

# The cosmic microwave background radiation fluctuations from H I perturbations prior to reionization

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## ABSTRACT

Loeb and Zaldarriaga have recently proposed that observations of the cosmic microwave background radiation (CMBR) brightness temperature fluctuations produced by H I inhomogeneities prior to reionization hold the promise of probing the primordial power spectrum to a hitherto unprecedented level of accuracy. This requires a precise quantification of the relation between density perturbations and brightness temperature fluctuations. Brightness temperature fluctuations arise from two sources: (1) fluctuations in the spin temperature, and (2) fluctuations in the H I optical depth, both of which are caused by density perturbations. For the spin temperature, we investigate in detail its evolution in the presence of H I fluctuations. For the optical depth, we find that it is affected by density perturbations both directly and through peculiar velocities which move the absorption features around in frequency. The latter effect, which has not been included in earlier studies, is similar to the redshift space distortion seen in galaxy surveys and this can cause changes of 50 per cent or more in the brightness temperature fluctuations.

**Key words:** cosmology: theory – diffuse radiation – large-scale structure of Universe.

## 1 INTRODUCTION

The possibility of probing the Universe at high redshifts using the H I 21-cm line has been the topic of extensive theoretical investigation. This is perceived to be the most promising window for studying the ‘dark ages’, the era between the decoupling of the CMBR from the primeval plasma at  $z \sim 1000$  and the formation of the first luminous objects at  $z \sim 20$  (Hogan & Rees 1979; Scott & Rees 1990; Madau, Meiksin & Rees 1997; Tozz et al. 2000; Barkana & Loeb 2001; Iliev et al 2002; Miralda-Escude 2003). After decoupling, the gas temperature  $T_g$  is maintained at the CMBR temperature  $T_\gamma$  through collisions of the CMBR photons with the small fraction of electrons that survive the process of recombination. The collision process becomes ineffective in coupling  $T_g$  to  $T_\gamma$  at  $z \sim 200$ . In the absence of external heating at  $z < 200$  the gas cools adiabatically with  $T_g \propto (1+z)^2$  while  $T_\gamma \propto (1+z)$ . The spin temperature  $T_s$  is strongly coupled to  $T_g$  through the collisional spin-flipping process until  $z \sim 70$ . The collisional process is weak at lower redshifts, and  $T_s$  again approaches  $T_\gamma$ . This gives a range of redshifts where  $T_s < T_\gamma$ . We then have a window in redshift  $30 \leq z \leq 200$ , or equivalently in frequency  $\nu = 1420 \text{ MHz}/(1+z)$  where the H I will produce absorption features in the CMBR spectrum.

In a recent paper Loeb & Zaldarriaga (2003) propose that observations of the angular fluctuations in the CMBR brightness temper-

ature  $T_b$  arising from the H I absorption can be used to study the power spectrum of density fluctuations at small scales to a level of accuracy far exceeding those achievable by any other means. The enormous wealth of information arises from the fact that observations at different frequencies which are sufficiently separated will provide independent estimates of the power spectrum at the same wavenumber  $k$ . These observations will probe the power spectrum before the epoch of structure formation, and they hold the possibility of revealing the entire primordial power spectrum down to very small scales. In another recent paper Gnedin & Shaver (2004) have studied the linear fluctuations in the 21-cm emission from the pre-reionization era. They show that it should be possible to detect this signal against the foreground contaminations in the frequency domain. This signal is expected to constrain the equation of state of the Universe at high  $z$ .

The CMBR brightness temperature is related to  $T_s$  and the H I number density  $n_H$  as  $T_b \propto (1 - T_\gamma/T_s) n_H$ . Fluctuations in  $T_b$  arise from fluctuations in  $n_H$  directly and also through fluctuations in  $T_s$  which in turn arise from fluctuations in  $n_H$ . In calculating the fluctuations in  $T_s$ , Loeb & Zaldarriaga (2003) consider only one process, namely the change in the collision rate arising from fluctuations in  $n_H$ . Perturbations in  $n_H$  will also produce perturbation in  $T_g$ , which in turn will affect  $T_s$ . This effect has not been taken into account in their work.

Density perturbations produce peculiar velocities. This causes the frequency of the H I absorption features to be shifted by the line-of-sight component of the peculiar velocity. This effect will rearrange

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the H I absorption features in frequency space where converging velocity patterns appear as enhancements in the H I number density and diverging velocity patterns appear as decrements in the H I number density. It may be noted that this is the familiar linear redshift space distortion (Kaiser effect, Kaiser 1987) seen in galaxy redshift surveys. This effect has been studied by Bharadwaj, Nath & Sethi (2001) in the context of cosmological H I emission from  $z \sim 3.5$ .

It is important to identify and take into account all possible contributions to the brightness temperature fluctuations, if these are to be used to extract precise information about the power spectrum and the equation of state of the Universe. In this paper we study two effects which will contribute to brightness temperature fluctuations, namely (1) perturbations in the gas temperature produced by density fluctuations, and (2) the effect of redshift space distortions. To the best of our knowledge, these effects have not been included in earlier work.

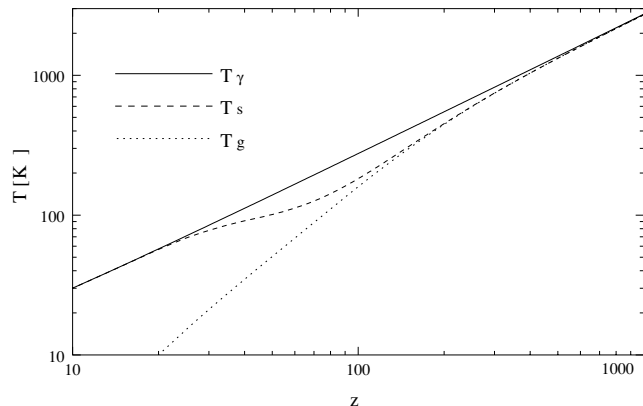
We next present an outline of the paper. In Section 2, we discuss the processes involved in determining the brightness temperature fluctuations and present the relevant equations. In Section 3, we present our results and discuss their consequences. It may be noted that we have used  $(\Omega_{m0}, \Omega_{\Lambda0}, \Omega_{b0}h^2, h) = (0.3, 0.7, 0.02, 0.7)$  whenever specific values have been needed for the cosmological parameters.

## 2 CALCULATING THE BRIGHTNESS TEMPERATURE FLUCTUATIONS

We first consider the evolution of the gas temperature after the recombination era ( $z \sim 1000$ ) when it becomes largely neutral. This is governed by the equation

$$\frac{\partial T_g}{\partial z} - \frac{2T_g}{3n_H} \frac{\partial n_H}{\partial z} = \frac{-9.88 \times 10^{-8}}{\Omega_b h^2} (1+z)^{3/2} (T_\gamma - T_g). \quad (1)$$

The third term in the above equation represents the energy transfer from the CMBR to the gas through collisions with the residual electrons (Peebles 1993). This term tries to maintain the gas temperature at the CMBR temperature as the Universe expands. The second term is the change in  $T_g$  in adiabatic expansion. If the H I is uniformly distributed, then  $n_H \propto (1+z)^3$  and in the absence of CMBR heating we have  $T_g \propto (1+z)^2$ . Collisions are able to maintain  $T_g = T_\gamma = 2.73 \text{ K}(1+z)$  up to a redshift  $z \sim 200$  (Fig. 1) after which  $T_g \propto (1+z)^2$ .



**Figure 1.** The evolution of the CMBR temperature, the gas temperature and the spin temperature as the Universe expands.

We next consider the evolution of the spin temperature  $T_s$  which is defined through the relation

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_s/T_s} \quad (2)$$

where  $n_1, n_0$  are the population densities and  $g_1 = 3, g_0 = 1$  the spin degeneracy factors of the excited and the ground states of the H I 21-cm transition. In this equation  $T_* = h_p \nu_e / k_B = 0.068 \text{ K}$ , where  $h_p$  is Planck's constant,  $\nu_e = 1420 \text{ MHz}$  is the frequency corresponding to the 21-cm line and  $k_B$  is Boltzmann's constant. The evolution of the ground-state population density is governed by two processes, one collisional and the other radiative:

$$\left( \frac{\partial}{\partial t} + 3 \frac{\dot{a}}{a} \right) n_0 = n_1 C_{10} - n_0 C_{01} + n_1 A_{10} + (n_1 B_{10} - n_0 B_{01}) I_{\nu_e} \quad (3)$$

where  $a(t)$  is the scalefactor,  $C_{01}$  and  $C_{10}$  are the collisional excitations and de-excitation rates of the hyperfine levels,  $A_{10}$  is the Einstein spontaneous emission coefficient,  $B_{01}$  and  $B_{10}$  are the Einstein  $B$  coefficients and  $I_{\nu_e}$  is the specific intensity of the background radiation at  $\nu_e$ .

In the regime of interest  $T_s, T_g, T_\gamma \gg T_*$  and we can use the approximation  $e^{-T_*/T} = 1 - (T_*/T)$  throughout. Also, the fact that in equilibrium the collisional processes and the radiative process are separately balanced gives us the relations  $C_{01} = 3(1 - T_*/T_g)C_{10}$  and  $B_{01} = 3 B_{10} = (3\lambda_e^3/2h_p c) A_{10}$  where  $A_{10} = 2.85 \times 10^{-15} \text{ s}^{-1}$  (Rybicki & Lightman 1979). The collisional de-excitation rate can be written as  $C_{10} = (4/3) \kappa(1 - 0) n_H$  where the values of  $\kappa(1 - 0)$  are tabulated as a function of  $T_g$  (Allison & Dalgarno 1969). Using these and equation (3) we obtain an equation for the redshift evolution of  $T_s$ :

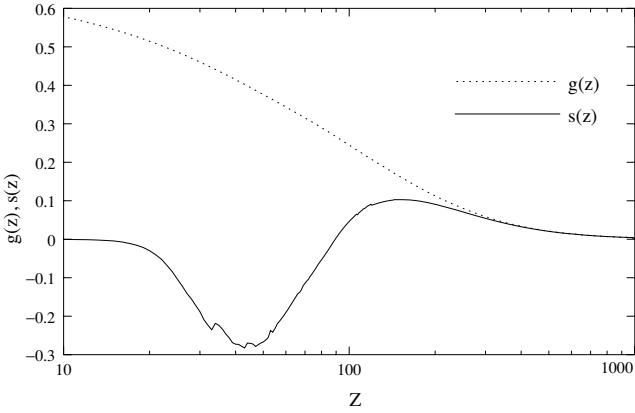
$$\frac{\partial}{\partial z} \left( \frac{1}{T_s} \right) = - \frac{4}{H(z)(1+z)} \times \left[ \left( \frac{1}{T_g} - \frac{1}{T_s} \right) C_{10} + \left( \frac{1}{T_\gamma} - \frac{1}{T_s} \right) \frac{T_\gamma}{T_*} A_{10} \right] \quad (4)$$

where  $H(z)$  is the Hubble parameter. The collisional term tries to set the spin temperature at the same value as the gas temperature while the radiative term tries to set it at the CMBR temperature, which process dominates being decided by the rate coefficients. At high redshifts the collisional process dominates and the spin temperature closely follows the gas temperature. At lower redshifts  $n_H$  falls substantially, the collisional process loses out to the radiative process and the spin temperature approaches the CMBR temperature. Fig. 1 shows the evolution of the spin temperature as the Universe expands.

We next shift our attention to the effect of H I density perturbations  $\Delta_H(\mathbf{x}, z) = [n_H(\mathbf{x}, z) - \bar{n}_H(z)]/\bar{n}_H(z)$ . These will produce fluctuations in the gas temperature. If the gas were undergoing adiabatic expansion, the fluctuations in the gas temperature  $\Delta_g(\mathbf{x}, z) = [T_g(\mathbf{x}, z) - \bar{T}_g(z)]/\bar{T}_g(z)$  would be related to  $\Delta_H$  through  $\Delta_g = (2/3) \Delta_H$ . This will be modified because of the energy that is pumped into the gas from the CMBR and  $\Delta_g = 0$  during the era when  $T_g = T_\gamma$ . We define a function  $g(z) = \partial \Delta_g / \partial \Delta_H$ , such that  $\Delta_g(z) = g \Delta_H(z)$ . Using this in equation (1) we obtain

$$\frac{dg}{dz} = \frac{9.88 \times 10^{-8} T_\gamma}{\Omega_b h^2 T_g} (1+z)^{3/2} g + \left( \frac{2}{3} - g \right) \frac{1}{\Delta_H} \frac{\partial \Delta_H}{\partial z}. \quad (5)$$

The first term on the right-hand side arises from the coupling of the gas to the CMBR and it tries to set  $g(z) \rightarrow 0$  while the second term corresponds to adiabatic expansion and it tries to make  $g(z) \rightarrow 2/3$ . The quantity  $g(z)$  is expected to evolve from  $g(z) = 0$  to  $g(z) = 2/3$



**Figure 2.** The evolution of the functions  $g(z)$  and  $s(z)$  defined in the text.

as the Universe expands and the contribution of the heat pumped into the gas decreases.

An interesting feature is that  $g(z)$  depends on the growth rate of density fluctuations. For example, it follows from equation (5) that a static density perturbation which does not evolve in time will not produce fluctuations in the gas temperature. Here we assume that  $\Delta_H$  follows the dark matter perturbation and grows as  $\Delta_H \propto a(z)$ . The result of integrating equation (5) is shown in Fig. 2. We see that  $g(z) \sim 0.3$  in the redshift range of interest ( $30 \leq z \leq 100$ ).

We finally come to the fluctuations in the spin temperature  $\Delta_s(\mathbf{x}, z) = [T_s(x, z) - \bar{T}_s(z)]/\bar{T}_s(z)$  produced by density perturbations. From equation (4) we see that fluctuations in  $T_s$  can arise from changes in  $T_g$  and from changes in the collision rate. The changes in collision rate  $C_{10} = (3/4) \kappa(1 - 0) n_H$  will come about directly through changes in  $n_H$  and also through changes in  $T_g$  which will affect the value of  $\kappa(1 - 0)$ . Taking into account both these effect we have  $\Delta C_{10} = (1 + (2/3) d \ln \kappa / d \ln T_g) C_{10} \Delta_H$ . Defining a function  $s(z) = \partial \Delta_s / \partial \Delta_H$ , such that  $\Delta_s(z) = s \Delta_H(z)$  and using equation (4) we obtain

$$\frac{ds}{dz} = -s \frac{1}{\Delta_H} \frac{\partial \Delta_H}{\partial z} + \frac{4}{H(z)(1+z)} \left\{ \left[ \frac{T_s}{T_g} (s - g) + \left( \frac{T_s}{T_g} - 1 \right) \left( 1 + \frac{d \ln \kappa}{d \ln T_g} \right) C_{10} + s \frac{T_s}{T_*} A_{10} \right] \right\}. \quad (6)$$

Here again, the evolution of  $s(z)$ , like that of  $g(z)$ , depends on the time evolution the density fluctuations. Fig. 2 shows the evolution of  $s(z)$  under the assumption  $\Delta_H \propto a(z)$ . We find that  $s(z) > 0$  for  $z > 90$ , i.e. a positive density perturbation causes the spin temperature to increase, and the effect is opposite at  $z < 90$ .

During the era when  $T_s < T_\gamma$  the H I along a line-of-sight  $\mathbf{n}$  reduces the CMBR brightness temperature at the frequency  $\nu$  by an amount

$$T_b(\mathbf{n}, \nu) = \frac{(T_s - T_\gamma)\tau}{1+z}. \quad (7)$$

Here  $\tau$  is the optical depth of the 21-cm H I absorption given by

$$\tau = \frac{3n_H h_p c^2 A_{10} a^2(z)}{32\pi k_B T_s \nu_e} \left| \frac{\partial r}{\partial \nu} \right| \quad (8)$$

where  $r$  is the comoving distance to the H I whose 21-cm absorption is redshifted to  $\nu$ .

We are interested in the angular fluctuations of the brightness temperature  $T_b(\mathbf{n}, \nu)$ . H I density fluctuations will produce fluctuations in  $T_b(\mathbf{n}, \nu)$  through the fluctuations in the spin temperature discussed earlier. Density fluctuations will also directly affect  $T_b$

through variations in the optical depth which we now calculate. The relation between the comoving distance  $r$  and the frequency  $\nu$  is given by

$$r = \int_{\frac{\nu}{\nu_e(1-\nu/c)}}^1 \frac{c da}{a^2 H(a)} \quad (9)$$

where  $\nu$  is the line-of-sight component of the peculiar velocity of the H I which produces the absorption. Density perturbations will, in general, be accompanied by velocity perturbations and these will move around the H I absorption features in frequency. Here we assume that the H I traces the dark matter and that we can use linear perturbation theory to relate the peculiar velocity to the density perturbations. Incorporating the effect of both the density fluctuations and the peculiar velocity we have

$$\tau = \frac{3\bar{n}_H h_p c^3 A_{10}}{32\pi k_B T_s \nu_e^2 H(z)} \left[ 1 + \Delta_H - \frac{1}{H(z)a(z)} \frac{\partial \nu}{\partial r} \right]. \quad (10)$$

Here we have dropped terms of order  $\nu/c$  in the final expression. Also, we have retained terms only to linear order in  $\nu$ . There is also the effect of our own motion which we have not included. These effects are not expected to be important. The term involving the derivative of the peculiar velocity is the dominant effect, particularly at the small scales of interest here.

Combining the effects of the fluctuations in the optical depth and in the spin temperature we can write the fluctuations in the CMBR brightness temperature as

$$\delta T_b(\mathbf{n}, \nu) = \bar{T} \left[ \left( 1 - \frac{T_\gamma}{T_s} \right) \left( \Delta_H - \frac{1}{Ha} \frac{\partial \nu}{\partial r} \right) + \frac{T_\gamma}{T_s} s \Delta_H \right] \quad (11)$$

where

$$\bar{T} = 2.67 \times 10^{-3} \text{K} \frac{\Omega_b h^2 (1+z)^{1/2}}{0.02 \Omega_{m0}^{1/2} h}. \quad (12)$$

It is convenient to express this in Fourier space where

$$\Delta_H(\mathbf{x}, z) = \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \Delta(\mathbf{k}, z) \quad (13)$$

and the Fourier transform of the peculiar velocity is given by  $\mathbf{v}(\mathbf{k}, z) = -iH(z) a(z) \mathbf{k} \Delta(\mathbf{k}, z)/k^2$ . Using this we express the fluctuations in brightness temperature as

$$\delta T_b(\mathbf{n}, \nu) = \bar{T} \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{r}\mu} \Delta(\mathbf{k}, z) \times \left[ \left( 1 - \frac{T_\gamma}{T_s} \right) (1 + \mu^2) + \frac{T_\gamma}{T_s} s \right] \quad (14)$$

where  $\mu$  is the cosine of the angle between the comoving wavevector  $\mathbf{k}$  and the line-of-sight  $\mathbf{n}$ .

We now calculate the angular power spectrum of the brightness temperature fluctuations resulting from the density fluctuations  $\Delta(\mathbf{k}, z)$ . The statistical properties of  $\Delta(\mathbf{k}, z)$  are specified through the 3D power spectrum defined as

$$\langle \Delta(\mathbf{k}, z) \Delta(\mathbf{k}', z) \rangle = (2\pi)^3 \delta_D^3(\mathbf{k} - \mathbf{k}') P(k, z) \quad (15)$$

where  $\langle \dots \rangle$  denotes ensemble average and  $\delta_D(\mathbf{l})$  is the Dirac delta function.

The angular power spectrum is calculated by decomposing the angular dependence of  $\delta T_b$  into spherical harmonics with expansion coefficients  $a_{lm}(\nu)$  and using these to calculate the angular power spectrum  $C_l(\nu) = \langle |a_{lm}|^2 \rangle$ . The angular power spectrum can be

expressed in terms of the 3D power spectrum as

$$C_l(\nu) = 4\pi\bar{T}^2 \int \frac{d^3k}{(2\pi)^3} P(k, z) \left[ \left(1 - \frac{T_\gamma}{T_s}\right) J_l(kr) + \frac{T_\gamma}{T_s} s j_l(kr) \right]^2 \quad (16)$$

where  $j_l(kr)$  are the spherical Bessel functions and

$$J_l(kr) = \left[ -\frac{l(l-1)}{4l^2-1} j_{l-2}(kr) + \frac{2(3l^2+3l-2)}{4l^2+4l-3} j_l(kr) - \frac{(l+2)(l+1)}{(2l+1)(2l+3)} j_{l+2}(kr) \right]. \quad (17)$$

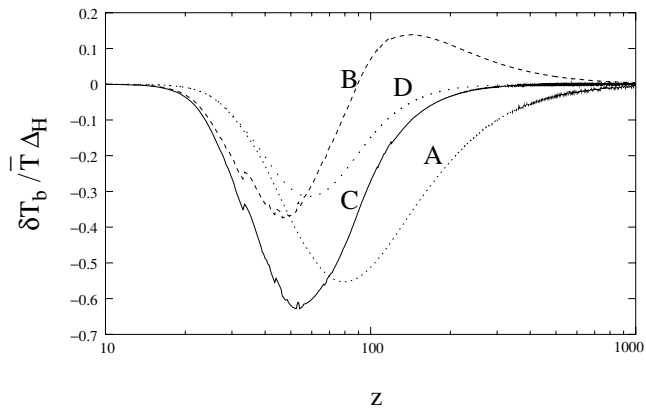
### 3 RESULTS AND DISCUSSION

In this paper we have investigated in detail the CMBR fluctuations produced by H I prior to the epoch of reionization. As proposed by Loeb & Zaldarriaga (2003), this holds the promise of allowing the power spectrum of density fluctuations to be probed to a high level of precision.

H I density perturbations produce fluctuations in the decrement of the CMBR brightness temperature by two means: (1) through fluctuations in the optical depth, and (2) through fluctuations in the spin temperature. The effect of changes in the optical depth in response to a positive density perturbation (curve A of Fig. 3) is such that it reduces  $T_b$  and enhances the decrement in the brightness temperature. This effect is maximum at  $z \sim 80$ . This effect is enhanced by peculiar velocities.

The change in brightness temperature produced by H I density perturbations through changes in the spin temperature varies with  $z$  (curve B of Fig. 3). Density perturbations increase the spin temperature and the brightness temperature in the redshift range  $z > 100$ . Here the collisional process is very efficient and  $T_s$  closely follows  $T_g$ . A positive density perturbation increases  $T_g$  which causes  $T_s$  to also increase. The effect of density perturbations on the brightness temperature acting through changes in the optical depth and through the spin temperature have opposite signs. The effect of changes in the optical depth is larger and the brightness temperature decrement is enhanced by a positive density perturbation (curve C of Fig. 3).

In the redshift range  $z < 100$  positive density perturbations lower



**Figure 3.** The fluctuations of the CMBR brightness temperature (in units of  $\bar{T}$ ) in response to H I density perturbations. (A) The response of  $(1 - T_\gamma/T_s)$  through changes in the optical depth, ignoring the effect of peculiar velocities. Peculiar velocities will enhance this effect. (B) The response of  $sT_\gamma/T_s$  through changes in the spin temperature. (C) The total response for  $\mu^2 = 2/3$ . (D) The same as (B) without the gas temperature fluctuations.

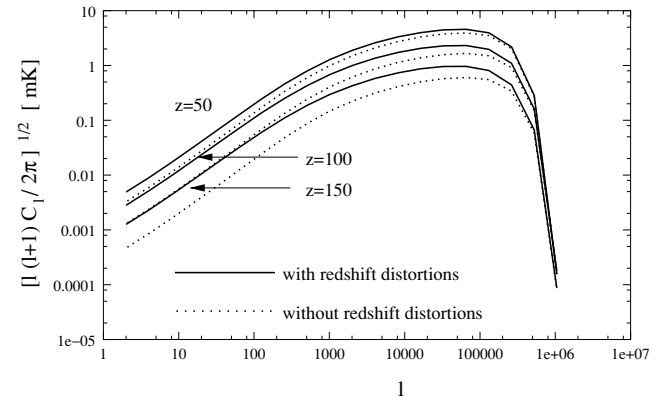
the spin temperature which enhances the brightness temperature decrement. In this regime the collisional process slowly loses out to the radiative process and  $T_s \rightarrow T_\gamma$ . A positive density perturbation enhances the collision rate which pulls the spin temperature down toward the gas temperature. The two processes which contribute toward brightness temperature fluctuations both act to enhance the decrement. Curve C of Fig. 3 shows the combined effect of both these processes for the value  $\mu^2 = 2/3$ . We find that the response of the brightness temperature to density perturbations peaks at  $z \sim 55$ . This is somewhat smaller than the value obtained by Loeb & Zaldarriaga (2003). Curve D of Fig. 3 shows the contribution to brightness temperature fluctuations arising from the spin temperature if the effect of density perturbations on the gas temperature is not taken into account (e.g. Loeb & Zaldarriaga 2003). We find that including the gas temperature makes a significant change, particularly at  $z > 100$  where there is a qualitative difference between curves B and D.

We have calculated the angular power spectrum  $C_l(\nu)$  of the brightness temperature fluctuations for the COBE normalized  $\Lambda$  CDM model (Bunn & White 1996). The power spectrum has been suppressed beyond the arbitrarily chosen value  $k = 14 h \text{ Mpc}^{-1}$  using a Gaussian cut-off. Our results are in qualitative agreement with those of Loeb & Zaldarriaga (2003). We find that the signal peaks at  $z \sim 50$  (Fig. 4) where the product of the growing mode of density perturbations and the response of brightness temperature to density perturbations is maximum.

To gauge the effect of peculiar velocities, we have calculated  $C_l(\nu)$  without incorporating this effect. This is easily done by replacing  $J_l(kr)$  with  $j_l(kr)$  in equation (16). The results are shown in Fig. 4. We find that the peculiar velocities increase  $\sqrt{C_l}$  by more than 50 per cent.

We have also quantified the effect of gas temperature fluctuations. We find that for  $z < 100$  the values of  $\sqrt{C_l}$  are  $\sim 10$  per cent lower if the gas temperature is not taken into account, and the effect is reversed at  $z > 100$  where  $\sqrt{C_l}$  is  $\sim 30$  per cent higher.

The ability to probe the dark matter power spectrum using the  $C_l(\nu)$ s will be restricted to scales  $k < k_J$  where  $k_J$  is the Jeans wavenumber. This has a nearly constant value  $\sim 500 h \text{ Mpc}^{-1}$  in the redshift range of interest. The H I power spectrum on scales smaller than the Jeans length-scale is interesting in its own right. The H I perturbations will undergo acoustic oscillations on these scales. The spin-temperature fluctuations and the peculiar velocities produced by density perturbations will be quite different from the situation considered here. On scales slightly larger than the Jeans



**Figure 4.** The angular power spectrum of the CMBR brightness fluctuations at various redshifts with and without the effects of peculiar velocities.

lengthscale ( $2\pi/k_j$ ) the density fluctuations are mildly non-linear with rms values in the range  $0.7 > \sigma > 0.1$ , and the Z'eldovich approximation may give a better description (e.g. Hui & Gnedin 1997).

The low-frequency cut-off imposed by the Earth's ionosphere restricts observations to frequencies more than  $\sim 10$ – $20$  MHz. Extracting the H I signal from the contaminations arising from the Galactic and extragalactic foregrounds is going to be a big challenge. The foregrounds are mostly continuum sources whose contribution varies slowly with frequency. It will be necessary to combine both the angular fluctuations and the frequency domain properties of the CMBR brightness temperature fluctuations in order to detect it (e.g. Shaver et al. 1999; Di Matteo et al. 2002; Oh & Mack 2003; Di Matteo, Ciardi & Miniata 2004).

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## REFERENCES

Allison A. C., Dalgarno A., 1969, ApJ, 158, 423  
Barkana R., Loeb A., 2001, Phys. Rep., 349, 125

Bharadwaj S., Nath B., Sethi S. K., 2001, JA&A, 22, 21  
Bunn E. F., White M., 1996, ApJ, 460, 1071  
Di Matteo T., Perna R., Abel T., Rees M. J., 2002, ApJ, 564, 576  
Di Matteo T., Ciardi B., Miniata F., 2004, MNRAS, submitted (astro-ph/0402322)  
Gnedin N. Y., Shaver P. A., 2004, ApJ, submitted (astro-ph/0312005)  
Hogan C. J., Rees M. J., 1979, MNRAS, 188, 791  
Hui L., Gnedin N., 1997, MNRAS, 292, 27  
Iliev I. T., Shapiro P. R., Farrara A., Martel H., 2002, ApJ, 572, L123  
Kaiser N., 1987, MNRAS, 227, 1  
Miralda-Escude J., 2003, Sci, 300, 1904–1909  
Loeb A., Zaldarriaga M., 2004, Phys. Rev. Lett., submitted (astro-ph/0312134)  
Madau P., Meiksin A., Rees M. J., 1997, ApJ, 475, 429  
Oh S. P., Mack K. J., 2003, MNRAS, 346, 871  
Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton, pp. 176, 177  
Rybicki G. B., Lightman A. P., 1979, Radiative Processes in Astrophysics. Wiley, New York, pp. 29–32  
Scott D., Rees M. J., 1990, MNRAS, 247, 510  
Shaver P., Windhorst R., Madau P., de Bruyn G., 1999, A&A, 345, 380  
Tozzi P., Madau P., Meiksin A., Rees M. J., 2000, ApJ, 528, 597

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