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The Critical Fluctuation of the Order Parameter in Type-II Superconductors

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We study here the effect of the critical fluctuation (i.e. the thermodynamical fluctuation) of the order parameter in dirty type-II superconductors in a magnetic field H slightly above the upper critical field. We find that the fluctuation gives rise to a singular contribution to various transport coefficients, which diverge like $(H-H_{c2})^{-1/2}$. It is shown at the same time that in the usual dirty superconductor (say, with the electronic mean free path $l\sim100$Å), the above effects will probably be unaccessible experimentally, because of the smallness of the coefficient in front of the singular terms. However, in extremely dirty superconductors (say with $l\sim$Å), we may expect significant modifications of the transport properties due to the fluctuation.

§ 1. Introduction

The critical fluctuation of the order parameter in the vicinity of the transition point has been well known in statistical mechanics. However, it has been generally believed that the critical fluctuation plays no important role in the superconducting transition, since the phase space for the fluctuation of the order parameter is quite limited (i.e. of the order $(\xi p_0)^{-2}$, where p_0 is the fermi momentum and ξ is the coherence distance of electron pairs).[1)-3)] Very recently Ferrell and Schmidt[4)] have suggested that in a dirty superconductor such fluctuation, in fact, gives rise to observable effects. Making use of a semi-phenomenological consideration as to the spatial behavior of the correlation function of the local order parameter, they were able to explain the critical behavior of the electric resistivity found experimentally by Glover.[5)]

In previous works,[6)] which we shall refer to as I, Caroli and Maki have shown that the (dynamical) fluctuation of the order parameter in type-II superconductors in high field region modifies significantly the electromagnetic conductivity, which becomes strongly anisotropic depending on the relative direction of the polarization vector of the electromagnetic wave to the applied magnetic field.

The purpose of this paper is to study the effect of the thermodynamical fluctuation of the order parameter on the properties of type-II superconductors at the critical region (i.e. in the magnetic field slightly above the upper critical

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field H_{c2}). In the following we shall restrict our consideration on dirty type-II superconductors, where the effect of the critical fluctuation is most prominent.[7)]

We can describe the fluctuation of the order parameter in field H larger than H_{c2}, by making use of the formalism developed in I. The collective modes associated with the fluctuation of the order parameter are classified into the longitudinal and the transversal mode and so on according to the helecity of the fluctuation along the axis parallel to the external field (we take it here as the z-axis). For example, we have in the presence of a magnetic field H[6)]

$$\delta\Delta(\mathbf{r},t)=\exp(-E_0(q)t)\exp(iqz)\exp(iky)\exp(-eH(x-k/2eH)^2), \tag{1}$$

and

$$E_0(q)=2eD(H-H_{c2})+Dq^2, \quad \text{for the longitudinal mode,}$$

$$\delta\Delta(\mathbf{r},t)=\exp(-E_1(q)t)\exp(iqz)\Pi^{+}\exp(iky)\exp(-eH(x-k/2eH)^2), \tag{2}$$

and

$$E_1(q)\simeq 4eDH_{c2}+Dq^2, \qquad \text{for the transverse mode,}$$

respectively, where k and q are constants and $D=lv/3$ is the diffusion coefficient of the dirty metal (i.e. v is the fermi velocity and l is the electronic mean free path). In the field $H>H_{c2}$ all these modes are damped exponentially in time. At $H=H_{c2}$ the longitudinal mode becomes of diffusion like and remains essentially undamped, while other modes are still damped exponentially. Therefore the critical fluctuation is essentially due to the longitudinal modes,*) which give rise to additional contribution to various transport properties at $H\gtrsim H_{c2}$. We stress here that our treatment of the fluctuation is carried out in the spirit of the random phase approximation (i.e. the fluctuations are essentially of classical type as in the theory of Ornstein and Zernike), which precludes a possible critical behavior as discussed by Ferrell and Schmidt.[4)] When the amplitude of the fluctuation is small, we can treat the effect of the fluctuation on the equilibrium as well as on the nonequilibrium properties of the system as a small perturbation. In § 2 we consider the modification of the electron Green's function in the presence of fluctuation field (in the magnetic field H slightly larger than H_{c2}). It is shown that such a fluctuation gives rise to a diverging contribution to the density of state like $(H-H_{c2})^{-1/2}$ at $H=H_{c2}$. This follows from the fact that in the linear approximation the average amplitude of the fluctuation diverges like $(H-H_{c2})^{-1/2}$. We study in § 3 the effect of the non-linear correction of the fluctuation field, which completely suppresses the divergence found in § 2. In fact, we show that the average of the amplitude of the fluctuation field at $H=H_{c2}$ is of the order $(l\xi(T)p_0^2)^{-1/3}$, where l is the electronic mean free path, $\xi(T)$ the coherence distance in the dirty superconductor and p_0 is the fermi momentum. In § 4 we shall discuss various transport properties

*) A slightly different formulation has been already given by Eilenberger,[3)] who discussed the singularity in the specific heat of type-II superconductors in the temperature T close to T_{c0}.

such as electrical conductivity and thermal conductivity in the presence of the fluctuation of the order parameter.

§ 2. Formulation

We shall limit here our consideration to dirty type-II superconductors. In order to describe the fluctuation of the order parameter and the interaction of the fluctuation field with electrons, we begin with the following hamiltonian:[6]

$$H_g = -|g| \int \psi_\uparrow^+(\boldsymbol{r}, t)\psi_\downarrow^+(\boldsymbol{r}, t)\psi_\downarrow(\boldsymbol{r}, t)\psi_\uparrow(\boldsymbol{r}, t)\, dr^3, \tag{3}$$

which can be written also as

$$H_g = -|g| \int \Psi^+(\boldsymbol{r}, t)\Psi(\boldsymbol{r}, t)\, dr^3, \tag{4}$$

where

$$\Psi^+(\boldsymbol{r}, t) = \psi_\uparrow^+(\boldsymbol{r}, t)\psi_\downarrow^+(\boldsymbol{r}, t)$$

and

$$\Psi(\boldsymbol{r}, t) = \psi_\downarrow(\boldsymbol{r}, t)\psi_\uparrow(\boldsymbol{r}, t). \tag{5}$$

Following the procedure given in I, we know that the fluctuation spectrum of the order parameter is determined by

$$(1 - |g| \langle [\Psi^+, \Psi] \rangle)\, \delta\Delta_{\omega, q} = 0, \tag{6}$$

where $\langle [\Psi^+, \Psi] \rangle_{\omega q}$ is the Fourier transform of the retarded product. $\langle [\Psi^+ \Psi] \rangle_{\omega q}$ has been obtained as,[6]

$$\langle [\Psi^+, \Psi] \rangle = \frac{1}{|g|} + N(0) \left\{ \ln \frac{T_{c0}}{T} + \psi(1/2) - \psi\left(\frac{1}{2} + \frac{-i\omega + DQ^2}{4\pi T} \right) \right\}, \tag{7}$$

where $|g|$ is the interaction constant, $N(0)$ is the density of states of the electron at the fermi level and $\psi(z)$ is the digamma function and $\boldsymbol{Q}$ is the gauge invariant momentum operator:

$$\boldsymbol{Q} = \boldsymbol{q} + 2e\boldsymbol{A}.$$

and $\boldsymbol{A}$ is the vector potential and D is the diffusion constant of the dirty metal. Making use of Eq. (7), Eq. (6) is transformed into

$$\left\{ \frac{\partial}{\partial t} - D(\boldsymbol{\nabla} - 2ie\boldsymbol{A}(x))^2 \right\} \delta\Delta(\boldsymbol{r}, t) = \varepsilon_0 \delta\Delta(\boldsymbol{r}, t), \tag{8}$$

where

$$\varepsilon_0 = 2DeH_{c2}.$$

The solutions of Eq. (8) is classified according to the helecity along the

z-axis[6] (here we take the direction of the external field along the z-axis).

a) the longitudinal mode ($n=0$):

$$\delta\Delta(\boldsymbol{r},t)=\exp(-E_0 t)\exp(iqz)\exp(iky)\exp\left\{-eH\left(x-\frac{k}{2eH}\right)^2\right\}$$

with

$$E_0=2eD(H-H_{c2})+Dq^2. \tag{9}$$

b) the transverse mode ($n=1$):

$$\delta\Delta(\boldsymbol{r},t)=\exp(-E_1 t)\exp(iqz)\Pi^+\exp(iky)\exp\left\{-eH\left(x-\frac{k}{2eH}\right)^2\right\},$$

with

$$E_1\simeq 4eDH_{c2}+Dq^2, \tag{10}$$

etc.

At $H=H_{c2}$ the longitudinal mode is of diffusion type, while the other modes (for example the transverse mode) are simply damped in this field region. Therefore the critical fluctuation is caused essentially by the longitudinal modes.

In order to consider the effect of the fluctuation field to the electronic properties, it is convenient to introduce the following Green's function, which describes the propagation of the fluctuation field:

$$\mathcal{D}(\omega_\nu,\boldsymbol{q})=(\eta_{\omega_n,\omega_n+\omega_\nu,\boldsymbol{q}})^2\left\{\frac{|g|^2\langle[\Psi^+,\Psi]\rangle}{1-|g|\langle[\Psi^+,\Psi]\rangle}\right\}_{\omega_\nu,\boldsymbol{q}}$$

$$\simeq\frac{(\eta_{\omega_n,\omega_n+\omega_\nu\boldsymbol{q}})^2}{N(0)\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu+DQ^2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho\right)\right]}, \tag{11}$$

where

$$\rho=\frac{DeH_{c2}}{2\pi T}\quad\text{and}$$

$$\eta_{\omega_n,\omega_n+\omega_\nu\boldsymbol{q}}=\left\{1-\frac{1}{\tau|2\omega_n+\omega_\nu|}\left(1-\frac{(\tau v)^2}{3}Q^2\right)\right\}^{-1},$$

$$\text{for}\quad \omega_n(\omega_n+\omega_\nu)>0$$

$$=1,\quad\text{for}\quad \omega_n(\omega_n+\omega_\nu)<0$$

and

$$\tilde{\omega}=\omega\left(1+\frac{1}{2\tau|\omega|}\right). \tag{12}$$

Here $\omega_n=2\pi(n+\frac{1}{2})T$ the frequency associated with electron Green's function and $\omega_\nu=2\pi\nu T$ where n and ν are integer. The factor $(\eta_{\omega_n,\omega_n+\omega_\nu,\boldsymbol{q}})^2$ in Eq. (11) comes from the renormalization of vertices,

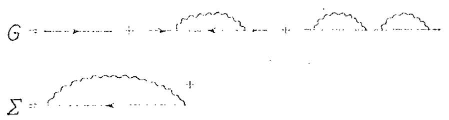

Fig. 1. The diagram for the renormalized Green's functions in the presence of the fluctuation field of the order parameter is given.

In terms of $\mathcal{D}(\omega_\nu, \boldsymbol{q})$ the Green's function for a single electron in a dirty metal in the presence of the fluctuation field is given by (see Fig. 1.)

$$G^{-1}(\omega_n, \boldsymbol{p}) = i\tilde{\omega}_n - \xi_{\boldsymbol{p}} - \sum(\omega_n, \boldsymbol{p}), \tag{13}$$

and

$$\sum(\omega_n, \boldsymbol{p}) = T\sum_\nu \int \frac{d^3q}{(2\pi)^3} \mathcal{D}(\omega_\nu, \boldsymbol{q}) \frac{1}{i(\tilde{\omega}_n + \tilde{\omega}_\nu) + \xi_{\boldsymbol{p}+\boldsymbol{q}}}. \tag{14}$$

In the presence of a magnetic field, we can decompose $\mathcal{D}(\omega_\nu, \boldsymbol{q})$ into one which describes the longitudinal mode and the transverse mode and so on. Since the transverse mode and other modes with larger helecity do not contribute to the critical fluctuation, we consider only the effect of the longitudinal fluctuation in the following.

Then Eq. (14) reduces to

$$\begin{aligned}\sum(\omega_n, \boldsymbol{p}) &= T\sum_\nu \frac{2eH}{(2\pi)^3} \int_{-\infty}^{\infty} dq\, \mathcal{D}_L(\omega_\nu, \boldsymbol{q}) \frac{1}{i(\tilde{\omega}_n + \tilde{\omega}_\nu) + \xi_{\boldsymbol{p}+\boldsymbol{q}}} \\ &\simeq T \frac{2eH}{(2\pi)^3} \int_{-\infty}^{\infty} dq\, \mathcal{D}_L(0, \boldsymbol{q}) \frac{1}{i\tilde{\omega}_n + \xi_{\boldsymbol{p}+\boldsymbol{q}}},\end{aligned} \tag{15}$$

where

$$\mathcal{D}_L(0, q) = \left(\frac{|\tilde{\omega}_n|}{|\omega_n| + \varepsilon_0}\right)^2 \frac{1}{N(0)\left\{\psi\left(\frac{1}{2} + \frac{2eDH + Dq^2}{4\pi T}\right) - \psi\left(\frac{1}{2} + \rho\right)\right\}}. \tag{16}$$

In the above treatment we have only retained the term with $\omega_\nu = 0$, since only this term gives rise to a diverging contribution. The factor $2eH$ in front of the integral comes from the available phase space corresponding to the choice of k in Eq. (9). The final integral over q is easily done and we find

$$\sum(\omega_n, \boldsymbol{p}) \simeq a(H, T)\left(\frac{|\tilde{\omega}_n|}{|\omega_n| + \varepsilon_0}\right)^2 \frac{1}{i\tilde{\omega}_n + \xi_{\boldsymbol{p}}} \tag{17}$$

and

$$\begin{aligned}a(H, T) &= \frac{2eH}{(2\pi)^2} \frac{1}{N(0)} \frac{4\pi T^2}{\psi^{(1)}(\frac{1}{2} + \rho)} \frac{1}{D\sqrt{2e(H - H_{c2})}}, \\ &= \frac{6\pi T^2}{l\xi(T)p_0^2} \frac{1}{\psi^{(1)}(\frac{1}{2} + \rho)} \frac{1}{\sqrt{h - 1}},\end{aligned} \tag{18}$$

where

$$h=\frac{H}{H_{c2}},\quad \xi(T)=(2eH_{c2}(T))^{-1/2}$$

and p_0 is the fermi momentum and $\psi^{(1)}(z)$ is the tri-gamma function.

Finally the Green's function is given by

$$G(\omega_n, \boldsymbol{p})=-\frac{i\tilde{\omega}_n+\xi_{\boldsymbol{p}}}{\tilde{\omega}_n{}^2+\xi_{\boldsymbol{p}}{}^2+a(H,T)\left(\frac{\tilde{\omega}_n}{|\omega_n|+\varepsilon_0}\right)^2}. \tag{19}$$

Making use of the above Green's function we can calculate the density of state for example. The density of state in the presence of the fluctuation field is given by

$$\begin{aligned}N(\omega)&=\frac{1}{2\pi}\operatorname{Im}\int\frac{d^3p}{(2\pi)^3}G(-i\omega, \boldsymbol{p}),\\ &=N(0)\operatorname{Re}\left\{\frac{\tilde{\omega}}{\tilde{\omega}^2-a(H,T)\left(\frac{\tilde{\omega}}{\omega+i\varepsilon_0}\right)^2}\right\}\\ &=N(0)\left\{1+\frac{a(H,T)}{2}\frac{\omega^2-\varepsilon_0{}^2}{(\omega^2+\varepsilon_0{}^2)^2}\right\}.\end{aligned} \tag{20}$$

Here $a(H,T)$ has been already defined in Eq. (18) and $\varepsilon_0=2eDH_{c2}$. The above expression is equivalent to the one we find in the gapless superconductor if we replace $a(H,T)$ by Δ^2 where Δ is the order parameter.[8] Furthermore, we note that $a(H,T)$ diverges like $(H-H_{c2})^{-1/2}$ at $H=H_{c2}$. As we shall soon see, at the field $H=H_{c2}$ the nonlinear effect of the fluctuation field is no longer negligible and an appropriate treatment of this effect will eliminate the spurious divergence found above.

§ 3. Nonlinear effect of the fluctuation

In the previous section we have seen that a simple treatment of the fluctuation results a diverging expression for the density of state at $H=H_{c2}$.

We shall consider here the equation which determines the amplitude of the fluctuation in the presence of the fluctuation. For this purpose we have to calculate $\langle[\Psi^+,\Psi]\rangle_{\omega_\nu,q}$ in the presence of the fluctuation field. Since the effect of the magnetic field is introduced in the theory by a gauge invariant generalization of the momentum operator[9] we restrict our calculation in the case $\boldsymbol{A}=0$ (although we maintain the classification of the spectrum as given in Eqs. (9) and (10) and consider the modification of Eq. (8) due to the presence of the longitudinal mode). In order to calculate $\langle[\Psi^+,\Psi]\rangle_{\omega,q}$, we should consider not only the renormalization of ω_n due to the fluctuation but also the renormalization of the vertex.

Making use of the simple rule found for the effect of the fluctuation it is easy to write down the retarded function*)

$$\langle[\Psi^{+},\Psi]\rangle_{0q}=N(0)\,2\pi T\sum_{\omega_n}\left\{\frac{1}{|\omega_n|+\dfrac{Dq^2}{2}}-\phi^2\frac{1}{\left(|\omega_n|+\dfrac{Dq^2}{2}\right)^3}\right\}$$

$$=N(0)\left\{\ln\frac{\gamma\omega_D}{\pi T}-\psi\left(\frac{1}{2}+\frac{Dq^2}{4\pi T}\right)-\frac{\phi^2}{2(2\pi T)^2}\psi^{(2)}\left(\frac{1}{2}+\frac{Dq^2}{4\pi T}\right)\right\},\qquad(21)$$

where ω_D is the Debye frequency and ϕ is a parameter (the effective amplitude of the fluctuation field, which should be determined self-consistently). Substituting Eq. (21) in Eq. (11) we have now for the longitudinal mode

$$\mathscr{D}(0,\boldsymbol{q})=\left(\frac{|\tilde{\omega}_n|}{|\omega_n|+\varepsilon_0}\right)^2\frac{1}{N(0)\left[\psi\left(\dfrac{1}{2}+\dfrac{DQ^2}{4\pi T}\right)+\dfrac{\phi^2}{2(2\pi T)^2}\psi^{(2)}\left(\dfrac{1}{2}+\dfrac{DQ^2}{4\pi T}\right)-\psi\left(\dfrac{1}{2}+\rho\right)\right]}.\qquad(22)$$

Finally the amplitude ϕ is determined by

$$\phi^2(h)=\frac{2eH}{(2\pi)^3}\frac{T}{N(0)}\int_{-\infty}^{\infty}dq\left\{\psi\left(\frac{1}{2}+\frac{2DeH+Dq^2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho\right)+\frac{\phi^2(h)}{2(2\pi T)^2}\psi^{(2)}\left(\frac{1}{2}+\rho\right)\right\}^{-1},$$

$$=\frac{2eH}{(2\pi)^2}\frac{4\pi T^2}{N(0)}\frac{1}{\psi^{(1)}(\frac{1}{2}+\rho)D\left\{2e(H-H_{c2})+\phi^2(h)\dfrac{\psi^{(2)}(\frac{1}{2}+\rho)}{4\pi TD\psi^{(1)}(\frac{1}{2}+\rho)}\right\}^{1/2}}.\qquad(23)$$

Equation (23) can be rewritten as

$$\left(\frac{\phi(h)}{\phi(1)}\right)^2=\left(\frac{h-1}{A}+\left(\frac{\phi(h)}{\phi(1)}\right)^2\right)^{-1/2},\qquad(24)$$

where

$$\phi(1)=2\pi T(l\xi(T)p_0^2)^{-1/3}\left\{\frac{9\rho}{\pi^2\psi^{(1)}(\frac{1}{2}+\rho)\cdot\psi^{(2)}(\frac{1}{2}+\rho)}\right\}^{1/6}.\qquad(25)$$

$$h=H/H_{c2}$$

and

$$A=\left[\frac{3}{8\pi l\xi(T)p_0^2}\frac{\rho^{-1}\psi^{(2)}(\frac{1}{2}+\rho)}{[\psi^{(1)}(\frac{1}{2}+\rho)]^2}\right]^{2/3}.\qquad(26)$$

*) The coefficient of ϕ^2 in Eq. (21) is obtained by following the similar procedure given in 9) and C. Caroli, M. Cyrot and P. G. de Gennes, Solid State Commun. 4 (1966), 17.

The above expression gives a finite $\phi(h)$ for $h=1$ as expected.

§ 4. Effect of the fluctuation on the transport properties

The formalism developed in preceding sections is easily applied to the calculation of the various transport properties of the system (i.e. the metal in the normal state in a magnetic field H slightly above the upper critical field).

We shall here present the calculation of the electrical conductivity as an illustration. Since the effect of the fluctuation of the order parameter gives rise to only a small conection to the conductivity we can treat these effects as a perturbation. The electric conductivity is expressed in terms of the retarded product of the current operators:

$$i\omega\sigma_{\mu\nu}(\omega)=Q_{\mu\nu}(\omega),$$

where

$$Q_{\mu\nu}(\omega)=\langle[j_\mu, j_\nu]\rangle(\omega), \tag{27}$$

and the current operator is given by

$$\boldsymbol{j}_\mu(\boldsymbol{r})=\sum_\sigma \frac{1}{2mi}(\boldsymbol{\nabla}_\mu'-\boldsymbol{\nabla}_\mu-2ei\boldsymbol{A}_\mu)\psi_\sigma^+(\boldsymbol{r}')\psi_\sigma(\boldsymbol{r})|_{\boldsymbol{r}'=\boldsymbol{r}}. \tag{28}$$

The above retarded product is obtained by making use of the thermal Green's function technique. Contribution to $Q_{\mu\nu}(\omega)$ due to the fluctuation comes from the terms given in the diagrams (Fig. 2). Here a wavy line represents the propagation of the fluctuation. Since the evaluation of each term corresponding to each diagram is similar to those in dirty type-II superconductors,[9] we can immediately write down the result here:

$$\begin{aligned}Q_{\mu\nu}(\omega_\nu)=\sigma\Bigg\{\omega_\nu+2\pi T\sum_{n=-\infty}^{\infty}\frac{2eH}{(2\pi)^3}T\int_{-\infty}^{\infty}dq\\ \times\Bigg[\frac{1}{2}\frac{\omega_n\cdot\omega_{n+\nu}}{|\omega_n|\cdot|\omega_{n+\nu}|}\left(\frac{1}{(|\omega_n|+\alpha(q))^2}+\frac{1}{(|\omega_{n+\nu}|+\alpha(q))^2}\right)\\ +\frac{1}{(|\omega_n|+\alpha(q))(|\omega_{n+\nu}|+\alpha(q))}\Bigg]\frac{1}{N(0)\left\{\psi\left(\frac{1}{2}+\frac{\alpha(q)}{2\pi T}\right)-\psi\left(\frac{1}{2}+\rho\right)\right\}}\Bigg\},\end{aligned} \tag{29}$$

where $\sigma=e\tau N/m$ the conductivity of the normal metal and $\alpha(q)=\frac{1}{2}\{2eDH+Dq^2\}$ and $\rho=DeH_{c2}(T)/2\pi T$. Here we consider only the contribution from the longi-

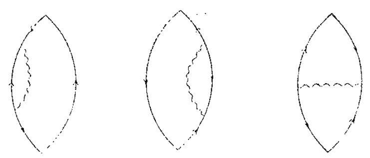

Fig. 2. Lowest order corrections due to the fluctuation of the order parameter to the conductivity.

tudinal mode (i.e. we retain only the contribution comes from $\mathcal{D}_L(\omega_\lambda, \boldsymbol{q})$ with $\omega_\lambda=0$).

The integration over q is easily done and, keeping only the most diverging terms in $(H-H_{c2})$, we have

$$Q_{\mu\nu}(-i\omega)=\delta_{\mu\nu}\sigma\left\{-i\omega+\frac{\phi^2(h)}{\pi T}\left[\psi^{(1)}\left(\frac{1}{2}-\frac{i\omega}{2\pi T}+\rho\right)\right.\right.$$
$$\left.\left.-\left(\frac{2\pi T}{i\omega}+\frac{2\pi T}{i\omega-\varepsilon_0}\right)\left(\psi\left(\frac{1}{2}-\frac{i\omega}{2\pi T}+\rho\right)-\psi\left(\frac{1}{2}+\rho\right)\right)\right]\right\}, \qquad (30)$$

where $\phi(h)$ has to be determined by Eq. (24).

It is easy to see that Eq. (30) is equivalent to one in the dirty type-II superconductors,[10] if $\phi(h)^2$ is replaced by $\langle|\Delta(\boldsymbol{r})|^2\rangle$, (more precisely speaking for one with $\mu=\nu=z$).[6] It is interesting to note that in the dirty metal the conductivity is always isotropic in the field H slightly larger than H_{c2}.*) Since $\phi^2(h)$ is of the order of $(l\xi(T)p_0^2)^{-1}$, the effect of the fluctuation in the usual type-II superconductor, say, with $l\sim 100$ Å is extremely small (i.e. $\sim 10^{-4}$). However in the extremely dirty material with $l\sim$Å, we may expect observable effects due to the critical fluctuation (i.e. of the order of 10^{-2}). For example, the measurement of the surface impedence seems very promising. For an electromagnetic wave in the microwave range, the surface resistence is given by

$$R_{cr}/R_n=\left(1-\frac{\phi^2(h)}{2\omega\pi T}\psi^{(1)}\left(\frac{1}{2}+\rho\right)\right), \qquad (31)$$

where R_n is the surface resistence in the normal state. In the region where $H-H_{c2}$ is not too small we can further simplify Eq. (31) as

$$R/R_n\simeq 1-\frac{3T}{4\omega l\xi(T)p_0^2}(h-1)^{-1/2}. \qquad (32)$$

The effect of the fluctuation is more important in the high temperature region, since the fluctuation treated here is of thermodynamical nature. We can discuss also the effect of the critical fluctuation to other transport properties, by simply replacing $\langle|\Delta|^2\rangle$ in the expressions for dirty type-II superconductors[11] by $\phi(h)^2$ defined in Eq. (24). For example, we have the following expressions for the thermal conductivity and the ultrasonic attenuation coefficient in the critical region respectively:

$$K_{cr}/K_n=1-\frac{3\phi(h)^2}{2(\pi T)^2}\rho\left[\rho\psi^{(2)}\left(\frac{1}{2}+\rho\right)+\psi^{(1)}\left(\frac{1}{2}+\rho\right)\right] \qquad (33)$$

and

*) Note added in proof: We become aware of the fact that we cannot neglect the contribution from a diagram having two fluctuation quanta in the intermediate state in the calculation of $Q_{\mu\nu}(\omega)$. An appropriate treatment of this term results, in fact, in an anisotropic conductivity.

$$\alpha_{cr}{}^L/\alpha_n{}^L = 1 - \frac{\phi^2(h)}{8(\pi T)^2}\left[\rho^{-1}\psi^{(1)}\left(\frac{1}{2}+\rho\right) - \psi^{(2)}\left(\frac{1}{2}+\rho\right)\right]. \quad (34)$$

§ 5. Concluding remarks

We have seen above that the critical fluctuation (or the thermodynamical fluctuation) of the order parameter will give rise to observable effects in the transport properties of the dirty type-II superconductors, if the electronic mean free path is extremely small.

The measurement of the various transport properties in the extremely dirty type-II superconductors in the field H slightly above H_{c2} will demonstrate the existence of the longitudinal modes discussed above if the experimental difficulty in the preparation of a homogeneous sample with extremely short electronic mean free path is overcome.

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