The Construction of Logical Space
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What is Logical Space?

1. *Truth*

   - When we assert something about the world: (1) we make a *distinction* amongst ways for the world to be, and (2) we commit to one side of the distinction.
   - For the assertion to be *true* is for the way the world actually is to be on the side of the distinction we are committed to.

2. *Logical Space*

   - *Logical space* is the set of distinctions amongst ways for the world to be.
   - To single out a *region* of logical space is to take sides with respect to some of these distinctions.
   - To single out a *point* in logical space is to take sides with respect to all such distinctions.

In What Sense is Logical Space Constructed?

1. *One distinction or two?*

   Water-Distinction
   The distinction between ways for the world to be whereby there is water and ways for the world to be whereby there is no water.

   H_2O-Distinction
   The distinction between ways for the world to be whereby there is H_2O and ways for the world to be whereby there is no H_2O.

   - Do *Water-Distinction* and *H_2O-Distinction* pick out one distinction or two?

   Here we are interested in distinctions amongst *ways for the world to be*, not distinctions amongst ways of *representing* ways for the world to be.

2. *'Just Is'-Statements*

   The question whether *Water-Distinction* and *H_2O-Distinction* pick out one distinction or two is the question whether to accept the following 'just is'-statement:

Water
For there to be water *just is* for there to be H$_2$O.

3. *The Main Thesis*

Our conception of logical space is determined by the 'just is'-statements we accept.

Some Additional 'Just Is'-Statements

Sibling
For Susan to be a sibling *just is* for her to share a parent with someone else.

Heat
For something to be hot *just is* for it to have high mean kinetic energy.

Physicalism
For such-and-such a mental state to be instantiated *just is* for thus-and-such brain states and environment conditions to obtain.

Properties
For Susan to instantiate the property of running *just is* for Susan to run.

Death
For Socrates's death to take place *just is* for Socrates to die.

Tables
For there to be a table *just is* for there to be some things arranged tablewise.

Dinosaurs
For the number of the dinosaurs to be Zero *just is* for there to be no dinosaurs.

How to Understand the 'Just Is'-Operator

1. *A Paraphrase*:

'For Susan to be a sibling *just is* for her to share a parent with someone else'

   might be paraphrased as:

'If Susan is a sibling, she *thereby* shares a parent with someone else; if she shares a parent with someone else, she is *thereby* a sibling'

2. *A Metaphor*

When God made the world and made it the case that Susan was a sibling, there was nothing *extra* God had to do (or refrain from doing) in order to ensure that Susan shared a parent.

And when God made the world and made it the case that Susan shared a parent, there was nothing *extra* God had to do (or refrain from doing) in order to ensure that Susan was a sibling.

3. *Facts*

Suppose that Susan is, in fact, a sibling. Then:

'For Susan to be a sibling *just is* for her to share a parent with someone else'

> *might be paraphrased as*

The sentences 'Susan is a sibling' and 'Susan shares a parent' describe the same *fact*.

Alternatively: 'Susan is a sibling' and 'Susan shares a parent' describe the same *feature of reality*.

4. *Symmetry*

'For Susan to be a sibling *just is* for her to share a parent with someone else'

> *is equivalent to*

'There is *no difference* between Susan's being a sibling and her sharing a parent with someone else'

When to Accept a 'Just Is'-Statement?

1. *Advantages and Disadvantages*

Accepting a 'just is'-statement has advantages and disadvantages:

- The *advantage* of accepting a 'just is'-statement is that one is left with fewer demands for explanation.
- The *disadvantage* of accepting a 'just is'-statement is that one has additional theoretical resources.

Example: the advantage of accepting *Heat* is that there is no need to answer a question like "I can see that this slab of metal has high mean kinetic energy. But is it hot?"

An disadvantage of accepting *Heat* is that there is no room for the view say that this slab of metal is hot because it contains large amounts of caloric, even though it has low mean kinetic energy.

2. *Finding a Balance*

In accepting a 'just is'-statement one should strive to find a balance between these competing considerations.

(More generally, the decision to accept a 'just is'-statement should be based on its ability to led to fruitful theorizing.)

3. *A Corollary*

Our conception of logical space is not independent of our best hypotheses about how the world is, since part of what we do

when we investigate the world is decide which 'just is'-statements to accept.

Summary: Carnap and Quine

- *Thesis (Carnap)*

  One's conception of logical space is determined by the set of sentences one treats as *analytic*.

  (A point in logical space can be modeled as a maximal *analytically consistent* set of sentences.)

- *Antithesis (Quine)*

  Our notion of analyticity is not robust enough to play that role.

- *Synthesis (Post-Quinean Carnapianism)*

  One's conception of logical space is determined by the set of 'just is'-statements one accepts.

  (A point in logical space can be modeled as a maximal *metaphysically consistent* set of sentences.)

Metaphysicalism

Metaphysicalism is the conjunction of a metaphysical thesis and a linguistic thesis.

1. *The Metaphysical Thesis*

   The world is *metaphysically structured*: there is a 'metaphysically privileged' way of carving up a fact into its constituent parts.

   Example:
   - *Proposal*
     The fact that Susan runs might be carved up into two parts: Susan and the property of running.
   - *Counterproposal*
     That same fact might also be carved up into: the property of running and the property of being instantiated by Susan.
   - *The Metaphysicalist*
     At most one of these carvings is 'metaphysically privileged'.

   Alternatively: At most one of them corresponds to the way the world is *at the fundamental level*; at most one of them corresponds to the worlds *metaphysical structure*.

2. *The Linguistic Thesis*

   In order for an atomic sentence to be true, the sentence's *logical form* must by in sync with the metaphysical structure of the relevant fact.

Example:

- *Logical Form*
  Logical form carves the sentence 'Susan runs' it into: (1) the singular term 'Susan', and (2) the predicate 'runs'.
- *Metaphysical Structure*
  The world's metaphysical structure carves the fact that Susan runs into: (1) the object Susan, and (2) the property of running.
- *Correspondence*
  In order for 'Susan runs' to be true, its logical form must be in sync with the metaphysical structure of the relevant fact: 'Susan' must refer to Susan, and 'runs' must pick out the property of running.

3. *A Consequence of Metaphysicalism*

- 'Socrates's death takes place' and 'Socrates dies' are atomic sentences distinct logical forms (or so I shall assume). So they cannot both be accurate descriptions of the same fact.

- In other words: *Death* must be false.

Compositionalism

Compositionalism is the conjunction of two linguistic theses. (One of them concerns singulartermhood, the other concerns reference.)

1. *Singular Terms*

   The following conditions are *jointly sufficient* for t to count as a singular term:

   - *Syntax*
     From a purely syntactic point of view, t behaves like a singular term.

   - *Truth Conditions*
     One has assigned truth-conditions to every sentence including t that one wishes to have available for use.

   - *Semantic Structure*
     One's assignment of truth-conditions respects any inferential connections guaranteed by logical form.

     Example:
       Introduce the new word 'direction*'.

   - *Syntax*
     From a purely syntactic point of view, 'the direction* of a' behaves like a singular term.
   - *Truth Conditions*
     The truth conditions of 'The direction* of a = the direction* of b' are, by definition: that line a be parallel to line b.[†]

[†]More generally:

- $[\ulcorner\text{the direction* of } a = \text{the direction*} \text{ of } b\urcorner]^N = \ulcorner a \text{ is parallel to } b\urcorner$.
- $[\ulcorner x_i = \text{the direction* of } a\urcorner]^N = \ulcorner z_i \text{ is parallel to } a\urcorner$.
- $[\ulcorner x_i = x_j\urcorner]^N = \ulcorner z_i \text{ is parallel to } z_j\urcorner$.
- $[\ulcorner\exists x_i(\phi)\urcorner]^N = \ulcorner\exists z_i([\phi]^N)\urcorner$.
- $[\ulcorner\phi \wedge \psi\urcorner]^N = \text{the conjunction of } [\phi]^N \text{ and } [\psi]^N$.
- $[\ulcorner\neg\phi\urcorner]^N = \text{the negation of } [\phi]^N$.

- *Logical Form*
  Our assignment of truth-conditions guarantees, for example,
  that 'there exists an *x* such that (*x* = the direction* of *a*)' is a
  consequence of 'the direction* of *a* = the direction* of *a*'.

2. *Reference*

   In order for the singular term *t* to be *non-empty* it is sufficient that
   the truth conditions that one has assigned to '$\exists x(x = t)$' (or some
   inferential analogue) be satisfied.

   Example:

   - The truth-conditions of:
        'There is an *x* such that (*x* = the direction* of line *a*)'
     are
        that there be something parallel to line *a*
   - Since *a* is parallel to itself, this is enough to guarantee that 'the
     direction*' is non-empty.

3. *Some Consequences of Compositionalism*

   - There is no *linguistic* obstacle for either of the following to be
     true:

   *Direction**
      For the direction* of *a* to be identical to the direction* of *b*
      just is for *a* and *b* to be parallel.

   Death
      For Socrates's death to take place *just is* for Socrates to die.