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The Conductivity of Spherically Symmetric Layered Earth Models determined by Sq and longer Period Magnetic Variations*

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Summary

An approach to the inverse problem of electromagnetic induction in spherically symmetric layered Earth models is described. In the first case single periodic variations alone are considered and it is assumed that the conductivity is uniform in a single thick shell which surrounds a perfectly conducting sphere. The conductivity and thickness of the shell are determined for each variation separately by using the observed value of the ratio of internal to external parts of the magnetic potential at the surface of the Earth. Results have been obtained by using a variational technique, for Sq variations, the 27-day variation and its harmonics, and the annual variation. In the second case several variations are used simultaneously to obtain a multilayered model. Finally the method is used to give an estimate of the maximum screening effect of the oceans.

1. Introduction

Time variations in the magnetic field observed at the surface of the Earth are produced by primary sources, which are located at both interior and exterior parts of the Earth. The magnetic variation field arising from external primary sources is mathematically separated into primary e and induced i components using the method of spherical harmonic analysis, or the surface integral formulae derived by Price & Wilkins (1963). Estimates of the radial distribution of electrical conductivity can then be obtained from a study of the relation between these two components (Lahiri & Price 1939; Banks (1969), Bailey (1970), Parker (1970, 1972)).

In the case of periodic primary fields, the induced currents are periodic and have the same frequency as the primary, but with modified amplitude and phase. The complex ratio $i(\omega)/e(\omega)$ for any particular spherical harmonic at angular frequency ω, is known as the response or transfer function, S. Bailey (1970) showed that with a complete knowledge of S at all frequencies in a spatial distribution represented by any one spherical harmonic, it is theoretically possible, with the assumption that the distribution of conductivity is spherically symmetric, to determine a unique conductivity profile for the Earth. Such complete data are impossible to obtain so that the problem is underdetermined, and a loss of uniqueness results.

The results of Banks (1969) for the $P_1^{\,0}$ harmonic have been used by Bailey (1970) and Parker (1970) to determine conductivity profiles in the mantle, and by Parker

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(1972) to obtain bounds on the conductivity in the mantle and to test the observational data for self consistency. The data so far available appears inadequate to yield a profile which can be confidently thought to be that of the Earth. However useful information can be derived from the data, and the method presented here uses observed Sq response ratios for various spherical harmonics to obtain information about the conductivity in the top 600 km or so of the mantle, and response ratios corresponding to longer period magnetic variations to obtain information as far down as 2500 km.

2. Available data

Several determinations of the response function have been made for variations at discrete frequencies, and these have been recently reviewed by Price (1970). It is only necessary to give a brief summary of the available results here.

The earliest discrete response functions were obtained for the quiet day daily variation (Sq). The amplitude ratios and phase differences were obtained by Chapman (1919) for the P_5^4, P_4^3, P_3^2, P_2^1, inducing fields; that is variation fields of period 6 hr, 8 hr, 12 hr, and 24 hr, respectively. Other more recent determinations of the response for the last three of these periods have been made by Hasegawa & Ota (1950), and Matsushita & Maeda (1965). There is some degree of variation, particularly as far as phase differences are concerned. In the numerical work which follows, the average for each harmonic has been taken to minimize errors as far as possible. The values used are shown in Table 1.

The difficulty, apart from the inadequate distribution of observatories, in analysing data at this high frequency end of the spectrum is that the data is affected to some degree by lateral inhomogeneities in conductivity, in particular because the oceans are highly conducting, any field induced in them from known primary fields, will in turn induce fields in the mantle which are not of the same form as the original inducing field.

Clearly, care must be taken in interpreting results obtained from Sq and its harmonics. The general world-wide pattern of the data obtained from observatories does suggest that the spurious effects, while important, are not serious enough to make the data useless. This is supported by the consistent results obtained in Section 4 for all determinations of the Sq responses. It seems natural to use the data that exists, bearing in mind that models which are derived need to be treated with caution. The alternative is to neglect Sq data until such time as the effect of the oceans and other lateral inhomogeneities can be removed from it. However, the importance of Sq in determining the features of the conductivity of the upper part of the mantle is too great for the Sq data to be neglected entirely.

Longer period responses have been obtained. Eckhardt, Larner & Madden (1963) and Banks (1969), assumed a P_1^0 inducing field and determined the response function for the 27-day variation, corresponding to the recurrence tendency of magnetic storms, and its harmonics. The assumption that this particular variation is in a P_1^0 mode was based on a study of the form of the current system causing the variation.

Table 1

Mean for each harmonic of the amplitude ratio S_n^m and phase arg S_n^m of the Sq responses

n	m	$\lvert S_n^m\rvert$	arg S_n^m
2	1	0·370	13°·4
3	2	0·444	15°·1
4	3	0·432	15°·3

In addition to the periods mentioned above, the P_1^0 response of the semi-annual variation, and the P_2^0 response of the annual variation due to ionospheric dynamo action have been obtained (Banks (1969, 1972)). The assumptions that these variations are P_1^0 and P_2^0 harmonics need further examination, the data is quite inadequate to test them since only one or two stations were used. It is difficult to obtain estimates for periods much longer than this because the Earth is generating its own internal low frequency signals by dynamo action in the core, which contaminate the data for the 11-year variation and its 5·5-year subharmonic (Currie 1966). These very long period variations are essential because of their greater penetration depth, for extending our knowledge of the conductivity profile to even greater depths in the mantle.

The magnetic variation spectrum in the frequency range 0·05–0·5 cpd has been analysed by Banks (1969) and the response function for this range calculated on the assumption that the variation field in this range is in a P_1^0 mode. Banks' determination appears to show that the phase of the response exceeds 180 degrees at frequencies greater than about 0·150 cpd. The fact that physically impossible values of the phase differences were found from the data indicates that this analysis is unsatisfactory. Probably the assumption of the form of the inducing field is invalid for the particular variations considered, and the variation field cannot be described by a P_1^0 harmonic alone. It would be worthwhile repeating these calculations with more comprehensive data to obtain satisfactory results.

3. Formulation of the problem

Within the Earth, assumed to be a sphere of radius a with vacuum permeability, the magnetic vector potential **A** is toroidal and can be represented in terms of a scalar function T which satisfies a diffusion equation. If T is represented as a series of the form

$$T = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} aR_n^m(\rho, t)\, P_n^m(\cos\theta) \exp(im\phi), \tag{1}$$

where (r, θ, ϕ) are spherical polar co-ordinates, $P_n^m(\cos\theta)$ are the associated Legendre polynomials of degree n and order m, and $\rho = r/a$, it is found that the radial function R_n^m satisfies the differential equation

$$\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial}{\partial \rho} R_n^m\right) = n(n+1)\, R_n^m + \rho^2 a^2 \mu\sigma \frac{\partial}{\partial t} R_n^m, \tag{2}$$

obtained first by Lahiri & Price (1939).

The conditions on **B** at the boundary of the sphere require that both the normal and tangential components be continuous. Eckhardt (1963) first pointed out that provided there are no radial electric currents, as in this case, it is possible to split the variation magnetic field into parts of external and internal origin for any subsphere of the Earth. Thus in virtue of the boundary conditions it can be shown that at the surface of the sphere and at any arbitrary radius within the sphere we have

$$\left.\begin{aligned} \frac{n(n+1)}{\rho} R_n^m &= ne_n^m - (n+1)\, i_n^m \\ \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho R_n^m) &= e_n^m + i_n^m. \end{aligned}\right\} \tag{3}$$

These equations may be solved for the response ratio as a function of radial distance and time, thus,

$$S_n^m(\rho, t) = i_n^m/e_n^m = \frac{n}{n+1}\left[\frac{\rho \dfrac{\partial}{\partial\rho} R_n^m - nR_n^m}{\rho \dfrac{\partial}{\partial\rho} R_n^m + (n+1) R_n^m}\right]. \tag{4}$$

There are two significant values that S_n^m may assume in its path in the complex plane; $S_n^m = 0$, and $S_n^m = n/(n+1)$. In the first case $\rho(\partial/\partial\rho) R_n^m = nR_n^m$ and it can be seen from equation (3) that $i_n^m = 0$ and the response ratio corresponds to that at the surface of an insulating sphere. In the second case $R_n^m = 0$, so that the radial component of magnetic field is zero, and the ratio corresponds to that at the surface of an infinitely conducting sphere.

4. Thick shell surrounding sphere

Initially a simple model is considered which consists of a single thick shell of uniform conductivity surrounding an inner sphere, periodic time variations alone are considered. The thickness and conductivity of the outer shell will be determined by the observed response at a single frequency. A series of such single layer over superconductor models is useful in suggesting what general form the conductivity profile must take in the mantle.

The problem is of the following boundary value type; the values of R_n^m and its first derivatives are specified at $\rho = 1$, and are obtained from equation (3). The radial differential equation contains unknown parameters σ the conductivity, and q the value of ρ at which the interface between the outer layer and the inner sphere occurs. The problem is to determine both σ and q such that the observed boundary values at the surface of the outer layer, and the appropriate values at the surface of the inner sphere are satisfied. In the numerical work which follows it was assumed that $R_n^m = 0$ at $\rho = q$ corresponding to a superconducting inner sphere. The problem is unusual in that normally the boundary positions are fixed and the unknown parameter σ has discrete values for which there are non-trivial eigensolutions, in this case the lower boundary position has to be determined as part of the solution.

Because the value of q for any particular variation is not known *a priori*, an initial value technique can be used in which the observed values of the radial function and its derivatives are used as initial values. The radial differential equation is integrated with an assumed conductivity, and the integration proceeds downwards until the real part of the response function exceeds $n/(n+1)$. An improved estimate of the conductivity is obtained by the method indicated below, and the iterative process is carried on until the required accuracy is obtained for the boundary conditions at $\rho = q$.

The following variational technique enables improved estimates of the conductivity to be obtained, it uses the fact that first order changes in the eigenfunction will give rise to only second order changes in the eigenvalue, so that with only a rough approximation to the eigenfunction R_n^m the eigenvalue σ can be obtained to a higher order of accuracy.

The variational principle may be obtained as follows (Jady 1969); write the differential equation in the form

$$\frac{d}{d\rho}\left(\rho^2 \frac{d}{d\rho} R_n^m\right) + \{k^2 a^2 \rho^2 - n(n+1)\} R_n^m = 0, \tag{5}$$

where $k^2 = -i\omega\mu\sigma$. Multiply by R_n^{m*}, the complex conjugate of R_n^m and integrate with respect to ρ from $\rho = q$, to $\rho = 1$,

$$\int_q^1 \left\{R_n^{m*} \frac{d}{d\rho}\left(\rho^2 \frac{d}{d\rho} R_n^m\right)\right\} d\rho + \int_q^1 k^2 a^2 \rho^2 |R_n^m|^2 d\rho - n(n+1) \int_q^1 |R_n^m|^2 d\rho = 0. \quad (6)$$

Integrate the first term by parts to give

$$\left[\rho^2 R_n^{m*} \frac{d}{d\rho} R_n^m\right]_q^1 - \int_q^1 \rho^2 \left|\frac{d}{d\rho} R_n^m\right|^2 d\rho - i\omega\mu\sigma a^2 \int_q^1 \rho^2 |R_n^m|^2 d\rho$$

$$-n(n+1) \int_q^1 |R_n^m|^2 d\rho = 0, \quad (7)$$

from which it can be seen that all the terms are real except the first and the one containing σ. The first term vanishes, or the imaginary part is zero, at the lower limit if the sphere is perfectly conducting or insulating respectively. In the former case $R_n^m = 0$ at $\rho = q$, and in the latter $(d/d\rho) R_n^m = nR_n^m/q$. Equating imaginary parts of equation (7) we have the following variational principle for σ

$$\sigma = \frac{im \left[R_n^{m*} \frac{d}{d\rho} R_n^m\right]^{\rho=1}}{\omega\mu a^2 \int_q^1 \rho^2 |R_n^m|^2 d\rho}. \quad (8)$$

Provided the conductivity in the layer is small, good approximations to σ can be obtained directly from this formula by using the radial function appropriate to a non-conducting layer to perform the integral. The values of q are obtained from the uniform core model of Chapman & Price (1930). In the case of a perfectly conducting inner sphere q is independent of m and is given exactly by the equation

$$S_n^m = \frac{n}{n+1} q^{2n+1}. \quad (9)$$

Even when the inner core is only finitely conducting this formula is still a good approximation and q is only weakly dependent on m. The values of q obtained for the P_4^3, P_3^2, and P_2^1 harmonics are shown below

	q
P_4^3	0·934
P_3^2	0·928
P_2^1	0·889.

In the non-conducting layer R_n^m satisfies

$$R_n^m = \alpha_n \rho^n + \beta_n \rho^{-(n+1)}, \quad (10)$$

where α_n and β_n are in general complex constants determined from observations at the surface of the sphere. The integral in equation (8) may now be evaluated explicitly thus:

$$\int_q^1 \rho^2 |R_n^m|^2 d\rho = \frac{\alpha_n^2}{2n+3}(1 - q^{2n+3}) - \frac{\beta_n^2}{2n-1}(1 - q^{-(2n-1)}) + \alpha_n \beta_n (1 - q^2). \quad (11)$$

The values of σ obtained by evaluating the Rayleigh quotient in this manner are shown in Table 2, where values of σ and q obtained numerically with the iterative scheme described above, are shown for comparison.

The computational procedure when the conductivity is not small, or the approximation obtained by applying equation (8) once is not good enough, requires the solution of the pair of simultaneous equations for the real and imaginary parts of the radial function

$$\left.\begin{aligned} \rho^2 \frac{d^2 P}{d\rho^2} + 2\rho \frac{dP}{d\rho} - n(n+1) P + a^2 \rho^2 \omega\mu\sigma Q = 0, \\ \rho^2 \frac{d^2 Q}{d\rho^2} + 2\rho \frac{dQ}{d\rho} - n(n+1) Q - a^2 \rho^2 \omega\mu\sigma P = 0. \end{aligned}\right\} \quad (12)$$

By defining new variables

$$y_1 = P,\ y_2 = Q,\ y_3 = P',\ y_4 = Q', \quad (13)$$

where dashes denote differentiation with respect to ρ, the equivalent set of first order initial value differential equations written in the form

$$y_r' = y_r(\rho, y_1, y_2, y_3, y_4)$$

becomes

$$\left.\begin{aligned} y_1' &= y_3, \\ y_2' &= y_4, \\ y_3' &= -2y_3/\rho + n(n+1)\, y_1/\rho^2 - a^2 \omega\mu\sigma y_2, \\ y_4' &= -2y_4/\rho + n(n+1)\, y_2/\rho^2 + a^2 \omega\mu\sigma y_1. \end{aligned}\right\} \quad (14)$$

The initial values of y_1, y_2, y_3, and y_4 are specified at $\rho = 1$. The equations are now in a form suitable for numerical integration by the Runge–Kutta method.

The iterative process converged rapidly. The starting value of σ was usually taken to be zero and in the case of Sq the value of σ which was obtained after one application of the variational principle was within at worst 9·5 per cent of its final value. The results which were obtained for the response functions of Sq and its harmonics are shown in Table 2.

Table 2

The conductivity σ and radius of inner core q for single layer models obtained with Sq responses

		(a)		(b)	(c)	
n	m	σ (mho/m)	q	σ (mho/m)	σ (mho/m)	q
2	1	0·0177	0·8685	0·0160	0·0175	0·8695
3	2	0·0230	0·9050	0·0209	0·0210	0·9040
4	3	0·0175	0·9185	0·0165	0·0164	0·9335
5	4					

		(d)		(e)	
n	m	σ (mho/m)	q	σ (mho/m)	q
2	1	0·0151	0·8645	0·0321	0·9050
3	2	0·0247	0·8960	0·0145	0·9110
4	3	0·0143	0·8995	0·0207	0·9250
5	4	0·0114	0·9065		

(a) Computed values using mean Sq responses.

(b) Approximate values for σ using equation (11).

(c) Matsushita & Maeda (1965).

(d) Chapman (1919).

(e) Hasegawa & Ota (1950).

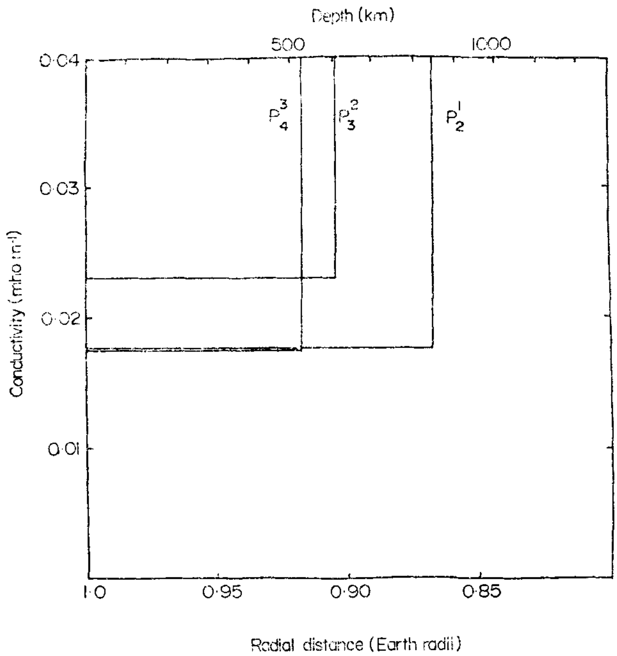

FIG. 1. Single layer over superconductor models corresponding to the $P_2{}^1$, $P_3{}^2$, and $P_4{}^3$ harmonics of Sq.

The response functions determined by Matsushita & Maeda seem to give results which agree most closely with the results for the mean values of the Sq responses. It is striking how well the values of σ accord with each other. These results indicate that on the basis of this single layer model the conductivity is low for at least 420 km depth from the surface, corresponding to the thinnest layer, and perhaps as far down as 850 km, corresponding to the thickest layer. The results for the mean Sq responses are shown in Fig. 1.

It is important to determine how both σ and q are affected by possible errors in the determination of the response functions. The results for the 12-hourly harmonic have been obtained assuming errors of 10 and 20 per cent in the magnitude of S and the phase of S separately. The results illustrated in Fig. 2 show that errors in the phase of the response are relatively unimportant to the values of σ that are obtained, whereas errors in magnitude make substantial differences. On the other hand the values of q are affected by changes in both by roughly corresponding amounts.

The conductivities determined by the longer period variations are shown in Fig. 3. The Sq results are included for comparison. It can be seen that the results derived for these longer period variations yields information about the conductivity to greater depths, and in the case of the annual variation, almost to the surface of the core. Results for the semi-annual variation using response estimates obtained by Banks from data of Eckhardt, Currie, and Banks, were difficult to obtain. It was necessary to assume the minimum value for the magnitude of the response in each case to obtain the results shown in Fig. 3. Even then the conductivity appears too high to be consistent with that required for the annual variation.

It is clear that the conductivity required in the layer by all these longer period variations is greater than for the Sq variations. To be compatible with the Sq results and the harmonics of the 27-day variation, the higher conductivity required by the latter must be located between about 600 and 950 km depth from the surface. This provides further independent evidence of the existence of the steep rise in conductivity

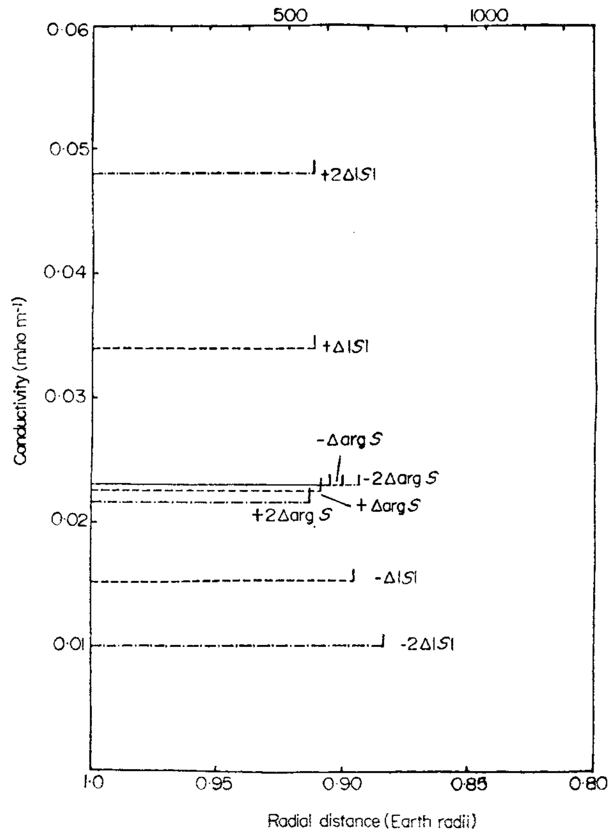

FIG. 2. Variation of the $P_3{}^2$ single layer over superconductor model assuming changes of 10 per cent (Δ), and 20 per cent 2(Δ) in phase, and magnitude of the response S. For clarity in the figure, only part of the vertical lines representing the superconductor have been drawn.

which has been the subject of much discussion, and confirms the suggestion made by Price (1970) that the rise occurs at greater depths than the 400 or 500 km which have been proposed by other writers.

The $|W_1{}^0|$ response curve obtained by Banks does show a change in slope at about 27/3 day period, possibly examination of single layer models for Banks' data around this period would enable reliable estimates to be made of the location and nature of the rise.

The path taken by the response function in the complex plane starting from the observed value at the surface of the Earth, and ending where it takes on its superconducting ratio is shown as a typical case (ii) for the mean $P_3{}^2$ variation in Fig. 4. The path is restricted to lie in the first quadrant and within a semicircle which passes through the origin and the point $n/(n+1)$, and is shown as a dashed line. Also shown is the path (i), calculated for the case of zero conductivity, the path in this case is a straight line moving directly away from the origin. The path for a conducting layer as it approaches the superconducting ratio, also tends to a straight line radiating from the origin. This is expected for as S approaches $n/(n+1)$ the governing differential equation becomes the same as that when the conductivity is zero.

5. Multilayer models

The variational technique so far developed can be extended to the case in which the sphere is separated into concentric layers, in each of which the conductivity is

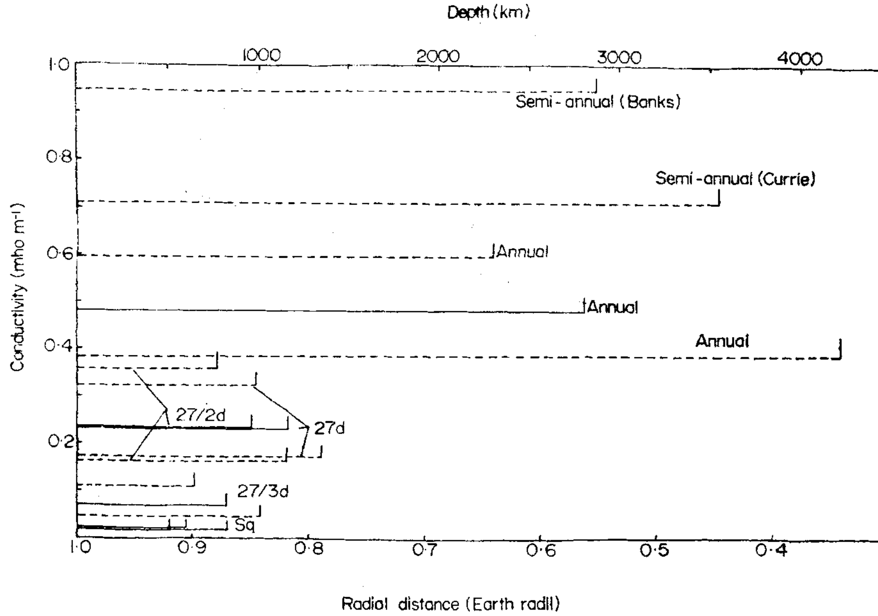

FIG. 3. Single layer over superconductor models corresponding to longer period variations, Sq results as in Fig. 1 are included for comparison. Dashed lines are results obtained with maximum estimated errors in $|S|$. For clarity in the figure, only part of the vertical lines representing the superconductor have been drawn.

supposed constant. It is assumed that as in the previous model there is a superconducting inner sphere. The modification required is straightforward; suppose that the layer of conductor just below the surface has conductivity σ_1 and its inner boundary is at ρ_1, the next layer has conductivity σ_2 and extends down to ρ_2 and so on. Equate the imaginary parts of equation (7) and we have for each discrete observed variation,

$$im\left(R_n^{m*}\frac{d}{d\rho}R_n^m\right)_{\rho=1}\Big/\omega\mu a^2 = \sigma_1\int_{\rho_1}^{1}\rho^2|R_n^m|^2\,d\rho+\sigma_2\int_{\rho_2}^{\rho_1}\rho^2|R_n^m|^2\,d\rho+\dots$$

$$+\sigma_N\int_{q}^{\rho_{N-1}}\rho^2|R_n^m|^2\,d\rho. \quad (15)$$

A one-layer model is determined by the observed response function of a single magnetic variation, a model with N layers requires N magnetic response functions to be specified. As the number of response measurements increases, the number of separate layers increases and the thickness of these layers becomes correspondingly less until in the limit the step profile becomes a smooth (unique) continuous profile.

The computational procedure is essentially as for the single layer model, except that the boundary positions which separate the layers have to be specified beforehand. The integrals in equation (15) are evaluated for the N variations, with assumed values of σ_i and the resulting simultaneous equations solved for the coefficients σ_i. The integrals are recalculated with the new values of σ_i and the iterative process is continued. The process will always give solutions, but for certain choices of boundary positions negative values of σ will be obtained, reflecting an inadmissable choice of boundary positions.

Applications of the method have been made to obtain the broad features of the conductivity structure of the mantle using response functions of three widely-spaced

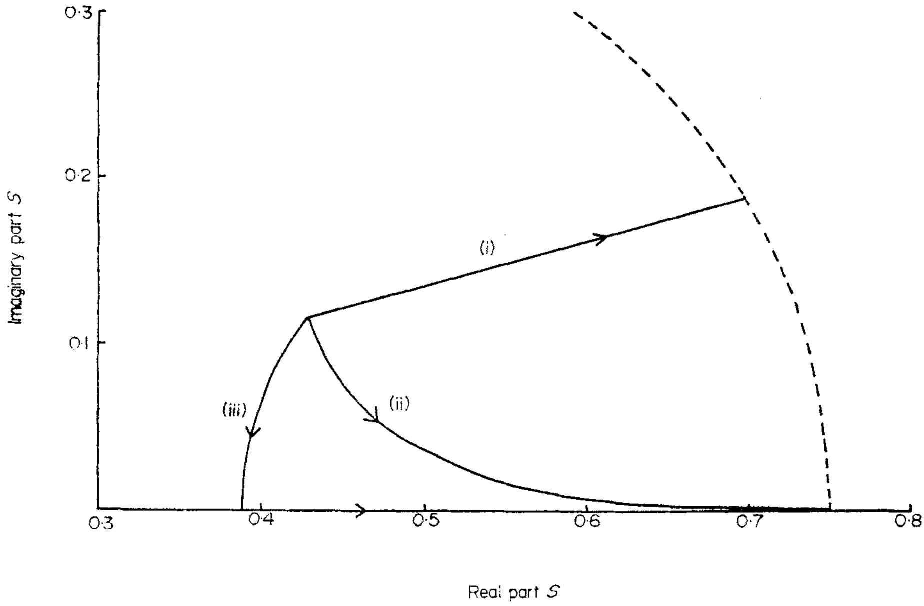

FIG. 4. Path of the $P_3{}^2$ response curve in the complex plane for (i) zero conductivity in the outer layer, (ii) single layer over superconductor model, (iii) maximum ocean effect. A portion of the restricting semicircle is shown with a dashed line.

variations; the daily variation $P_3{}^2$, 27 day $P_1{}^0$, and the annual variation $P_2{}^0$. Firstly the latter two were considered together, with the boundary between the two layers chosen arbitrarily at $\rho = 0{\cdot}8$, roughly 1300 km depth from the surface, and then at $\rho = 0{\cdot}9$, roughly 650 km deep. The results are shown in Fig. 5. Also shown is the effect of taking into account the 12-hourly harmonic of Sq. The introduction of the low conductivity demanded by Sq leads to an increase in conductivity between $\rho = 0{\cdot}8$ and $\rho = 0{\cdot}9$ and a reduction in conductivity lower down.

At first sight this is a suprising result, but it can be understood when it is realized that if the low conductivity indicated by the Sq variations extended to a depth nearly as great as that of the superconducting basement obtained in the single layer model for the 27-day variations, an almost infinite conductivity would have been required to satisfy the boundary conditions at the superconducting depth, and below this, because of the shielding effect of this high conductivity, the conductivity would have to be low, possibly zero to satisfy the boundary conditions for the annual variation. This demonstrates clearly, through the multiplicity of choice in making the divisions between the conducting layers, the non-uniqueness of the problem. It is also easy to see how spikes can arise in conductivity modelling problems, as for example in seeking the conductivity profile of the Moon.

A model determined by the three harmonics of Sq simultaneously could not be obtained. Whether the data at such close frequencies is imcompatible, or whether a special choice of boundary divisions is required, is not certain. Such a model would give useful information about the details of the conductivity profile within about the top 600 km or so of the Earth's radius, in particular evidence of conductivity changes associated with the seismic low velocity layer might be found. It was possible to produce models with any two out of the three Sq responses and several sets have been obtained, but as yet no obvious features from which conclusions might be draw have emerged.

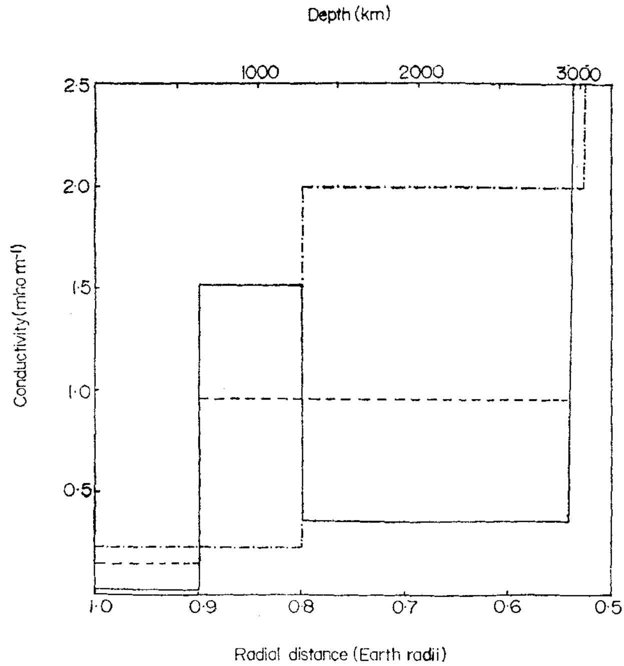

FIG. 5. Two layers over superconductor models determined by the 27 day P_1^0 and annual P_2^0 variations, and a three layer model (solid line) determined with the addition of the P_3^2 harmonic of Sq.

6. Ocean effect

The effect of the oceans on the magnetic response functions is difficult, because of the departure from spherical symmetry, to determine accurately, but will be greatest on high frequency variations. Their average world-wide effect can be represented approximately by replacing them in the model with a uniformly conducting thin shell. The maximum integrated conductivity may then be found for each magnetic variation by supposing that the conductivity underlying the shell is zero down to the superconducting depth. Equation (15) enables the conductivity in the shell to be found readily. The values for the three harmonics of Sq are shown in Table 3 with the integrated conductivities of curve *d* and curve *e*, obtained by Lahiri & Price, for comparison.

The maximum ocean effect determined by Sq are all less than curve *e*, but greater than curve *d*, and it would seem that $2{\cdot}9\ 10^3$ mho represents the maximum that would satisfy all three responses. This corresponds to a spherical shell with say the thickness of the mean depth of the oceans, but with conductivity 0·97 mho/m, which is about a quarter that of sea water. The path of the P_3^2 response function with the maximum oceanic shell compatible with it is shown in Fig. 4. From the value observed at the surface of the Earth, the path describes a nearly circular arc until it reaches the real axis, and then moves along the real axis towards the point $n/(n+1)$.

7. Conclusions

An eigenvalue approach to the problem of the conductivity of the mantle has enabled a simple model consisting of a single uniformly conducting layer overlying a superconducting basement to be derived for each discrete observed magnetic response function, including the annual variation, and the Sq variations. The use of the latter has to be defended against the objection that lateral inhomogeneities in conductivity contaminate the response measurements at this high frequency end of the variation spectrum and so render the data useless. It is true that the data is contaminated to an unknown extent, but the consistency of the results for all the determinations of Sq responses is evidence that this data is usable, and that it is important in determining the conductivity of the upper part of the mantle.

Table 3

Maximum integrated conductivities of an oceanic shell compatible with each Sq response. Curve d and curve e are the integrated conductivities obtained by Lahiri & Price (1939)

	P_4^3	P_3^2	P_2^1	curve *d*	curve *e*
Integrated conductivity (10^3 mho)	2·9	4·4	4·8	2·0	5·1

More accurate data would naturally enable features of the profile that have been suggested from other studies to be distinguished with confidence, e.g. the conductivity change associated with the seismic low velocity layer. Nevertheless, even though as has been shown the conductivity is sensitive to changes in the magnitude of the response function, the conclusion from the Sq results is that at least the top 600 km of the mantle has low conductivity overall. The exact location and nature of the rise in conductivity which takes place below this cannot be determined with certainty by the longer period variations, except that a lower limit of 950 km is required by the 27-day variations. At depths greater than this seemingly no spectacular changes take place and there is even a possibility of a fall in conductivity.

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