REVIEW

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The Costs and Benefits of Price Stability: An Assessment of Howitt's Rule

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The central banks of New Zealand, Canada, and the United Kingdom have recently decided to make price stability the overriding goal of monetary policy. Similar proposals in the United States have received a lukewarm reception. Although some opponents have argued that moderate inflation is beneficial, many concede its effect on economic welfare is detrimental.[1] Instead, they argue that a price stability policy is suboptimal because, once inflation is under way, it is better to tolerate some moderate inflation than to bear the cost necessary to achieve price stability. Howitt's Rule is the clearest statement of the proposition that the benefits resulting from reducing inflation must be weighed against the cost of reducing it.[2] (See the shaded insert, "Howitt's Rule.")

Assuming inflation affects both the level and growth rate of output, I state Howitt's Rule and explain how it argues for a continued policy of moderate inflation. Next, I analyze alternative estimates of the costs of achieving price stability and compare these costs with estimates of the gain from achieving price stability when inflation reduces the level or growth rate of output. Finally, I briefly review the cross-section and time-series evidence on the effects of inflation on output growth. In the final section I offer a summary and some concluding observations.

HOWITT'S RULE

In order to estimate the *optimal target rate of inflation,* one must somehow balance the gains from reducing inflation against the costs of doing so. The reduction in inflation should continue as long as the present discounted value of the benefits to society from a further small reduction exceeds the present discounted value of the cost. The optimal target rate is the rate at which the benefit of further reduction just equals the cost of raising unemployment by the required amount above the natural rate. (Howitt, 1990, p. 104, italics added.)

THE COST-BENEFIT INFLATION TRADE-OFF: HOWITT'S RULE

Howitt (1990) argues that, although inflation is costly, once it is under way, society is better off to tolerate a little inflation than to bear the cost necessary to achieve price stability. Howitt (1990, p. 103) notes, "There are a host of reasons why the best *average rate of inflation, ignoring costs of getting there*, *is zero* . . ." (italics added). Arguing that the cost of achieving zero inflation may be substantial, however, Howitt suggests his rule can be used to determine the "optimal" inflation rate.

Howitt argues that the transitions cost "will not be negligible, so that it will be optimal to stop disinflation, even when the gain from further reduction is still positive." Therefore, Howitt (1990, p. 104) concludes, "*The optimal target is probably somewhere above zero*" (italics added).

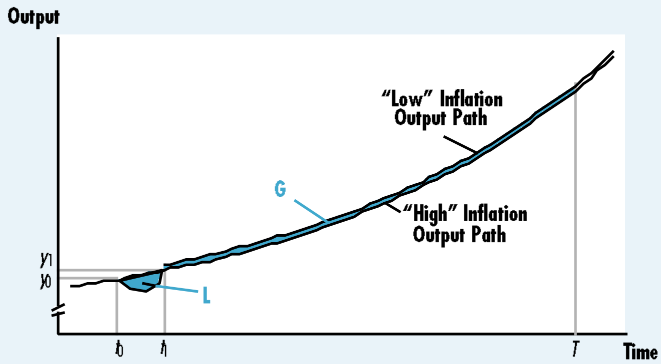
The cost-benefit trade-off Howitt alludes to is represented in Figure 1 (under the assumption that inflation reduces the level of output). At time t_0 policymakers decide to pursue a policy that will reduce the steady-state inflation rate from its current level to a lower level,

[1] See DeLong and Summers (1992) and Summers (1991). Marty and Thornton (1995) provide an analysis of several arguments asserting that the economy benefits from moderate inflation.

[2] See Howitt (1990).

Figure 1

Cost-Benefit Trade-Off



Note: Drawn to scale, with y_0 = \$5.5 trillion, $\alpha = 0.025$, $\theta = 0.100$, $t_0 = 0$, $t_1 = 6$, and $T = 60$.

perhaps zero. This change in policy causes output to fall below its steady-state path until time t_1, when the economy achieves its new lower inflation rate and a permanently higher level of output.[3] The permanent increase in the level of output is taken to be some proportion, θ, of current period output, that is, the increase is equal to θy_0. The shaded area marked *G* represents the permanent gain to output associated with achieving a lower rate of inflation for a given time horizon (*T*). The shaded area marked *L* denotes the temporary output loss associated with reducing the steady-state inflation rate.

Discounting Future Output

Howitt's Rule states that disinflation should continue until the present value of *G* equals the present value of *L*. Hence, the output levels in Figure 1 must be discounted at some discount rate, β. Figure 1 can be easily modified to show the effect of discounting. This is done in Figure 2, which shows the present (time t_0) value of the output streams in Figure 1 discounted at the rate β. Figure 2 assumes that the discount rate is larger than the growth rate of output, α, so the present value of future output always lies below current output.

Under this assumption, the present value of future output approaches zero as the time horizon approaches infinity, $T \rightarrow \infty$. This is true for both the high- and low-inflation output paths. Consequently, the present value of the gain from reducing inflation, *pvg* in Figure 2 (the shaded area between the present values of these alternative output paths), is finite. Howitt's Rule is to cease the disinflation process when *pvg* equals *pvl*. Hence, it is possible to choose a positive inflation rate as the "optimal target inflation rate."

The sizes of *pvg* and *pvl* depend on the level of current output, y_0, so it is convenient to express the present value of the gain and present value of the loss from reducing inflation as a proportion of current output. That is,

$$PVG = pvg/y_0$$
and
$$PVL = pvl/y_0,$$

respectively.

When Inflation Affects Output Growth

Figures 1 and 2 are drawn under the assumption that inflation affects only the level of output. Increasingly, economists have paid attention to the possibility that inflation may affect the growth rate of output.[4]

The behavior of output associated with reducing the steady-state inflation rate when inflation reduces output growth is shown in Figure 3, as well as the output paths for the high- and low-inflation alternatives and the present values of these paths. It is assumed that inflation reduces output growth, that is, the growth rate of output in the low-inflation state, μ, is larger than the growth rate of output in the high-inflation state, α. Figure 3 is drawn under the assumption that the rate at which future output is discounted is greater than the low-inflation output growth rate, $\beta > \mu$. In this case, the present value of future output approaches zero as $T \rightarrow \infty$ for both the low- and high-inflation states. Consequently, *PVG* is finite for any value of *T*.

[3] In Figure 1, once output has returned to its previous trend level, the time it takes to get to the new higher level is so short that it is inconsequential. If this is not the case, the estimates of the gains from reducing inflation (presented later) are overstated. Moreover, if the period to achieve the higher level of output is very long, it will be extremely difficult empirically to differentiate between level and growth rate effects of inflation because output would grow at a rate above its trend level for a long period before reaching its permanently higher level.

[4] See Barro (1995 and forthcoming); Bruno (1995); Clark (1993); Ericsson, Irons, and Tryon (1993); Fischer (1993); Dotsey and Ireland (1993); Briault (1995); Grier and Tullock (1989); Kormendi and Meguire (1985); Levine and Renelt (1991 and 1992); Logue and Sweeney (1981); Orphanides and Solow (1990); Ireland (1995); Jones and Manuelli (1995); Chari, Jones, and Manuelli (1995); King and Rebelo (1990); Golob (1993).

AN ASSESSMENT OF HOWITT'S RULE

A number of conditions must be fulfilled to apply Howitt's Rule. Some, like determining the relative sizes of *PVG* and *PVL*, are obvious. Others are subtle, yet just as important. I begin my assessment of Howitt's Rule with a discussion of the less obvious conditions upon which it depends.

Output as the Appropriate Measure of the Benefits and Costs

In Figures 1, 2, and 3, I implicitly assumed that the costs and benefits could adequately be represented by output gains and losses. This practice is common in much theoretical and virtually all applied work.[5] Two important limitations on this practice should be noted, however. One is that measured output does not conform perfectly with the theoretical measures. For example, measured output does not include home production. Hence, applications of Howitt's Rule to real-world data are problematic. This is a problem for virtually all applied work, so I do not discuss it further.

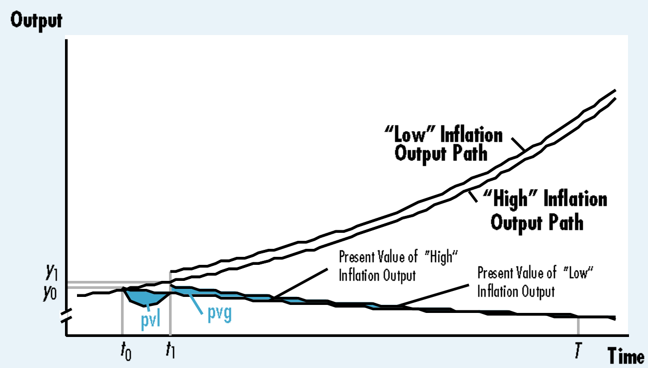
A more important limitation is that output is an imperfect measure of economic welfare. In economics, welfare is measured more abstractly, but more correctly, by the concept of utility. If there were a one-to-one correspondence between output and utility, there would be no problem, but this is not the case.

In theory or in practice, output does not include the utility individuals obtain from leisure. When more time is spent in production, less time is available for leisure. Consequently, it is possible for output to increase while economic welfare declines.[6] This consideration is particularly important for assessing the costs of inflation (and, consequently, the benefits from disinflation) because inflation is believed to distort the allocation of resources, in particular, the distribution of time between work and leisure.

The precise rationale for inflation's distorting effect varies from model to model.

Figure 2

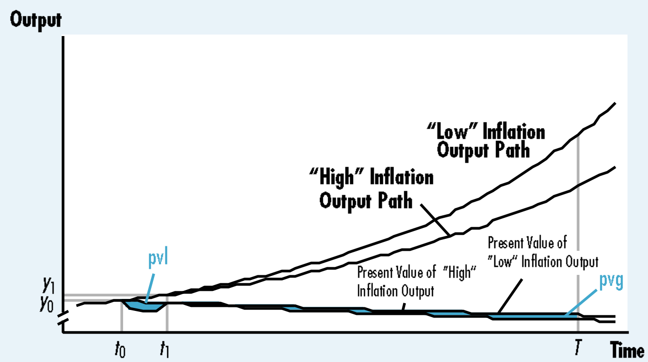
Effect of Discounting on Cost-Benefit



Note: Drawn to scale, with y_0 = \$5.5 trillion, α = 0.025, θ = 0.200, β = 0.040, t_0 = 0, t_1 = 6, and T = 60.

Figure 3

Behavior of Output



Note: Drawn to scale, with y_0 = \$5.5 trillion, α = 0.025, β = 0.040, μ = 0.030, t_0 = 0, t_1 = 6, and T = 60.

Many economists, however, believe that money is held to facilitate trade. Money is costly to hold, with the annual marginal cost per real dollar held equal to the nominal interest rate. The nominal interest rate is equal to the real interest rate (determined by real factors, such as productivity and thrift) and the expected rate of inflation. In the long run, actual and expected interest rates should be equal, regardless of how expectations are formed. Inflation is costly because the nominal interest rate becomes higher as the steady-state inflation rate rises. Consequently, so too is the cost of holding money. Indeed, the nominal

[5] See Phelps (1979), Taylor (1983), Aiyagari (1990), and Ball (1994b). Aiyagari (1990, p. 2) discusses why real output is an appropriate measure of welfare for governmental policy analyses.

[6] For example, during times of war, output typically increases; however, most people would consider themselves to be worse off.

interest rate increases approximately point for point with the steady-state inflation rate.

The higher the nominal interest rate, the fewer real money balances individuals wish to hold. In an attempt to hold less money, individuals reallocate their resources. The result may be reduced output, less leisure time, or both. Regardless of the outcome, economic welfare is reduced. For example, if individuals have less time for leisure, but output is unchanged, welfare falls. Consequently, associating the costs of inflation with output tends to bias Howitt's Rule toward a higher inflation rate than would be optimal if the welfare costs were correctly identified and measured. More generally, unless inflation is somehow beneficial, welfare must be lower at the higher inflation rate—even if the measured level of output is unchanged.[7] Hence, when output is the measure of welfare, an inflation bias is imparted to Howitt's Rule.

Discounting Output or Utility

Howitt's Rule calls for discounting the cost and benefit at a positive discount rate. Discounting is critical to Howitt's Rule. The reason is simple: Without discounting, Howitt's Rule would always choose zero as the optimal inflation target. The cost of reducing inflation (the area represented by *L* in Figure 1) is finite. This is true whether or not the cost of reducing inflation is discounted. In contrast, the benefit from reducing inflation is finite only if it is discounted. (Alternatively, the time horizon, *T*, is finite. I say more about this later.) Consequently, without discounting, Howitt's Rule will always choose zero inflation no matter how small the effect of inflation on either the level or growth rate of output.[8]

Discounting future income streams is such an integral part of financial decision making that the need to discount future income is taken for granted. Discounting utility is fundamentally different, however. It is important to distinguish it from discounting commonly used in finance.

Frequently, individuals are assumed to maximize the expected value of their lifetime utility from consumption and leisure, given their expected lifetime earnings (or resource endowments).[9] The discount rate has a specific interpretation in such analyses. The greater an individual's preference for current consumption (relative to future consumption), the larger the rate at which future income is discounted. Individuals who are impatient to consume have higher discount rates; those who are less anxious to consume have lower discount rates.[10] Consequently, the discount rate can be interpreted as an individual's rate of time preference.

Although it may be reasonable to assume that individuals discount their expected future utility relative to their current utility in making optimal plans, the role of discounting in evaluating transfers between individuals or between generations is controversial. Indeed, the mathematical economist Frank P. Ramsey (1928) considered it "ethically indefensible" for the current generation to discount the utility of future generations. At issue is whether it is appropriate for the current generation to assert that the welfare of future generations is less important than its own when making policy decisions that will affect the welfare of future generations forever.

Despite Ramsey's moral indignation, the practice of discounting utility or output is commonplace in theoretical, as well as practical, intergenerational public policy analyses. Perhaps intergenerational discounting is nonchalantly invoked because it appears to be consistent with human behavior.[11] Nevertheless, it is important to note that if you believe, as some economists do, that intergenerational discounting is morally reprehensible, you must conclude that the optimal inflation policy is price stability. If intergenerational discounting is inappropriate, so too is Howitt's Rule. In this event, the only basis for favoring a policy of moderate inflation is to argue that moderate inflation actually enhances economic welfare.

The Discount Rate

Discounting utility or output is necessary but not sufficient to cause the benefit

[7] To argue that welfare is somehow higher is to argue that inflation is beneficial. See Marty and Thornton (1995) for an analysis of four arguments that moderate inflation is beneficial.

[8] Although it is possible that Howitt's Rule would choose a negative rate of inflation in this case, I will not consider it here. Friedman (1969) suggested that the optimal monetary policy was to set the inflation rate to the negative of the real interest rate, so that the nominal interest rate would be zero. Consistent with the above analysis, a zero nominal interest rate would induce individuals to hold the maximum amount of real money balances (since the nominal interest rate cannot be negative) and, hence, derive the maximum benefit from holding money. Friedman's analysis assumes, among other things, a zero cost of maintaining the real money stock.

[9] If the time horizon is infinite, for such problems to have a solution, utility must be discounted at a positive rate. For example, in the case of continuous time, this is necessitated by the fact that the objective functional is of the general form $\int_0^\infty F(t, y, dy/dt)dt$. If this objective functional is not finite, there may be several paths that have infinite values. Although there are methods for isolating the optimal path in cases where the objective functional does not converge, these methods are difficult to apply. Therefore, it is common to express the objective functional in discounted form, that is, $\int_0^\infty F(t, y, dy/dt)e^{-\beta t}dt$. Discounting is necessary, but not sufficient to guarantee the convergence of this improper integral, however. Discounting is sufficient only if the function that is being discounted is

from reducing inflation to be finite. The discount rate must also be larger than the growth rate of output under stable prices.[12] If β is not larger than μ, the present value of the benefit of reducing inflation increases without bound as $T \rightarrow \infty$. Hence, Howitt's Rule will always choose zero as the optimal inflation rate regardless of the cost of getting there.

Given the critical nature of this requirement, it is natural to ask, Is it reasonable to assume that β is larger than μ? Unfortunately, there is no definitive answer. Nothing in economic theory ensures that this condition will hold. Likewise, nothing prevents it from holding. In simple models of intertemporal utility maximization, optimization requires that the rate of time preference equal the real interest rate. Theoretical models of this sort are usually solved by imposing a strong form of rational expectations called *certainty equivalence*. For this reason, the rate on very long-term, default-risk-free Treasury securities averaged over a long period is frequently taken as a proxy for the rate of time preference.

The *ex post* real rates on 10-year and 30-year Treasury bonds over the period of available data are presented in Table 1, along with the growth rates of real gross domestic product (GDP) for the same periods. These figures are generally consistent with the assumption that the rate of time preference exceeds the growth rate of output. In the case of the 10-year bond rate for the period 1954-94, however, the reverse is true. Moreover, the difference by which the long-term Treasury bond rate exceeds the growth rate of output is, at most, approximately 1.3 percentage points.

Hurd's (1989) attempts at estimating the rate of time preference directly have generally produced much smaller estimates of the rate of time preference. Although Hurd's estimates vary with the specification of the model and the estimation technique, they are generally too small to justify the assumption that $\beta > \mu$. Without this condition, however, Howitt's Rule will always choose price stability as being optimal.

Table 1

Real Long-Term Treasury Bond Rates and Real GDP Growth Rates

Period	10-Year Rate	30-Year Rate	GDP
1954-94	2.66	NA[1]	2.88
1978-94	3.74	3.84	2.48

[1] Data not available.

The Time Horizon

We have implicitly assumed that the time horizon, *T*, is infinite. But since it is possible to make the output gain from disinflation finite simply by assuming that *T* is finite, this possibility should be considered. The analysis of the discount rate applies equally well to the choice of time horizon. The assumption that the time horizon is finite is tantamount to stating that, beyond some point, all output or utility is discounted to zero. Hence, choosing a finite time horizon is analogous to imposing an arbitrarily high discount rate on all gains beyond some point. It is just as difficult to rationalize a finite time horizon as it is to rationalize an infinitely high discount rate. Consequently, if you believe the time horizon should be finite, you might just as well argue that the discount rate should be large.

APPLYING HOWITT'S RULE

If the conditions needed to yield a nonzero optimal inflation target when applying Howitt's Rule are accepted, it is still necessary to calculate the loss associated with the disinflation policy and the gain associated with achieving the lower inflation rate. Neither task is easy.

The Costs of Disinflation Policy

The costs of disinflation policy are associated with the stickiness of prices and wages.[13] In models where prices and wages are completely flexible, it is difficult for disinflationary mone-

bounded, that is, has a finite limit. If the function is not bounded, the objective functional will only converge if the discount rate is sufficiently large.

[10] Thus, some economists argue that relative savings rates are evidence of relative rates of time preference. Individuals or countries with high savings rates have low rates of time preference and vice versa.

[11] If concern for future generations is a strong motivation, we should expect to find a strong bequest motive in individuals' consumption and saving behavior. Hurd (1989) finds desired bequests to be all but nonexistent, however. He concludes, "apparently most bequests are accidental, the result of uncertainty about the date of death."

[12] There is also a question of whether output should be put on a per capita basis. Most theoretical work is done in per capita terms. Most applied work in this area has been done using aggregate output. Since there is no compelling reason to prefer one over the other, the practical convention is followed here, except where explicitly noted.

[13] See, for example, Aiyagari (1990).

tary policy to have adverse short-run output effects. Even with sticky prices, however, there is general agreement that, if the monetary authority's disinflation policy is credible, the cost of disinflation can be significantly reduced and perhaps eliminated.[14] For such reasons, there is no consensus about the cost of disinflation. Instead, I present estimates from a procedure for estimating the cost of disinflation frequently used by advocates of a policy of moderate inflation.[15]

The Cost of Reducing Inflation

I estimate the present value of the output loss that has accompanied significant periods of disinflation. The estimates are for the United States and are explicitly based on the work of Ball (1994a) and Howitt (1990), but the antecedent is Okun (1978).

The first step is to identify periods of significant disinflation. Ball defines a period of significant disinflation as a period when the long-run or trend inflation rate falls by one percentage point or more. The next step is to measure the output loss. Both Ball and Howitt consider the output lost during disinflation episodes to be the difference between what output would have been in the absence of the disinflation and actual output. The approaches differ only in the way they calculate what output would have been.

Ball does this by connecting output when the trend inflation rate is at its peak with output four quarters after the trend inflation rate reaches its trough. He does this on the assumption that the effects of disinflation on output continue for a while after disinflation ends.

Howitt uses the unemployment rate to gauge output loss. For Howitt, the disinflation period ends when the unemployment rate returns to the level it was when the disinflation episode began. The output loss is calculated by using Okun's Law. Howitt assumes that each percentage point rise in the unemployment rate for a year costs society the equivalent of 2 percent of real GDP.[16]

A significant difficulty with this general approach to measuring the output loss of disinflationary policy is that all output losses during these periods are attributed to disinflation. It is certainly possible—indeed, many would claim likely—that some part of the output decline is cyclical, that is, the economy's dynamic response to past shocks—monetary or real.

Perhaps more important, this approach assumes that all disinflation is attributable to monetary policy actions and not to the simultaneous occurrence of other shocks. This is particularly troubling since no attempt is made to connect the disinflation experience directly to policy actions. Consequently, there is no way to determine the extent to which the output loss during these disinflation experiences may be a result of other factors.[17] Because of these difficulties, it is safe to say that this approach overstates the output loss associated with disinflation to a greater or a lesser degree.[18]

The Benefit of Lower Inflation

Estimating the benefit of a lower inflation rate is no easier than estimating the cost of disinflation policy. Part of the difficulty stems from the fact that money affects economic welfare in many and complex ways. No general model of the welfare benefit of money and, hence, no model of the welfare cost of inflation, exists. Rather, a large body of literature outlines the potential benefits of price stability.[19]

Nevertheless, an approach to estimating the welfare cost of inflation, suggested by Bailey (1956) and Friedman (1969), is frequently used. This method assumes that the benefit of money accrues solely to those who hold it. Consequently, Bailey and Friedman assume there is no external benefit from money's use. Under this assumption, the benefits from holding money can be represented by what economists refer to as *consumer's surplus* or, more esoterically, *Harberger triangles.* The idea is simply that money is barren in that it pays no explicit interest, so the value of money's services, that is, the welfare

[14] See, for example, Taylor (1981 and 1983).

[15] See, for example, Fischer (1984) and Blinder (1989).

[16] Some estimates of Okun's coefficient put real GDP at 2.5 percent. Based on a study by Fortin and Bernier (1988), however, Howitt uses a lower figure (2 percent) to take account for the effect on leisure.

[17] For a more detailed discussion of these and other problems with Ball's approach, see Cecchetti (1994).

[18] A methodology that mitigates against these problems is vector autoregession (VAR). Unfortunately VAR results are very sensitive to the degree of difference in the model's specification (see Cecchetti, 1994, for an illustration of this point) and to the variables that are included in the specification, including the variable used to identify monetary policy shocks. VARs are subject to other criticisms as well (see, for example, Cecchetti, 1995, and Zellner, 1992).

[19] See, for example, Aiyagari (1990), Briault (1995), Lucas (1994), Tatom (1976), Garfinkel (1989), Fischer (1981), and Howitt (1990).

benefit of money, is equal to the area under the demand curve for real money.[20] The welfare cost of inflation is the amount by which this area shrinks when the inflation rate, and hence, the nominal interest rate, rises. Such an approach has been used often.[21] Most estimates of the cost of inflation range from about 0.0001 to 0.0005 of GDP per percentage point reduction in the long-run inflation rate.

Lucas (1994) has recently presented convincing evidence that welfare cost of inflation estimates based on Bailey's and Friedman's approach may be too low. Arguing that the data are better represented by the double-log specification of money demand, rather than the semi-elasticity specification commonly used and first suggested by Cagan (1956), Lucas estimates the annual welfare cost of inflation to be much higher than previous studies. He estimates the benefits from reducing inflation to be about 0.002 of GDP per percentage point reduction in the inflation rate.[22]

Lucas's estimates understate the welfare cost of inflation if there are social benefits from money in addition to money's private benefits. This possibility has been suggested by Brunner and Meltzer (1971) and Laidler (1990). Elsewhere I explicitly identified the social benefits from money's use.[23] In addition, Briault (1995) and I assert that if the social benefits of money are significant, disruptions of the monetary system because of inflation are potentially much more serious and costly than suggested by estimates based solely on money's private benefits. If social benefits from money exist, the usual estimates of the cost of inflation understate the welfare cost of inflation.

Moreover, as Briault (1995) points out, arguments for why inflation and inflation uncertainty lead to a misallocation of resources suggest that inflation may not only affect the level of output, but may affect the growth rate of output as well. Indeed, there has been increasing interest—both theoretical and empirical—in the possibility that inflation reduces output growth.

Comparing the Costs and Benefits

The estimates of the cost of disinflation, *PVL*, and the benefit, *PVG*, are for the three disinflation episodes Ball (1994a) identifies. They are 1969:4-1971:4, 1974:1-1976:4, and 1980:1-1983:4. The average rate of output growth for the United States was 3.01 percent from 1960:1 to 1995:3. Consequently, the estimates presented in Tables 2, 3, and 4 are based on the assumption that the growth rate in the "high" inflation state, α, is 3 percent (0.03). The estimates of the effect of inflation on the level of output as a proportion of current period income per percentage point reduction in the inflation rate, θ, reflect estimates obtained in the literature and range from $\theta = 0.0001$ to 0.002. The estimates of *PVG* are calculated using the formula presented in Appendix A. To make the estimated cost and benefit as comparable as possible, *PVG* is based on the length of each disinflation episode and the corresponding reduction in the inflation rate identified by Ball (1994a).

A comparison of *PVL* and *PVG* in Table 2 shows that *PVL* > *PVG* only if the effect of inflation on the level of output is assumed to be relatively small and the discount rate is taken to be relatively large. Indeed, the importance of the discount rate in deciding that moderate inflation is optimal is apparent. Table 2 assumes there is no growth rate effect, $\mu = \alpha$. When the discount rate is only slightly larger than the growth rate of output, *PVG* exceeds *PVL* even for an extremely small effect of inflation on output. For example, if the discount rate is assumed to be about half a percentage point larger than the growth rate of output, *PVG* > *PVL* for each of the disinflation episodes that Ball has identified, even if the percentage point reduction in the level of output is only 0.0001. When the likelihood that estimates of *PVL* overstate the welfare cost and estimates of *PVG* understate the welfare benefits of disinflation policy are taken into consideration, these estimates suggest that a policy of moderate inflation is optimal only if the effects of inflation on

[20] Failure to understand the nature of the welfare costs of inflation has led Aiyagari (1990) to suggest that they could be reduced simply by allowing "more forms of money that are used in transactions to earn market rates of interest." This recommendation fails to recognize that an asset which provides multiple services has a return that reflects the marginal value of each of the services rendered. This insight is the basis for the Divisia monetary aggregates, or monetary services indexes, pioneered by Barnett (1980). Inflation would cause the rates on assets that provided relatively more monetary services to rise less than those which provide fewer monetary services. As a result, inflation would continue to induce individuals to reduce their use of monetary services.

[21] See, for example, Cooley and Hansen (1989), Fischer (1981), and Lucas (1981 and 1994).

[22] Lucas (1994) estimates that at a 10 percent nominal interest rate, the output loss per annum is equal to 1.3 percent of current output. Assuming the real interest rate is 3 percent, a nominal interest rate of 10 percent is equivalent to a steady-state inflation rate of 7 percent. The proportionate decline in output per percentage point drop in the inflation rate is therefore equal to 0.0018 [0.0130/7] ≈ 0.002.

[23] See Thornton (1995).

Table 2

Costs and Benefits of Disinflation: Ball's Approach

θ	β	PVL			PVG[1]		
		1969:4-1971:4	1974:1-1976:4	1980:1-1983:4	1969:4-1971:4	1974:1-1976:4	1980:1-1983:4
0.0001	0.0305	0.0118	0.0477	0.0973	0.4276	0.7989	1.7627
0.0001	0.0350	0.0118	0.0474	0.0964	0.0424	0.0789	0.1733
0.0001	0.0400	0.0117	0.0471	0.0955	0.0210	0.0389	0.0851
0.0001	0.0450	0.0117	0.0469	0.0945	0.0138	0.0256	0.0556
0.0002	0.0305	0.0118	0.0478	0.0973	0.8551	1.5978	3.5254
0.0002	0.0350	0.0118	0.0474	0.0964	0.0847	0.1578	0.3466
0.0002	0.0400	0.0117	0.0471	0.0955	0.0420	0.0778	0.1701
0.0002	0.0450	0.0117	0.0469	0.0945	0.0277	0.0512	0.1113
0.0010	0.0305	0.0118	0.0478	0.0973	4.2757	7.9890	17.6269
0.0010	0.0350	0.0118	0.0474	0.0964	0.4237	0.7891	1.7332
0.0010	0.0400	0.0117	0.0471	0.0955	0.2098	0.3891	0.8505
0.0010	0.0450	0.0117	0.0469	0.0945	0.1385	0.2559	0.5565
0.0020	0.0305	0.0118	0.0478	0.0973	8.5514	15.9780	35.2538
0.0020	0.0350	0.0118	0.0474	0.0964	0.8475	1.5782	3.4664
0.0020	0.0400	0.0117	0.0471	0.0955	0.4195	0.7783	1.7010
0.0020	0.0450	0.0117	0.0469	0.0945	0.2769	0.5118	1.1129

[1] The estimates are based on the duration of the disinflation episode and the reduction in the inflation rates for each episode as reported by Ball (1994a). The length of the disinflation episode, (that is, $t_1–t_0$, are 2.00, 2.75, and 3.75, respectively). The decline in the inflation rates are 2.14, 4.00, and 8.83 percent, respectively.

output are extremely small or the discount rate is large.

Estimates based on Howitt's procedure are reported in Table 3. These estimates are only presented for the third disinflation episode. The reason is that Howitt's procedure requires the unemployment rate to return to the level at the beginning of the disinflation episode. The unemployment rate was 3.6 percent in 1969:4 and 5.1 percent in 1974:1. The unemployment rate has yet to return to either of these levels, so by Howitt's criterion, these disinflation episodes have yet to end. Of course, the problem is that the unemployment rate is affected by factors other than disinflation. The secular effects on the unemployment rate clearly have been dominant. Because of this, the unemployment rate has yet to return to levels reached in the late 1960s. This fact serves to underscore a problem with both Howitt's and Ball's procedures. Namely, the methods assume that all the decline in employment and output is because of disinflation and not because of other factors.

A comparison of *PVL* and *PVG* in Table 3 yields conclusions similar to those reached in Table 2. The *PVLs* in Table 3 are considerably larger than those of the corresponding disinflation in Table 2 primarily because the length of the disinflation episode was much longer 7.25 years (compared with 3.75 years in Table 2), and output was assumed to be below its trend level during the entire period.

If Inflation Affects Output Growth

The conclusion reached by applying Howitt's Rule changes dramatically if inflation affects the growth rate of output. Estimates of *PVG* under the assumption that inflation reduces the growth rate of output are presented in Table 4. Since the discount rate, β, must be larger than the low-inflation output growth rate, μ, the range of values considered for β is smaller than that considered in Tables 2 and 3. Also, since estimates of *PVG* are not greatly affected by changes in the length of the disinflation episode, the values reported are for a disinflation episode of 3.75 years.

These estimates show that the gain from reducing inflation is very large, even if inflation reduces output growth by as little as one-thousandth of 1 percent (0.00001). Other things being equal, Howitt's Rule moves decidedly in the direction of choosing a lower inflation target if inflation affects the growth rate of output.

The level and growth rate effects are additive. Hence, if inflation reduces both the level and growth rate of output, even by relatively small amounts, the *PVG* from reducing inflation could be very large, depending on the discount rate used. For example, assume that the inflation rate is cut from its current rate of about 3 percent to zero and this action raises the level of output by 0.0003 (a 0.0001 per percentage point drop in the inflation rate) and increases output growth by 0.00001. Assuming disinflation takes 3.75 years to be achieved and that the discount rate is a fairly large 4 percent, the *PVG* from achieving price stability would equal 0.125 (the sum of a level effect of 0.0289 and a growth rate effect of 0.0963). If we use the estimates of *PVL* per percentage point reduction in inflation using Howitt's method from Table 3, the *PVL* would only be 0.0772. The difference is 0.0478. Annualized output in the second quarter of 1995 was approximately $5.5 trillion. Consequently, the net present value of the gain from reducing inflation from its

Table 3

Costs and Benefits of Inflation: Howitt's Approach[1]

θ	β	PVL[2]	PVG[3]
0.0001	0.0305	0.2341	1.7596
0.0001	0.0350	0.2309	0.1703
0.0001	0.0400	0.2273	0.0821
0.0001	0.0450	0.2238	0.0528
0.0002	0.0305	0.2341	3.5192
0.0002	0.0350	0.2309	0.3406
0.0002	0.0400	0.2273	0.1642
0.0002	0.0450	0.2238	0.1056
0.0010	0.0305	0.2341	17.5961
0.0010	0.0350	0.2309	1.7031
0.0010	0.0400	0.2273	0.8212
0.0010	0.0450	0.2238	0.5280
0.0020	0.0305	0.2341	35.1922
0.0020	0.0350	0.2309	3.4063
0.0020	0.0400	0.2273	1.6425
0.0020	0.0450	0.2238	1.0560

1 The disinflation period is 1980:1–1987:2.

2 The PVL does not change with θ.

3 The length of the disinflation episode was 7.25; however, the decline in the steady-state inflation rate was 8.83 percent (as assumed by Ball, 1994a).

Table 4

PVG When Inflation Affects Output Growth

μ	β	PVG[1]
0.03001	0.035	0.3933
0.03001	0.040	0.0963
0.03001	0.045	0.0420
0.03010	0.035	4.0051
0.03010	0.040	0.9723
0.03010	0.045	0.4223
0.03100	0.035	49.0644
0.03100	0.040	10.6955
0.03100	0.045	4.4950

1 These estimates are based on the assumption that $\alpha = 0.03$ and $t_1 - t_0 = 3.75$.

current level of about 3 percent to zero would be about $0.27 trillion, or about $1,000 per person.

EMPIRICAL EVIDENCE ON INFLATION AND GROWTH

The above analysis shows that an important factor in determining whether the benefit of price stability exceeds the cost is whether inflation affects output growth. If it does, it is very difficult to argue that a policy of maintaining moderate inflation is socially optimal by appealing to Howitt's Rule. Hence, it is not surprising that a relatively large effort has been devoted to this question.[24]

Although there is evidence that inflation reduces output growth, it appears to be the case only at relatively high inflation rates. It is becoming increasingly clear that the effect of inflation on growth is statistically significant only at relatively high rates of inflation.[25] For example, Barro finds a statistically significant effect only when inflation is 15 percent or higher. Similarly, Bruno and Easterly find a statistically significant inflation effect only when the inflation rate exceeds 40 percent for two or more years.[26]

The evidence is controversial; however, the important point is that this evidence cannot support or reject the idea that moderate inflation affects the growth rate of output. The reasons are twofold.

First, the data contain relatively few observations where the inflation rate is in the low to moderate range and even fewer observations where inflation is zero. Extrapolating statistical results outside the region for which the data are relevant is risky. Consequently, using the results of such data to infer the effect of moderate inflation on growth is tenuous at best. This is especially true if the effect of inflation on growth is believed to be nonlinear, that is, the effect of inflation on growth is disproportionately larger or smaller at lower rates of inflation.

It is commonly believed that the effects of inflation will be proportionately smaller at lower rates of inflation than at higher rates.[27] Recently, however, Lucas (1994) presented evidence that the effect of inflation on output is proportionately larger at lower rates of inflation. It is safe to say that any statement about the exact nature of the nonlinearity at low rates of inflation is conjectural. There is some evidence, however, that the effects at high rates of inflation are bounded. That is, beyond some point, higher inflation rates have no further effect on either the level or growth rate of output. If the effects are bounded at high inflation rates and one believes that the effects of inflation are proportionately smaller at lower inflation rates, then there must be an inflection point—the effects of inflation first increase at an increasing rate and then increase at a decreasing rate.

Second, because an economically relevant growth rate effect can be very small, it may be difficult to find an economically relevant growth rate effect that is also statistically significant. For example, Barro (1995) investigated the nonlinearity of the inflation effect by partitioning the median inflation rates into those up to 15 percent, 15 percent to 40 percent, and above 40 percent. His estimates of the effect of inflation on growth for the three partitions were –0.00016, –0.00037, and –0.00023, respectively. Only the last two coefficients were significantly different from zero. My previous analysis showed that the estimate for the first partition (–0.00016) is large enough to be economically relevant. Its absolute value, however, is less than one-half its estimated standard error of 0.00035. Given the amount of variability in data like these, it may be very difficult to obtain an economically relevant, statistically significant estimate of inflation's effect on output growth.

Time-Series Evidence

Recently, Ericsson, Irons, and Tryon (1993) criticized cross-sectional studies of the effects of inflation on growth, arguing that the results are very sensitive to a few very high inflation countries and that time-averaged data can give misleading

[24] See, for example, Wallich (1969); Barro (1995 and forthcoming); Bruno (1995); Jones and Manuelli (1995); Mankiw, Romer, and Weil (1992); Logue and Sweeney (1981); Kormendi and Meguire (1985); Grier and Tullock (1989); Bruno and Easterly (forthcoming); Levine and Renelt (1992); Levine, Renelt, and Zervos (1993); and Fischer (1993). All these empirical studies use cross-sectional or panel data.

[25] See Bruno and Easterly (forthcoming) and Barro (1995 and forthcoming).

[26] Although these results are broadly consistent with those of Levine and Renelt (1992); Levine, Renelt, and Zervos (1993); and Fischer (1993), who showed that the effect of inflation on growth is due to the presence of a few high inflation countries, Barro and Bruno and Easterly present evidence that their results are not sensitive to a few high inflation countries.

[27] See, for example, Howitt (1990) and DeLong and Summers (1992).

results about the causal relationship between inflation and output growth. Instead they use time-series data for the United States for the period 1953-90. Using a statistical procedure called *co-integration analysis*, Ericsson, Irons, and Tryon find evidence that inflation does not affect the long-run rate of output growth.[28] Instead, they find that the level of output is positively related to the long-run inflation rate. Their result is circumspect, however. Taken literally, it implies that not only is a little inflation good for the economy, but that a lot of inflation is even better.

More important, their finding depends critically on their assumption about the time-series properties of the price level. It can be shown that different assumptions about the time-series properties of the price level yield considerably different results. Appendix B gives the details. In particular, it can be shown that long-run inflation and output growth are negatively related. The estimated long-run relationship between per capita output and the price level is:

$$(1) \qquad X_t = -0.0695P_t + 0.0217T,$$

where X is the natural log of real per capita output, P is the natural log of the implicit price deflator, and T is a deterministic time trend. Equation 1 suggests that the long-run growth rate of output is equal to 0.0217 less 0.0695 times the inflation rate. The inflation rate over this period was 4.37 percent. Using this figure, per capita output growth is estimated to be 1.87 percent, somewhat larger than the actual growth rate of per capita output of 1.68 percent over this period.

This analysis is very simple, including only output and the price level. Moreover, co-integration analysis cannot indicate whether the negative long-run relationship between the price level and output is a result of the effects of inflation on output growth, the effect of output growth on inflation, or the effect of some other variable(s) on both output and prices. Nevertheless, it is interesting to note that a very reasonable change in the specification of Ericsson, Irons, and Tryon's model dramatically alters their result. The data suggest that, for the United States (over this period), inflation could have had a detrimental effect on output growth. Hence, the time-series evidence is at least consistent with the proposition that inflation reduces the growth rate of output.

IS PRICE STABILITY THE BEST POLICY?

Most policymakers and economists agree that very high rates of inflation are detrimental. So much so, that there is broad agreement that the gain from reducing inflation from "high" levels to "moderate" levels warrants the temporary loss of employment and output that many believe is necessary to achieve this goal. Despite the fact that most economists believe inflation is detrimental to economic welfare, many believe that, once under way, it is better to live with some moderate inflation than to bear the costs necessary to achieve stable prices. Peter Howitt (1990) succinctly states the proposition that the cost of going from moderate to zero inflation does not warrant the benefits from price stability. Howitt's Rule states that a policy of disinflation should be continued until an inflation rate is reached where the present value of the costs of further disinflation are just equal to the present value of the gains from a further reduction in the inflation rate.

Does economic analysis support or refute the idea that it is desirable to establish a policy of price stability? Unfortunately, there is no definitive answer to this question. The analysis presented here, however, suggests that a popular and (for many) a compelling economic argument for a policy of positive inflation requires several controversial conditions. If one believes that the welfare gains of future generations should not be discounted or that the discount rate is low, it is difficult to argue for a policy of moderate inflation.

Moreover, this argument is weakened significantly if inflation reduces the growth

[28] See Dickey, Jansen, and Thornton (1991) for a discussion of co-integration and unit roots.

rate and not simply the level of output. The economic case for a policy of price stability based on the effects of inflation on output growth may come down to whether one finds compelling various theoretical arguments of why even moderate inflation should affect output growth.[29] If you believe it is likely that inflation reduces the growth rate of output, you are much more likely to believe that price stability is the best monetary policy.

[29] Marty and Thornton (1995) have argued that monetary policies motivated by concerns for transitional unemployment are likely to lead to an inflationary bias that makes targeting any moderate inflation rate difficult.

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Appendix A

THE BENEFITS OF PRICE STABILITY

Let the level of steady-state output at each point in time at the current "high" inflation rate be given by:

$$\text{(A1)} \qquad y_t = e^{\alpha t},$$

where α is the growth rate of real output given the high inflation state.

When calculating the effect of a permanent reduction in the inflation rate, assume the monetary authority implements the deflationary policy at time t_0, when the level of output is y_0, and that the new lower inflation policy is achieved at time t_1. We consider the possibility that reduced inflation raises the level of output and the possibility that it permanently raises the growth rate of output, once the new lower inflation rate is achieved. For the level effect, output is unchanged until time t_1 but is higher by an amount δ for all $t > t_1$. It is convenient to express δ as a proportion of output at the time that the disinflation policy is implemented, that is, $\delta = \theta y_0$. For the growth rate effect, output is assumed to grow at the rate α until time t_1 and at the rate μ for all $t > t_1$, where $\mu > \alpha$. Under these assumptions, the low-inflation output path is given by:

$$\text{(A2)} \quad y_t^* = \begin{bmatrix} y_0 e^{\alpha t}, & \text{for} \quad t_0 \le t \le t_1 \\ [y_0 e^{\alpha(t_1 - t_0)} + \delta] e^{\mu t}, & \text{for} \quad t > t_1 \end{bmatrix}.$$

If output is discounted at the rate $\beta > \mu$ for all t greater than t_0, the difference between low- and high-inflation output, yields the present value of the gain,

$$\text{(A3)} \quad pvg = \int_0^T e^{\beta t}(y_t^* - y_t)dt = y_0 e^{(\alpha-\beta)(t_1-t_0)} \left[\int_{t_1}^T e^{(\mu-\beta)t}dt - \int_{t_1}^T e^{(\alpha-\beta)t}dt\right] + \delta \int_{t_1}^T e^{(\mu-\beta)t}dt.$$

Dividing through by y_0 and performing the integration, yields:

$$\text{(A4)} \quad PVG = e^{(\alpha-\beta)(t_1-t_0)} \left[\frac{e^{(\mu-\beta)T}}{(\mu-\beta)} - \frac{e^{(\alpha-\beta)T}}{(\alpha-\beta)} + \frac{(\mu-\beta)e^{(\alpha-\beta)t_1} - (\alpha-\beta)e^{(\mu-\beta)t_1}}{(\mu-\beta)(\alpha-\beta)}\right] + \theta\left[\frac{e^{(\mu-\beta)T} - e^{(\mu-\beta)t_1}}{\mu-\beta}\right]$$

Because we have no interest in the case where the time horizon is finite, take the limit of the above expression as T goes to infinity. This yields:

$$\text{(A5)} \quad \lim_{T\to\infty} PVG = e^{(\alpha-\beta)t_1}\left[\frac{(\mu-\beta)e^{(\alpha-\beta)t_1} - (\alpha-\beta)e^{(\mu-\beta)t_1}}{(\mu-\beta)(\alpha-\beta)}\right] - \theta\left[\frac{e^{(\mu-\beta)t_1}}{\mu-\beta}\right]$$

The first term on the right side of equation A5 is the effect of a change in the growth rate. Hence, it is zero if $\mu = \alpha$ (that is, there is no growth-rate effect). The second term on the right side is the effect of a shift in the level of output. This term is zero only if $\theta = 0$. The first term is an increasing function of $\mu - \alpha$ and $\mu - \beta$ (recall that $\mu - \beta$ is strictly negative under the assumptions stated) and a decreasing function of t_1. The second term is an increasing function of θ and $\mu - \beta$ and a decreasing function of t_1. The importance of the size of the discount rate relative to the growth rate of output is clear from this expression. Both terms on the right side of equation A5 approach +∞ as β approaches μ. If β is only slightly larger than μ, *PVG* gets very large.

Appendix B

TIME-SERIES EVIDENCE ON INFLATION'S EFFECTS ON GROWTH

Ericsson, Irons, and Tryon (1993) recently criticized cross-sectional studies of inflation's effects on growth. They argue that the results are very sensitive to a few very high inflation countries and argue that time-averaged data can give misleading results about the causal relationship between inflation and output growth. Moreover, using tests for unit roots and co-integration, they find evidence that generally does not support the theory that inflation has an effect on growth. Indeed, their results for the United States suggest that, in the long run, inflation raises the level of output without affecting the growth rate of output. Taken literally, Ericsson, Irons, and Tryon's results suggest that not only is a little inflation good for the economy, but that a lot of inflation is even better.

TESTS FOR A UNIT ROOT

Their analysis, however, depends critically on tests for a unit root, supporting the notion that the real per capita output, *X*, is integrated of order 1, *I*(1), while the price level, *P*, is *I*(2). These tests have difficulty rejecting the null hypothesis of a unit root when the estimated root is close to, but perhaps different from, unity, as illustrated in Table B1. It shows the results from Augmented Dickey-Fuller (ADF) tests of a unit root for *X*, Δ*X*, *P*, and Δ*P* and the corresponding estimates of the root, λ, using annual data for the period 1953-90. As did Ericsson, Irons, and Tryon, I included a constant and a deterministic time trend in each case and used ADF tests with two lags.

The null hypothesis of a unit root is not rejected even when the estimated root is quite different from zero, as shown in Table B1. This is true for *X* and Δ*P*, where the estimated roots are 0.7006 and 0.8262, respectively. This is because of the well-known fact that such tests lack the ability to reject the null hypothesis when it is false.[1]

Table B1

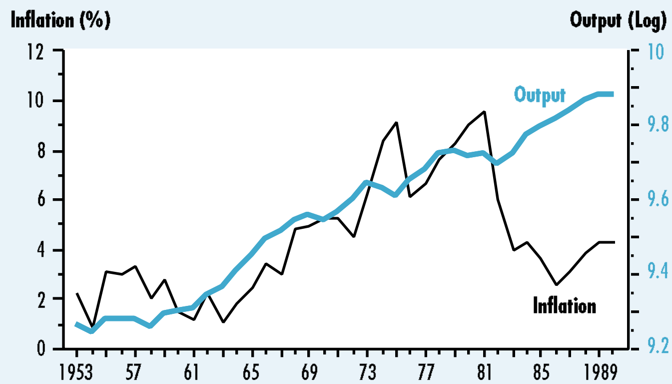
Results of Augmented Dickey-Fuller, Unit Root Tests

Variable	Estimate Root	ADFt-Statistic	Critical Value, 5% Level
X	0.7006	–2.452	–3.461
ΔX	–0.0548	–3.680[1]	
P	0.9508	–2.565	
ΔP	0.8262	–1.496	

[1] Indicates the null hypothesis of a unit root is rejected at the 5 percent significance level.

Figure B1

Per Capita Output and Inflation



NONSTATIONARY VARIABLES

How this problem affects the determination of whether *X* or Δ*P* is nonstationary is illustrated in Figure B1, which shows *X* and Δ*P* over the sample period. The ADF test cannot reject the null hypothesis of a unit root for both *X* and Δ*P*. In the case of *X*, Figure B1 makes it clear that this variable is nonstationary in the sense that it rises fairly steadily. Hence, both the test and an inspection of the data suggest that the series is nonstationary.[2]

From a visual inspection of Δ*P*, it is much less clear that the inflation rate is nonstationary in the sense that it will wander off indefinitely. The problem is

[1] This is referred to as the power of the test. Hence, the Dickey-Fuller test is said to lack power.

[2] Some care must be exercised here, however, because the data could be stationary around a deterministic time trend. Because our test included a constant and a trend variable, we are more inclined to accept the test results in this case.

Table B2

Estimate Eigenvalues and Co-Integrating Vectors for Two Specifications of Equation B2

Eigenvalue	X	ΔP	Trend	Eigenvalue	X	P	Trend
0.4399	–1.0000	1.0581	0.0179	0.4502	–1.0000	–0.0695	0.0217
0.0900	–0.3757	–1.0000	0.0075	0.1851	–0.2424	–1.0000	0.0615

that there is a lot of persistence in the inflation rate. When the inflation rate increases, it continues to increase for a period before it begins to fall. When falling, it continues to fall for a period before it begins to rise. It is this persistence in the inflation rate that accounts for the large estimated root. Consequently, the test results that suggest that ΔP is nonstationary must be viewed with some skepticism.[3]

CO-INTEGRATION ANALYSIS

Given the uncertainty of the order of integration of *P*, it is important to investigate whether Ericsson, Irons, and Tryon's results are sensitive to their claim that *P* is $I(2)$. To investigate this, I performed their co-integration analysis with their specifications and using both *X* and ΔP and *X* and *P*.

The method of testing for co-integration is that of Johansen (1988). This approach starts from a very general vector autoregression representation of the form

$$Y_t = \beta(L)Y_{t-1} + \varepsilon_t, \qquad \text{(B1)}$$

where *Y* is a two-by-one vector, $\beta(L)$ is a polynomial in the lag operator, *L*, that is, $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_k L^k$ and $Lz_t = z_{t-1}$. Equation B1 can be rewritten as,

$$\Delta Y_t = \Gamma(L)\Delta Y_{t-1} + \Pi Y_{t-k} + \varepsilon_t. \qquad \text{(B2)}$$

If the elements of the *Y* vector, y_1 and y_2, are $I(1)$, they are co-integrated, that is, there exists a linear combination of y_1 and y_2 that is $I(0)$, if the rank of the two-by-two matrix Π is 1.[4] The Johansen test for co-integration is a test of the rank of Π.

Given the uncertainty about whether *P* is $I(1)$ or $I(2)$, tests for co-integration were performed for two specifications of Y', namely, $Y' = (X\ \Delta P)$ and $Y' = (X\ P)$. In both cases, a constant term and a deterministic time trend were included and *k* was set at 2. The results are summarized in Table B2. Regardless of the specification of *Y*, there is evidence of a single co-integrating vector.[5]

In the case where $Y' = (X\ \Delta P)$, the estimated normalized co-integrating vector was $X = 1.0581\Delta P + 0.0179T$, where *T* denotes the deterministic time trend. This estimate is very close to that reported by Ericsson, Irons, and Tryon. They interpret the estimated co-integrating vector as the long-run relationship between output and inflation. The implication of this co-integrating vector—that output could be increased simply by increasing the inflation rate—is preposterous.

In contrast, the estimate of the co-integrating vector when $Y' = (X\ P)$ is $X = -0.0695P + 0.0217T$. This estimate suggests that the level of output falls as the price level rises. Because of constant coefficients and the log-linear specification, this result implies that the rate of output growth falls as the inflation rate rises. The estimate suggests that a reduction of the inflation rate from 3 percent to zero would increase the per capita output growth rate by 0.00209, an amount that, though small, is economically significant.

[3] The same conclusion applies to accepting the hypothesis that X is nonstationary as opposed to having a deterministic trend. The null hypothesis of a unit root is rejected because there is considerable persistence in the swings of X about its trend rather than getting farther and farther from its estimated trend. Plotting X about its trend over this period makes this point clear.

[4] Note that the rank of Π could only be two if the elements of Y are I(0). If the rank of Π is zero, there would be no linear combination of the elements of Y that is stationary [that is, I(0)]. See Dickey, Jansen, and Thornton (1991) for a more detailed discussion of co-integration and the Johansen and other tests for co-integration.

[5] The level specification [$Y' = (X\ P)$] is preferable on theoretical grounds because economic theory suggests that there should be a contemporaneous relationship between X and P, that is, output and the price level are determined simultaneously. Indeed, equation B1 can be viewed as the reduced form of a structural model of the form, $AY_t = B(L)Y_{t-1} + v_t$, so that, $\beta(L) = A^{-1}B(L)$ and $e_t = A^{-1}v_t$. The problem is that it is difficult to obtain estimates of the structural parameters in A without imposing some rather severe restrictions. See Keating (1992) for a discussion of these restrictions.