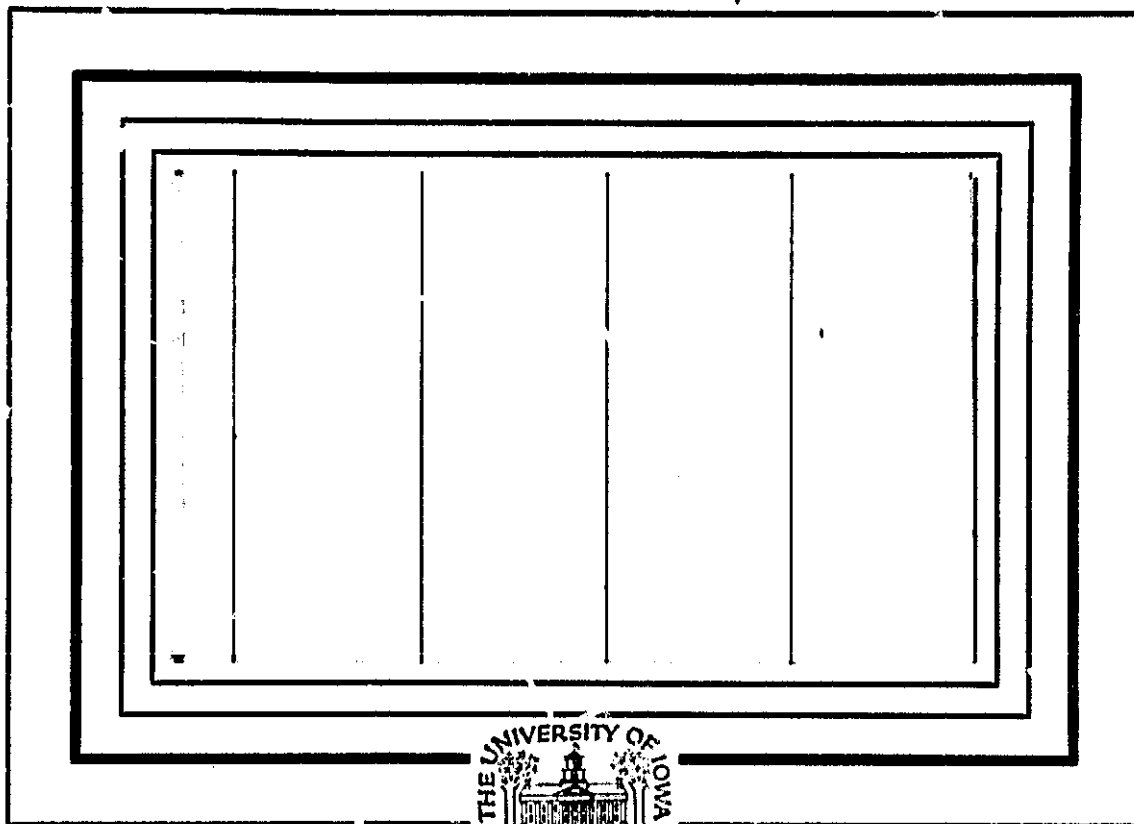


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THE CURRENT SHEET IN JUPITER'S MAGNETOSPHERE

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October 1975

## ABSTRACT

We present a theoretical model for the plasma in the Jovian magnetosphere whose pressure is comparable to the corotational energy density. The model predicts a thin current sheet of  $1 R_J - 2 R_J$  half-thickness. The current sheet lies almost precisely in the magnetic equatorial plane and is not appreciably warped as has been suggested previously.

## I. INTRODUCTION

Observations of the magnetic field structure and energetic particle fluxes in the Jovian magnetosphere have been interpreted by many authors as indicative of thin disk-like magnetosphere [e.g., Smith et al., 1974; Van Allen et al., 1974; Goertz et al., 1975]. This topology was first suggested by Gledhill [1967]. The Jovian magnetosphere contains a thin current sheet which is presumably due to the rapid rotation of the planet. It has been proposed by Van Allen et al. [1974] and by Goertz et al. [1975] that the current sheet lies almost precisely parallel to the magnetic equatorial plane which is inclined with respect to the rotational equatorial plane by an angle  $\alpha \approx 10^\circ$ . It has been suggested that this may not be the case for Jupiter because the centrifugal force will warp the current sheet so that it becomes parallel to the rotational equator [see, e.g., Hill et al., 1974]. We will show below that this is not true for the case of Jupiter where the plasma pressure in the current sheet is comparable to or larger than the corotational energy density. (This possibility was also mentioned by Hill et al. [1974] but not studied in detail.) In fact, the deviation of the current sheet from the magnetic equatorial plane may be very small ( $< 1 R_J$ ) even at a distance of  $100 R_J$ .

In section II we will review some observations relevant to the problem. In section III we will develop a model illustrating

the effects of rotation on the Jovian magnetosphere. This model is very similar to that of Axford and Gleeson [1975] but differs in so far as it allows for the temperature of the plasma to be large whereas Axford and Gleeson restrict their treatment to the case of low-energy plasma.

## II. OBSERVATIONS

By now it seems to be an accepted conclusion that the Jovian magnetosphere corotates with the planet. Due to this corotation and thus wobbling of the magnetosphere (due to the tilt of the dipole) the two spacecrafts Pioneer 10 and 11 sampled different regions of latitude as the planet rotated beneath the spacecraft. The energetic particles seem to be confined to the vicinity of a current sheet (or neutral sheet). Also, the magnetic field is strongly distended which is indicative of a strong azimuthal current confined to a narrow region in latitude. The current sheet is presumably neither precisely parallel to the magnetic (dipole) equator nor precisely parallel to the rotational equator. It lies somewhere between these two planes. Figures 1a and 1b show two models for the shape of the current sheet. Figure 1a depicts a warped current sheet tending to become parallel to the rotational plane [Smith et al., 1974; Hill et al., 1974]. Figure 1b shows an essentially rigid sheet aligned with the magnetic equatorial plane [Van Allen et al., 1974]. There are two observations which may help to decide which configuration is actually realized in the Jovian magnetosphere.

1. Smith et al. [1974] calculate the angle between the perturbation field  $\vec{b}$  (observed field minus dipole field) and the axis of rotation  $\vec{R}$  as well as the angle between  $\vec{b}$  and the magnetic

dipole axis  $\vec{M}$ . They find that  $\cos^{-1}(\vec{R} \cdot \hat{b})$  does not vary appreciably along the trajectory of Pioneer 10 except for short times when the spacecraft presumably dips into the current sheet and encounters a rapidly varying field. The angle  $\cos^{-1}(\vec{M} \cdot \hat{b})$ , however, shows a more or less sinusoidal variation except for the short times when the spacecraft dips into the current sheet. The angle  $\cos^{-1}(\vec{R} \cdot \hat{b})$  tends to approach  $90^\circ$  as the spacecraft moves away from the planet. Smith et al. conclude correctly that the perturbation field in the limited region covered by Pioneer 10 becomes parallel to the rotational equator. But they further conclude, not necessarily correctly, that this proves that the current sheet itself becomes parallel to the rotational equator.

We think that this conclusion is not necessarily correct for the following reason: As the current sheet is rotated around the planet it also moves up and down (see Figure 1a). Thus the perpendicular distance  $z$  of the spacecraft from the current sheet varies in time. A constant angle  $\cos^{-1}(\vec{R} \cdot \hat{b})$  would hence imply that the direction of  $\vec{b}$  does not change with perpendicular distance  $z$ . In other words

$$\frac{\partial}{\partial z} \left( \frac{b_z}{b} \right) = \frac{b_r}{b^2} \left( b_r \frac{\partial b_z}{\partial z} - b_z \frac{\partial b_r}{\partial z} \right) = 0 \quad ,$$

where  $\vec{b} = b_z \hat{z} + b_r \hat{r}$  and  $\hat{z}$  is either along  $\vec{R}$  or  $\vec{M}$ .



At large distances from the current sheet the current density  $\partial b_r / \partial z$  is essentially zero and the angle  $\cos^{-1} (\vec{R} \cdot \hat{b})$  should only be a constant if

$$\frac{\partial b_z}{\partial z} = -\frac{1}{r} \frac{\partial}{\partial r} (r b_r) = 0.$$

Thus the fact that the  $\cos^{-1} (\vec{R} \cdot \hat{b})$  is a constant is consistent with the idea of a thin current sheet parallel to the rotational axis only if  $b_r$  decreases as  $1/r$ . This, however, does not seem to be the case (see the magnetic field data of Smith et al. [1974]).

On the contrary, it seems much more plausible that  $b_z/b$  should vary with  $z$  in some regular way. Thus the fact that  $\cos^{-1} (\vec{R} \cdot \hat{b})$  is a constant and  $\cos^{-1} (\vec{M} \cdot \hat{b})$  is not, does neither prove that the current sheet is parallel to the rotational axis nor parallel to the magnetic axis. We conclude that the observation of Smith et al. is inconclusive as to whether or not the current sheet is warped. This conclusion is supported by the fact that the model field of Goertz et al., which is based on a current sheet precisely parallel to the magnetic equatorial plane, reproduces the observations of Smith et al. quite satisfactorily. Figure 2 displays the angles  $\cos^{-1} (\vec{R} \cdot \hat{b})$  and  $\cos^{-1} (\vec{M} \cdot \hat{b})$  calculated from the model; the dashed lines shows the observations of Smith et al.

2. The second observation is already contained in the first. From the drops in magnetic field strength and field direction,

Smith et al. concluded that Pioneer 10 dipped into the current sheet on many occasions (e.g., when it was at  $86 R_J$  during the outbound pass) actually passed through the current sheet. A passage through the current sheet is only possible if the Joviographic latitude of the current sheet is larger than the spacecraft's latitude. At  $86 R_J$  the latitude of P10 was  $8.6^\circ$ . Thus the current sheet must have been at a latitude of at least  $8.6^\circ$ . Or, the magnetic latitude of the current sheet must have been less than  $\alpha - 8.6^\circ \sim 1.4^\circ$ . As Figure 1a clearly shows, any appreciable warping of the current sheet will increase its magnetic latitude above this value. From the fact that Pioneer 10 actually crossed the current sheet several times during the outbound pass we conclude that the warping must everywhere be less than about  $1^\circ$ , i.e., nowhere near the  $8^\circ$  implied by the warped model of Figure 1a.

Finally we point out that the model field of Goertz et al. reproduces not only the observed directions of  $\hat{b}$  but also the magnitude of the observed fields to better than 5% accuracy (figure 3). (It should, however, be noted that their model applies only to the outbound pass of Pioneer 10, i.e., to Jupiter's magnetotail.) Although the good agreement between the model field and the observations is not a proof that the current sheet lies in the magnetic equatorial plane, it tends to support that assumption.

In conclusion we would like to say that no observation clearly distinguishes between the two models, although the rigid

disk model of Van Aller seems more in agreement with the data than the warped model. In the following section we will show that theory predicts very little warping.

## III. THE MODEL

Consider a cylindrical coordinate system  $(r, \varphi, z)$  with the  $z$  axis parallel to the magnetic dipole axis. Let the whole magnetosphere rotate with an angular velocity  $\Omega$  about an axis inclined at an angle  $\alpha$  with respect to the  $z$  axis. We assume  $\alpha$  small but not zero. Initially we assume that a thin plasma sheet lies in the plane  $z = 0$ . It is thin enough so that its magnetic field is equivalent to that of an azimuthal current sheet. We also assume that only meridional field components exist, i.e.,  $B_\varphi = 0$ . The last two assumptions are identical to those made by Gleeson and Axford. Following them we also assume that close to the current sheet

$$\frac{\partial}{\partial r} \ll \frac{\partial}{\partial z} \quad . \quad (1)$$

Thus  $\nabla \cdot \vec{B} = 0$  reduces to

$$\frac{\partial B_z}{\partial z} = 0 \quad B_z = B_{z0}(r) \quad . \quad (2)$$

Then:

$$\vec{j} = \frac{c}{4\pi} \frac{\partial B_r}{\partial z} \hat{\varphi} \quad .$$

The force equations for the plasma sheet in equilibrium are then

$$-\rho\Omega^2(r \cos \alpha + z \sin \alpha) \cos \alpha = -\frac{\partial p}{\partial r} + \frac{1}{c} j_{\phi} B_z - \frac{GM_J \rho}{r^2} \cos \beta \quad (4)$$

$$-\rho\Omega^2(r \cos \alpha + z \sin \alpha) \sin \alpha = -\frac{\partial p}{\partial z} - \frac{1}{c} j_{\phi} B_r - \frac{GM_J \rho}{r^2} \sin \beta \quad (5)$$

$$\tan \beta = z/r \quad , \quad (6)$$

where  $\rho$  is the plasma density,  $p$  is pressure and  $GM_J = 1.267 \times 10^{23} \text{ cm}^3/\text{s}^2$  [Anderson et al., 1974]. We now restrict ourselves to small values of  $\beta$ , i.e., regions close to the magnetic equator. We also adopt  $\sin \alpha \ll \cos \alpha$ . Then near the equator ( $z \ll r$ ) the equations reduce to

$$-\rho\Omega^2 r = -\frac{\partial p}{\partial r} + \frac{1}{c} j_{\phi} B_z - \frac{GM_J}{r^2} \rho \quad (7)$$

$$-\rho\Omega^2 r \cos \alpha \sin \alpha = -\frac{\partial p}{\partial z} - \frac{1}{c} j_{\phi} B_r \quad . \quad (8)$$

We know that

$$B_r = B_r^D + b_r \quad B_z = B_z^D + b_z \quad , \quad (9)$$

where the superscript D refers to the dipole field. We now write:

$$p = k(r, z) \frac{\rho \Omega^2 r^2}{2} . \quad (10)$$

We recover the result of Gleeson and Axford if  $k \ll 1$ . However, if the magnetosphere plasma is supplied by the Jovian ionosphere (as in the models of Ioannidis and Brice [1971] and Goertz [1975])  $k \gg 1$ . Also Frank et al. [1975] find thermal plasma in the Jovian magnetosphere with  $p \approx 10 \frac{\rho \Omega^2 r^2}{2}$ . Thus  $k \ll 1$  does not seem to be a valid assumption for the Jovian magnetosphere.

Equation (8) can be rewritten as

$$\left( -\rho \Omega^2 r \cos \alpha \sin \alpha + \frac{1}{4\pi} \frac{\partial b_r}{\partial z} B_r^D \right) = - \frac{\lambda}{\partial z} \left( p + \frac{b_r^2}{8\pi} \right) . \quad (11)$$

In the symmetry plane of the current sheet the right-hand side of this equation is identically equal to zero, because both the kinetic pressure  $p$  and magnetic pressure  $b_r^2/8\pi$  are symmetric about the current sheet. Since the first term of the left-hand side is non-zero we must have a deviation of the current sheet from the plane  $z = 0$ , where  $B_r^D = 0$ . Let us assume that the current sheet is symmetric about the surface  $z = d(r)$ . We require then that

$$\rho^2 r \cos \alpha \sin \alpha = \frac{1}{4\pi} \frac{\partial b_r}{\partial z} B_r^E |_{z=d(r)} \quad (12)$$

For Jupiter

$$B_r^D = \frac{12zr}{(z^2 + r^2)^{5/2}} [G]$$

(where  $z$  and  $r$  are expressed in units of Jovian radii). In the current sheet we then have, assuming the left-hand side of Eq. (11) to be zero,

$$p + \frac{b_r^2}{8\pi} = C(r) \quad (13)$$

Goertz et al. [1975] have shown that close to the magnetic equator ( $z \ll r$ ) and for distances  $r > 0$

$$b_r \approx \frac{b_0}{r^{a+1}} \text{th} \left( \frac{z}{D} \right) \quad , \quad b_0 = 10^{-1} [G] \quad , \quad a = 0.7 \quad ,$$

$$D \approx 1 R_J \quad (14)$$

We also require that  $p \rightarrow 0$  for  $z \rightarrow \infty$ . Thus

$$p = \frac{1}{8\pi} \frac{b_0^2}{\text{ch}^2 \left( \frac{z}{D} \right)} \frac{1}{r^{2(a+1)}} \quad (15)$$

Inserting Eqs. (14) and (15) into Eq. (12) yields

$$d = \frac{D}{k} \frac{\cos \alpha \sin \alpha}{3 \frac{B_z^D}{r^3}} b_0 \frac{(r^2 + d^2)^{5/2}}{r^{a+3}} \quad (16)$$

where  $B_z^D r^3 = 4G$ .

Combining Eqs. (7), (10), (14), and (15) gives

$$\frac{D}{k} = -r^{a+2} \frac{B_{z0}}{b_0} \frac{1}{\left[ k(a+1) + \left( 1 - \frac{GM_J}{\Omega^2 r^3} \right) \right]} \quad (17)$$

Goertz et al. (1975) have shown that the perturbation field close to the magnetic equator ( $z \ll r$ ) has the form

$$b_z \approx \frac{acb_0 D}{r^{a+2}} \quad (18)$$

where  $c \approx 10$ . Since  $B_{z0} = B_z^D + b_z$  we find

$$\frac{D}{k} = r^{a-1} \frac{\left( \frac{B_z^D r^3}{b_0} \right)}{ack + k(a+1) + \left( 1 - \frac{GM_J}{\Omega^2 r^3} \right)} \quad (19)$$

With the values of Goertz et al. (1975) for the constants  $a$ ,  $c$ , and  $b_0$  we find



$$D = \frac{40}{r^{0.5}} \frac{1}{8.7 + \frac{1}{k} \left(1 - \frac{GM_J}{\Omega^2 r^3}\right)} < \frac{4.6}{r^{0.5}} \quad (20)$$

where the inequality holds for  $r > 2$ , i.e. for  $1 > GM_J/\Omega^2 r^3$ . Equation (20) tells us that even for a high-energy plasma ( $k \gg 1$ ) the current sheet is thin. In fact, for  $k = \infty$ ,  $D$  varies from  $D \approx 1.6$  at  $r = 20$  to  $D = 1$  at  $r = 100$ . This is in excellent agreement with the value of  $D = 1$  that was assumed in the magnetic field model of Goertz et al. at  $r > 20$ .

Finally we calculate the deviation of the current sheet from the magnetic equatorial plane by inserting equation (19) into equation (16). For  $d \ll r$ , which should be the case for  $r > 10$ , we find

$$\frac{d}{r} \approx \frac{1}{3} \cos \alpha \sin \alpha \frac{1}{k(1 + a + ac) + \left(1 - \frac{GM_J}{\Omega^2 r^3}\right)} \quad (21)$$

We recover the result of Hill et al. (1974) for  $k = 0$  and  $1 > GM_J/\Omega^2 r^3$ .  $1 > GM_J/\Omega^2 r^3$  was also assumed by Hill et al. (1974). It is certainly true in the regions where equation (21) is valid, namely  $r \geq 20$ . For  $k \approx 1$  the deviation of the current sheet from the magnetic equatorial plane is about 10 times smaller than the value predicted by Hill et al. (1974). It should be noted that  $k \gg 1$  corresponds precisely to the case, mentioned by Hill et al.,

where the magnetic mirror force equals or exceeds the centrifugal stresses.

We are now in the position to calculate the density  $\rho_0$  in the current sheet. By using equations (10) and (15) we find

$$\rho_0 = \frac{b_0^2}{4\pi\Omega^2 r^2} \frac{1}{kr^{2(a+1)}} \quad (22)$$

The plasma density for a hydrogen plasma is then

$$N_0 \approx \frac{3 \times 10^8}{k} \frac{1}{r^{5.4}} \quad (23)$$

The results of Frank et al. (1975) indicate a value for  $k \sim 5$  at  $r \approx 10$  which is, however, uncertain to at least a factor of 2. Equation (23) predicts a value of  $N \sim 10^3/k \text{ cm}^{-3}$  at  $r = 10$ , which for  $k \approx 5$  is an order of magnitude larger than the plasma density observed by Frank et al. We regard this discrepancy not as too serious for two reasons: i) equation (23) is valid only for  $r \geq 20$ , where  $b_r$  and  $b_z$  are well described by equations (19) and (18), ii) the temperatures and densities reported by Frank et al. may well be uncertain by factors of 2-5 each.

Finally we note that the self-consistent current density calculated from the drift velocities

$$\vec{V}_D = \frac{1}{qB^2} \vec{F} \times \vec{B}$$

is numerically consistent with the value of

$$j = \frac{c}{4\pi} \nabla \times B = \frac{c}{4\pi} \frac{\partial b_r}{\partial z}$$

to within a factor of 1.5. The slight discrepancy is purely due to uncertainties in the numerical values of the parameters  $a$  and  $c$  and the neglect of the current density due to  $\partial b_z / \partial r$ .

## IV. DISCUSSION

The treatment of the current sheet above has two major shortcomings.

(i) It is based on a magnetic field model which describes well only the magnetic field observations obtained on the outbound leg of Pioneer 10. It is by no means clear that a similar analysis can be performed for the front side of Jupiter's magnetosphere. Clearly, an analysis of the magnetic field data obtained by Pioneer 10 and 11 while they traversed the front side magnetosphere, similar to that by Goertz et al. (1975) is needed.

(ii). The solutions had to be left in terms of an unspecified parameter  $k$ . Unfortunately no plasma measurements are available in those regions where the analysis is valid.  $k$  must thus be determined from some kind of model calculation. Quantitative models for the plasma temperature and density are available only for the inner magnetosphere ( $r < 15$ ).

We now present a crude calculation to obtain a value for  $k$ . The observed spiralling of the magnetic field lines (Smith et al. 1974, Northrop et al. 1974) has been frequently interpreted in terms of a radial outflow model (see e.g. Kennel and Coroniti 1975). The spiral angle can be related to an outflow velocity very much in the same way that the solar wind garden-hose angle is related to the solar wind speed. Since the spiralling angle is proportional to  $r$

(Northrup et al., 1975) the outflow velocity should be constant. In this case the Alfvén Mach number of the flow is

$$M_A = \frac{V_A}{V} \approx \frac{4 \times 10^7}{r^{0.5}} \approx 0.4 \frac{k^{1/2}}{r^{0.5}}$$

where we have used the estimate of Kennel and Coroniti

$V \approx 10^5$  km/s. We require that  $M_A$  is of order unity. (If  $M_A$  were much larger a shock would form and decelerate the flow to sub-Alfvénic velocities.) Then  $k \approx 0r^{0.6}$  and the density decreases as

$$N_O \approx \frac{5 \times 10^7}{r^{0.6}}$$

At  $14R_J$  the density would be about  $0.6 \text{ cm}^{-5}$  which is not too different from the value  $1 \text{ cm}^{-5}$  quoted by Frank et al.

## ACKNOWLEDGMENTS

I would like to thank M. F. Thomsen for many helpful discussions and the calculations on which figures 2 and 3 are based. This work was supported in part by NSF Grant Number GA-31676 and in part by the National Aeronautics and Space Administration under contracts NAS2-5603 and NAS2-0553.

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## LIST OF FIGURES

Figure 1a. The warped current sheet model (after Smith et al. [1974] and Hill et al. [1974]).

Figure 1b. The rigid current sheet model (after Van Allen et al. [1974]).

Figure 2a. The angle between the model perturbation field

$$\vec{b} = \frac{b_0}{(r^2 + z^2)^{(a+1)/2}} \text{th}(z/D) \hat{r} + \frac{a D b_0}{(r^2 + z^2)^{(a+2)/2}} (\log_{10} (z/D) + C) \hat{z}$$

and the rotational axis  $\vec{R}$ . The dashed line represents the observations of Smith et al. (1974).

Figure 2b. The angle between the model perturbation field and the magnetic dipole axis  $\vec{M}$ . The dashed line represents the observations of Smith et al. (1974).

Figure 3. The magnitude of the model magnetic field strength  $B = |\vec{B}^D + \vec{b}|$  along the outbound trajectory of Pioneer 10. The dots represent 1 hour averages published by Smith et al. (1974).

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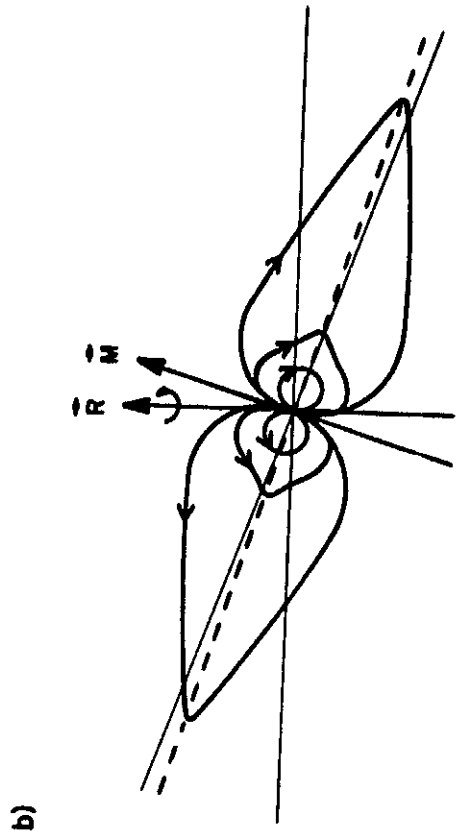
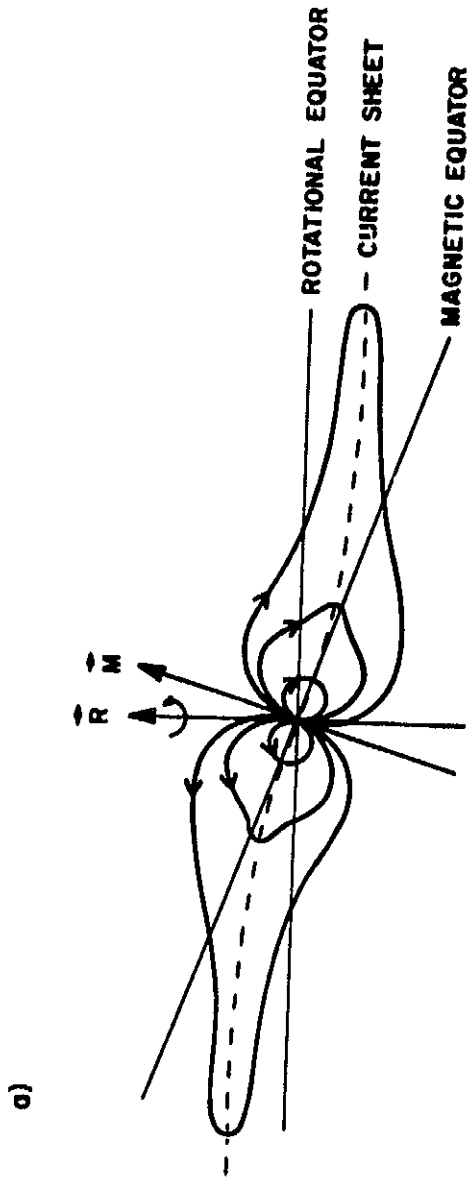


Figure 1

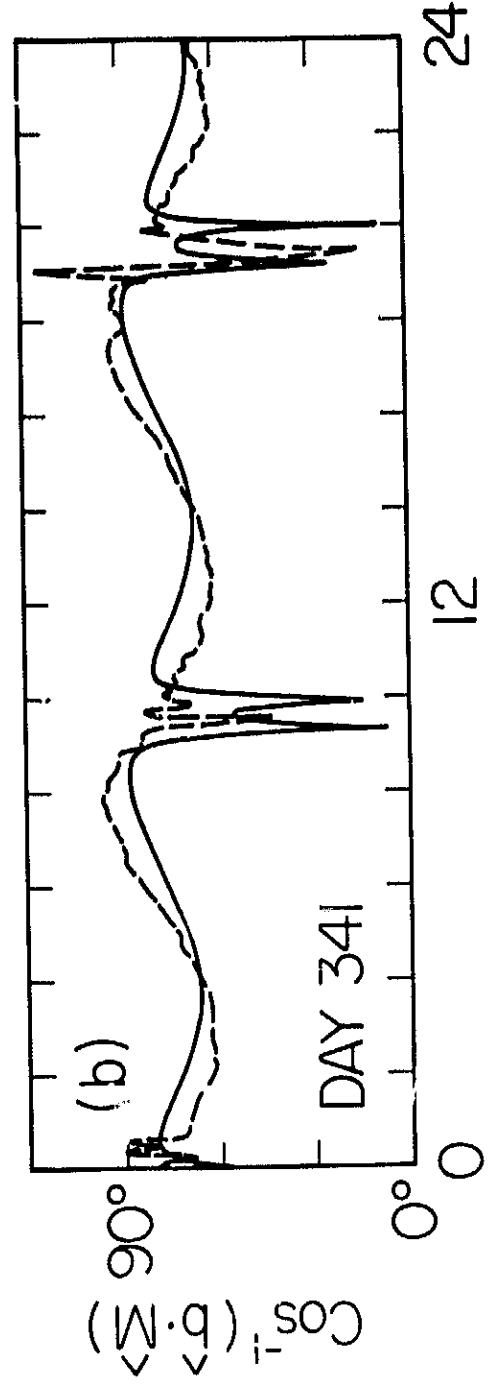
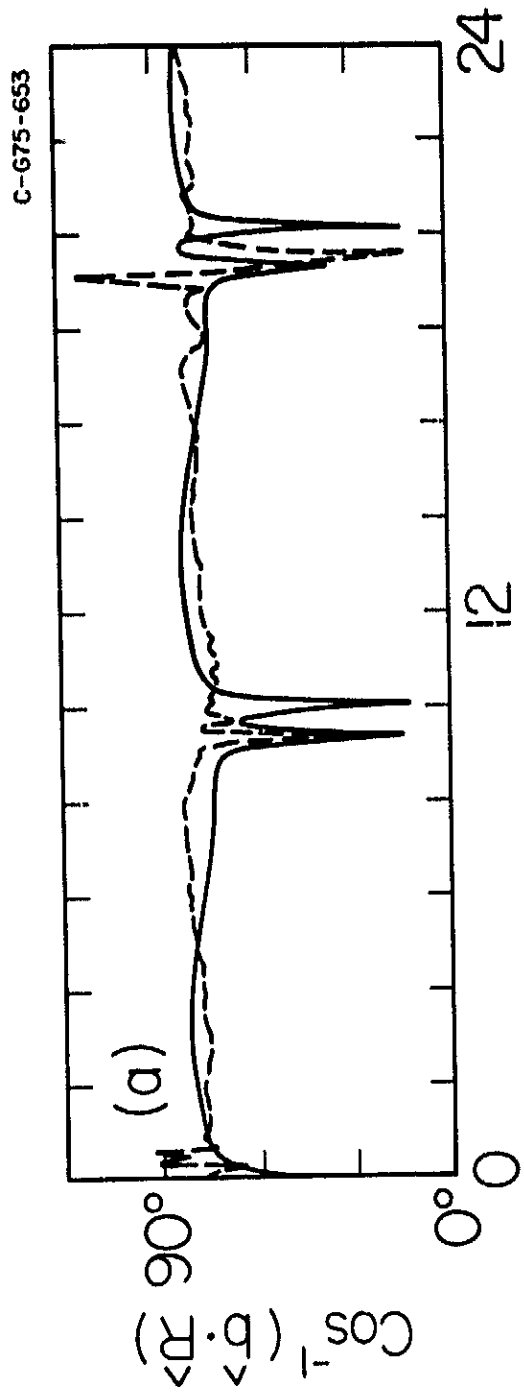


Figure 2

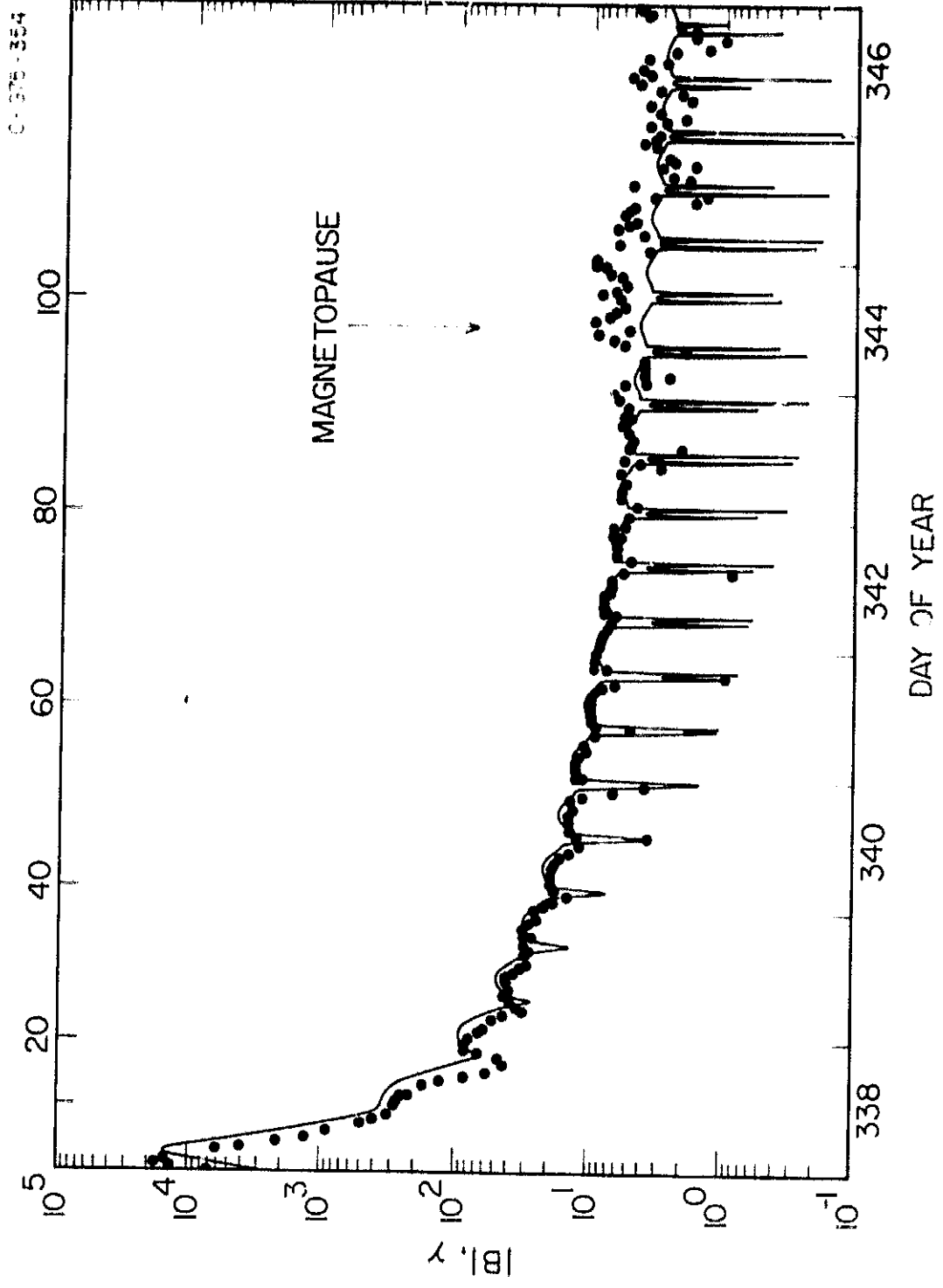


Figure 3