VOL. 83, NO. B4 JOURNAL OF GEOPHYSICAL RESEARCH APRIL 10, 1978

The Control of Volcanic Column Heights by Eruption Energetics and Dynamics

L. WILSON,[1] R. S. J. SPARKS, T. C. HUANG, AND N. D. WATKINS[2]

Graduate School of Oceanography, University of Rhode Island, Kingston, Rhode Island 02881

The height reached by a volcanic eruption column, together with the atmospheric wind regime, controls the dispersal of tephra. Column height is itself a function of vent radius, gas exit velocity, gas content of eruption products, and efficiency of conversion of thermal energy contained in juvenile material to potential and kinetic energy during the entrainment of atmospheric air. Different heights will be attained for the same total energy release depending on the style of the eruption: a discrete explosion produces a transient plume, whereas a prolonged release of material forms a maintained plume. A maintained eruption plume will also be formed if discrete explosions occur within a few minutes of one another, and eruptions producing large volumes of tephra commonly lead to maintained plume formation. Observed eruption columns from eight eruptions with cloud heights in the range 2–45 km and volume rates of magma production in the range 10 to 2.3×10^5 m^3/s are compared with predicted values deduced from theoretical relationships for fluid convection. Theoretical model heights were calculated in two ways: first, for a wide range of eruptive conditions by using a dynamic model of eruption column formation and second, by using a theoretical formula relating height to rate of thermal energy release. Results from the two calculations were found to agree well and furthermore showed satisfactory agreement with the eight observations. Expected cloud heights can be usefully expressed as a function of heat release rate, expressed as the equivalent volume eruption rate of magma, for three different values of the efficiency of heat use. The results imply that many eruptions involve highly efficient use of the released heat, which indicates that the particle sizes in these eruptions are sufficiently small to allow rapid heat transfer to air entrained into the column. For certain combinations of vent radius, gas exit velocity, and gas content, column collapse to form pyroclastic flows should occur. Cloud heights have been calculated for a wide range of permutations of these parameters corresponding to the onset of collapse. The maximum theoretical height expected for a stable maintained plume is about 55 km, corresponding to a volume eruption rate of 1.1×10^6 m^3/s.

INTRODUCTION

Discrete volcanic ash layers and dispersed ash zones have now been documented as a minor but significant component of all deep-sea sediments. Because tephra are often highly distinctive, at least in Pleistocene sediments, single eruptive episodes can frequently be identified by using a variety of parameters such as ash morphology, glass chemistry, the glass refractive index, and mineralogy. This characteristic, and the fact that ash horizons are generally distributed over very large areas during single eruptions, is the basis of the well-known value of tephra deposits in defining stratigraphic relationships.

Ash horizons are, however, potential indicators of several other important geological processes. A substantial proportion of the ejecta from explosive volcanic eruptions is deposited in the oceans either by direct fallout or by reworking of subaerial tuffs. Generally, it is difficult to trace all but the largest subaerial deposits to distances greater than 150 km even in depositional environments which favor preservation. In contrast, tephra in deep-sea cores often appear to be well preserved. Therefore crucial information on the productivity of individual volcanoes, volcanic regions, and global volcanism is more likely to be found in abyssal sediments. Abyssal tephra can also yield potential information on the volcanic cloud height and net paleowind velocity for an eruption or eruptive series.

Two of the authors have studied the downwind dispersal of volcanic ash from several sources by systematic analyses of the dispersed ash in downwind sediment core traverses [*Huang et al.*, 1975; *Watkins and Huang*, 1977]. Cloud heights for the eruptions represented were computed by using a simple model which relates cloud heights and net paleowind velocity profiles to the downwind sorting of volcanic ash particles. In principle, the cloud height can be expressed in terms of the total energy which is required to inject material to the height deduced and the ambient conditions in the atmosphere by using the theoretical treatment of *Morton et al.* [1956], which considers turbulent gravitational convection of both a continuous plume and an instantaneous source. The relationship used in our earlier papers assumes, however, an instantaneous explosion and yields the minimum explosive energy required to attain a given cloud height. Were an eruption to take place over many days, for example, the column height would depend on the rate of energy release rather than the total energy, and an instantaneous explosion would not be an appropriate model.

The four principal factors which control atmospheric dispersal of ash particles are the wind velocity profile in the atmosphere during the entire period of the eruption, the variation of the wind velocity during the eruption, the eruption column height, and the spatial distributions of particles of different sizes within the column. The growth of eruption columns and the way in which eruption cloud height is controlled by the rate of release of kinetic and thermal eruption energies are the principal concerns of this paper.

Models of the structure of eruption columns have recently been presented by *Wilson* [1976] and *Sparks and Wilson* [1976]. On the basis of both theoretical [*Wilson*, 1976] and empirical [*Blackburn et al.*, 1976] evidence they proposed that eruption columns can be conveniently considered in two regions. In the lower part of the column, where gas and ejecta are discharged at a high velocity, the momentum (kinetic

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[1] Also at Lunar and Planetary Unit, Environmental Science Department, Lancaster University, Lancaster, LA1 4YQ, United Kingdom.

[2] Now deceased.

Paper number 7B1126.
0148-0227/78/047B-1126$03.00

energy) of the mixture dominates the motion. This region can be modeled as a high-speed gas jet, or alternatively, the mixture can be considered as a projected slug. The choice of model for this region depends on the style of activity, principally the frequency of explosions feeding the column. During this stage the mixture decelerates rapidly as it interacts with the atmosphere and entrains air. As the column decelerates, buoyancy forces increase, air is heated, and momentum is decreased until eventually, buoyancy becomes dominant. In the upper part of the column, where buoyancy dominates the motion, the column essentially behaves as a convective plume. Uprise velocities are moderate, and decelerations are slow.

In most eruption columns the lower gas jet region makes up less than 10% of the total column height. Thus as a first approximation, models of convective plumes may be appropriate for understanding how cloud height is determined. In this paper we investigate various theoretical treatments of atmospheric convection and apply them to the problem of the relationship between eruption energetics and cloud height. We also compare the theoretical models with observational data.

ERUPTION CLOUD HEIGHT

Heights of eruption columns have been used as indices of the relative 'intensity' or 'explosivity' of different eruptions [*Knox and Short,* 1964; *Shaw et al.*, 1974]. It is therefore important to develop a detailed understanding of the factors which control eruption column height.

Observations suggest that two common modes of eruption can be distinguished which lead to eruption columns [*McBirney,* 1973]. One involves the occurrence of sudden discrete explosions which generally last only a few minutes or hours. Examples of such canonlike explosions [*McBirney,* 1973] are the 1938 and 1947 eruptions of Asama [*Minakami,* 1950], explosions during the 1976 eruption of Augustine volcano [*Hobbs et al.*, 1977], explosions during the 1975 eruption of Ngauruhoe [*Nairn and Self,* 1978], and the 1968 explosions of Arenal volcano [*Fudali and Melson,* 1972].

The other style of eruption is characterized by a continuous discharge of gas and entrained ejecta which has been compared to a fire hose by *McBirney* [1973]. Such eruptions may pulsate and may even be the result of closely spaced explosions, but the eruption column fed by the explosions can be regarded as a maintained plume. Such eruptions typically last for tens of minutes to several hours. Examples of such events are the opening Plinian phase of the 1947 Hekla eruption [*Thorarinsson,* 1954] and phases of the 1975 Ngauruhoe eruption [*Nairn and Self,* 1978].

The two contrasting styles of eruption clearly must be represented by different physical models. In a sudden discrete explosion the height of the column is related to the total energy released, whereas in a maintained, continuously fed plume the column height is related to the rate at which energy is supplied to the plume. Thus, as was stressed earlier, if the column heights are deduced from geological data on tephra, the way in which they are related to eruption energetics will be dependent on whether a maintained plume or instantaneous explosion model is assumed.

The criterion for an instantaneous explosion is that the duration of the explosion is short in comparison with the rise time of the cloud. The criterion for a maintained plume is that there is a continuous supply of material at the base of the column. A maintained plume model is still valid even if the supply does not continue indefinitely, provided it is maintained for a time comparable to that needed for the cloud to reach its maximum potential height. We emphasize that the two styles of activity are end-members of a continuous spectrum of eruption styles and thus the division of our modeling into two types is arbitrary but convenient.

A fundamental difference between the two models is in the quantity of energy expended in reaching a given height. In the instantaneous sudden explosion the released cloud has to do work in displacing the atmosphere as it rises as well as lose energy owing to turbulent mixing with the atmosphere at the column sides. In the maintained plume the atmosphere has already been displaced, so the only source of energy loss is at the column sides. We shall compare the two models for eruption cloud growth with relevant data for several historic eruptions.

MAINTAINED PLUMES

Theoretical calculations. There have been many attempts to relate heights of convective clouds to conditions at their bases, often in connection with the release of effluent gases from industrial processes. The application of these treatments has been summarized by *Briggs* [1969] and *Settle* [1978]. Settle draws attention to the need to distinguish between formulae based on the initial kinetic energy supply and those based on the initial thermal energy. The observed convective structure of many such columns suggests that models based on the rate of thermal energy release are likely to be applicable to most explosive volcanic eruptions, where the degree of magma fragmentation is sufficient to yield high efficiency of heat release.

Wilson [1976] attempted to take account of both factors in deriving heights of maintained (Plinian) eruption columns, using dynamic equations to follow motions in the lower gas jet part of the column, where the initial kinetic energy is rapidly lost. He also used arguments based on conservation of energy to deduce final cloud heights. It was demonstrated that the main effect of the high-speed gas thrust part of the column was to initiate the rapid entrainment of atmospheric air. In all cases the initial density of the erupted fluid was greater than that of the atmosphere, and mixing and heating of air within the gas thrust region was therefore required to produce a mixture less dense than the atmosphere so that thermal convection could be initiated. These facts suggest that the heights of column rise calculated by Wilson should be compatible with the results of calculations based on thermal energy release alone. Such a correspondence would be of great value in simplifying the use of these calculations, since the rate of thermal energy release at a vent can be easily specified in terms of the average radius of the vent and the velocity, density, and temperature of the erupting fluid, together with an efficiency factor to account for the thermal energy lost to the system when coarse hot clasts leave the column. This efficiency factor is related to the degree of fragmentation, as will be discussed later.

There are a number of simple formulae available to relate cloud height to thermal energy release. The discussion of *Briggs* [1969] shows that the formula proposed by *Morton et al.* [1956] is essentially identical to that derived by Briggs and other authors to give the total rise height H of a buoyant plume in a stable atmosphere. The Morton et al. formula is expressed as

$$H = 31(1 + n)^{-3/8}\dot{Q}^{1/4} \qquad (1)$$

where H is measured in meters, $\dot{Q}$ is the rate of production of thermal energy at source in kilowatts, and n is the ratio of the vertical gradient of absolute temperature (the environmental

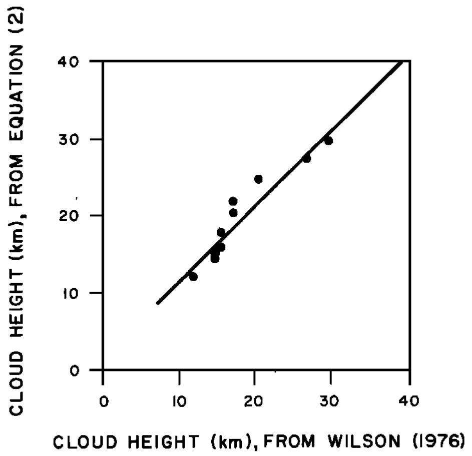

Fig. 1. Comparison of cloud heights deduced from the theoretical treatment of *Wilson* [1976] and from (2), derived from the work of *Morton et al.* [1956].

lapse rate) to the adiabatic decrease in temperature with height (the adiabatic lapse rate). This formula gives the final height that a buoyant plume will reach in a stable density-stratified fluid. A number of simplifying assumptions are made by *Morton et al.* [1956], but the formula satisfactorily agrees with observations [*Briggs,* 1969]. For an environmental lapse rate in a standard atmosphere of 6.5°C/km and an adiabatic lapse rate of 9.8°C/km, (1) can be reexpressed as

$$H = 8.2\dot{Q}^{1/4} \tag{2}$$

where H is in meters and $\dot{Q}$ is the steady rate of release of thermal energy in watts. $\dot{Q}$ is related to the conditions at the vent by

$$\dot{Q} = \beta u \pi r^2 s(\theta - \theta_a)F \tag{3}$$

in which β, u, s, and θ are the bulk density, velocity, specific heat, and temperature of the erupting fluid; θ_a is the temperature to which the eruption products ultimately cool ($\sim$270 K in most cases), r is the vent radius, and F is an efficiency factor of heat usage. The bulk density β is related to the density of the magmatic gas ρ_g, the density of the pyroclasts ρ_m, and the weight fractions of gas and pyroclasts N and x_m:

$$\frac{1}{\beta} = \frac{x_m}{\rho_m} + \frac{N}{\rho_g} \tag{4}$$

If it is assumed that the predominant gas is water and that the erupting fluid is at atmospheric pressure, then for $\theta = 1200$ K, ρ_g is 0.18 kg/m³. The thermal properties of the magma are dominated by the solid phase for gas contents of a few percent by weight, and so s, the specific heat, is taken as 1.1×10^{-3} J kg^{-1} K^{-1}.

TABLE 1. Heat Loss Related to Tephra Fragment Size

Gas Velocity u, m/s	Particle Diameter d,* cm	Time t,† s
10	0.06	0.04
30	0.4	1.6
100	3.0	90.0
300	50.0	2.5×10^4
500	220.0	4.8×10^5
700	600.0	3.6×10^6

*Values of particle diameter d for which the terminal velocity in steam at 1200 K is equal to the given gas velocity u when the particle density is 1000 kg/m³.

†Times t needed for substantial heat loss from particles with the stated diameters.

Equation (2) applies, strictly, only to the vertical rise of an eruption column in a still atmosphere (no wind). A number of formulae for the rise of industrial plumes [*Briggs,* 1969] include a dependence of cloud height on wind speed. However, (2) should be applicable to most large explosive eruptions, since the upward velocity of the cloud is commonly much greater than the transverse wind velocity over much of the column height and since the rate at which a particle-rich plume (eruption column) is bent over by the wind should be much smaller than the corresponding rate for a particle-poor plume because the initial upward momentum is less rapidly dominated by the addition of horizontal momentum from the entrained air. For strong winds and moderate- to small-sized eruption columns the effect of wind on column height may be significant and is discussed in detail by *Briggs* [1969] and *Settle* [1978].

We have compared (2) to *Wilson*'s [1976] cloud heights by taking 10 sets of values of u, r, and N for which Wilson gives heights and inserting these values into (3) and (4) to give $\dot{Q}$, using an efficiency factor $F = 0.7$. These values of $\dot{Q}$ were inserted into (2) to find H. Figure 1 shows the results of this method with a least squares regression line. The two methods give closely comparable estimates of cloud height, and there is a strong correlation with small variance about the best fit line derived from different principles.

Factors controlling efficiency of heat use. The relationship between the height reached by an eruption cloud and the controlling conditions at the vent is critically dependent on two factors: the ratio of kinetic to thermal energy released (per unit time and per unit mass of erupted material) and the extent of magma fragmentation (essentially the particle size distribution of the ejecta in the column). In explosions with a high content of nonjuvenile lithics, for example, in some maar eruptions, the amount of thermal energy may be substantially reduced. For sufficiently high initial velocities of gas and fragments the rate of kinetic energy release may exceed that of thermal energy release. However, for most eruptions where predominantly high temperature juvenile lava and gases are produced, thermal energy will be shown to be the main factor in controlling cloud heights. Thus F in (3) is primarily controlled by the degree of magma fragmentation.

In some subaerial eruptions, high-temperature lava fragments and gases are injected into the atmosphere at high velocity. In the lower part of an eruption column, pyroclastics and gas are rapidly decelerated as work is done against gravity and air friction. The subsequent convective rise of the column is influenced by the particle size distribution in two ways. Large clasts have a high ratio of inertia to drag force and can decouple relatively easily from the gas motion. The time needed for thermal waves to cross such particles may be greater than their residence time in the column, so little of their heat can be utilized in raising the temperature of the surrounding gases and driving convective movement [*Sparks and Wilson,* 1976]. Small clasts, however, may be essentially frozen into the gas motion, so that the gas and fine particle mixture can be considered as a compound fluid and the particles can readily supply heat to drive convection on the time scales of most eruptions.

A convenient measure of the ease with which fragments can decouple from the gas movements is the ratio of the terminal velocities of such fragments in the gas to the absolute gas

TABLE 2. Calculations of Kinetic Energy Contribution to Total Energy in Explosive Volcanic Eruptions

Initial Velocity U_0, m/s	Kinetic Energy as Percent of Total Energy
10	0.01
30	0.09
100	0.9
300	7.9
500	19.2
700	31.8

velocities. If this ratio is much less than unity, particles and gas will travel together. Table 1 shows the diameters of clasts of density of 1.0×10^3 kg/m^3 for which the terminal velocity in hot steam at 1200 K (calculated by the method of *Walker et al.* [1971]) is equal to the absolute gas velocity for a range of gas velocities. Clasts much smaller than the size shown will not decouple easily from the gas motion. Also given are the times needed for thermal waves to pass from the center to the edge of such particles and hence for significant heat loss to occur.

It is clear that the efficiency of heat use is of fundamental importance in driving a convective eruption column, and its role can be considered in two extreme examples. If the pyroclasts consist largely of coarse ejecta where the ratio of terminal velocity to absolute gas velocity in Table 1 is greater than unity, little of the heat will be utilized to drive a convective column. Hawaiian lava fountains which produce predominantly large spatter pieces have poorly developed columns. The height of lava fountains is mainly controlled by the kinetic energy of the burst. Similarly, Strombolian explosions generally are poorly fragmented. For example, *Self et al.* [1974] estimate that only 5% of the 1973 ejecta in the 1973 eruption of Heimaey were finer than 1 mm. *Walker* [1973] also documents the relatively low degree of fragmentation in many cinder cones formed from Strombolian activity.

On the other hand, if the ejecta consist largely of ash-sized material such that the ratio in Table 1 is less than unity, virtually all the magmatic heat can be converted into mechanical energy to drive the convective column. Plinian eruption columns can be so modeled because the magma is usually sufficiently fragmented that much of the thermal energy can be utilized in driving convection. For example, *Susuki et al.* [1973] show that at least 55% of the ejecta in the 1667 Tarumai Plinian deposit are finer than 1 mm. On the basis of the grain size characteristics of the deposits, *Sparks and Wilson* [1976] estimate at least 70% efficiency in the use of the heat in selected Plinian columns.

Table 2 shows the relative importance of kinetic and thermal energy in many eruptions. The available kinetic energy per unit mass of erupted material is taken as $\frac{1}{2}U_0^2$, where U_0 is the mean initial velocity of the eruption products. This energy is essentially derived by expansion of the gas phase. The available thermal energy is taken to be 70% of the heat which would be liberated in cooling magma from 1200 K to 300 K. No allowance has been made in Table 2 for the latent heat liberated by magma crystallization or vitrification or the minor amount of thermal energy in the gas, and so the relative importance of kinetic energy may be overestimated by as much as 10%. It is clear therefore that thermal energy release is completely dominant in all eruptions involving highly fragmented magma.

INSTANTANEOUS EXPLOSIONS

Certain types of volcanic activity appear to meet the criterion of an instantaneous explosion. Often such eruptions consist of short detonations in which a cloud of ejecta is discharged into the atmosphere. Examples of this type of activity, often described as vulcanian explosions, have been documented to be the characteristic recent activity of Asama in Japan [*Minakami,* 1950] and to have occurred during the activity of Augustine volcano in Alaska in January 1976 [*Hobbs et al.,* 1977]. The best definition of an instantaneous explosion is one which supplies ejecta to the cloud for a time that is short in comparison with the time required to reach the maximum cloud height. The cloud from one explosion should be substantially dissipated before the next explosion. Eruptive clouds typically take only a few minutes to rise to close to their maximum height. For example, the 1947 eruption cloud of Hekla rose to 20 km above the vent in 5 min, a rise rate of 67 m/s [*Thorarinsson,* 1954]. Similarly, the August 14, 1947, explosion of Asama rose to over 3.5 km in 100 s, a rise rate of 35 m/s. Clearly, an eruption can be modeled as an instantaneous event only if the explosion or explosions occur over a discrete period of less than a few minutes. Many eruptions, such as Hekla 1947, are either continuous discharges of ejecta or repeated explosions lasting many minutes, hours, or days, and the maintained plume model is regarded as being more applicable.

Morton et al. [1956] give the height of a cloud in a standard atmosphere from an instantaneous event as

$$H = 1.37Q^{1/4} \tag{5}$$

TABLE 3. Data on Historic Eruptions

Eruption	Average Volume Eruption Rate,* m^3/s	Cloud Height,† km	Duration, hours	Source
Hekla 1947	17,000	24	0.5	*Thorarinsson* [1954, 1968]
Hekla 1970	3,333	14	2	*Thorarinsson and Sigvaldason* [1972]
Soufriere 1902	11,000–15,500	14.5–16	2.5–3.5	*Anderson and Flett* [1903], *Carey and Sigurdsson* [1977]
Bezymianny 1956	230,000	36–45	0.5	*Gorshkov* [1959, 1961]
Fuego 1971	640	10	10	*Rose et al.* [1973], *Bonis and Salazar* [1973]
Heimaey 1973‡	50	2–3		*Self et al.* [1974]
Ngauruhoe 1974	10	1.5–3.7	14	*Self* [1974]
Santa Maria 1902	17,000–38,000	27–29	24–36	*Rose* [1972], *Sapper* [1904]

*Data on volume rate of eruption are given in terms of the dense rock equivalent of magma.
†Cloud heights are above the top of the volcano, not sea level.
‡The data on Heimaey refer to the first weeks of the eruption.

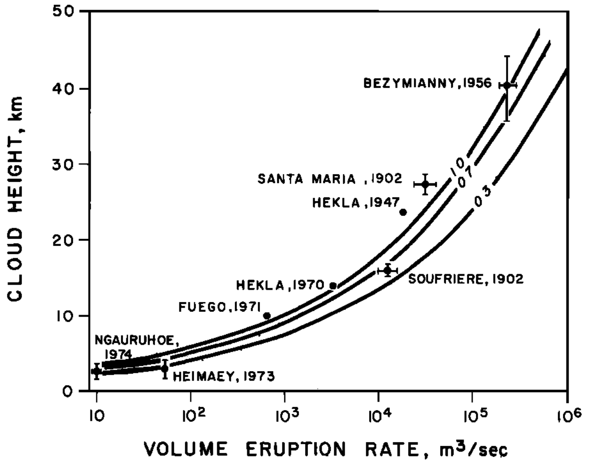

Fig. 2. Plot of observed eruption cloud heights against volume eruption rate for eight explosive eruptions. Error bars have been included where there is sufficient information (Table 3) to assess variations in both parameters. Three theoretical curves are shown for F values (see text) of 1.0, 0.7, and 0.3 using (2).

where H is in meters and Q is in joules. Data for the few eruptions where such a model may apply are compared with this formula later.

THE ROLE OF EXTERNAL WATER ON COLUMN DYNAMICS AND CLOUD HEIGHT

The models developed thus far assume that the gas driving the column into the atmopshere was originally dissolved in the magma and constitutes only a few percent of the erupted mass. The bulk of the heat exchange was considered to occur between air entrained into a column and the hot pyroclasts. Another important class of explosive eruption, however, occurs when abundant groundwater or seawater mixes with the erupting products before they are discharged into the atmosphere. In the case of phreatomagmatic activity a great deal of the heat exchange may be between magma and water, and the column dynamics controlling cloud height may be very different. In this situation for mixing ratios of water to magma of less than about 0.335, a large proportion of the thermal energy is used to transform the water into steam. Because the heat of vaporization of water is 580 cal g^{-1}, cooling of the magma can be substantial for mixing ratios in the range 0.1–0.3. If the mixture of pyroclasts and steam then rises, the thermal energy used in vaporization can only be recovered by the condensation of the steam. If condensation occurs, the phase change from steam to water droplets requires a large change in volume. This will be partially compensated by the mixing of air into the column during condensation, but nonetheless, a substantial increase in density must occur. It is under such conditions that base surge clouds are believed to form in both nuclear explosion clouds and phreatomagmatic eruptions [*Moore*, 1967]. In such a situation a substantial proportion of the thermal energy is not used to drive convection. Consequently, cloud heights should be lower in a phreatomagmatic eruption than in a magmatic eruption with the same volume rate of production.

COMPARISONS WITH OBSERVATIONS

For comparison of the theoretical heights with heights observed in actual eruptions it is useful to express the average rate of release of thermal energy in terms of the average rate of magma production $\dot{v}$, given as the equivalent volume of dense rock (density σ = 2500 kg/m³) erupted per second, since this is the parameter that is most commonly quoted in the literature. The appropriate modification to (2) gives

$$\dot{Q} = \sigma \dot{v} s(\theta - \theta_A)F \qquad (6)$$

Thus three measurements are required for each eruption: the volume of ejecta, the time interval during which it is erupted, and the corresponding cloud height. Most commonly, only overall average values of $\dot{v}$ are quoted from total volumes of ejecta, and total eruption durations and maximum cloud heights are generally given. Unfortunately, there are very few eruptions for which all three of these observations are accurately known.

Table 3 shows data for eight eruptions in which reasonable confidence can be placed. The two Hekla eruptions are both Plinian events in which there was a continuous ejection of material into the atmosphere [*Thorarinsson*, 1954, 1968; *Thorarinsson and Sigvaldason*, 1972]. The Soufriere, Santa Maria, Ngauruhoe, and Fuego eruptions were also recorded as continuous discharges of tephra and gas mixtures into the atmosphere from andesitic stratovolcanoes [*Anderson and Flett*, 1903; *Rose*, 1972; *Sapper*, 1904; *Carey and Sigurdsson*, 1977; *Self*, 1974; *Rose et al.*, 1973; *Bonis and Salazar*, 1973]. The Fuego eruption produced minor volumes of pyroclastic flows, but these volumes are not included in our calculations of $\dot{v}$. The explosion of Bezymianny in 1956 also produced pyroclastic flows [*Gorshkov*, 1959, 1961], but these volumes are not included in our calculations of $\dot{v}$; again, only the air fall volume is utilized here. Little of the heat contained in the ejecta which formed the pyroclastic flows is used to drive the column. The 1973 Heimaey eruption consisted of Strombolian explosions in which the frequency of explosion (one per second) in the early phases of the activity was such that the plume above the vent could be regarded as being continuously fed. Generally, on windy days the maximum column height was difficult to estimate, but on still days, column heights of 2–3 km were recorded [*Self et al.*, 1974].

Figure 2 shows the relationships between maintained cloud height and volume eruption rates calculated from (2)–(4). Considering the simple theoretical basis of the calculations and the vagaries of the field estimates, the calculations coincide well with the recorded cloud heights. The theoretical lines are calculated for efficiency factors F = 1.0, 0.7, and 0.3. For all the eruptions except Heimaey the observed cloud heights

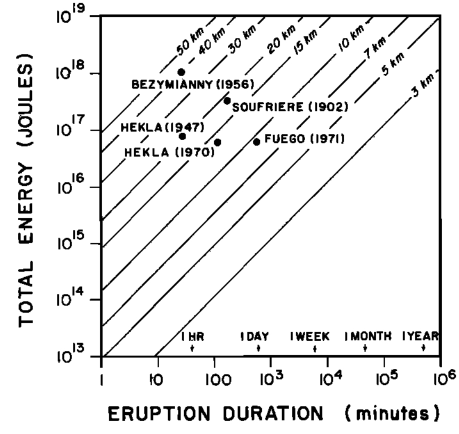

Fig. 3. Diagram of selected volcanic eruptions illustrating the relationships of cloud height (diagonal lines) to eruption duration and total energy. The total energy estimated assumes use of all the available thermal energy (F = 1.0).

coincide with theoretical curves for which the efficiency factor is high. *Settle* [1978] also shows similar relationships, utilizing formulae for plume rise and comparing them with some recorded eruption cloud heights.

There is a tendency for the larger eruptions, notably the 1902 Santa Maria and the 1947 Hekla columns, to be somewhat higher than would be predicted by using an F value of 1.0. This observation is even more striking when it is considered that an F value of 1.0 is most improbable. One reason for this discrepancy is that the cloud heights are the maximum observed whereas the volume rates of eruption are time averaged. In these two cases and probably in many other eruptions there is evidence that the volume rate of eruption can fluctuate considerably. This has been documented for the Hekla eruption and inferred for other eruptions, such as Santa Maria, by reversely graded pyroclastic fall deposits. The maximum cloud heights may reflect the peaks in volume eruption rate rather than the average. Another factor is that there may be significant departures from the value of n in (1). Vertical temperature gradients also have a role in controlling cloud height, as is illustrated by *Settle* [1978]. The data in Figure 2 indicate that the assumption of a standard atmosphere is reasonable, but the scatter in data may partly reflect variations in vertical temperature gradients in each case.

The Heimaey eruption illustrates the importance of the mechanism and type of eruption in cloud formation. Another column was produced by lava entering the sea during this eruption. This column consisted largely of white steam and reached much greater heights than the eruptive column. The Strombolian activity at Heimaey did not fragment the magma greatly, less than 5% of the ejecta being finer than 1 mm [*Self et al.*, 1974] and the median size being greater than 1 cm. In such a situation the efficiency factor could be expected to be low, and indeed, the best match of data with theory is for a low value of F.

We have shown above that for most maintained plumes the thermal energy release exceeds the kinetic energy release, so that $\dot{Q}$ in (2) can be regarded as a good approximation to the total energy release rate. If H is interpreted as the average cloud height during an eruption, then $\dot{Q}$ can be taken as the total energy released divided by the eruption duration. Figure 3 shows the result of plotting total energy released against eruption duration. In logarithmic coordinates, lines of constant slope represent constant values of $\dot{Q}$ and hence particular cloud heights, as indicated. Analyses of particle size data in traverses of abyssal cores can in principle yield estimates of cloud heights for eruptions in the Pleistocene and Pliocene [*Huang et al.*, 1975]. These heights, on the maintained plume model, can be directly expressed in terms of energy release rate and hence volume eruption rate. Furthermore, if estimates of the duration of any eruption can be made, the total energy released and volume erupted can be ascertained; alternatively, if the total volume of an ash horizon can be estimated, the energy released and eruption duration can be calculated. It is strongly emphasized that the estimate of energy released does not correspond to the total thermal and kinetic energy of an eruption unless the efficiency factor is 1.0. In the eruptions considered in Table 3 a value of F close to 1.0 seems acceptable, but there are other types of eruption, notably phreatomagmatic eruptions, where F will be much less than 1.0.

Three eruptions of short duration for which data are available are those of Bezymianny in 1956 [*Gorshkov*, 1959, 1961] and Asama in June 1938 and August 1947 [*Minakami*, 1950]. If we use the quoted total volumes of material liberated and assume a temperature of 1200 K, the values of Q for these three events are about 10^{18}, 3.3×10^{14}, and 3.3×10^{14} J, respectively. Corresponding calculated cloud heights, when (5) is used, are 59, 8, and 8 km. The observed heights were 43, 3.5, and 7–9 km. In two cases therefore the cloud heights are significantly different from theoretical estimates based on the formula for a discrete explosion. There is an obvious discrepancy in the Asama cases, where similar volumes of material led to very different cloud heights. Possible reasons for the differences include the presence of cold country rock or the presence of coarse clasts in the ejecta, since either factor would lead to our having overestimated Q. Finally, substantial departures from the standard atmosphere used in computing the constant in (4) may also result in different heights [*Settle*, 1978]. In the Bezymianny case a maintained plume model appears to match the data much better (Figure 2). Until many more data are recorded from discrete explosions, the use of (5) must be treated with caution or at least qualified as involving a minimum energy release.

THEORETICAL LIMITS ON CLOUD HEIGHT

According to the simple model presented thus far there should be no limit to the height of an eruption column if large enough volume eruption rates occur. In practice, there is a limit to the volume rates of eruption which can produce high eruption columns.

It was noted above that all eruptive mixtures are discharged into the atmosphere with a density greater than that of their surroundings. If sufficient air can be entrained as a column rises, the column eventually becomes less dense than the atmosphere, and convection ensues. For certain combinations of vent radius, gas content, and gas velocity, however, the column is still more dense than the atmosphere after all the initial kinetic energy is expended, and the collapse of the column and the formation of pyroclastic flows result. These relationships have been investigated by *Sparks and Wilson* [1976] and *Wilson* [1976], who found that the conditions leading to collapse involve large vent radii, low gas velocities, and low gas contents. Generally, column collapse occurs at high volume rates of production because $\dot{v}$ increases with increasing vent radius and decreasing gas content. But $\dot{v}$ decreases with decreasing gas velocity, and so the relationships are not simple. Sets of permutations of vent radius, gas content, and eruption velocity are given by *Sparks and Wilson* [1976] and *Wilson* [1976] for eruption columns on the point of collapse for $F = 0.7$. We used (3) and (4), together with $\sigma = 2500$ kg/m^3 for dense rock, to deduce the corresponding volume rates of eruption, and we then applied (2) to obtain cloud heights just before collapse.

Figure 4 summarizes these results, showing maximum volume eruption rate, and hence maximum cloud height, as a function of vent radius for three values of eruption velocity. At the point of collapse, only two of the three variables (gas content, gas velocity, and vent radius) are independent, the third being controlled by the other two. The gas contents implied are therefore also shown in Figure 4. At any point in the diagram, collapse of the column would be expected when any combination of an increase in vent radius, a decrease in gas velocity, or a decrease in gas content occurred. Various calculations and observations imply that eruption velocities in excess of 700 m/s are unlikely to occur on earth [*McGetchin and Ullrich*, 1973; *Wilson*, 1976], and so Figure 4 implies that the largest volume eruption rate that can lead to a maintained eruption column is 6.5×10^6 m^3/s, corresponding to a column height of 48 km. Greater heights should occur only very rarely,

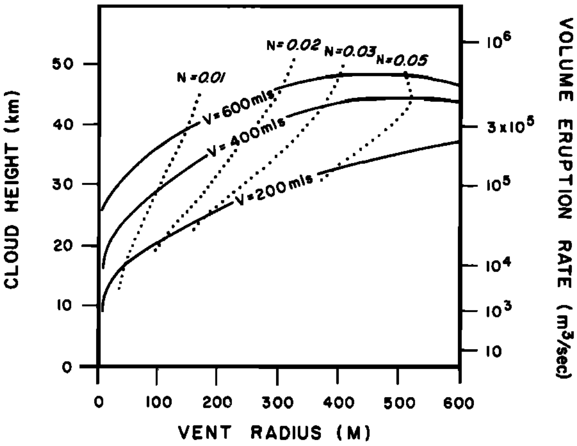

Fig. 4. Combinations of vent radius, gas velocity, and gas content which are predicted to be at the critical boundary between column collapse and column convection according to the model of *Sparks and Wilson* [1976]. Three curves are shown for gas velocities of 200, 400, and 600 m/s. The maximum cloud height and implied volume eruption rate are shown as a function of vent radius for each gas velocity. The value of the weight fraction of gas N is uniquely fixed at this boundary, and so the implied gas fractions are shown as dotted curves.

for any increase in volume eruption rate would lead to column collapse. Even if the factor F in (3) is increased to 1.0 and a velocity of 700 m/s is used, the maximum cloud height in a standard atmosphere is increased to 55 km, and the maximum eruption rate to 1.1×10^6 m³/s.

CONCLUSIONS

1. A model relating eruption column height to the release rate of thermal energy (proportional to volume rate of eruption of magma) has been investigated and found to be reasonably consistent with field observations. Cloud heights ranging from 2 to 45 km from eight eruptions fit a formula for a steady maintained plume in a standard atmosphere and a model based on conservation of energy.

2. For 'instantaneous' explosions there is insufficient data to confirm available relationships between explosion yield and cloud height. Instantaneous models can be applied only to eruptions in which there is a single discrete explosion lasting less than a few minutes because clouds initially rise at tens of meters per second.

3. There are some types of eruption in which the magma heat utilization in driving the column is not efficient. These include Strombolian eruptions where the fragmentation is poor and little heat is given to the column because of large fragment size, some vulcanian explosions in which the magma may have cooled to a low temperature or a large proportion of cold country rock is ejected, and phreatomagmatic eruptions in which much of the thermal energy is used in vaporizing external waters and recovery of this energy can occur only if condensation occurs.

4. For magmatic eruptions, a theoretical limit to cloud height of 55 km on the earth in a standard atmosphere is predicted because at high volume rates of eruption, column collapse occurs to form pyroclastic flows rather than high convective columns. The maximum theoretical volume rate of production that can lead to the formation of a stable eruption column is 1.1×10^6 m³/s.

Acknowledgments. Norman Watkins lost his life in November 1977 as a result of a tragic illness. He was the creator and driving force of the recent research program on volcanic ash in deep-sea sediments at the University of Rhode Island. This program has generated this and many other contributions, and Norman Watkins created and inspired many new approaches and concepts through this program. His colleagues and a great many earth scientists will miss this dynamic and versatile man.

Research for this study was supported by the Division of Earth Sciences, National Science Foundation, grant EAR76-22319, and the Program of Submarine Geology and Geophysics, National Science Foundation, grant OCE-74-22347-A03. Research was also supported by NATO research grant 1222. M. Settle and S. Self are gratefully acknowledged for helpful discussions concerning parts of this study.

REFERENCES

Anderson, T., and J. S. Flett, Report on the eruptions of the Soufriere in St. Vincent in 1902, and on a visit to Montagne Pelee in Martinique, 1, *Phil. Trans. Roy. Soc. London, 200,* 353-553, 1903.

Blackburn, E. A., L. Wilson, and R. S. J. Sparks, Mechanisms and dynamics of Strombolian activity, *J. Geol. Soc. London, 132,* 429-440, 1976.

Bonis, S., and O. Salazar, The 1971 and 1973 eruptions of volcan Fuego, Guatemala, and some socio-economic considerations for the volcanologist, *Bull. Volcanol., 37*(3), 394-400, 1973.

Briggs, G. A., Plume rise, Critical Review Series, *Rep. TID-25075,* pp. 1-81, At. Energy Comm., Washington, D. C., 1969.

Carey, S., and H. Sigurdsson, The eruptions of Soufriere and Mt. Pelee in 1902 and a mixed surface tephra layer in the Tobago trough (abstract), *Eos Trans. AGU, 58,* 528, 1977.

Fudali, R. F., and W. G. Melson, Ejecta velocities, magma chamber pressure and kinetic energy associated with the 1968 eruption of Arenal volcano, *Bull. Volcanol., 35,* 383-401, 1972.

Gorshkov, G. S., Gigantic eruption of the volcano Bezymianny, *Bull. Volcanol., 20,* 77-109, 1959.

Gorshkov, G. S., The Kamchatka Valley of Ten Thousand Smokes, *Proc. Pac. Sci. Congr. 9th,* no. 12, 237-239, 1961.

Hobbs, P. V., L. F. Radke, and J. L. Stith, Eruptions of the St. Augustine volcano: Airborne measurements and observations, *Science, 195,* 871-873, 1977.

Huang, T. C., N. D. Watkins, and D. M. Shaw, Atmospherically transported volcanic glass in deep-sea sediments: Volcanism in sub-Antarctic latitudes of the South Pacific during late Pliocene and Pleistocene time, *Geol. Soc. Amer. Bull., 86,* 1305-1315, 1975.

Knox, J. B., and N. M. Short, A diagnostic model using ash fall data to determine eruption characteristics and atmospheric conditions during a major volcanic event, *Bull. Volcanol., 27,* 5-24, 1964.

McBirney, A. R., Factors governing the intensity of explosive andesitic eruptions, *Bull. Volcanol., 37,* 443-453, 1973.

McGetchin, T. R., and W. G. Ullrich, Xenoliths in maars and diatremes with inferences for the moon, Mars, and Venus, *J. Geophys. Res., 78,* 1833-1853, 1973.

Minakami, T., On explosive activities of andesitic volcanoes and their fore-running phenomena, *Bull. Volcanol., 10,* 59-87, 1950.

Moore, J. G., Base surge in recent volcanic eruptions, *Bull. Volcanol., 30,* 337-363, 1967.

Morton, B. R., G. Taylor, and J. S. Turner, Turbulent gravitational convection from maintained and instantaneous sources, *Proc. Roy. Soc., Ser. A, 234,* 1-23, 1956.

Nairn, I., and S. Self, Explosive eruptions and pyroclastic avalanches from Ngauruhoe in February 1975, *J. Volcanol. Geotherm. Res.,* in press, 1978.

Rose, W. I., Jr., Notes on the 1902 eruption of Santa Maria volcano, Guatemala, *Bull. Volcanol., 36*(1), 29-45, 1972.

Rose, W. I., Jr., S. Bonis, R. E. Stoiber, M. Keller, and T. Bickford, Studies of volcanic ash from two recent Central American eruptions, *Bull. Volcanol., 37*(3), 338-364, 1973.

Sapper, K., Die vulkanischen Ereignisse in Mittelamerika in Jahre 1902, *Neues Jahrb. Mineral. Geol. Palaeontol.,* 39-90, 1904.

Self, S., R. S. J. Sparks, B. Booth, and G. P. L. Walker, The 1973 Heimaey Strombolian scoria deposit, Iceland, *Geol. Mag., 111,* 539-548, 1974.

Settle, M., Volcanic eruption clouds and the thermal power output of explosive eruptions, *J. Volcanol. Geotherm. Res.,* in press, 1978.

Shaw, D., N. D. Watkins, and T. C. Huang, Atmospherically transported volcanic glass in deep-sea sediments: Theoretical considerations, *J. Geophys. Res., 79*(21), 3087-3094, 1974.

Sparks, R. S. J., and L. Wilson, A model for the formation of ignimbrite by gravitational column collapse, *J. Geol. Soc. London, 132,* 441-451, 1976.

Susuki, T., Y. Katsui, and T. Nakamura, Size distribution of the Tarumai Ta-b pumice fall deposit, *Bull. Volcanol. Soc. Jap.*, *18*, 47–63, 1973.

Thorarinsson, S., *The Tephra Fall from Hekla on March 29th, 1947, The Eruption of Hekla 1947–48*, vol. II, part 3, pp. 1–78, H. F. Leiftur, Reykjavik, 1954.

Thorarinsson, S., On the rate of lava and tephra production and the upward migration of magma in four Icelandic eruptions, *Geol. Rundsch.*, *57*(3), 705–718, 1968.

Thorarinsson, S., and G. E. Sigvaldason, The Hekla eruption of 1970, *Bull. Volcanol.*, *36*(2), 269–288, 1972.

Walker, G. P. L., Explosive volcanic eruptions—A new classification theme, *Geol. Rundsch.*, *62*, 431–446, 1973.

Walker, G. P. L., L. Wilson, and E. L. G. Bowell, Explosive volcanic eruptions, I, The rate of fall of pyroclasts, *Geophys. J. Roy. Astron. Soc.*, *22*, 377–383, 1971.

Watkins, N. D., and T. C. Huang, Tephras in abyssal sediments east of the North Island, New Zealand: Chronology, paleowind velocity, and paleoexplosivity, *N. Z. J. Geol. Geophys.*, *20*, 179–198, 1977.

Wilson, L., Explosive volcanic eruptions, III, Plinian eruption columns, *Geophys. J. Roy. Astron. Soc.*, *45*, 543–556, 1976.

(Received June 24, 1977;
revised November 14, 1977;
accepted November 28, 1977.)