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The Coupling of the Core to the Precession of the Earth

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Summary

The core follows the precession of the mantle by virtue of coupling to it. A simple model is presented which allows quantitative consideration of a superposition of both inertial and dissipative (electromagnetic) coupling torques. With the preferred value of the dissipative coupling coefficient, the dissipative mechanism accounts for only 2 or 3 per cent of the precessional coupling torque, but the dissipation amounts to about 3×10^{10} Watts, which may suffice for a geomagnetic dynamo driven by precession. The dissipation itself is only weakly dependent upon the coupling coefficient and no assumption can lead to dissipation exceeding 10^{11} W. This is much smaller than the loss of rotational energy by tidal friction; also it is hardly a significant contribution to the total core-to-mantle heat flux if we suppose that an adiabatic temperature gradient is maintained in the core, thus supporting the contention that the core contains potassium with a radioactive heat generation of order 10^{13} W. Motion of the core is found to contribute to the semi-annual terms of precession and nutation.

Introduction

The Earth's rotational axis is inclined to the pole of the ecliptic (normal to the orbital plane) by $23\frac{1}{2}°$ and precesses about it with a period of 25 800 years. The torques which cause this precession are due to the gravitational fields of the Moon and Sun acting on the Earth's equatorial bulge and are well understood. The response of the fluid core has been less obvious. By virtue of its higher density it is less elliptical than the Earth as a whole and so is subjected to luni-solar precessional torques which are insufficient to maintain precession at the observed rate for the whole Earth. As Bullard (1949) noted, the effects of allowing the core to lag the mantle by more than a very modest angle are so violent as to be inadmissible, so that precession of the core must be maintained by coupling to the mantle. The problem has been to identify the mechanism or mechanisms of this coupling.

The effectiveness of inertial coupling has been recognized for many years and is associated particularly with the name H. Poincaré. An illuminating review is given by Toomre (1966), who considered a simple mechanical analogue, a particle or small marble sliding or rolling without friction around a slightly oblate spheroidal cavity. If the initial trajectory follows the equator and the axis of the cavity is then turned through a small angle, the particle will continue initially to orbit in the original plane now inclined to the equator of the cavity. Its path is then slightly elliptical and the effect of the purely normal force of the cavity wall is to make the orbit of the particle precess in a retrograde sense at an angular rate equal to the cavity ellipticity times the orbital angular frequency. The significance of the precessional nature of this conserva-

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tive, inertial torque appears not to have received due emphasis. In Section 3, Toomre's model is adopted directly for a simple quantitative picture of the inertial coupling of the Earth's core and mantle.

Alternatively, dissipative torques have also been examined many times. Toomre (1966) found them all to be inadequate, which was his essential reason for favouring inertial coupling. Viscous or turbulent coupling can readily be dismissed for any reasonable value of core viscosity. The estimated viscosity, of order 0·1 poise (Gans 1972), is about 10^7 times too small. A more serious contender is magnetic coupling, which Rochester (1960) and Roden (1963) estimated to be adequate to couple the core and mantle axial rotational rates; the coupling time constant is estimated from length-of-day fluctuations (Munk & MacDonald 1960) to be about 1·6 y (Stacey 1969). However magnetic coupling appears inadequate to excite the Chandler wobble by coupling to irregularities in core motion (Rochester & Smylie 1965) (this subject is reviewed by Rochester 1968). Malkus (1963, 1968) postulated that a precessional torque of magnetic origin exerted on the mantle had the effect of stirring the core and constituted a plausible driving mechanism for the geomagnetic dynamo. This idea has an obvious attractiveness, but Toomre's (1966) argument appears to stand in the way. What is lacking is a discussion of the effects of superimposed inertial and dissipative torques operating simultaneously. The present paper suggests a simple approach to that problem.

Dissipative coupling—an energy argument

If the precessional coupling of the core to the mantle is entirely dissipative, i.e. due to electromagnetic or viscous torques, and the core is assumed to rotate as a rigid body, then we can assign a lag angle ε_d, being the angle between the instantaneous rotational axes of the mantle and core, as in Fig. 1(*a*). Using Malkus's (1968) value for the dynamical ellipticity of the core, $2{\cdot}45\times10^{-3}$, the luni–solar torques account for 75 per cent of the precessional torque on the core, the balance arising from interaction with the mantle. Now, imagine that the luni–solar torques are switched off; the core will continue its precessional motion in trying to catch up with the mantle, but at only 25 per cent of the present average rate since only the mantle interaction torque is available. Thus the rate at which ε_d decreases is 25 per cent of the angular rate of motion of the Earth's axis in precession and if the precession period, τ, is 25 800 y, the initial rate of restoration of the coincidence of core and mantle axes is

$$-\frac{d\varepsilon_d}{dt} = 0{\cdot}25\frac{2\pi\sin\theta}{\tau} = 7{\cdot}7\times10^{-13}\ \text{radian s}^{-1}. \tag{1}$$

By bringing the core and mantle axes to coincidence from a small angular difference, ε_d, with conservation of total angular momentum, the rotational energy destroyed is

$$U = \tfrac{1}{2}\frac{C_m C_c}{C_m + C_c}\,\omega_m^2\,\varepsilon_d^2 = 2{\cdot}11\times10^{28}\,\varepsilon_d^2\ \text{Joules} \tag{2}$$

where C_m, C_c are the axial moments of inertia of the mantle and core and ω_m is the axial rotation rate. Thus the instantaneous rate of destruction of rotational energy by dissipative coupling is

$$-\frac{dU}{dt} = \frac{C_m C_c}{C_m + C_c}\,\omega_m^2\,\varepsilon_d\left(-\frac{d\varepsilon_d}{dt}\right) = 3{\cdot}25\times10^{16}\,\varepsilon_d\ \text{W} \tag{3}$$

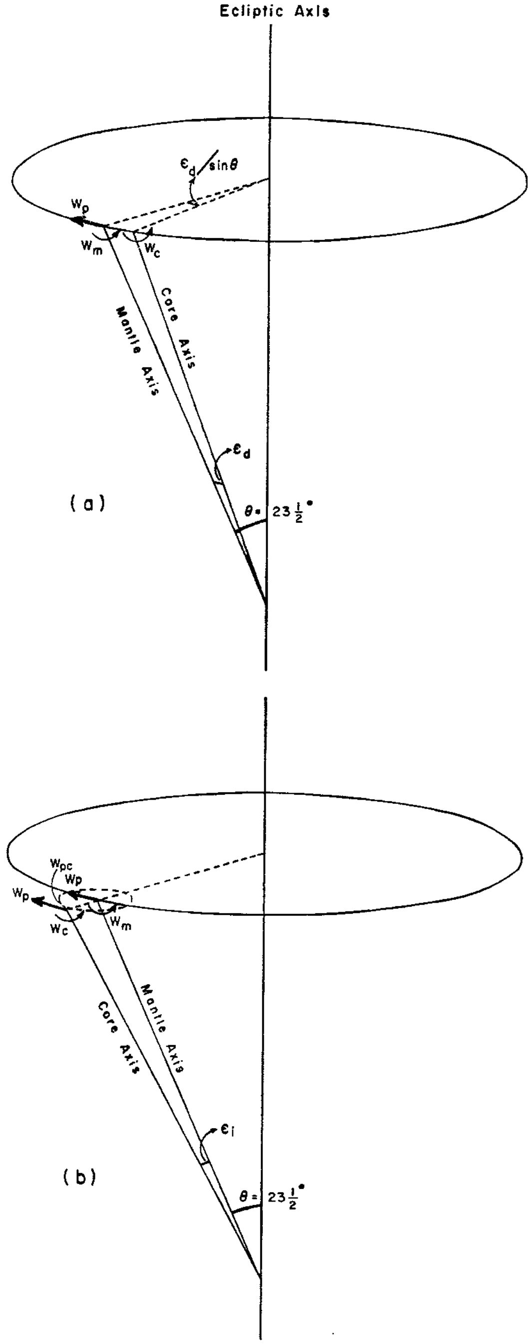

FIG. 1. Orientations of the rotational axes of the core (ω_c) and mantle (ω_m) for (*a*) dissipative, and (*b*) inertial coupling of the core to the precession (ω_p) of the mantle.

We may note at this point that a maximum plausible value of ε_d is imposed by the fact that no more than perhaps $1 \cdot 5 \times 10^{12}$ W of the astronomically indicated rotational energy loss is unaccounted for by marine tides (Miller 1966), so that we must have $\varepsilon_d < 5 \times 10^{-5}$ radian, which is the upper limit imposed by Toomre (1966). An even tighter constraint is imposed by considerations which follow.

We can also compare the limit thus imposed with the value deduced by supposing that the coupling coefficient is the same as that for length-of-day fluctuations. The calculations of Rochester & Smylie (1965) encourage the view that this supposition is reasonable. [W. V. R. Malkus has drawn the author's attention to the fact that his experiments with fluids in rotating ellipsoidal cavities give ' spinover ' times (i.e. relaxation times for axial misalignment) at least three times shorter than ' spinup ' times (i.e. relaxation times for angular velocity of axial rotation). However it is assumed here that for the short times considered the core can be treated as a rigid body (having a rigidity imposed by the magnetic field) and that hydrodynamic relaxations within the core are not relevant in the present context. It is acknowledged that this assumption will bear further examination.] The time constant for restoration of equilibrium axial rotation, τ_R, may be estimated from the spectral cut-off in length-of-day fluctuations for which Munk & MacDonald (1960) give 0·1 cycle per year, so that $\tau_R \simeq 1{\cdot}6$ y (Stacey 1969), but with a substantial uncertainty. Then the coupling coefficient (torque per unit angular velocity difference) is

$$K_R = \frac{C_m C_c}{C_m + C_c} \cdot \frac{1}{\tau_R} \simeq 1{\cdot}57 \times 10^{29}\ \text{J sec} \tag{4}$$

The supposition that this is valid also for axial misalignment means that τ_R is also the time constant for axial readjustment so that

$$\varepsilon_d = \left(-\frac{d\varepsilon_d}{dt}\right)\tau_R = 3{\cdot}9 \times 10^{-5}\ \text{rad}. \tag{5}$$

Since this coincides, within the uncertainties, with the upper bound imposed by the energy argument, it is apparent that dissipative coupling cannot be dismissed. In particular the coupling time constant may be shorter than the estimate from the work of Munk & MacDonald, for example if the spectral cut-off in length-of-day variations is characteristic of the core motions which excite them rather than the core–mantle coupling. However, no conclusion about dissipative coupling is possible without considering also inertial coupling.

Inertial coupling

Now suppose instead that the coupling is entirely inertial, i.e. non-dissipative, and that once again the luni-solar torques are switched off, leaving the core rotating about an axis at an angle ε_i to the rotational axis of the mantle. The torque exerted on it by the mantle is in a sense which would align a stationary core with the mantle axis, but in the circumstance that the core is rotating the torque causes the core axis to precess in a retrograde sense about the mantle axis, maintaining constant the angle ε_i between them. We can see immediately that this must be so from the dual requirements that angular momentum and rotational energy must both be conserved. In fact the whole core behaves like the marble in Toomre's (1966) analogy. The angular rate of the precession of the core axis about the mantle axis will be close to

$$\omega_{pc} = -f\varepsilon_c\,\omega_c = -1{\cdot}25 \times 10^{-7}\ \text{rad sec}^{-1} \tag{6}$$

where e_c is the surface ellipticity of the core, assumed equal to its dynamical ellipticity, $2{\cdot}45 \times 10^{-3}$, ω_c is the angular velocity of axial rotation and f is a factor to allow for elastic yielding of the mantle in response to the normal stresses imposed on it by the core, that is a partial adjustment of the core–mantle boundary to the rotational equator of the core. We may take as a sufficient approximation for the present purpose $f = 0{\cdot}70$ which is the reduction in the frequency of the Chandler wobble relative to Eulerian free precession of a rigid earth (Munk & MacDonald 1960) by elastic yielding of the mantle to the gyroscopic torque (i.e. the elastic response of the mantle to the core precessional torque is presumed to be similar).

Now when the core has reached the position relative to the mantle axis indicated in Fig. 1(*b*), let the luni-solar torques be switched on again, causing the Earth to precess at an angular rate ω_p but the core only at a rate $0{\cdot}75\,\omega_p$. But the value of ε_i can be so chosen that the precession of the core axis about the mantle axis will just make up the difference. This coupling contribution to the precession of the core is

$$\Delta\omega_p = 0{\cdot}25\,\omega_p = \omega_{pc}\frac{\varepsilon_i}{\sin\theta} = 3{\cdot}15\times10^{-7}\,\varepsilon_i \qquad (7)$$

so that $\varepsilon_i = 7{\cdot}9\times10^5\,\omega_p = 6{\cdot}07\times10^{-6}$ rad. At this angle to the mantle axis, the core will continue to precess in unison with it about the pole of the ecliptic, rather than precess about the mantle axis. The requirement that the core keep up with the mantle therefore ensures that the angle between the axes will be self-adjusted to this value.

The combination of dissipative and inertial couplings

It must however be supposed that both inertial and dissipative (electromagnetic) coupling mechanisms are operative. The geometry as seen end-on to the rotational axes of the core and mantle, is then as represented in Fig. 2, in which the axes are now separated by a general angle ε. The symbols ω_i and ω_d are used to represent the contributions to the total precession of the core by the two mechanisms, noting as before that dissipative coupling tends to bring the axes together and inertial coupling to make the core precess about the mantle axis, so that the two effects are mutually perpendicular. We may therefore treat ω_i and ω_d as vectors. The inertial coupling argument above gives a satisfactorily specific estimate of ω_i:

$$\omega_i = 3{\cdot}15\times10^{-7}\,\varepsilon. \qquad (8)$$

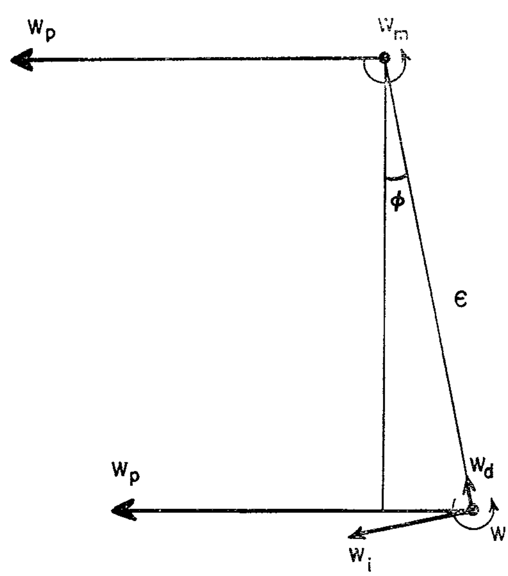

FIG. 2. Combination of contributions ω_i and ω_d to the total precession of the core ω_p due to inertial and dissipative couplings respectively. ω_i and ω_d must combine to give $0{\cdot}25\,\omega_p$ in the direction indicated by ω_p.

The dissipative coupling argument leaves more uncertainty in ω_d, but allowing validity of the estimate of K_R and its applicability to the problem, we have

$$\omega_d = \frac{\varepsilon}{\tau_R \sin\theta} = 4{\cdot}97\times 10^{-8}\,\varepsilon. \tag{9}$$

For the vector sum of ω_i and ω_d to equal $0{\cdot}25\,\omega_p$, two conditions must be satisfied:

$$\omega_i \cos\phi + \omega_d \sin\phi = 0{\cdot}25\,\omega_p = 1{\cdot}94\times 10^{-12}\,\text{rad sec}^{-1} \tag{10}$$

and

$$\omega_i \sin\phi = \omega_d \cos\phi. \tag{11}$$

Solution of these equations gives $\varepsilon = 6{\cdot}08\times 10^{-6}$ rad, $\tan\phi = 0{\cdot}158$. Then the fraction of the coupling which is provided by dissipative processes is

$$\frac{\omega_d \sin\phi}{\omega_i\cos\phi + \omega_d \sin\phi} = \frac{\tan^2\phi}{1+\tan^2\phi} = 0{\cdot}024 \tag{12}$$

and the energy dissipation due to the ω_d component of core motion is

$$-\frac{dU}{dt} = \frac{C_m C_c}{C_m + C_c}\,\omega_m^{\,2}\,\varepsilon\left(0{\cdot}25\frac{2\pi\sin\theta}{\tau}\right)\sin\phi = 3{\cdot}25\times 10^{16}\,\varepsilon\sin\phi$$

$$= 3{\cdot}1\times 10^{10}\,\text{W}. \tag{13}$$

This is a significant dissipation and compels serious consideration of the geomagnetic dynamo driven or at least strongly influenced by precessional torques (Malkus 1963, 1968).

The estimate of dissipation in equation (13) supposes that the dissipative coupling coefficient is known from the length-of-day fluctuations. This is subject to doubt because the relative motion of core and mantle considered here is due to axial misalignment, whereas the length-of-day variations involve different angular speeds about a common axis. It may even be that the estimated time constant refers to the exciting core motions rather than to the coupling coefficient. We can accommodate an unknown dissipative coupling coefficient by introducing an arbitrary constant into equation (9):

$$\omega_d = C\,\varepsilon. \tag{14}$$

Then solution of equations (10) and (11) with substitutions (8) and (14) instead of (8) and (9) gives

$$\varepsilon\sin\phi = 1{\cdot}94\times 10^{-12}\left(\frac{C}{(3{\cdot}15\times 10^{-7})^2 + C^2}\right). \tag{15}$$

By equation (13) this is proportional to the energy dissipation, so that we may select the value of C for maximum dissipation by taking $d\,(\varepsilon\sin\phi)/dc = 0$ which gives $C = 3{\cdot}15\times 10^{-7}\,\text{sec}^{-1}$ and hence $\tan\phi = 1$, i.e. $\phi = \pi/4$. With these values $\varepsilon = 4{\cdot}3\times 10^{-6}$ rad and $-dU/dt = 9{\cdot}9\times 10^{10}$ W. Thus the dissipation cannot exceed 10^{11} W even with the most extreme estimate of the dissipative coupling.

Coupling of the core to nutation and irregularity of precession

The precession is not a steady phenomenon but proceeds in semi-annual and semi-monthly ' bursts ', the lunar (or solar) torque being zero each time the Moon (or Sun) crosses the equatorial plane. Accompanying the irregularity of precession is a nutation or ' nodding ' of the rotational pole towards and away from the ecliptic pole. We must therefore enquire how the core responds to these irregularities. The magnitudes of the torques (inertial and dissipative) between the core and mantle are proportional to the angular departure of the axes, ε, and so the coupling torques are not switched off when the luni-solar torques become zero. The response of the core coupling can be assessed qualitatively in terms of the relaxation time, τ_c, represented by the ratio of the angle ε to the average angular rate of motion of the core axis due to the coupling (i.e. 25 per cent of the total angular rate). Thus

$$\tau_c = \frac{\varepsilon}{0{\cdot}25\times 2\pi \sin\theta/\tau} = 7{\cdot}9\times 10^6 \text{ sec} = 91 \text{ days}. \tag{16}$$

This is substantially longer than the semi-monthly period, so that we expect the core motion essentially to smooth out slightly the semi-monthly motion. However, the relaxation time is shorter than the semi-annual period, so that the semi-annual irregularity is substantially transmitted to the core.

If the coupling were purely inertial, the core axis would simply swing backwards and forwards on its precessional path about the mantle axis, and since no energy is dissipated no energy would be fed to the Chandler wobble or any other dissipative motion. The motion of the mantle resulting from the oscillation of inertial torques preserves the mutual alignment of the axis of rotation and axis of figure. But the superposition of dissipative, electromagnetic coupling tends to separate the axes of rotation and figure and thus causes torques of the kind required to excite the wobble. (The motions due to the two mechanisms are mutually perpendicular in the sense indicated in Fig. 1). Thus some excitation of the Chandler wobble arises in this way. However, quantitatively the effect is inadequate to excite a wobble of the observed amplitude. The amplitude of core motion ($\sim 10^{-5}$ rad) appears superficially to be adequate to excite a 5×10^{-7} rad wobble of the mantle, but the dissipative coupling time constant preferred here is too long and the 6 months periodicity of the excitation is unfavourable to a build-up of wobble to the observed amplitude. Core excitation of the wobble could only arise from *large* internal irregularities in the core motion.

Possibly the most important implication of the oscillatory nature of the precessional torque and of the consequent oscillatory core–mantle torque is that it provides the sort of oscillatory motion which Bullard & Gubbins (1971) suggest may constitute the actual driving mechanism for the geomagnetic dynamo. They contemplated a mechanism which could operate in a stably stratified core because calculations by Higgins & Kennedy (1971) indicated that the core temperature gradient is sub-adiabatic and therefore cannot support convective or stirred motion, which has conventionally been assumed to be a prerequisite for a geomagnetic dynamo. This may not be the only way to avoid the Higgins–Kennedy paradox (Stacey 1972) but the possibility of dynamo action in a stratified core introduces a new dimension into the conjectures on the physics of the core. If it is valid then the oscillatory dynamo mechanism appeals to the 3×10^{10} W of dissipation estimated above.

Discussion

The significant conclusion derived here is that although dissipative (electromagnetic) coupling of the core and mantle provides only 2 to 3 per cent of the mutual torque required to maintain the precession of the core, the balance being inertial, the dissipation amounts to about 3×10^{10} W of rotational energy. This is clearly important to the prospect of a geomagnetic dynamo driven by precessional torques (Malkus 1963, 1968). This result is not very dependent upon the acknowledged uncertainty in the strength of the electromagnetic coupling. Thus if it is increased to the implausible degree that it accounts for 50 per cent of the precessional coupling, the dissipation is maximized, but still amounts to no more than 10^{11} W. The estimated dissipation is so much smaller than the loss of rotational energy which is indicated astronomically ($2 \cdot 7 \times 10^{12}$ W), most of which is due to tidal friction, that an attempt to estimate the core dissipation in terms of a shortfall in dissipation by tidal friction appears forlorn. It is also significant that the core dissipation is smaller by a large factor than the heat flux from an adiabatic core, which Stacey (1972) estimated to be not less than 4×10^{12} W, this being the value for thermal conduction alone without allowing for convective heat transport. The present conclusions thus reinforce the argument (Goles 1969; Lewis 1971; Hall & Murthy 1971) that the core contains potassium with a total heat generation of the order of 10^{13} W. But both the convective and precessional geomagnetic dynamos remain in contention.

Two assumptions which are made here need specific mention. First the dynamical and figure ellipticities of the core were assumed to be the same, which is equivalent to assuming a uniform core. The numerical estimates are in error to the extent that this is not correct, but the error cannot be large. Second, it is here supposed that the core behaves coherently, deforming only to the extent that, in rotating about an axis different from that of the mantle, it conforms to the ellipticity of the core–mantle boundary—a matter of a few centimetres. This is a limitation which precludes examination of the model for finer details, such as the relative axial rotations of the core and mantle. From a similar rigid core model, but with dissipative coupling only, Aoki (1969) concluded that a differential rotation sufficient to cause the westward drift of the geomagnetic field resulted from the misalignment of core and mantle axes. The misalignment estimated in the present paper is, however, much smaller than that required by Aoki, but in any case this problem requires an analysis of core motions in which the fluidity is recognized (such as by Aoki & Kakuta 1972), because differential rotations within the core become important. Relaxation of the assumption that the core is essentially rigid introduces changes in the geometry envisaged in Fig. 1, but does not appear to alter the conclusions drastically. In particular a very slight relative motion between the inner and outer cores must be expected from precessional torques, by virtue of the density contrast, but its effect is very slight.

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