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THE COST OF ANNUITIES: IMPLICATIONS FOR SAVING BEHAVIOR AND BEQUESTS

Benjamin M. Friedman

Mark Warshawsky

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Cambridge, MA 02138
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ABSTRACT

The fact that most elderly individuals in the United States choose to maintain a flat age-wealth profile, rather than buy individual life annuities, stands in contrast to central implications of the standard life-cycle model of consumption-saving behavior. The analysis in this paper lends support to an explanation for this phenomenon based either on the cost of annuities, importantly including the element of that cost due to adverse selection, or on the interaction of that cost and an intentional bequest motive.

Expected yields offered on individual life annuities in the United States are lower by some 4-6%, or 2 1/2-4 1/2% after allowing for adverse selection, than yields on alternative long-term fixed-income investments. Simulations of an extended model of life-cycle saving and portfolio behavior, allowing explicitly for uncertain lifetimes and Social Security, show that yield differentials in this range can account for the observed behavior, even in the absence of a bequest motive, during the early years of retirement. By contrast, at older ages the combination of yield differentials in this range and a positive bequest motive is necessary to do so.

Benjamin M. Friedman
Harvard University
Littauer Center 127
Cambridge, MA 02138
(617) 495-4246

Mark Warshawsky
Federal Reserve Board
20th and Constitution Streets, N.W.
Washington, D.C. 20551
(202) 452-3722

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Benjamin M. Friedman and Mark Warshawsky*

> It is common experience that as we grow older and nearer to eternity we become more, not less, anxious about money. Derek Brewer, Chaucer and His World (p. 213).

It is startling, at least for economists who view consumption-saving behavior within the framework of the familiar life cycle model, to confront the fact that in the United States few individuals purchase life annuities. According to the life cycle model, the chief principle governing individual saving behavior is the desire to smooth consumption patterns over one's lifetime, within the constraints imposed by limited lifetime resources.[1] Despite ample evidence of such smoothing behavior with respect to short-run income fluctuations, age-wealth profiles show little if any tendency for elder individuals to dissave out of available resources as their remaining life expectancy shortens.[2] This behavior would be understandable in terms of the standard life cycle model with risk-averse individuals uncertain about their length of life, only if annuity markets did not exist. Without access to life annuities, elder individuals would conserve wealth to self-insure against the risk of having to reduce consumption in later years if their life span turns out to be unexpectedly long.[3] Well-developed markets for life annuities do exist in the United States, however. The puzzle is that so few people choose to use them.[4]

Some of the potential answers to this puzzle bear important implications not only for the theory of consumption-saving behavior but also for major issues of public policy. Perhaps the most obvious possibility is that most people save (or, in old age, choose not to dissave) not for motives related to the usual life cycle reasoning but, instead, to leave bequests to their heirs.[5] Although at a formal level it is easy enough to modify the life cycle model to incorporate a bequest motive, the two rationales for saving have strongly differing implications. If saving to fund intentional bequests accounts for a large share of actual wealth holding, then familiar life cycle conclusions, on issues as diverse as the efficacy of tax incentives for capital formation and the economic effects of Social Security, may no longer hold.

An alternative explanation for the lack of participation in the annuity market is that most individuals automatically receive life annuities from Social Security and, for an increasing fraction of the labor force, employer-sponsored pension plans. Nevertheless, it is difficult to believe that the combination of Social Security and private pension coverage so precisely matches each individual's preferences for annuity holding as to leave only minimal individual annuity demand remaining. Moreover, although both Social Security and private pension coverage have grown enormously just within recent decades, the minimal extent of individual life annuity purchases in the United States is apparently a phenomenon of long standing.

A more plausible explanation, which again relegates intentional bequests to a minor role and thereby rescues the life cycle model and its implications, is that people shun individual annuities because they are not priced "fairly" in the actuarial sense. A "load factor," depressing the yield on an annuity below the corresponding actuarially fair yield, could reflect ordinary transactions costs (including taxes and a competitive return to the annuity issuer's capital at risk), monopoly profits earned by annuity issuers in imperfectly competitive markets, or — as discussed at some length below — adverse selection among annuity purchasers.

Whatever its source, such a differential between the implicit expected yield on annuities and the available yield on alternative forms of wealth holding would clearly discourage annuity purchases. In the limit, if the differential were large enough, standard models of consumption-saving behavior without annuity markets could again apply.

The object of this paper is to examine the pricing of individual life annuities in the United States, and to infer from the observed market price structure the respective roles of actuarially unfair annuity prices and the intentional bequest motive in explaining the puzzle that so few individuals actually purchase such annuities. More specifically, the questions addressed here are, first, how large the yield differential actually is on readily available individual life annuities and, second, whether the observed yield differential is large enough to discourage a typically risk-averse elder individual from buying annuities in the absence of a positive bequest motive. In light of the great attention devoted to annuities in the theoretical literature of consumption-saving behavior, together with the potential importance of actuarially unfair annuity prices in explaining the observed behavior, it is surprising that (to the authors' knowledge) no one has previously examined these data and their implications.

Section I presents data on the implicit yield on individual life annuities in the United States during 1968-83, and compares these yields to those on alternative long-term fixed-income investment vehicles to measure the effective cost of annuities. Section II, drawing on the work of Fischer (1973), describes a model representing the consumption-saving and portfolio allocation behavior of an individual with uncertain lifetime, who has access to a life annuity market but (in general) also values bequests. Section III uses simulations of this model, based on observed annuity yield differentials, to draw inferences about the respective roles of annuity costs and a bequest motive in accounting for the typical individual's preferences for maintaining a flat age-wealth profile instead of buying annuities. Section IV briefly summarizes the paper's principal findings.

I. Yields on Individual Life Annuities

Table 1 presents calculations of the expected yields on individual life annuities offered in the United States, during 1968-83, based on annuity premiums quoted in successive issues of the A.M. Best Flitcraft Compend.[6] The premium underlying each reported calculation is that quoted for a single-premium immediate annuity for 65-year-old males. The assumed mortality probabilities underlying these calculations are the general population mortality probabilities for 65-year-old males reported in the 1970 and 1980 U.S. Life Tables, adjusted by a factor of .985 to reflect the 1.5% annual improvement in U.S. male mortality probabilities that has occurred over the last two decades, and by a further factor of .9925 to reflect the assumption of a future 0.75% annual improvement in male mortality probabilities for all ages.[7]

The first column of Table 1 indicates the expected yield calculated from the mean premium charged on this basic annuity contract by the ten largest insurance companies in the United States. These data are probably the most relevant for analyzing economy-wide individual behavior. The largest insurers usually do business in all regions of the country, so that the typical 65-year-old U.S. male has access to annuities at this mean premium with little or no search costs. The associated expected yield has risen over time as market interest rates have risen, although the expected annuity yield has consistently remained well below the contemporary yields on readily available fixed-income investments. This simple comparison does not indicate that annuities are a "dominated asset," however, because the lifetime guarantee provided by an annuity is not available from other investment vehicles.[8]

The remaining columns of Table 1 indicate the potential returns to

TABLE 1

YIELD EQUIVALENT OF INDIVIDUAL LIFE ANNUITIES

	Ten Largest Insurers			Complete Sample
	Mean	High	Low	High
1968	2.25%	2.60%	1.90%	2.75%
1969	2.58	3.03	2.18	3.18
1970	2.91	3.76	2.31	4.06
1971	3.27	4.27	2.42	4.27
1972	3.36	4.11	2.51	4.11
1973	3.62	4.22	2.82	4.22
1974	3.85	4.50	3.25	4.55
1975	4.29	4.89	3.74	4.89
1976	4.61	5.21	3.86	5.66
1977	4.67	5.07	4.07	5.57
1978	4.73	5.13	4.13	5.68
1979	4.78	5.23	4.18	6.18
1980	5.29	6.29	4.54	6.84
1981	5.92	6.72	5.02	8.42
1982	6.57	9.37	5.07	12.17
1983	6.80	8.85	5.13	10.65

Notes: Calculations assume general population mortality probabilities.
Calculations are for 65-year-old-males.

market search by showing the dispersion of expected yields calculated from the premiums charged for this same basic contract by different insurers.[9] The second and third columns show data for the highest and lowest expected yields on this contract offered by any of the ten largest insurers. Presumably most 65-year-old males have access to the highest yields in this group at only modest search cost. The final column of the table shows the highest expected yield on this contract offered by any of the fifty-odd insurers in Best's sample. Because the smaller companies in the sample do not necessarily maintain sales forces in all parts of the country, however, there is no presumption that the typical 65-year-old male has ready access to this complete-sample highest yield.

If all individuals had identical mortality probabilities, the spread between the calculated expected yields shown in Table 1 and some reference yield reflecting the typical individual's opportunity cost would indicate the effective cost to the individual of the risk avoidance that annuities provide. In fact, however, many individuals have information that leads them to expect either a shorter or a longer life than the population-wide average. By contrast, insurers typically charge a uniform premium to all individuals of the same age and sex, presumably because information about individual mortality probabilities is too costly, or perhaps even impossible, to obtain and use. Individuals expecting longer (shorter) than average lifespans will therefore perceive that life annuities bear higher (lower) expected yields, and hence will be more (less) likely to buy them.[10] This adverse selection — adverse from the viewpoint of the insurer, that is — will lead to underwriting losses if the insurer continues to charge a premium that is actuarially fair to the population as a whole.

Table 2 therefore presents analogous calculations of the expected yields on the same basic annuity contract for a 65-year-old male, based on the same annuity premiums as before, but now based on alternative mortality probabilities compiled from the actual company experience on individual life annuity contracts issued in the United States during 1971-75, again adjusted as indicated above to reflect the improvement in mortality probabilities.[11] Figure 1 indicates the extent to which the sub-population who choose to buy annuities in fact have a greater survival probability than the general population. Because of this greater life expectancy, the expected yields shown in Table 2 are greater than the corresponding values shown in Table 1, based on general population mortality probabilities. Nevertheless, even these greater expected yields resulting from the actual company experience mortality probabilities are still lower than the contemporary yields on readily available alternative forms of wealth holding. Hence even the sub-population who voluntarily buy annuities still face a negative yield differential representing the cost of the protection from risk that the annuity provides.

Table 3 summarizes the cost of this basic annuity contract by showing the 1968-83 mean values of the differential between the expected yields shown in Tables 1 and 2 and two different market interest rates: the 20-year U.S. government bond yield and the average yield on corporate debt directly placed with insurance companies. From the standpoint of an individual's opportunity cost of funds, the (lower) yield on U.S. Government bonds is more relevant if the individual has no better investment vehicle available. Increasingly, however, U.S. financial intermediaries have offered individuals ways of buying shares in pools of less liquid but higher-yielding assets.

TABLE 2

YIELD EQUIVALENT OF LIFE ANNUITIES: ANNUITY PURCHASERS ONLY

	Ten Largest Insurers			Complete Sample
	Mean	High	Low	High
1968	4.25%	4.60%	3.85%	4.70%
1969	4.53	4.98	4.13	5.13
1970	4.81	5.66	4.21	5.96
1971	5.17	6.07	4.27	6.07
1972	5.21	5.96	4.36	5.96
1973	5.47	6.07	4.67	6.07
1974	5.65	6.35	5.05	6.35
1975	6.04	6.69	5.49	6.69
1976	6.36	6.96	5.61	7.41
1977	6.42	6.82	5.82	7.32
1978	6.48	6.88	5.83	7.38
1979	6.43	6.93	5.88	7.88
1980	6.94	7.94	6.19	8.49
1981	7.57	8.37	6.62	10.12
1982	8.17	11.12	6.67	13.82
1983	8.40	10.45	6.75	12.30

Note: Calculations are for 65-year-old males.
Calculations assume company experience mortality probabilities.

FIGURE 1

PROBABILITY OF MALE AT AGE 65 SURVIVING TO AGE X

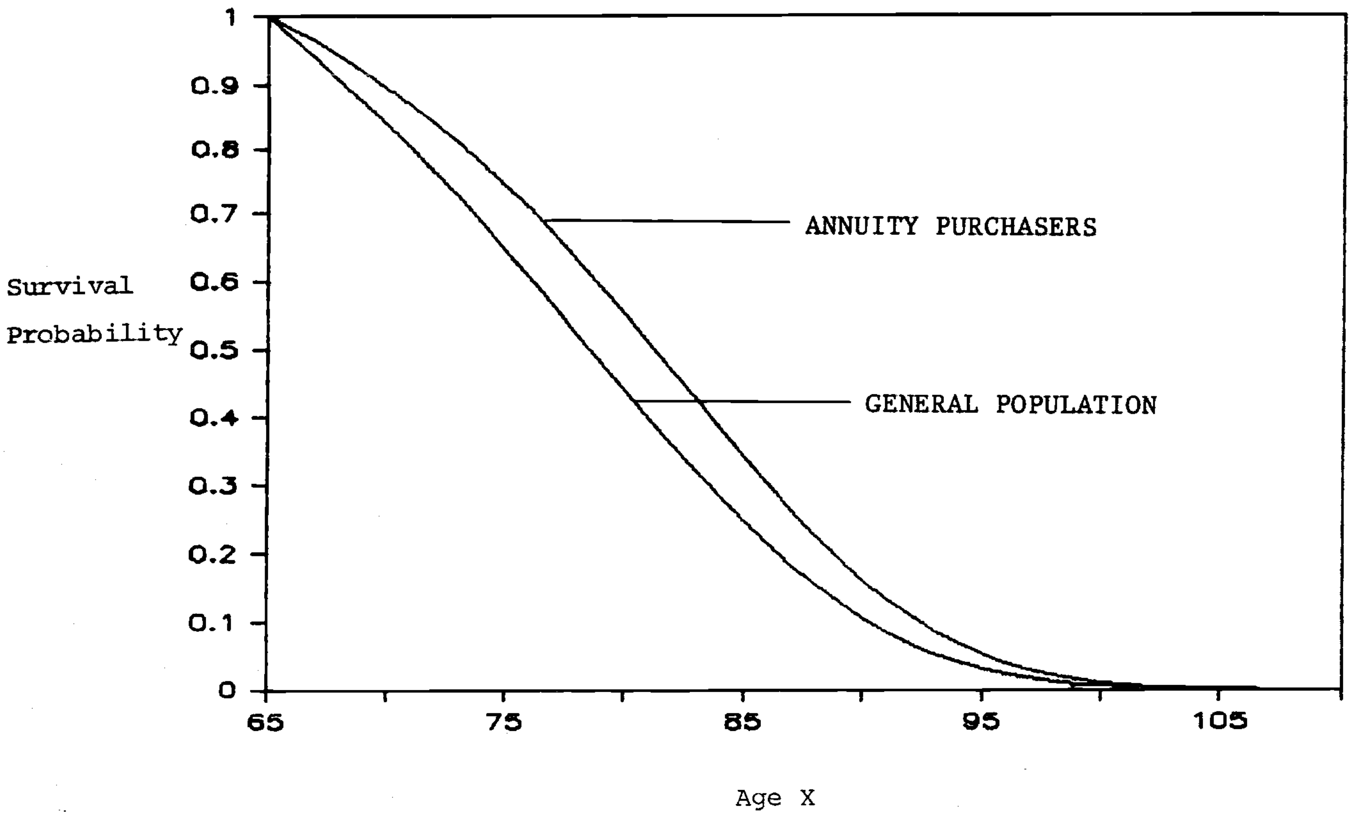

TABLE 3

MEAN YIELD DIFFERENTIALS ON INDIVIDUAL LIFE ANNUITIES, 1968-83

Mortality Probabilities:	General Population		Annuity Purchasers	
Base Interest Rate:	Government Bonds	Direct Placements	Government Bonds	Direct Placements
Insurer:				
Ten-Largest Mean	-4.21%	-6.13%	-2.43%	-4.35%
Ten-Largest High	-3.35	-5.27	-1.56	-3.48
Ten-Largest Low	-4.98	-6.90	-3.21	-5.13
Complete-Sample High	-2.73	-4.65	-.95	-2.87

Note: Calculations are for 65-year-old males.

The resulting average yield differentials shown in Table 3 range from a minimum (in absolute value) of -.95% for the highest expected yield offered by any company in Best's sample, compared to the Government bond yield and based on actual company experience mortality probabilities, to a maximum of -6.90% for the lowest yield offered by any of the ten largest insurers, compared to the direct placement yield and based on general population mortality probabilities. The differentials in the table that are probably most relevant for studying economy-wide individual behavior are those shown in the first row, for the mean expected yield offered by the ten largest insurers, compared to the yield on either Government bonds or direct placements, and based on either general population or company experience mortality probabilities. These differentials range from -2.43% to -6.13% — large values from some perspectives, but not, for example, in comparison to the yield differential that most individuals incur in holding money balances.

The question at issue here is whether these differentials are sufficient to account for the preference for maintaining a flat age-wealth profile over purchasing life annuities observed among the elderly population in the United States. Since it is impossible to answer this question on a purely empirical basis, some more formal analytical approach is necessary. Section II develops a suitable framework for analysis, and Section III applies this framework in the context of the observed yield differentials reported here.

II. A Model of Saving and Annuity Demand

The model developed here to analyze the demand for individual life annuities in the context of life cycle saving and a bequest motive is an annuity analog of Fischer's (1973) model of the demand for life insurance, generalized to incorporate fixed mandatory holdings of socially provided annuities.[12] Following Fischer, it is useful to represent the individual's decision problem in this expanded life cycle context as the maximization of expected lifetime utility

$$E(U) = \sum_{t=0}^{w-x-1} [p_t U_t(C_t) + p_t q_{t+1} V_{t+1}(G_{t+1})] \quad (1)$$

where w is the assumed maximum length of life, x is the individual's age as of time t=0, p_t is the probability that an individual of age x at t=0 will be alive at any time $t \geq 0$, q_{t+1} is the (conditional) probability that such an individual who was alive at time t would die at time t+1.[13] $U_t(C_t)$ is utility received from consumption C at time t, and $V_{t+1}(G_{t+1})$ is utility received from (anticipation of) a bequest G at time t+1. Again following Fischer, it is convenient to specify the two utility functions in the iso-elastic form

$$U_t(C_t) = \frac{C_t^{1-\beta}}{1-\beta} \cdot \alpha^t \quad (2)$$

$$V_t(G_t) = \frac{G_t^{1-\beta}}{1-\beta} \cdot b_t \quad (3)$$

where β is the Pratt-Arrow coefficient of relative risk aversion, α is the time preference parameter, and b_t (in comparison to α^t) indicates the relative utility attached to bequests left in period t.[14]

The usual life cycle specification of behavior with no bequest motive is therefore just the special case of this model with $b_t=0$ for all $t>0$. In general, however, people may value bequests, and they may value them differently at different times. For example, Yaari (1965) has suggested that b_t follows a hump-shaped pattern, with higher values during the years when family dependency is important, and either level or declining values during retirement years when children have typically become financially independent.

The individual's problem is then to maximize (1), subject to a given initial wealth position and to a nonnegativity constraint on wealth in each subsequent time period, given the menu of available investment opportunities (including any mandatory holding of socially provided annuities) and their respective yields.[15] In each period the individual must decide not only how much of current wealth to consume but also how to allocate the remainder among the available investment vehicles. The specific asset menu considered here includes a riskless one-period bond bearing gross rate of return R_t, a one-period social annuity bearing gross rate of return Q_t^S to survivors, and a one-period market annuity bearing gross rate of return Q_t^A to survivors.[16] Both annuities are actuarially fair — that is, there is no expected yield differential for either — if

$$(1-q_{t+1})Q_t^S = (1-q_{t+1})Q_t^A = R_t. \tag{4}$$

With little relevant loss of generality, it is also convenient to set R_t constant at R for all $t \geq 0$.

The dynamic programming solution to this problem proceeds from the final period t=w-x-1, in which the certainty of death at the end of the period ($q_{w-x}=1$) simplifies the problem of an individual who has survived to that date to merely choosing C_{w-x-1} to maximize the sum of utility from current consumption $U_{w-x-1}(C_{w-x-1})$ and utility from bequests $V_{w-x}(G_{w-x})$, subject to then-remaining wealth W_{w-x-1} and the constraint

$$G_{w-x} = R \cdot (W_{w-x-1} - C_{w-x-1}). \tag{5}$$

Given the iso-elastic utilities assumed in (2) and (3), the solution is just

$$C_{w-x-1} = k_{w-x-1} \cdot W_{w-x-1} \tag{6}$$

where

$$k_{w-x-1} = \frac{R(Rb_{w-x})^{-1/\beta}}{1+R(Rb_{w-x})^{-1/\beta}} \tag{7}$$

and the corresponding indirect utility function

$$J_1[W_{w-x-1}] = \max_{C_{w-x-1}} \{U_{w-x-1}(C_{w-x-1}) + V_{w-x}(G_{w-x})\} \tag{8}$$

is

$$J_1[W_{w-x-1}] = \delta_{w-x-1} \cdot \frac{W_{w-x-1}^{1-\beta}}{1-\beta} \tag{9}$$

where

$$\delta_{w-x-1} = k_{w-x-1}^{-\beta}. \tag{10}$$

The consumption decision (5) represents the entire solution for $t=w-x-1$, since in that period the availability of annuities is irrelevant to the analysis.

The dynamic programming solution next proceeds to the individual's optimal consumption and portfolio decisions for the immediately prior period, given wealth remaining at that time. An individual alive at $t=w-x-2$ will die at the end of that period with probability q_{w-x-1}. Hence the relevant maximand governing the decisions to be taken as of t=w-x-2 is $U_{w-x-2}(C_{w-x-2})$ plus the bequest motive $V_{w-x-1}(G_{w-x-1})$ with probability q_{w-x-1} and the indirect utility function in (9) with probability $(1-q_{w-x-1})$. The indirect utility function for t=w-x-2 is therefore

$$J_2[W_{w-x-2}] = \max_{C_{w-x-2},A_{w-x-2}} \{\frac{C_{w-x-2}^{1-\beta}}{1-\beta} + (1-q_{w-x-1})\alpha\delta_{w-x-1}\cdot\frac{(W_{w-x-2}-C_{w-x-2})^{1-\beta}}{1-\beta}$$

$$\cdot [R(1-A_{w-x-2}-S_{w-x-2}) + Q^A_{w-x-2}A_{w-x-2} + Q^S_{w-x-2}S_{w-x-2}]^{1-\beta}$$

$$+ q_{w-x-1}\, b_{w-x-1}\cdot\frac{(W_{w-x-2} - C_{w-x-2})^{1-\beta}}{1-\beta} \cdot [R(1-A_{w-x-2} - S_{w-x-2})]^{1-\beta} \qquad (11)$$

where A and S are the proportions of saving (W-C) invested in market annuities and (mandatorily) in social annuities, respectively. The usual life cycle model with no market for annuities is therefore just the special case represented by $A_t=0$ for all $t\geq 0$ (and, if there are no social annuities either, $S_t=0$ for all $t\geq 0$ also).[17]

The first-order conditions for (11) then give the optimal values of consumption and purchases of market annuities at t=w-x-2 as

$$C_{w-x-2} = \delta_{w-x-2}^{-1/\beta} \cdot W_{w-x-2} \qquad (12)$$

$$A_{w-x-2} = \frac{R\left[\left(\frac{Rb_{w-x-1}\, q_{w-x-1}}{(1-q_{w-x-1})\alpha\delta_{w-x-1}(Q^A_{w-x-2} - R)}\right)^{-1/\beta} -1\right]}{Q^S_{w-x-2} + R\left[\left(\frac{Rb_{w-x-1}\, q_{w-x-1}}{(1-q_{w-x-1})\alpha\delta_{w-x-1}(Q^A_{w-x-2} - R)}\right)^{-1/\beta} -1\right]} - S_{w-x-2} \qquad (13)$$

and the corresponding value of the indirect utility function as

$$J_2[W_{w-x-2}] = \delta_{w-x-2} \cdot W_{w-x-2} \tag{14}$$

where

$$\delta_{w-x-2} = \left[\frac{k_{w-x-2}}{1+k_{w-x-2}}\right]^{-\beta} \tag{15}$$

and

$$k_{w-x-2} = [\alpha\delta_{w-x-1}(R(1-A_{w-x-2}-S_{w-x-2}) + Q^{A}_{w-x-2}A_{w-x-2} + Q^{S}_{w-x-2}S_{w-x-2})^{1-\beta}$$

$$+ b_{w-x-1}q_{w-x-1}(R(1-A_{w-x-2}-S_{w-x-2}))^{1-\beta}]^{-1/\beta} . \tag{16}$$

The remainder of the dynamic programming solution proceeds backward to the initial period $t=0$ in an analogous way. The expressions for each period's optimal consumption and purchases of market annuities, and for each period's value of the indirect utility function, are of the same form (but with subscripts adjusted accordingly) as (12), (13) and (14), respectively. Because the analytical properties of the model are not sufficient to address the more quantitative questions that are the focus of this paper, however, Section III proceeds with numerical simulations of the model under several different sets of assumptions about the crucial bequest motive parameter and the cost of private annuities.

III. Simulation Results

The model developed in Section II generates lifetime streams of consumption and annuity purchase values that are optimal for given values of parameters describing preferences (β, α and b), the market environment (R, Q^A and Q^S), and mortality probabilities (p and q). The principal focus of interest in this paper is on one aspect of preferences and one aspect of the market environment — the bequest motive and the cost (yield) of market annuities, respectively.

The strategy adopted here for representing the bequest motive is simply to asumme that the correspondence between b_t in (3) and α^t in (2) is just

$$b_t = \alpha^t \cdot \theta, \qquad t=0,\ldots,35 \tag{17}$$

where θ is a non-age-specific parameter indicating the individual's life-long preference for bequests relative to current consumption, given the other parameters of the model, including in particular the interest rate (R), the curvature of the utility function (β), and — because θ implicitly gives the relative weight of a stock (the bequest) versus a flow (consumption) — the assumed time unit of analysis. Given θ, the bequest amount is larger as R is higher, and smaller as β is higher. For example, from (5)-(7) and (17), θ takes the value $(G_{w-x}/C_{w-x-1})^{\beta} \cdot \alpha^{35}/R$, where (G_{w-x}/C_{w-x-1}) is just the ratio of the final-period bequest to the prior-period consumption. The normally limiting case for altruistic bequests, in which an individual provides for his heirs' consumption at the same level as his own, indicates $(1/R-1)^{\beta} \cdot \alpha^{35}/R$ as the logical upper bound on θ.[18] In the simulations reported below, the strength of the bequest motive is indicated by θ and the corresponding bequest/consumption ratio (G_{w-x}/C_{w-x-1}) that would result under access to fair annuities, given the other assumed parameters.

The strategy used to represent the market for private annuities is simply to assume that such annuities are readily available at some expected yield differential

$$D = Q_t^A(1-q_{t+1}) - R. \tag{18}$$

In the simulations reported below, the values used for D include zero, all four values shown in the top line of Table 3 for the expected yield differential offered by the ten largest insurers (ranging from -2.43% to -6.13%), and the smallest differential shown in Table 3 (-.95%, for the highest yield offered by any company in Best's sample, compared to the Government bond yield and based on actual company experience mortality probabilities). Because these yield differentials are large enough in some cases to induce an individual to want to act as if he were the insurer by issuing rather than holding annuities against his own life — that is, in effect short-selling the annuity contract by issuing debt on which repayment (adjusted by his survival probability) is contingent on his own survival — a nonnegativity constraint is imposed in all simulations.

Table 4 summarizes the results of three sets of simulations of the model, based on different combinations of these assumed values of θ and D. In each simulation the assumed time preference parameter is $\alpha = .99$, the assumed market interest rate is $R = 1.01$, the assumed coefficient of relative risk aversion is $\beta = 4$, the assumed fraction of wealth mandatorily invested in actuarially fair social annuities is $S = .5$, and the assumed

TABLE 4

THE DEMAND FOR PRIVATE ANNUITIES

		$\frac{G_{w-x}}{C_{w-x-1}}$	Expected Yield Differential (D) 0.00%	-.95%	-2.43%	-4.21%	-4.35%	-6.13%
Age 65:	θ=0	.00 %	50.00%	50.00%	50.00%	.00%	.00%	.00%
	1	.89	42.79	42.24	39.39	.00	.00	.00
	5	1.34	39.53	38.71	34.50	.00	.00	.00
	10	1.59	37.75	36.77	31.80	.00	.00	.00
	25	2.00	34.99	33.76	27.59	.00	.00	.00
	50	2.38	32.57	31.11	23.84	.00	.00	.00
	100	2.83	29.84	28.09	19.56	.00	.00	.00
Age 70:	θ=0	.00%	50.00%	50.00%	50.00%	.00%	.00%	.00%
	1	.89	41.45	41.07	39.94	.00	.00	.00
	5	1.34	37.67	37.09	35.39	.00	.00	.00
	10	1.59	35.63	34.94	32.93	.00	.00	.00
	25	2.00	32.48	31.62	29.09	.00	.00	.00
	50	2.38	29.75	28.72	25.72	.00	.00	.00
	100	2.83	26.71	25.45	21.92	.00	.00	.00
Age 75:	θ=0	.00%	50.00%	50.00%	50.00%	50.00%	50.00%	.00%
	1	.89	39.71	39.43	38.75	36.99	36.75	.00
	5	1.34	35.28	34.85	33.83	31.21	30.84	.00
	10	1.59	32.93	32.41	31.17	28.07	27.65	.00
	25	2.00	29.34	28.67	27.11	23.23	22.71	.00
	50	2.38	26.26	25.46	23.60	19.02	18.39	.00
	100	2.83	22.86	21.89	19.68	14.30	13.57	.00

Notes: Values are present expected values as percentages of initial wealth.
Calculations are for males.
Assumed values are α=.99, R=1.01, β=4 and S=.5.

mortality probabilities are those reported in the 1980 U.S. Life Tables.[19] The upper panel of the table presents results based on mortality probabilities for 65-year-old males, while the middle and lower panels present analogous results for 70- and 75-year-old males, respectively.

For each set of assumed values, the solution of the model yields an entire time series representing each year's consumption and each year's division of privately held wealth (that is, wealth not mandatorily held in social annuities) between bonds and private annuities.[20] Given the objective of this paper, however, Table 4 reports for each simulation only the optimal holding of private annuities at three specific ages, expressed in each case as a percentage of total wealth.[21] With one-half of total wealth assumed to be held in social annuities, and with the nonnegativity constraint imposed, the range of possible values of the share of wealth held in private annuities is from zero to 50%.

The results shown in Table 4 suggest that, within the empirically relevant range of variation for either D or θ, demand for individual life annuities is sensitive both to the cost of annuities and to the bequest motive. If annuities are actuarially fair and there is no bequest motive, (that is, with $D = \theta = 0$), the optimal solution to the maximization problem posed in Section II calls for stabilizing the lifetime consumption stream completely by investing all of privately held wealth in private annuities, so that A = 50.00%. Either a negative yield differential or a positive bequest motive makes annuities less attractive. The results summarized in the table show how variations in D or θ, or both, reduce A from the 50.00% reference point.

These results suggest not only that the cost of annuities is a key part of the explanation for the small participation in the market for individual life annuities, but also that the adverse selection element is a crucial part of that cost. Given the near absence of individual annuity purchases, the real question at issue in these simulations is what combinations of D and θ are sufficient to drive optimal annuity holdings to no more than a trivially small percentage of initial wealth. For 65-year-old males, for example, none of the values of θ shown in Table 4 does so in the absence of a negative expected yield differential. By contrast, for each of the three largest (in absolute value) values of D considered, the optimal portfolio allocation decision is to buy no private annuities at all — regardless of the presence or absence of a bequest motive. Reference to Table 3 indicates that D = -2.43%, the largest value considered at which it is optimal to hold any private annuities at all, is the differential faced by an individual who knows that his mortality probabilities are like those of other annuity purchasers rather than the general population, and for whom the Government bond yield represents the opportunity cost of funds. By contrast, D = -4.21%, the smallest value considered at which it is optimal to hold no private annuities, is the differential faced by a comparable individual without specific knowledge of his mortality probabilities.

The results based on mortality probabilities for 70-year-old males are essentially identical to those for 65-year-olds. For 75-year-olds, however, the results differ in two interesting respects. First, for this group positive private annuity holdings are optimal even at D = -4.21% or -4.35% (although still not at -6.13%). Second, at these larger yield differentials, a bequest motive corresponding to a bequest/consumption ratio of nearly three is sufficient to reduce optimal private annuity holdings to less than 15% of initial wealth. A more modest bequest motive still leaves the optimal demand for private annuities implausibly large, however.

Although the simulation results summarized in Table 4 are sensitive in at least some regard to each of the underlying assumptions, it is especially interesting in this context to consider the implications of an alternative assumption about the strength of individuals' risk aversion. Because the basic purpose of an annuity is to insure against the consumption risk associated with an uncertain lifetime, greater risk aversion unambiguously implies a larger demand for annuities.

Table 5 therefore presents a corresponding set of results, identical to those in Table 4 in every way except that the assumed coefficient of relative risk aversion is $\beta = 2$.[22] As in the results based on $\beta = 4$ in Table 4, the demand for private annuities with $\beta = 2$ appears to be sensitive both to the yield differential and to the strength of the bequest motive. For 65-year-olds, for example, at the same $D = -2.43\%$ considered above, the optimal holding of private annuities varies from 50% of total wealth for $\theta = 0$ to only 1% for $\theta = 10$, corresponding to a bequest/consumption ratio of 2.5. With a zero yield differential, $\theta = 50$ is sufficient to reduce private annuity demand to only 12% of total wealth, but the corresponding bequest/consumption ratio is implausibly high.[23] The results for 70-year-olds are again essentially identical. Finally, again as in the results in Table 4, for 75-year-olds the combination of $D = -2.43$ and $\theta = 10$ (bequest/consumption ratio of 2.5) is sufficient to reduce private annuity demand to just over 15% of wealth. Similarly, with a bequest motive of this strength private annuity purchases are barely positive for $D = -4.21\%$ and are zero for $D = -4.35\%$.

In sum, during the early retirement years the cost of annuities, as represented by the expected yield differential, appears to be a

TABLE 5

THE DEMAND FOR PRIVATE ANNUITIES WITH LOW RISK AVERSION

		$\frac{G_{w-x}}{C_{w-x-1}}$	Expected Yield Differential					
			.00%	-.95%	-2.43%	-4.21%	-4.35%	-6.13%
Age 65:	θ=0	.00%	50.00%	50.00%	50.00%	.00%	.00%	.00%
	1	.80	42.77	41.34	32.88	.00	.00	.00
	5	1.79	34.93	31.88	14.01	.00	.00	.00
	10	2.53	29.72	25.55	1.28	.00	.00	.00
	25	4.01	20.75	14.60	.00	.00	.00	.00
	50	5.67	12.41	4.36	.00	.00	.00	.00
	100	8.02	3.09	.00	.00	.00	.00	.00
Age 70:	θ=0	.00%	50.00%	50.00%	50.00%	.00%	.00%	.00%
	1	.80	41.53	40.50	37.38	.00	.00	.00
	5	1.79	32.60	30.43	23.86	.00	.00	.00
	10	2.53	26.80	23.85	14.95	.00	.00	.00
	25	4.01	17.10	12.77	.00	.00	.00	.00
	50	5.67	2.80	.00	.00	.00	.00	.00
	100	8.02	.00	.00	.00	.00	.00	.00
Age 75:	θ=0	.00%	50.00%	50.00%	50.00%	50.00%	50.00%	.00%
	1	.80	39.92	39.12	37.25	32.16	31.43	.00
	5	1.79	29.68	28.00	24.10	13.56	12.02	.00
	10	2.53	23.24	20.97	15.71	1.61	.00	.00
	25	4.01	12.77	9.52	1.96	.00	.00	.00
	50	5.67	.00	.00	.00	.00	.00	.00
	100	8.02	.00	.00	.00	.00	.00	.00

Notes: Values are present expected values as percentages of initial wealth.
Calculations are for males.
Assumed values are α=.99, R=1.01, β=2 and S=.5.

sufficient explanation for the observed general absence of participation in the individual life annuity market in the United States, for either level of risk aversion. By contrast, at older ages a combination of the cost of annuities and a positive bequest motive, acting in conjunction, is necessary to provide an explanation for this phenomenon within the context of the analytical framework applied here.

IV. Conclusions and Caveats

The fact that most elderly individuals in the United States choose to maintain a flat age-wealth profile, rather than buy individual life annuities, stands in contrast to central implications of the standard life-cycle model of consumption-saving behavior. The analysis in this paper lends support to an explanation for this phenomenon based either on the cost of annuities, importantly including the cost element due to adverse selection, or on the interaction of that cost and an intentional bequest motive.

Expected yields offered on individual life annuities in the United States are lower by 4.21-6.13%, or 2.43-4.35% after allowing for adverse selection, than yields on alternative long-term fixed-income investments. Simulations of an extended model of life-cycle saving and portfolio behavior, allowing explicitly for uncertain lifetimes and Social Security, show that yield differentials in this range can account for the observed behavior, even in the absence of a bequest motive, during the early years of retirement. By contrast, at older ages, the combination of yield differentials in this range and a positive bequest motive can also do so.

As is usually the case with any initial analysis, caution is appropriate in relying on these results without further research. Although the model used in this analysis does generalize the standard life cycle model in several potentially important ways, it still limits the conclusiveness of the results by in effect excluding a priori three further possible explanations for the observed behavior.[24]

First, informal discussions with insurers and financial advisors to individuals suggest that many people choose stable age-wealth profiles, rather than either buying annuities or simply consuming out of wealth, from fear of the consequences of catastrophic illness. Although this explanation seems to imply (counter to experience) that the introduction of Medicare should have changed the typical behavior in a readily visible way, it is difficult to judge how much knowledge people actually have about the Medicare program — or, indeed, how much confidence they have in it.

Second, the entire analysis here, as in most of the literature to date, is implicitly in real terms. By contrast, the individual life annuities available in U.S. markets guarantee specified nominal payments. Insurers often argue that they would find little market for a "real annuity" if they marketed such an instrument, but (to the authors' knowledge) the experiment is untried.

Third, while the analysis here employs the standard theory of expected utility maximization, evidence exists that apparently contradicts this theory as a description of decision making under risk, especially when the prospect of rare events is involved. In particular, individuals appear systematically to overweight the probability of rare events.[25] The more familiar implication of this psychological tendency is that it contributes to the attractiveness of both gambling and insurance. In reverse, however, it would also make annuities less desirable.

These further possible explanations for the fact that most of the retired elderly choose flat age-wealth profiles over purchases of individual life annuities remain as objects for future research, so that the explanation provided here in terms of actuarially unfair pricing (either with or without a positive bequest motive) is necessarily tentative. Within the limits of the analysis carried out here, however, the conclusions reported in this paper point clearly toward the importance of actuarially unfair annuities.

Footnotes

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1. The standard references are Modigliani and Brumberg (1954) and Ando and Modigliani (1963).

2. See, for example, Mirer (1979) and Hubbard (1983).

3. See, for example, Davies (1981).

4. The Retirement History Survey indicates that only 2% of the elderly population own individual annuities of any sort; see for example, Friedman and Sjogren (1980). A more specific example may further help to illustrate the low level of activity in the individual life annuity market: One large insurer, considered to be more active than average in this market, sold an average of $18.5 million of individual life annuities each year during 1980-83. By contrast, the same company's annual sales of individual life insurance averaged $18.1 billion during this period.

5. Kotlikoff and Summers (1981), for example, have argued that intergenerational transfers account for "the vast majority" of aggregate U.S. savings, while life cycle saving accounts for "only a negligible fraction" of the aggregate.

6. The expression used to calculate these yields is

$$P = \sum_{t=1}^{w-x-1} (1+r)^{-t} p_{xt}$$

where P is the quoted per-dollar premium, w is the assumed maximum length of life (here taken to be 110 years), x is the age at the date of issue (here 65 years), r is the relevant interest rate, and p_{xt} is the probability that an individual of age x at time t=1 will survive to any year $t \geq 1$. These annual calculations are then converted to a monthly basis. The Appendix to Friedman and Warshawsky (1985) shows that calculations for 65-year-old females, or for 70-year-old or 75-year-old males, lead to results roughly similar to those for 65-year-old males.

7. The calculations rely on the 1970 tables for years 1968-70, on the 1980 tables for years 1980-83, and on both tables (weighted) for years 1971-79. See Faber (1982) for a complete description of the U.S. Life Tables, and Wetterstrand (1983) for a discussion of improvements in mortality probabilities.

8. The value of a 35-year certain annuity, calculated using the long-term U.S. Government bond rate, exceeded the mean annuity premium charged to 65-year-old males in all years of the sample except 1980, 1981 and 1982; see Friedman and Warshawsky (1985).

9. This dispersion probably reflects search costs; see, for example Pratt et al. (1979). Alternatively, it could reflect different marketing choices by different insurers.

10. See Rothschild and Stiglitz (1976) for an analysis of the behavior underlying this kind of adverse selection.

11. See Society of Actuaries (1983) for the actual company experience tables.

12. In fact, Fischer's model is really an annuity model, despite his application of it to the demand for life insurance.

13. Probabilities p_t and q_{t+1} are, of course, conditional on initial age x. Writing them as $p(x)_t$ and $q(x)_{t+1}$ would be appropriate but would clutter an already cumbersome notation. Conditionality on x is to be understood, here and below.

14. See Hakkanson (1969) for a discussion of the iso-elastic utility function.

15. In a more general context it would also be necessary to take account of labor income. The focus of this paper, however, is on the elderly retired population.

16. As in Fischer (1973), the assumption of one-period annuities makes the analysis tractable. The annuities actually available for purchase in the United States are instead life annuities.

17. The model as written here imposes no nonnegativity constraint on choice paramter A — that is, it does not explicitly preclude short sales of annuities. The simulations reported in Section III below impose $A \geq 0$.

18. It is also possible, of course, to posit a variety of circumstances in which an altruistic individual may value his heirs' consumption more than his own, so that θ need not be bounded. In addition, individuals may value bequests for non-altruistic reasons; see, for example, Bernheim et al. (1984).

19. The assumptions $\alpha = .99$ and $R = 1.01$ follow Kotlikoff and Spivak (1981). The assumption $\beta = 4$ follows Bodie et al. (1985), and corresponds to the evidence found by Grossman and Shiller (1981). One-half is about the fraction of total wealth constituted by Social Security for the average wealth constituted by Social Security for the average retired elderly individual in the United States; see Kotlikoff and Smith (1983), Table 3.7.19 (p.127).

20. See Friedman and Warshawsky (1985) for illustrations of these time paths in the case of fair annuities.

21. Wealth held in annuity form is valued at the present expected value.

22. Friend and Blume (1975) found evidence indicating a relative risk aversion coefficient roughly equal to 2.

23. For males Menchik and David (1982, p. 193, Table 1) reported a median bequest equal to 2.1 times median annual labor income (defined as one-fortieth of average annual labor earnings), and a mean bequest equal to 4.2 times mean annual labor income.

24. An additional factor explored by Kotlikoff and Spivak (1981), intergenerational risk sharing within families could also possibly explain the absence of individual life annuity purchases, but not the flat age-wealth profile.

25. See, for example, Kahneman and Tversky (1979).

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