nu_e}$). The evolution with time, during the course of their state function, ψ_ν, is given by that of the amplitudes (or coefficients) associated with the states of the superposition. As a consequence, the proportion of the three mass states varies during the propagation. As these mass states can themselves be put in the form of superpositions of the leptonic states, it entails that the initial neutrino (ν_e) transforms partly into neutrinos of the other species (ν_μ and ν_τ), with a given "oscillation length" (or "wave length").[49] Such effects (such *phenomena*) are actively searched for by physicists for the three types of neutrinos.[50]

It is generally considered that physical neutrinos are those characterized by their properties in the (weak) interaction[51] through which they are produced or destroyed (interactions with other particles or eventual decays), that is to say that they are the "leptonic" neutrinos ν_e, ν_μ and ν_τ. Nevertheless, in the propagation of one or the other of these neutrinos, the effective *physical state* would, at any instant of time, under the considered hypothesis (of non-zero masses, and of some degree of leptonic numbers violation), be due to the mentioned transformations, a *linear superposition of these states*, evolving in time in a determined way. The detection by (weak) interaction of one of the states allows, by comparison with the initial state (given by the choice of one of the three types of neutrinos), the physical state at a chosen place on the covered distance to be determined (i.e. at a given time of flight). This detection is based on reactions of interaction where a neutrino transforms into the corresponding charged lepton ($\nu_e + n \rightarrow e^- + p$ and, similarly, $\nu_\mu + \ldots \rightarrow \mu^- + \ldots$, $\nu_\tau + \ldots \rightarrow \tau^- + \ldots$). For production, these reactions require enough energy to create the mass of the charged leptons.

In the case of neutrinos originated from nuclear reactions, the energies are insufficient to create masses larger than that of the electron. The neutrinos ν_e, transformed during their travel into ν_μ or in ν_τ, will therefore not produce reactions that would detect them and remain sterile. If one finds less ν_e than there were at the beginning, it might well be that the pure initial state has been transformed into a superposition of different neutrinos, of which only the projection on the ν_e-state is detected. This is, for example, what is supposed to happen with solar neutrinos, whose proportion received on Earth is far less than what is expected if neutrinos continued on their way remaining identical to themselves.[52] We would have there again (actually, the oscillation phenomenon has recently been definitely proven experimentally), an indubitable direct effect of the *physical character of a linear superposition state*.

The example (be it a real phenomenon or a simple possibility) gives indeed also evidence that the thought of such *states of superposition* is hereafter familiar to physicists. A superposition of states has to be understood as a simple change of basis relative to another set of mutually compatible physical magnitudes, corresponding to one of the possible "preparations." The physical state that physicists consider is not restricted *to that after the measurement* (otherwise it would only be the incident deficient neutrino); it is the state that is revealed to them by this measurement, and that also contained another undetected component that can immediately be reconstituted. Recent observations (in 2002) on neutral currents induced by solar neutrinos, which are not dependent on mass threshold effects (as the neutrino is simply scattered by the nucleon target), have yielded the expected rate, confirming that the neutrino beam arriving on Earth is in a superposition state of all the neutrino leptonic states. Of course, all these phenomena are studied with great numbers of "particles," but their description and their explanation must be understood in terms of properties of individual "particles," for the same reasons as those considered previously.

7. BEFORE DECOHERENCE, SUPERPOSITION

It remains to evoke another type of phenomenon of recent production and observation, "*decoherence*." We will not undertake a thorough discussion of its implications and its interpretation here. In particular, we will not pronounce (reserving the discussion for another opportunity) whether this phenomenon gives a solution to the problem of measurement of quantum systems, or whether it brings new views on the relationships between the "classical" and the "quantum." At least it illustrates an important aspect, to my eyes, through its "visualization": it makes us *see* a state of superposition propagating and thus allows us to better *conceive* the possibility and the *physical reality* of such states.[53]

The metastable state of superposition that has been observed recently for "mesoscopic systems"[54] is an "*entangled*" state made by coupling a Rydberg atom in a two-energy states superposition with an electromagnetic field (of few photons) in a two components superposition state. The field is a physical system that plays the role of the Schrödinger's cat of the famous thought experiment (Schrödinger 1935). The overall system is entangled (not factorizable in its various components), and this entanglement (that constitutes the "coherent state") is further multiplied through successive interactions with the various (quantum) elements of the environment (such as those that constitute the observation apparatus), so that in the end the initial coherence does not show anymore, the effect being absorbed rapidly ("decoherence"). In such a production experiment of a coherent entangled state, one can vary the parameters which determine the degree of coherence of the system: these parameters are the number of photons that make the electric field, and the time of propagation of the entangled system (which is the time elapsed between its production and its analysis to determine whether it is still in a coherent state). The coherent state itself manifests as such by some interference which can be observed through a correlation between pairs of the atom-analyzers at detection. Coherence can then be controlled, and the condition and time when coherence ceases marks the shift from quantum to "classical-type" behavior of the system. This shift is attributed to the many interactions occurring between the system and the quantum components of the environment. The simple original entangled system combines itself with the states of the latter (each one being itself in a linear superposition), giving rise to a further entanglement: as the process is going further, it leads in an irreversible way to a many component entangled system. Quantum non-separability forbids going back to the original components simply entangled, and that original entanglement is lost in the end, as it becomes diluted in the multiple entangled overall system, and has become definitely inappreciable. In the end, the quantum character of the state under study has been lost, although the whole process has been considered from a purely quantum point of view. So to speak, a "classical" behavior (a non quantum one) has been generated from quantum states merged inside entangled multiplicities.

It is clear that the process of decoherence is not identifiable with that of measurement, for it happens softly through the quantum interactions themselves, whereas measurement is a process which immediately chooses one of the final states by suppressing the others: the continuous soft (natural) process is (artificially) interrupted by the arrangement of apparatus itself, which favours at random only one of the components of the final state and destroys the superposition. So to speak, measurement is decoherence plus projection (reduction) on only one of the components of the initial state of the physical system under consideration. Nevertheless, decoherence helps to understand the initial stage of such a transition, which seems, in the final stage, to be purely of a statistical mechanics and thermodynamics nature. But I do not want to comment further on this, leaving it for another opportunity.[55] I rather content myself with observing that evidence for the process of coherence to decoherence is *per se* evidence for the physical character of the coherent, entangled, i.e. quantum linear superposition state, shown as propagating in space and time.

8. CONCLUSION

All the physical phenomena examined so far persuade us that the state function ψ represents (or describes) the state of the physical system completely. I mean by "complete representation" adequacy and covering: there is nothing more in the physical system than what is comprised in its theoretical representation by the state function.

If we restrict the question of the theoretical representation of quantum systems to the mere quantum level where these systems exhibit properties and interact with others systems of a similar nature, the concepts of state function, quantum system, quanton, quantized field, with the magnitudes that qualify them, are self-sufficient: for conception and handling in theoretical work, they do not ask for any physical or conceptual underlying classical basis such as that of a undulatory or corpuscular substance, distinguishable and localized. For the quantum physics of atomic and subatomic phenomena and quantized fields the "quantum level" where these concepts operate is the fundamental level, and, in particular, physical systems are effectively represented by their "state functions," and physical magnitudes by their "operators." At this level of representation, it is not necessary to go back, for each magnitude and each state, to the practical circumstances of their determination that refer ultimately to observations with the help of classical apparatuses.

For theoretical thought at the quantum level, the classical systems constituting these apparatuses are only intermediary instances in the process of the constitution of data that are in the end translated into quantum terms. The data being acquired, the quantum domain allowed itself to be conceived and explored in full conceptual and theoretical independence with respect to the classical domain.

This consideration does not diminish the problem of the quantum-to-classical relationship: it simply puts it aside, provisionally, as a fundamental problem. It is an epistemological and philosophical decision, taken in order to give the quantum domain and its theoretical representation the largest autonomy with respect to particular philosophical perspectives on knowledge. It has often been considered that (physical) knowledge is to be referred to observation, in the name of a primacy of perception in characterizing phenomena. However, contemporary reflection on science, and particularly on the various areas of physics, has led to conceive of the relations of concepts and theories to *perception* as most indirect. The demand for *intelligibility* requires, as I suggested in the beginning, a direct and close connection with the *understanding* that undertakes its theoretical elaboration by following a process of *rational construction* that is linked only in an intermediate manner with the forms of *perception*. Regarding the conceptualization and the theoretical insight obtained from them, the phenomena under consideration are first brought to the understanding and secondly to the perception. If we refer these *phenomena* to (quantum) *objects*, that means that the latter are rationally constructed before being secondarily and indirectly perceived.

The question of the *physical meaning of magnitudes*, among which the representative state function of a system is foremost, is henceforth more directly illuminated than by the current ("orthodox") interpretation, conceiving this meaning *through reference to measurement*. The reference, according to the view proposed here, is to quantum phenomena, whose access is indirect but recognizable by a rational and consistent construction, that is supported by data coming in the last instance from the perceptual (observation and experiment). Consequently, there is nothing to oppose considering the state function in the form of a superposition (but basis-invariant) *describing effectively the state* of a physical system evolving in the course of time.

The notion of *quantum physical state* differs from the current idea of a *physical state*, referring generally to magnitudes that are directly observable through instruments ruled by the laws of classical physics. The difference between a physical phenomenon (or system) at the quantum level and a phenomenon (or a system) at the classical level is that *the second is closer (if not homogeneous) to its conditions of observation referred to perception*, whilst the first remains radically *distant from them* and is definitely *heterogeneous* to them.[56] This formulation of the difference between the classical and the quantum domains is free of philosophical bias about knowledge: it has the advantage of not arbitrarily limiting the capacity of the quantum to be intelligible. If they are dissimilar in their relationship with *perception*, their links to *understanding* are not of a different nature: all concepts of physics, classical as well as quantum, are expressed by magnitudes that are constructed (by man) and abstract.[57]

That a *quantum state* be accessible to experiment only indirectly does not affect the possibility *to acquire knowledge of it. Magnitudes* that characterize it are also not directly accessible, since they are not endowed with numerical values. To take into account all the elements considered in what precedes, we must therefore conceive an *extension of meaning* to the quantum domain, of the notions of *physical magnitude* and of *physical state*, beyond the meaning usually accepted for them in classical physics (including the theory of relativity). This extension, legitimated by the *phenomena* (with a sense of this term that does not reduce them to mere objects of perception but that conceives them according to their capacity to be brought to *knowledge*), is actually already realized in practice by the main properties of the very formalism of quantum theory.[58]

If we look back to their history, such extensions of meaning have been a common procedure in mathematics as well as in physics: an example among many others in mathematics is the extension of the concept of number from integer to fractional, to irrational and then to imaginary and complex numbers; as for physics, consider only motion, force, energy and also the extension of finite to differential magnitudes. In all cases, such extensions were not the least obvious and led to hard scientific and philosophical debates and controversies.

By proposing this extension of meaning for the concept of physical magnitude to forms that are not endowed with numerical values, to states that are linear superpositions of eigenstates, in order to ensure epistemological *aseity* (self-contentedness) for the quantum domain and its theoretical representation, we give primacy to understanding over perception, which is driven to an ancillary status. This is a pragmatic decision that avoids deciding on the fundamental problem that still remains open to the relationship between the classical and the quantum, but that allows us at the same time to consider with full legitimacy a wide range of phenomena that might well be the base of all others. But, considering the present state of our knowledge, we cannot be sure of this. We can only relate it to the more fundamental and general question, still standing and in evolution, of the unity of physical phenomena and of a unified approach to them. But precisely this kind of approach might still be doomed to remain out of reach for present theories, until a deeper penetration of the unity of physical phenomena is obtained through a sound unification of the fundamental interaction fields of matter.

To find a solution outside this perspective, if proven possible, the "quantum problem of measurement," that is to say the nature of the relationship of the quantum and the classical, would be finally only of limited interest. With the practical rule connecting, through probabilities, quantum magnitudes and their state functions with the corresponding classical entities determined from measurement devices, we have the minimal algorithm needed to place on a pragmatic basis the quasi autonomous existence of two coherent and intelligible domains of physical reality, with reference to their proper and specific phenomena and objects: the classical and the quantum.

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NOTES

1. Respectively: Einstein, letter to Karl Popper, 11.9.1935, published as an appendix in (Popper 1959; Einstein 1953, 6-7).
2. On the history of quantum electrodynamics, see (Schweber 1994).
3. Cf., for example, (Bimbot and Paty 1996).
4. This term has been introduced by Mario Bunge (Bunge 1973).
5. See (Paty, 2000a).
6. Bell (1973, 1987a). See the complete quotation in the epigraph.
7. (Einstein, Podolsky, and Rosen 1935; Einstein 1948; 1949). Cf. (Paty 1988a; 1986; and in press).
8. In particular in (Einstein, Podolsky, and Rosen 1935; Einstein 1948; 1949). See (Paty 1986; 1988b; 1995a; and in press).
9. The two systems U and V form at initial time one only system $U \oplus V$ and are allowed thereafter to move away from each other at arbitrary distances (for instance, two photons emitted in correlation by an atom).
10. For instance, the overall momentum, $\vec{P} = \vec{P}_u + \vec{P}_v$, or the overall spin, $\vec{J} = \vec{J}_u + \vec{J}_v$ (its modulus J and one of its components, $J_1 = J_{u1} + J_{v1}$).
11. Measurement of magnitude A for the system V determines its state function, $\psi_V{}^A$, and the correlation gives the corresponding magnitude for the system U, and therefore the state function of the latter, $\psi_U{}^A$.
12. Let $\psi_V{}^B$ be determined by the measurement of B, from which we deduce, without measuring, $\psi_U{}^B$. *A priori*, $\psi_U{}^A$ and $\psi_U{}^B$ are different, although they were not, *in principle*, perturbed, as they have not been directly measured.
13. In particular in (Einstein 1948; 1949). See (Paty 1995; and in press).
14. Ensembles of systems can admit a non-biunivocity of their state function, if the latter is only about mean values.
15. They are called “Bell’s inequalities”: see (Bell 1964; 1966; 1987b). They are relevant for the property of locality generally speaking, independently of they being or not related to determinist hidden variables, to which they had been linked in a first period. More general relationships have been obtained since then: Bell’s theorem for locality without hidden-variables (Bell 1971; Eberhard 1977; Peres 1978; Stapp 1980), and for more than two quantum correlated particles (Greenberger, Horne, and Zeilinger 1989; 1990; Mermin 1990).

16. (Freedman and Clauser 1972; Aspect, Grangier, and Roger 1981; 1982; Aspect, Dalibar, and Roger 1982; Aspect 1983). See the following reviews and analyses of the experimental results: (Bell 1976a; Paty 1977; 1986; Clauser and Shimony 1978).
17. See (Bohm and Hiley 1975, in Lopes and Paty 1977, 222; Paty 1988a, ch. 6, and Paty 1986).
18. The use of this word, coined by Schrödinger in 1935 (Schrödinger 1935; 1984), has been reactivated recently (Shimony 1993; d'Espagnat 1994; Cohen, Horne, and Stachel 1997a, b, etc.).
19. This aspect has been emphasized in the article quoted (Paty 1986), and already in 1980 (Paty 1981; 1982). Bernard d'Espagnat seems to come also to the same conclusion in one of his recent books (d'Espagnat 1994, 430).
20. I remember my discussions with him: we disagreed on this point. This question represented to him an intellectual challenge whose difficulty remained untouched.
21. (Bell 1987b). I have quoted elsewhere (Paty 1988a, 245), a letter in this sense of John Bell to Alain Aspect.
22. This word is inadequate if by individuality one means a unity. Undifferentiated quantum systems can be counted: they keep *cardinality*.
23. Poincaré (1912). See (Paty 1996).
24. See (Paty 1993, ch. 5 and 7).
25. See, in particular (Pflegor and Mandel 1967; Grangier 1986). The concepts of quantum theory of field, that permit the definition of states with a given number of particles, underlie these experiments. It is necessary, for example, to prepare one-photon states of the electromagnetic field (Grangier 1986).
26. $P = |\psi|^2$.
27. Consider an initial individual system crossing a diaphragm with two slit a and b, and whose state is represented by $\varphi(x) = \frac{1}{\sqrt{2}}[\psi_a(x) + \psi_b(x)]$. Let z be the variable corresponding to various localizations on the screen, placed at a distance x from the diaphragm. The state $\varphi(x)$ of the individual interfering system can be considered as a linear superposition of states prepared along the values z_i of the variable (or magnitude) z : $\varphi(x) = \sum_{z_i} \alpha_i \vartheta(z_i)$. The probability of an impact on the screen in z_i is $|\alpha_i|^2$.
28. By the square of its modulus.
29. Cf. (Paty 1990).
30. See (Paty 1988b; 1993, ch. 10; and in press).
31. Cf. (Paty 1995a; in press). See above.
32. Already in 1911-1912, Ladislas Natanson and Paul Ehrenfest had diagnosticated the non-classical character of the statistics corresponding to Planck's radiation law. See, for instance, (Kastler 1981; Darrigol 1988; 1991; Pesic 1991).
33. This includes the invariant characteristics shared by the various possible states of a system, that contribute to define the system and its particular states corresponding to given magnitudes.
34. Consider, in effect, a system of two identical quantum particles *1* and *2*, each in its state, represented by the state functions ψ_1 and ψ_2. The state function of their coupled system is symmetrical for the permutation of the particles in the case of Bose-Einstein statistics, hence: $\psi_{12} = \frac{1}{\sqrt{2}}(\psi_1 \otimes \psi_2 + \psi_2 \otimes \psi_1) = \psi_{21}$. Nothing forbids identical (indistinguishable) particles *1* and *2* from being in the same state inside the system (identical bosons can accumulate in the same state inside a system). For the case of Fermi-Dirac statistics, the coupled state function is antisymmetric: $\psi_{12} = \frac{1}{\sqrt{2}}(\psi_1 \otimes \psi_2 - \psi_2 \otimes \psi_1) = -\psi_{21}$. If the identical fermions *1* and *2* were totally indistinguishable, occupying the same state in the system, then one would have: $\psi_{12} = -\psi_{12} = 0$: two

identical fermions cannot occupy the same state inside a quantum system (exclusion principle).

35. (Cornell and Wiemann 1998). Cf. (Griffin et al. 1995).
36. The mechanism was proposed by R. H. Fowler as soon as he knew the statistics studied by Paul Dirac, who was his student (cf. Doncel et al. 1987, 274).
37. From the point of view of arithmetic, concerning how to count or to identify by a number, such objects are characterized by cardinality, but not by ordinality. It has been proposed from a logic point of view to describe them with a set theory whose elements would possess this property, different from that of Zermelo-Frenkel (cf., for instance, French and Krause 1996).
38. Or, according to the usual terminology, “a complete set of observables that commute.”
39. Strictly speaking, the representation of these “particles” makes use of the quantum theory of fields. However, the features of their properties that we discuss here are only those of the basic formalism of quantum mechanics (the definition of a state from physical magnitudes and the principle of superposition for the state functions).
40. The magnitudes (the “observables,” in the quantum jargon), H_s, M and S commute between themselves $([H_s, S] = 0, \text{etc.})$ and have the same eigenstates.
41. H_F and S do not commute $([H_s, S] \neq 0)$.
42. Let us recall that in quantum theory the mathematical form of *physical magnitude* is a linear *operator* acting on the state function.
43. Due to Gerhart Lüders, Wolfgang Pauli, and Julian Schwinger, who established it around 1952-1955 (see Lüders 1952 and especially 1954; Pauli 1955; Schwinger 1951-1953). See comments in (Enz 1973; Doncel et al.1987; Yang 1982).
44. In fact, weak interaction does not conserve CP in these processes.
45. The whole thought of “elementary particles” physics is, as quantum physics in general, ruled by the superposition principle. We could have taken other examples of state mixtures as superpositions: the neutral states of “vector mesons” (ω^0, φ^0, with spin-parity $J^P = 1^-$) under the conservation of a given magnitude (for example under SU_2 “isospin” symmetry or SU_3 “unitary spin” symmetry), or the state superpositions of the neutral “intermediate bosons” (γ and B) of the gauge symmetry electroweak theory of A. Salam, S. Weinberg, and S. Glashow (cf., p. ex. Paty 1970; 1985). These bosons, and also the charged “intermediate bosons” ($W^{\pm}$), are initially supposed to have a vanishing mass as the photon, and their mixture, or superposition, is characterized by a coefficient (ϑ_{W-S}) called “Salam-Weinberg mixing angle,” that is the parameter of the theory. The symmetry breaking generates the finite masses of the “physical” “intermediate bosons” $(W^{\pm}, W^0)$, related to the mixing parameter (see, f. ex., H. Pietschmann and D. Haidt, in Gaillard and Nikolic 1977; Paty 1985). All this however is happening inside the limits of the range of weak interaction, that is extremely small. The examples that we have presented in the text are more striking for our purpose, insofar as they correspond to phenomena that are manifested on large spatial distances, covered during the propagation, and for which one hardly could refrain to speak of *physical states*, beyond the mere mathematical formalism of the theory.
46. One example, hypothetical but theoretically founded, would be eventual oscillations of neutrons into antineutrons $(n \rightarrow \bar{n})$, through an interaction field violating baryonic number (such as required by the “Grand Unification” theories).
47. Or “heavy lepton” (with mass 1777 MeV, the muon mass being 106 MeV, and the electron mass 0,5 MeV; the mass unit is MeV, million of electron-volts, in the appropriated unit system commonly used in subatomic physics).
48. See, f. ex., (Paty 1995b); Alexei Smirnov in (Nguyen-Khac and Lutz 1994). Leptonic numbers are no more completely conserved, and the heavier neutrinos can decay into a lighter neutrino together with other particles (a different process than “oscillations” considered here).
49. “Oscillations” are a function of neutrino mass differences, energies and covered distances.

50. These experiments concern, besides nuclear reactor or solar neutrinos (essentially $\overline{\nu}_e$), atmospheric neutrinos (and antineutrinos) (mainly ν_μ) and those produced at particle accelerators (ν_μ and ν_τ).
51. Neutrinos interact only through "weak interactions."
52. ν_e neutrinos are detected by their capture by a nucleus with emission of an electron (or of a position in the case of $\overline{\nu}_e$ antineutrinos). Neutrinos of other kinds resulting from oscillation are sterile for this type of reaction, and escape detection. But they are indeed part of the incident flux.
53. On the theoretical interpretations of the phenomena and of the experiments, see notably (Zurek 1982; 1991; d'Espagnat 1994; Omnès 1994a and b).
54. In the experiment performed at the Laboratoire de physique de l'École Normale Supérieure, Paris: (Haroche, Brune and Raimond 1997).
55. For a reflection on this state of things, see (Paty, in 2000a).
56. There still remains, anyhow, between a *physical system* qualified as such, be it a classical or a quantum one, and its *conditions of observation*, a difference of nature. I want only to underline here that the working modes of measurement devices are referred to classical phenomena.
57. Cf. (Paty 1988a, and 2000a).
58. Intuitively perceived by such theoreticians as Dirac, who extended the notion of commutative magnitudes expressed by ordinary numbers (*c*-numbers), to non-commutative ones (*q*-numbers) (Dirac 1926a and b; 1928). Cf. (Mehra and Rechenberg 1982, vol. 4, 162 sq.; Darrigol 1992), it has not, however, been explicitly legitimated as such, which ensured the permanence of the dominant philosophical interpretation (cf. Paty, 2000a).

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