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The Cyclical Behavior of Optimal Bank Capital

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Abstract

This paper presents a dynamic model of optimal bank capital in which the bank optimizes over costs associated with failure, holding capital, and flows of external capital. The solution to the infinite-horizon optimization problem is related to period-by-period value-at-risk (var) in which the optimal probability of failure is endogenously determined. Over a cycle, var is positively correlated with optimal flows of external capital, but negatively correlated with optimal net changes in capital and the optimal level of total capital. Thus, a regulatory minimum requirement based on var, if binding, is likely to be procyclical. The model suggests several ways of reducing this problem. For example, a var-based requirement makes more sense if it is applied to external capital flows than if it is applied to the total level of capital. Call report data suggest that U.S. commercial bank behavior since 1984 is consistent with the model.

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1. Introduction

The main purpose of this paper is to investigate the extent to which bank capital requirements based on value at risk (var) are likely to exacerbate an economic or financial cycle. For instance, would such requirements be binding on banks in economic recessions, leading to a reduction in lending as the economy is slowing down, and non-binding in economic expansions, perhaps encouraging excessive lending? Earlier research has inquired whether the 1988 Basel Accord, in which minimum capital requirements are somewhat responsive to risk, contributed to an economic slowdown – a "credit crunch" – in the United States in the early 1990s.[1]

With the introduction of a new set of risk-sensitive Basel requirements in 2001 (Basel Committee 2001a), concerns about the so-called procyclicality of capital requirements have again surfaced. For instance, Bank for International Settlements General Manager Andrew Crockett (2000) has warned that: "Indicators of risk tend to be at their lowest at or close to the peak of the financial cycle, i.e., just at the point where, with hindsight, we can see that risk was greatest."

The new Basel requirements are not based on var. They rely on external or internal credit ratings to assign risk weights to instruments in banks' portfolios. Nevertheless, the Basel Committee (2001b) "believes that improvements in risk measurement and management will pave the way to an approach that uses full credit models as a basis for regulatory capital purposes." Therefore, it is not too early to consider the consequences of using credit risk models to formulate capital requirements, and it is likely that those requirements would make use of model-based var or some component thereof (see Basel Committee 1999).[2]

To examine the issue of the procyclicality of var-based requirements, the strategy of this paper is to construct a formal model of how a forward-looking bank with rational expectations would choose its optimal level of capital in a dynamic setting. It is important that the model be dynamic because a cycle is a predictable pattern that unfolds over time. To the extent that this pattern is anticipated, forward-looking banks and supervisors should factor it into their decision-making about capital.

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[1] The conclusions of this research are mixed. See, e.g., Bernanke and Lown (1991), Lown and Peristiani (1996) and Jackson et al. (1999).

[2] Note also that Gordy (2000) argues that a mapping may be constructed between var and the minimum requirements in the new Basel Accord (Basel Committee 2001a), and that the mapping reconciles the two approaches under certain conditions. Thus, the analysis of this paper could be useful in the implementation of the new Accord, though differences between the Accord and var must be carefully considered.

The bank faces three types of capital-related costs. First, it is costly for the bank to hold capital, and the cost is proportional to the level of capital of an operating bank. Second, the bank faces a cost of failure, which is proportional to the absolute value of the negative net worth of a bank that fails. This measure corresponds, for example, to the costs of bankruptcy, loss of charter value, reputational loss, and legal costs.

The third type of cost facing the bank is associated with net changes in external capital. In the model, it is costly to adjust the level of external capital in either direction, that is, whether the bank raises new external capital or sheds external capital on net. The sources of these entry and exit costs include asymmetric information, market signaling, taxes, and supervisory and market pressure on the bank.

The objective of the bank is to minimize a function of the three types of costs over an infinite horizon, given some dynamic identities. The bank is assumed to have rational expectations, and it selects a stream of net external capital flows over the infinite horizon. The solution allows for the expression of the current level of capital and the current external capital flow as functions of current and expected future net losses to the bank.

To investigate cyclical behavior, losses are then assumed to follow a second-order autoregressive process (AR(2)) with complex roots, so that expected future losses cycle around the unconditional mean of the distribution of losses. This assumption makes it possible to calculate all expected future losses and to solve explicitly for the current level and flow of capital in terms of observable losses. The theoretical implications of these results are used to examine the relationship between var-based requirements and the unconstrained optimum path of capital. In addition, some theoretical implications of the model are used to test its empirical plausibility when confronted with data for U.S. banks since 1984.

The findings show that, over the cycle, optimal flows of external capital are positively correlated with the var measure, but optimal net changes in capital and the optimal level of total capital are negatively correlated with var. These results suggest that a regulatory minimum requirement based on var, if binding, is likely to be procyclical.

However, the model also suggests several ways of dealing with procyclicality. First, the minimum requirement can be calibrated so as to minimize the conflict between var and optimum capital in economic downturns. Second, supervisory review may be used to insure that banks maintain sufficient capital above the minimum in economic expansions. Third, a var-based capital requirement may be applied to the flow of external capital. This approach is more in line with optimal behavior in the model than if the minimum is applied to the total level of capital.

These results differ from the conventional wisdom regarding capital and risk because the standard approach to optimal capital is essentially static and not sufficiently forward-looking. The model of this paper takes explicit account of stocks and flows of capital over time and incorporates dynamic adjustment costs that are likely to exist in practice.

Section 2 of the paper presents the model and its solution in general terms. Section 3 presents the central results of the paper. It introduces a cyclical process for losses and examines its implications for the cyclical behavior of optimal capital levels and flows. In addition, Section 3 provides numerical examples to illustrate the properties of the model and the relationship between var, net income, and capital stocks and flows. Some empirical evidence is provided in section 4, based on call report data for FDIC-insured U.S. commercial banks, which suggests that behavior since 1984 is consistent with the model. Section 5 discusses policy implications.

2. Model description and general solution

The question of this paper is dynamic in a very essential way. We would like to examine the cyclical behavior of optimal capital for a bank, where a cycle is defined as a particular predictable pattern that unfolds over time. The cycle has an expansion phase, in which the bank's profits are higher than the unconditional average, followed by a contraction phase, in which profits are lower than average. Since the pattern is predictable, it can and should be anticipated in the bank's decision-making, in particular in those decisions affecting capital. A static model may be able to identify some features of these optimal decisions, but it is necessarily myopic and any conclusions about dynamics are bound to be ad hoc.

In making its decisions about capital, the bank in this paper faces three types of costs: the cost of capital itself, the cost of bank failure, and the cost of adjusting the bank's level of capital by raising or shedding external capital. These types of costs have been analyzed earlier in the literature, but by and large the analysis has been based on essentially static models.

For instance, the key distinction between flows of external and internal capital has been made in papers by Froot et al. (1993), and Froot and Stein (1998). These papers develop a three-period model of bank capital with costs analogous to those of this paper, including costs of capital, failure, and adjustment. Since the model is moderately dynamic – it has three periods – it is possible to make distinctions between capital stocks and flows, and to speak of costs of adjustment. However, the dynamic structure is not sufficiently rich to allow questions about cyclical patterns.

Winter (1994) and Cummins and Danzon (1997) construct dynamic models of insurance company balance sheets in which capital plays an important role. They also model costs analogous to those of this paper. The model building strategy of these papers is to construct a static theoretical model of an insurance company, which is then embedded in a dynamic empirical model. In the context of the empirical model, the authors are able to draw important distinctions between stocks and flows of internal and external capital.

Some earlier papers have raised the issue of the cyclical behavior of capital explicitly, albeit within the framework of static models. These include Winter (1991), who concludes that solvency regulation exacerbates the cycle in property-liability insurance and Blum and Hellwig (1995), who argue that binding capital requirements increase the amplitude of macroeconomic cycles. In both cases, static models are used to perform comparative static thought experiments involving the degree to which capital or solvency requirements are binding.

Hoggarth and Saporta (2001) and Heid (2000) study the impact of the new capital accord of the Basel Committee (2001a). Both papers are based on extensions of the static Blum and Hellwig (1995) model, and the latter also uses elements of a model by Bernanke and Blinder (1988). The closest precedents for the formal dynamic structure of the present model are not in the financial capital literature, but in the literature on business inventories (for instance, Blanchard (1983) and Blanchard and Fischer (1989)).

Turning to the model itself, define first a simple but important dynamic identity:

$$K_t = K_{t-1} + R_t - L_t \,. \tag{1}$$

The bank's capital this period (K_t) equals capital the previous period plus net new external capital raised (R_t) minus net losses sustained (L_t). Capital here represents equity capital only. Including other elements of bank capital is possible, but it would complicate the dynamics without adding much to the intuition derived from the model. Net external capital raised is a combination of inflows (new external capital) and outflows (dividends, stock buybacks). In normal circumstances, these flows are dominated by dividend payments in the case of U.S. banks, as we will observe in section 4.

Net losses are simply the negative of net income for the bank. The sign is reversed to clarify the relationship between this variable and var and to emphasize the potential for negative net income to affect capital adversely. Note, however, that in normal circumstances, net income would be positive and net losses negative. Note also that losses as defined here are different from the accounting losses of a bank that give rise to provisions and reserves for loan losses. In the accounting definition, the losses are nonnegative amounts that are deducted from the positive nominal payments associated with loans. Losses here correspond to the total net income of the bank, regardless of the sign or the source of those losses. In particular, the losses are not necessarily tied to credit risk.

Assume now that losses are not known with certainty at the beginning of the period, but that the stochastic distribution of losses is known to have a time-invariant cumulative distribution function *F*. We assume that *F* is continuous and that its mean is well-defined. K_{t-1} is predetermined as of time *t* and R_t is under the bank's control. Taking expectations in equation (1), we obtain

$$E_t K_t = K_{t-1} + R_t - E_t L_t \,. \tag{2}$$

Then, subtracting equation (2) from (1), we have that

$$K_t - E_t K_t = -(L_t - E_t L_t) \equiv -u_t \,, \tag{3}$$

so that the shock to capital is exactly the opposite of the shock to losses.

For expositional purposes, we first consider the model in the absence of adjustment costs, in which case it is essentially myopic. We then turn to the full dynamic model with adjustment costs.

A. Model with no adjustments costs

As noted, the bank faces three types of capital-related costs. First, it is costly for the bank to hold capital, and the cost is proportional to the level of capital of an operating bank. This cost is not necessarily a simple nominal return on capital, but may be the difference between the cost of capital funding and funding through other means such as debt. Theoretical analysis (for instance, Myers and Majluf (1984)) has argued that equity is more costly than other forms of corporate liabilities. We express this cost as

$$C_c = [c_c K_t]_+ \,, \tag{4}$$

where the plus sign subscript indicates that the cost is the amount within the brackets, if positive, and zero otherwise. The expected value of this cost at time t is given by

$$E_t C_c = c_c \int_{-\infty}^{K_{t-1}+R_t-E_tL_t} (K_{t-1} + R_t - E_t L_t - u_t) F'(u_t) du_t \ . \tag{5}$$

Second, the bank faces a cost of failure, which is proportional to the absolute value of the negative net worth of a bank that fails. This measure corresponds to the costs of bankruptcy, including loss of charter value, reputational loss, and legal costs. It is represented as

$$C_f = [-c_f K_t]_+ \tag{6}$$

and its expected value at time t is

$$E_t C_f = c_f \int_{K_{t-1}+R_t-E_tL_t}^{\infty} (-K_{t-1} - R_t + E_t L_t + u_t) F'(u_t) du_t \ . \tag{7}$$

In the absence of adjustment costs, the bank may fully adjust to the desired level of capital in the current period. Thus, the bank's objective is to select a level of external capital raised so as to minimize the expected value of the costs of capital and failure. Symbolically, the bank's problem is

$$\min_{\{R_t\}} C = E_t(C_f + C_c) \ . \tag{8}$$

This problem may be solved by substituting (5) and (7) into (8) and solving the first order conditions.

<u>Proposition 1</u>. (a) C is a "U-shaped" convex function of R_t.

(b) C has a global minimum at $R_t{*} = K{*} - K_{t-1} + E_t L_t$, where K* is defined implicitly by

$$P(K_t < 0) = P(u > K{*}) = 1 - F(K{*}) = \frac{c_c}{c_f + c_c} \ . \tag{9}$$

(c) The optimal expected level of capital is constant:

$$E_t K_t = K_{t-1} + R_t{*} - E_t L_t = K{*} \ . \tag{10}$$

Proposition 1(b) states that the solution to the model with no adjustment costs is equivalent to a var approach to risk in which the probability of failure, which is generally seen as a subjective parameter, is determined endogenously. Var is generally defined as the level at which the probability that losses will exceed the var is no greater than, say, α. From (9), we have that

$$P(u > K^*) = P(L_t > E_t L_t + K^*) = c_f / (c_f + c_c), \quad (11)$$

which implies that if we set $\alpha = c_f / (c_f + c_c)$, then

$$\mathrm{var}_t = E_t L_t + K^*. \quad (12)$$

Thus, a problem in which the objective function includes a desire to minimize a conditional tail expectation corresponding to bank failure is seen to be equivalent to setting var at a specific level, once the costs of capital are taken into account.[3]

Note also that var differs from the optimal level of capital by the predictable component of losses. Using (10) and (12), we obtain that

$$R_t^* = \mathrm{var}_t - K_{t-1}. \quad (13)$$

Thus, although the optimal expected level of capital is constant and hence uncorrelated with var, the optimal flow of external capital varies one-for-one with var, given the previous period's level of capital.

B. Full model with adjustment costs

We now bring adjustment costs explicitly into the model. These costs apply both when the firm is raising new external capital and when it is shedding external capital. Why do these entry and exit costs arise?

Several reasons have been given for entry costs in the corporate finance literature. For instance, these costs may be associated with asymmetric information. Equity is a form of capital for which monitoring costs are high, and the firm has an informational advantage over public investors as to the value of its own equity (Myers and Majluf (1984)). A related reason is that the issuance of equity may send a signal to the market that it is being done at time most advantageous for the company and not necessarily for outside investors (Winter (1994)). A specific example of this type of signal occurs when a bank is attempting to replenish capital after suffering severe losses. A third reason is the "trapped equity effect" of dividend taxation (Winter

[3] A recent paper by Artzner et al. (1999) argues that var is not – by their definition – a coherent measure of risk. They argue instead for another class of measures of risk, which includes the conditional tail expectation. The reader is referred to the mathematically rigorous treatment in Artzner et al. (1999) for details of these distinctions. Note,

(1994)). Once equity is raised, it is costly for investors to obtain returns in the form of dividends, which are subject to high taxes as compared with other forms of income.

Exit costs may arise for various reasons as well. For instance, the firm may be reluctant to shed equity if there is a good chance that it may have to bear round trip costs of raising the equity again in the short term (Winter (1994)). This rationale is particularly important in models with uncertainty. A second reason is the so-called stock repurchase premium (McNally (1999)). If the company opts for shedding equity through a stock repurchase, the market may interpret this as a signal that stock is undervalued, and the cost of the buyback may increase. Finally, an important cost of shedding equity comes from pressure from regulators, supervisors and the markets to maintain clearly sound levels of capital.

For all of the foregoing reasons, we model adjustment costs in the following simple form

$$C_a = \frac{1}{2} c_a R_t^{\,2} . \tag{14}$$

Of course, there is no assurance that the costs will be symmetrical, as assumed in (14). For instance, one might suspect that the costs of raising new capital are larger than the costs of shedding external capital. Nevertheless, since the objective is to study cyclical patterns and not to measure these costs precisely, (14) seems like a reasonable approximation that preserves the qualitative behavior while allowing for an explicit solution to the general model.

In fact, we adopt a similar approximation to the cost function C of the previous section before constructing the full model. The reason is that we would like to have linear-quadratic form for which the optimal solution may be computed explicitly. Thus, we use a second-order Taylor approximation to the U-shaped function C around the optimum value of R in the case with no adjustment costs:

$$C \approx \frac{1}{2}(c_f + c_c) F'(K^*)(K_{t-1} + R_t - E_t L_t - K^*)^2 . \tag{15}$$

Note that the constant term is irrelevant for optimization and that the first-order term disappears because the approximation is taken around the optimum.

Combining these approximations in an infinite-horizon objective function with time-discount factor β, the bank's intertemporal problem becomes

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however, that var – which Artzner et al. (1999) argue is not coherent – is equivalent to a conditional tail expectation (equation (7)) – which they classify as coherent – when the cost of capital is brought into the picture.

$$\min_{\{R_{t+i}\}} \quad E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2}(K_{t+i-1} + R_{t+i} - E_{t+i} L_{t+i} - K^*)^2 + \frac{a}{2} R_{t+i}^2 \right\}, \tag{16}$$

subject to (1), where $a = \dfrac{c_a}{(c_f + c_c)F'(K^*)}$.

One strategy for solving this optimization problem is to use equation (1) to solve for R_t and substitute the result in (16). Taking derivatives with respect to K_{t+i}, $i = 0,1,\ldots,\infty$, we obtain the first order conditions (or Euler equations)

$$E_t \left\{ K_{t+i+1} - \frac{\gamma}{\beta} K_{t+i} + \frac{1}{\beta} K_{t+i-1} - \frac{1}{\beta} L_{t+i} + L_{t+i+1} + \frac{K^*}{\beta a} \right\} = 0, \tag{17}$$

$i = 0,1,\ldots,\infty$, where $\gamma \equiv \dfrac{1}{a} + 1 + \beta$. These allow for the solution of *K* in terms of the *L*.

First note that there are two solutions to the characteristic polynomial $\lambda^2 - \dfrac{\gamma}{\beta}\lambda + \dfrac{1}{\beta} = 0$ corresponding to *K* in (17), with properties $0 \le \lambda_1 < 1$ and $\lambda_2 = \dfrac{1}{\beta\lambda_1} > 1$. As is customary in rational expectations models, the first root it solved backward and the second root is solved forward in terms of *K* and *L*. In addition, the second root may be expressed in terms of the first, so that only the first is needed to write the solution.[4] The resulting optimal paths of the level of capital and net new external capital are given in the following result.

<u>Proposition 2</u>. The solution to the optimization problem in expression (16), subject to (1), satisfies:

$$K_t = (1-\lambda_1)K^* + \lambda_1 \left\{ K_{t-1} + E_t \sum_{i=0}^{\infty} (\lambda_1\beta)^i (\beta L_{t+i+1} - L_{t+i}) \right\} - u_t \tag{18}$$

and

$$R_t = (1-\lambda_1)\left(K^* - K_{t-1}\right) + E_t \sum_{i=0}^{\infty} (1-\lambda_1)\lambda_1^i \beta^i L_{t+i}. \tag{19}$$

[4] See appendix for an exact expression for λ_1.

Note also that the root λ_1 is a function of the adjustment cost parameter *a* such that $a = 0 \Rightarrow \lambda_1 = 0$ and $a \to \infty \Rightarrow \lambda_1 \to 1$.

Equations (18) and (19) contain an infinity of expectational terms. Until the expectations are defined, these equations do not constitute observable reduced form relationships between losses and the capital levels and flows. Nevertheless, the expressions may be used to gain some intuition into the rational expectations solution.

For instance, recall that when there are no adjustment costs, the expected level of capital every period is set to *K**, and new capital raised is whatever it takes to bring expected capital to that desired level. Note that (18), apart from the disturbance term, is a weighted average of *K** and of a term involving initial capital and the present values of expected future losses. These expected losses enter as present values of generalized first differences of losses. When $a = \lambda_1 = 0$, the first term receives the full weight. As adjustment costs increase, the second term receives greater weight and the optimal level tends to converge to *K** only with a lag.

The second term in equation (19) is a long-run weighted average of discounted expected losses with weights $(1\text{-}\lambda_1)\lambda_1^i$, which add up to 1. When *a* = 0, all the weight is on contemporaneous expected losses and the equation reduces to $K^* - K_{t-1} + E_t L_t$. As *a* goes to infinity, the weights on the individual discounted losses become more uniform and also much smaller. The second term gains in importance as *a* increases from zero, but as *a* approaches infinity, the cost of raising external capital becomes prohibitive, and both terms go to zero.

3. Cyclical behavior

To model cyclical behavior, we need to assume that losses tend to behave cyclically in a predictable way. Thus, we feed a process L_t that looks like a wave into the optimal solutions given by (18) and (19), and we see what happens to K_t, R_t and ΔK_t over time. More specifically, we assume that losses follow a second-order autoregressive process with complex roots, which means that expected losses exhibit a sinusoidal pattern over time around an unconditional mean. Furthermore, assume that the amplitude of that pattern is constant to facilitate tracing and visualizing the processes over time.

The assumption of a sinusoidal pattern is justified by the fact that any stationary time series may be expressed as a linear combination of sinusoids (see, for example, Jenkins and Watts (1968)). Using multiple sinusoids could lead to more realistic patterns, but would also complicate the analysis and obscure the intuition behind the results that follow.

Thus, let losses be represented by the process

$$L_t = \left(2\cos\omega\right) L_{t-1} - L_{t-2} + u_t \tag{20}$$

with $0 < \omega < \pi$, where u_t is white noise. The roots of this equation are $\exp(\pm i\omega)$. Expected losses then follow a cycle of frequency ω with constant amplitude. It is normally desirable that the roots of a process like (20) lie outside the unit circle, which implies that the process is stationary. For present purposes, it is more convenient to use the limiting case in which the roots lie on the unit circle so that cycles remain undampened and their properties are clearer to examine. In addition to having convenient cyclical properties, expected future losses under this process have a particularly simple form,

$$E_t L_{t+n} = \frac{\sin((n+2)\omega)}{\sin\omega} L_{t-1} - \frac{\sin((n+1)\omega)}{\sin\omega} L_{t-2}, \tag{21}$$

which makes it possible to transform (18) and (19) into observable equations. For simplicity, we assume for the time being that the unconditional mean of *L* is zero. Since *L* represents the negative of net income, its unconditional mean is likely to be negative. We return later to the implications of a nonzero unconditional mean for expectations of the variables in the model.

Before examining the behavior of the full model under this process for losses, consider the implications of the process in the case with no adjustment costs.

A. Cyclical behavior in model with no adjustment costs

If there are no adjustment costs, we saw in Proposition 1(c) that the optimal expected level of capital is constant (*K**). Thus, there is no forecastable cyclical pattern to the optimal level of capital. We also saw in section 2.A. that the behavior of var in this case is determined by the predictable component of losses, which from (21) is clearly cyclical. Thus, optimal capital and var follow different paths as long as the probability distribution of losses (*F*) is constant.

Suppose, alternatively, that the distribution *F* varies over time in such a way that the probability of extreme events is related to the mean of the distribution. For example, L_t could follow a normal distribution in which the variance of L_t is a monotonic function of $E_t L_t$. In this formulation, the cyclical pattern of expected losses would be reflected in the variance of the distribution, and the optimal buffer *K** would vary over time in a cyclical way.

If K_t^* varies directly with $E_t L_t$, the optimal level of capital would move in the same direction as var. However, if K_t^* varies inversely with $E_t L_t$, the optimal level of capital would move in the opposite direction from var. Which of these two alternatives is more likely to occur?

Accurate data to estimate this type of relationship are difficult to find. In section 4.A., we will examine the path of real gross domestic product (GDP) as a proxy for the business cycle and conclude that evidence of comovement between the mean and variance of GDP is not at all strong. The hypothesis of homoskedasticity cannot be rejected and, in the analysis of the full model, we will maintain the assumption of a constant distribution *F*.

B. Cyclical behavior in full model with adjustment costs

Suppose that losses are cyclical in the manner defined in equation (20). Since expectations of losses at all horizons can be derived, as in equation (21), we can replace the expectation terms in expressions (18) and (19) to obtain equivalent equations based only on observable variables. In particular, we have:

<u>Proposition 3</u>. Suppose that losses follow the cyclical pattern (20), where the shock u_t has a constant distribution *F*. Then the optimal level of capital and the optimal net flow of external capital are given by

$$K_t = (1-\lambda_1)K^* + \lambda_1 K_{t-1} - L_t + \delta_1 L_{t-1} + \delta_2 L_{t-2} \tag{22}$$

and

$$R_t = (1-\lambda_1)(K^* - K_{t-1}) + \delta_1 L_{t-1} + \delta_2 L_{t-2}\,, \tag{23}$$

respectively, where $\delta_i = \delta_i(a, \beta, \omega)$. Exact expressions for the δ_i are given in the appendix.

The results of Proposition 3 may be used to construct numerical illustrations and also to examine in more detail the cyclical relationships among the variables. We start with the illustrations.

Extreme values of the adjustment cost parameter *a* have simple predictable effects on equations (22) and (23). For instance, if there are no adjustment costs ($a = \lambda_1 = 0$), then $R_t = K^* - K_{t-1}$ and $K_t = K^* - u_t$. Adjustment to past shocks occurs immediately by raising the amount of capital needed to achieve $E_t K_t = K^*$. Thus, expected capital is constant and actual capital deviates from its expectation by an amount corresponding to the unexpected shock to losses. In the other extreme case, adjustment is infinitely costly and $\lambda_1 = 1$. Then $R_t = 0$ and $K_t = K_{t-1} - L_t$. No external capital is raised and the level of capital is completely at the mercy of fluctuations in losses.

Values of *a* between these extremes are more realistic and interesting. Figure 1 considers the case where *a* = 5 and shows the effects of a shock to L_t of size $\sin\omega$. A shock of this magnitude produces a cycle in losses of amplitude 2 (losses are in the range of ± 1), which is a convenient benchmark. The choice of *a* is somewhat arbitrary, but it has some useful features. First, it is roughly consistent with empirical estimates presented later in section 4. Second, this value creates cycles in var and *K* of approximately the same amplitude, which is helpful when we try to compare these series graphically. The starting values for all variables in Figure 1 are their respective unconditional means.

In Figure 1, the pattern of var is essentially the same as that of losses, except that var is transposed upward by a constant amount *K** = 3. This illustrative value of *K** is consistent with an infinity of parametric assumptions and distributions. It is obtained, for example, by assuming that the innovation u_t follows a normal distribution with standard deviation 1.46, and that $c_f / (c_f + c_c) = .02$.

The capital variables follow cyclical patterns that are somewhat different from the cycle for losses. The optimal amount of external capital raised (*R*), for example, rises sharply in the first quarter in response to the shock in losses. Within a few quarters, it falls into a cyclical pattern whose timing is very close to that of losses, but with smaller amplitude. The net result is that the change in capital follows a cyclical pattern that is negatively related to the cycle of losses.

Finally, the level of capital moves in a cycle that is out of phase with the cycle of losses.

We will see shortly that the lag in the cyclical movements of the level of capital, relative to losses, bears a consistent relationship to the length of the cycle. This lag is easy to see in Figure 1, particularly because of the choice of adjustment cost coefficient $a = 5$, but it holds in general.

We can also observe in Figure 1 a conflict between var, interpreted as a minimum capital requirement, and the optimal level of capital. The initial shock to losses raises the value of var much more sharply than the optimal level of capital (which also rises). The value of var continues to exceed the optimal level of capital over approximately one half of the cycle, with the opposite relationship holding over the other half. This result suggests that a minimum capital requirement based on var would be very binding in contractionary periods, in which losses are cyclically high, and very loose in expansionary periods, when losses are cyclically low.

In Figure 1, the level of capital sometimes moves in the same direction as var, and sometimes in the opposite direction. On balance, this relationship is negative, as we will see shortly. This negative relationship is another indication that using var as a minimum for optimal capital is likely to be problematical.

Some intuition for the foregoing results may be obtained by considering two opposing influences on capital. Suppose that the economy is in a downturn. The first influence is simply that losses tend to be higher in a downturn, and that these losses tend to deplete capital. The second influence is that measures of risk tend to indicate a greater need for capital, and that the natural reaction is for the bank to raise more capital. In the absence of adjustment costs, the optimal reaction is to fully offset losses, which tends to keep capital at a constant level (abstracting from surprises). However, if there are adjustment costs, the reaction is somewhat delayed, and it is optimal to allow capital to decline in a downturn and to accumulate in a subsequent upturn.

Thus, without adjustment costs, capital is expected to remain stable. With adjustment costs, it will tend to vary in a pattern that is not exactly synchronized with measures of risk such as var. The result in either case is that there tends to be a conflict between the behavior of an essentially myopic measure of risk, like var, and the behavior of optimal bank capital over the cycle.

C. Covariances and lags between optimal capital and var

The important questions regarding the procyclicality of var-based requirements focus on the relationship between optimal capital and var at cyclical frequencies. Since we have assumed that our cycle has frequency ω, this is the relevant frequency for the analysis of covariation. This section uses frequency domain techniques to examine the relevant relationships for in-phase components of frequency ω and to compute the exact lag between optimal capital and var, which was observed in Figure 1.

First, we compute the equivalent of regression coefficients for each of the key optimal variables with respect to var. These coefficients are obtained from the lag structures of equations (22) and (23). Thus, let Y represent one of R, ΔK or K and let

$$Y_t = h_{YX}(L)X_t . \tag{24}$$

Note that equations (22) and (23) are of this form. Note also that a constant may be added to the right-hand side of equation (24) without fundamentally affecting the results. Then the coefficient of a regression of the frequency-ω component of Y on the in-phase frequency-ω component of X is given by

$$b_{YX}(\omega) = \mathrm{Re}(h_{YX}(e^{-i\omega})) , \tag{25}$$

where "Re" indicates the real part of the complex number in parentheses.[5] These spectral regressions are "analytical" in the sense that no data are needed to calculate the coefficients and the "standard errors" are zero.

<u>Proposition 4</u>. In spectral regressions of the sort described above,

(a) the coefficient of R regressed on L is nonnegative for any frequency ω,

$$0 \le b_{RL}(\omega) \le 1 ,$$

(b) the coefficient of ΔK regressed on L is nonpositive for any frequency ω,

$$-1 \le b_{\Delta KL}(\omega) \le 0 , \text{ and}$$

(c) the coefficient of K regressed on L is nonpositive for any frequency ω

$$b_{KL}(\omega) \le 0 .$$

Expressions for the coefficients are found in the appendix.

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[5] See, e.g., Jenkins and Watts (1968, section 8.3.1).

Now, from equation (12) we can determine that

$$L_t = \text{var}_t - K^* + u_t , \tag{26}$$

so that L and var differ by a constant plus an innovation that is uncorrelated with var. Thus, if the regressor in Proposition 4 were var instead of L, we have essentially an errors-in-variables problem in which the regression coefficient is biased toward zero, but has the same sign. Therefore, we also have the following.

Proposition 4a. In spectral regressions of the sort described above,

(a) the coefficient of R regressed on var is nonnegative for any frequency ω,

(b) the coefficient of ΔK regressed on var is nonpositive for any frequency ω, and

(c) the coefficient of K regressed on var is nonpositive for any frequency ω.

The signs of the relationships between R or ΔK, on one hand, and either L or var on the other are easily verified in Figure 1. The sign of the coefficient of K is harder to visualize because the pattern in K is similar to those of L and var, only transposed a few periods to the right. This lag relationship may be confirmed by calculating the phase between K and L, which is defined as:

$$\phi_{KL}(\omega) = \arg(h_{KL}(e^{-i\omega})) . \tag{27}$$

Here, "arg" is the argument of the complex number in parentheses, that is, the angle that appears in the exponent of the polar form of the number. The ratio

$$-\phi_{KL}(\omega)/\omega \tag{28}$$

is a measure of the phase lag between K and L, measured in periods.[6]

Proposition 5. (a) The phase lag between K and L lies in the interval

$$\frac{\beta \sin\omega}{1-\beta\cos\omega} \le -\phi_{KL}(\omega)/\omega \le \frac{\sin\omega}{1-\cos\omega} . \tag{29}$$

(b) Since $\beta \approx 1$,

$$-\phi_{KL}(\omega)/\omega \approx \frac{\sin\omega}{1-\cos\omega} = \frac{q}{4} - \frac{1}{2} , \tag{30}$$

[6] Sargent (1979, section XI.6) provides a helpful discussion of phase lags.

where $q = 2\pi / \omega$ is the length of the cycle of frequency ω, in periods.

In Figure 1, q = 20 and the phase lag is about 4½ periods.

D. Implications of a nonzero unconditional mean of L_t

As noted earlier, the analysis so far has assumed that the unconditional mean of the loss process is zero. Since losses are defined as the negative of net income, it is more realistic to assume that losses have a negative unconditional mean. The effects of that assumption are explored in this section.

Thus, suppose that equation (20) is instead

$$L_t = c_0 + \left(2\cos\omega\right)L_{t-1} - L_{t-2} + u_t, \tag{31}$$

where $c_0 < 0$, but that the rest of the model is the same. Since the dynamic model is linear, the shape of the impulse responses remains as illustrated in Figure 1. However, the unconditional means of the variables, which are used as starting points in Figure 1, may be different. Specifically,

$$\begin{aligned} \bar{L} = \bar{R} &= \frac{c_0}{2(1-\cos\omega)} < 0, \\ \bar{K} &= K^* - \frac{1-\delta_1-\delta_2}{1-\lambda_1}\bar{L}, \text{ and} \\ \overline{\text{var}} &= \bar{L} + K^* < K^*. \end{aligned} \tag{32}$$

The unconditional means of all the variables are lower than with a homogeneous process for *L*, with the exception of the mean of *K*, for which the change is ambiguous.

For low values of ω, that is, for cycles with relatively low frequency, the unconditional mean of *K* is higher than before. More precisely, there is a neighborhood of $\omega = 0$ in which the mean of *K* is higher. For ω outside that neighborhood (higher than some value between 0 and π), the mean of *K* is lower. In the case of the 20-quarter cycles illustrated in Figure 1, the frequency is sufficiently close to zero so that the unconditional mean of *K* increases. The intuition is roughly as follows.

When net income tends to be positive on average, losses have less of a detrimental effect on the level of capital. If the stochastic distribution of losses around the conditional mean remains the same, the risk of loss – and therefore var – is also reduced. The bank finds itself accumulating more capital at a time when ideally it would like to cut back. However, the costs of adjusting capital, in particular the costs of shedding capital, make it suboptimal to fully cut back in the short run, resulting in a higher average level of capital.

The foregoing effects are illustrated in Figure 2, which is analogous to Figure 1, but with the assumption that $c_0 = -.05$, which means that the unconditional mean loss is -.51. One salient feature of Figure 2 is that the direct conflict between var and the optimal level of capital is reduced. The proportion of the cycle during which var is higher than optimal capital is more limited, and the amount by which var exceeds optimal capital is less than in Figure 1. Hence, the propensity toward the creation of a credit crunch is moderated. However, the potential for moral hazard in good times is increased, since it may be easier for banks to skimp on capital and still exceed the low var-based minimum.

E. Discussion of results

The conflict between optimal capital and a var-based minimum requirement that we see in Figure 1 and in Propositions 4(c) and 4a(c) may manifest itself in practice in two forms, depending on whether the economy is expanding or contracting. In the contraction phase of the cycle, the potential is for a credit crunch, in which bank capital falls below the var level and the institution may be driven to cut back on its holdings of risky assets, such as commercial loans. The potential for this conflict may be reduced by judicious calibration of the var-based minimum requirement. It is important to consider not only the unconditional average relationship between var and the level of capital, but also the relationship conditional on an economic downturn.

Proposals to use var in capital requirements could stipulate that the minimum be calibrated by taking some function of var, rather than var itself. Calibration of this scaling function would be important because it can affect the extent of the conflict between var and optimal capital. An attempt to use the function to bring the unconditional means of optimal capital and var in line – making the requirement “more accurate” – runs the risk of making the requirement more procyclical. In contrast, a function that produces a low requirement lessens the danger of procyclicality, but it can lead at times to a requirement that falls far below the level of optimal capital.

Whether or not a scaling function is applied to var, the level of optimal capital during the expansion phase of the cycle may greatly exceed the var minimum. This creates a potential moral hazard problem in that the level of optimal capital may be so far above the var minimum that the bank may be tempted to let capital fall toward the minimum. Pillar 2 (supervisory review process) of the new Basel Accord (Basel Committee 2001c) contains important provisions that could be very useful in dealing with this type of moral hazard problem.

Specifically, Principle 3 states that "Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum." The motivation that the Basel Committee gives for this principle is close to the issues raised in the present paper. For instance, the Committee refers to adjustment costs ("It may be costly for a bank to raise additional capital, especially if this needs to be done quickly or at a time when market conditions are unfavourable.") and to macroeconomic cycles ("There may be risks … to an economy at large … that are not taken into account in Pillar 1 [quantitative requirements].").

4. Empirical evidence

In this section, we consider the empirical plausibility of the theoretical model presented earlier. Ideally, one would estimate the model directly using data for individual banks. However, that strategy is unavailable because of a lack of appropriate data. Accounting data on banks' net income is not reflective of the true stochastic distribution of losses, which plays a central role in the model. The level of income may be accurately represented on average, but the volatility of income and the likelihood of a large loss tend to be greatly understated. Therefore, this section focuses on the empirical verification of a few assumptions and empirical implications of the model.

A. Cyclicality and predictability of economic performance

The model assumes that the economic performance of the bank is to some extent predictable and that it follows a cyclical pattern. Since accounting data for individual banks are not conducive to the types of estimates we need, we present evidence with regard to the business cycle in general and to the credit quality of individual banks.

For example, credit ratings are known to have predictive power for the credit quality of bond issuers and in particular for defaults.[7] Nickell et al. (2000) examine how rating transition matrices – including the transition to default – vary over the course of a business cycle.[8] They find that rating transitions follow a systematic pattern, and that downgrades and failures are more likely to occur at the trough of the cycle.

Other research has shown that business cycles are somewhat predictable. Stock and Watson (1989, 1993) have developed statistical techniques for identifying and predicting business cycles, and they show that the techniques have significant predictive power. Estrella and Mishkin (1998) look at the out-of-sample predictive power of a broad array of indicators for predicting NBER-dated recessions. They find that some of these indicators have strong predictive power, including the term-structure spread, stock prices, and the Stock-Watson leading indicators.

In section 3.A, we noted how systematic variations in the stochastic distribution of losses could determine the cyclicality of optimal capital in the absence of adjustment costs. Specifically, if the probability of large deviations of losses from the conditional mean is greater when expected losses are larger, var-based minimum requirements could follow a pattern consistent with optimal capital. Having made the connection between financial performance and the business cycle, we can test whether the volatility of gross domestic product (GDP) varies systematically with its conditional mean.

A straightforward test is based on a model in which the conditional variance of the GDP gap is a function of its conditional expectation. Thus, let y_t represent the output gap – the difference between the logarithms of real and potential GDP. We use U.S. chain-weighted GDP and the Congressional Budget Office's measure of potential GDP, both in 1992 dollars. A model with an AR(2) component is defined as:

$$\begin{gathered} y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t \\ h_t = \gamma_0 \exp\left(\gamma_1 \left(\alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2}\right)\right) \\ \varepsilon_t \sim N(0, h_t)\,. \end{gathered} \tag{33}$$

[7] See, for example, Estrella et al. (2000).

[8] The (*i*,*j*) element of a rating transition matrix contains the probability that a company with rating *i* ends up with rating *j* after a given period (for instance, one, three or five years). Transition matrices based on long data samples are published by the major credit rating agencies. See, for instance, Estrella et al. (2000).

The sign of the relationship between the conditional volatility of GDP and its conditional mean is given by the coefficient γ_1. As argued in Section 3.A. (reversing the sign because losses are inversely related to output), if this coefficient is negative, the level of optimal capital would follow a cyclical pattern consistent with var-based requirements. Results using quarterly data, as described above, for the period from 1966 to 2000 are presented in Table 1.

The results in Table 1 are not supportive of a relationship between the conditional mean and variance. The estimates of the AR(2) portion of the model are very significant, lending some support to the theoretical specification of this paper.[9] For the variance, only the estimate of the multiplicative constant is significant. The coefficient of the conditional mean, γ_1, has a negative sign, but it is not significantly different from zero.

Price (1994) finds analogous results with U.K.data for the period from 1957 to 1992 using a GARCH-M specification.[10] He estimates a model for the logarithm of GDP, rather than the GDP gap. The low-frequency components of these two series are bound to be different, since log(GDP) is bound to contain some type of trend, but the fluctuations at business cycle frequencies should be similar. The key cyclicality result is the same as in Table 1: Price (1994) reports that the coefficient that relates the conditional mean and variance has a *t* statistic of 0.0. A GARCH-M model applied to our U.S. data gives a qualitatively similar result (t = -1.12).

An alternative approach to the issue of changes in variance over the cycle is based on examining changes in credit ratings at different points in the business cycle. The pattern of these changes is frequently summarized in a rating transition matrix and, as noted, Nickell et al. (2000) find that transition matrices vary over the business cycle in predictable ways.

Furthermore, they construct a multinomial ordered probit model that seems to capture cyclical fluctuations in transition matrices, and which allows them to condition on whether the economy is in one of three cyclical states (peak, trough, and normal). The probit model is consistent with the assumption that ratings are driven by a continuous latent variable whose stochastic distribution is homoskedastic. The assumption of homoskedasticity is not easily

[9] Note, however, that the roots of the empirical AR(2) component are real. In contrast, the theoretical model's assumption of complex roots gives rise to explicit cycles in expectations. The empirical result probably reflects the difficulty in forecasting accurately over a horizon that is long enough to encompass a full business cycle. The hump-shaped impulse response implicit in the empirical estimates is consistent with foreseeing only the near-term portion of a long cycle.

[10] The GARCH-M model (Engle and Bollerslev 1986) is similar to the model in (33) in that the conditional mean and variance are functionally related. In the GARCH-M case, however, the mean is a function of the variance.

testable in this context, but the success of the model in capturing empirical patterns is at least suggestive that the mean and the variance may be unrelated.

B. Time-series behavior of bank capital

Although full estimation of the theoretical model of this paper is problematical, a look at time series data over a period containing at least one business cycle can help determine whether some features of the model are consistent with the empirical evidence. Detailed data on changes in capital for all FDIC-insured U.S. commercial banks are available at an annual frequency from 1984 to 1999 (from the FFIEC call reports).

For instance, we can calculate net losses, L_t, which corresponds to the negative of net income, net new external capital raised, R_t, which corresponds to net new external capital minus dividends paid, and the level of capital, K_t, which corresponds to total equity capital. Two elements of capital are excluded from the analysis, namely changes incident to business combinations and unrealized gains from available-for-sale securities. We exclude the first because it would lead to double counting when we look at aggregate data for all banks, and the second because it tends to add only short-term volatility, whereas we wish to focus on medium-frequency cyclical fluctuations.

Consider the implications of two features of the model. First, the model assumes that adjustment costs are associated with changes in external capital in either direction. Thus, we would expect that external capital should be close to zero unless net income or losses are very large. If net income is large, there may be a tendency to shed external capital, which is replaced with cost-effective internally generated capital. If losses are very large, there may be a need to raise external capital to return to prudent levels of capitalization. Second, we expect from Proposition 4 that net new external capital raised (R_t) should move in the opposite direction from net income ($-L_t$), while the net change in capital (ΔK_t) should move with net income.

Figure 3 shows the values of these three variables over the sample period, which tend to confirm the regularities described in the previous paragraph. As expected, net external capital raised is relatively close to zero until 1992. After that, the economy entered a fairly vigorous economic expansion and net income grew year after year. The banks then shed some external capital, mainly by expanding their dividend payments. The figure also suggests that net external capital and the net change in capital bear the expected relationships to net income. We return to this question shortly.

A few exceptional spikes are noteworthy in Figure 3. For example, spikes occur in both the net income and net change in capital series in 1987. In that year, several internationally-active banks made substantial provisions for loans to "less developed countries" (LDCs), which had been deteriorating since the LDC crisis of the early 1980s. The net external capital series shows an upward spike in 1992, most likely as a result of the 1988 Basel Accord and the 1991 U.S. banking legislation, both of which came fully into effect in that year. Other than a few points such as these, banks seem to have behaved generally as suggested by the theoretical model for optimal capital.

C. Relationships of capital stocks and flows to net income

The mathematical relationship between the contemporaneous levels of R_t and L_t in equation (23) is not particularly simple, even though it assumes a relatively simple process for L_t. The spectral regression discussed in Proposition 4(a) (with coefficient given by equation (36) in the appendix) corresponds to a linear relationship, but may not represent the full story. For instance, the true relationship may be nonlinear, or there may be a significant disturbance term, and the relationship applies strictly only to the in-phase components of the variables.

In practice, the spectral regression coefficient captures most of the relationship between R_t and L_t, and the same is true of the relationship between ΔK_t and L_t, which is captured well by the regression coefficient in equation (37). We can see this in the first three numerical columns of Table 2, which show the theoretical values of the spectral regression coefficients $b_{RL}(\omega)$ and $b_{\Delta KL}(\omega)$, as well as numerical estimates obtained by regressing values of R_t and ΔK_t, respectively, on L_t. These values were obtained by using the model to generate steady state values over one full cycle and then running simple regressions with those values. Recall that in Figure 1, we assume that $\beta = 1/1.01$, a = 5 and the cycle length is 20 quarters (periods). Here we make the same assumptions but also allow a also to take on the values 1 and 10.

Note that the values of $b_{RL}(\omega)$ in Table 2 are substantially larger than those of 1-λ_1. Equation (19) suggests that the optimum flow of external capital with adjustment costs is obtained by scaling down the level corresponding to no adjustment costs by a factor of 1-λ_1.

However, the effect of the other elements within brackets in (19) is that the scaling down is not as large as indicated by the outside multiplier. The actual scaling down factor is more like the theoretical spectral coefficient $b_{RL}(\omega)$.

In table 2, the numerical estimates that are drawn from the theoretical model could differ in principle from the values of the spectral coefficients, but they are the same to three decimal places for all three values of *a*. In addition, the very high R^2 s in the numerical regressions indicate that the relationship is almost exact and linear.

The final column in Table 2 presents regression estimates using the data described above for U.S. banks from 1984 to 1999. The fit of this equation is, to be sure, not as tight as that of the theoretical equations, but the R^2 is fairly high at 76 percent. In addition, the coefficient estimate is significantly positive, as the theory implies.

Estimates of $b_{\Delta KL}$ obtained in a similar way are presented at the bottom of Table 2. Once again, the spectral regression produces an almost complete representation of the contemporaneous relationship between the two variables. The empirical estimates are also significant and in agreement with the implications of the model, although the R^2 of 51 percent is a bit lower.[11]

The empirical estimates are very similar to those obtained from the model with *a* = 5. In fact, we can use the empirical regression estimate of b_{RL}, together with expression (36) and the definition of λ_1, to solve for empirical estimates of λ_1 and *a*, given β and ω.[12] The estimates, followed in parentheses by standard errors computed by the delta method, are 1-λ_1 = .349 (.061) and *a* = 5.26 (2.33). The *t* statistic for *a* is 2.26, suggesting that adjustment costs play a significant role in the model.

Figure 4 shows visually that the linear relationships in the empirical regressions are fairly accurate approximations of the data for banks from 1984 to 1999.The figure shows a scatter plot of the data for both external capital raised and net change in capital, plotted against net income. Also shown in the figure are the fitted values from the regressions in Table 2. As in Figure 3, the

[11] Note that in the empirical estimates we do not observe b_{RL} - $b_{\Delta KL}$ = 1 exactly, as the model suggests. The reason is that some minor components of the change in capital have been excluded, as explained earlier. The difference between the two coefficients is 1.02.

only large outliers correspond to the large loss provisions in 1987, which led to very low net income, and to the effects in 1992 of the 1988 Basel Accord and of the 1991 banking legislation.

5. Policy implications

This paper investigates the issue of whether var-based minimum capital requirements are procyclical by comparing the dynamic path of optimum bank capital with the path of var. The model of the paper assumes that the bank optimizes by minimizing three costs: the cost of capital, the cost of failure, and the cost of adjusting capital. Minimum requirements are not explicitly imposed on the bank, but we focus instead on the identification of periods of conflict, in which var exceeds the optimum, and periods of laxity, in which the optimum is well above var.

The conflict between var and the optimum level of capital exists with or without adjustment costs. Without adjustment costs, the conflict occurs strictly in the negative phase of the cycle, when losses are above their unconditional average. With adjustment costs, the maximum gap between optimal capital and var tends to be larger, and the timing of the occurrence of the gap is somewhat delayed.

When the optimum level of capital is compared with the var over a cycle in the presence of adjustment costs, the optimum level tends to lag var by about one quarter of a cycle. If the average level of optimum capital differs from the average var by an amount comparable to the amplitude of the cycle, or less, conflicts between the two are likely at some point along the cycle because of the phase lag between the two. Contemporaneous movements in these two variables are sometimes positively, sometimes negatively related, but on average they are negatively related over the cycle.

Like the optimum level of capital, the change in the optimum level is negatively related to var, though in this case the relationship is clear and contemporaneous. In contrast, a positive relationship exists between the optimum flow of net new external capital raised and var. These regularities suggest that a var-based minimum requirement is likely to be procyclical if it is applied to the level of capital, but the analysis also points to various solutions to the problem of procyclicality.

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[12] It is customary in empirical estimates of these types of models to take the value of β as given because it can only be estimated very imprecisely and the other estimates are not very dependent on its particular value. See West

First, the danger of causing a credit crunch in an economic downturn may be reduced by judicious calibration of a var-based minimum requirement. In the calibration process, the key is to focus on the relationship between var and optimum capital during an economic downturn, rather than simply looking at the unconditional average relationship between these variables.

Second, supervisory review may be very helpful in dealing with the moral hazard problem that faces banks as the gap between var and optimum capital increases in an economic expansion. At times, the gap may be so large that there is a temptation to follow a shortsighted approach and to let capital fall toward the var. Supervisory review may be used to insure that the bank maintains an adequate buffer between minimum and actual capital even in such times. This type of supervisory strategy is employed in Pillar 2 of the new Basel Accord.

Third, var may be the basis for a non-distortionary capital requirement – one that does not conflict with optimum levels – if it is applied to net external capital raised. In a scheme of this sort, the minimum capital raised could be some fraction of the amount that would be raised in the absence of adjustment costs. In that case, minimum requirements would not tend to conflict with optimum capital, and would be less likely to add to normal cyclical fluctuations.

(1999).

Appendix: Exact expressions with references from the text.

1. Coefficients of dynamic equations when losses follow AR(2) process with frequency ω (Section 3.B.).

$$\delta_1 = \frac{(1-\lambda_1)(2\cos\omega - \lambda_1\beta)}{1-2\lambda_1\beta\cos\omega + \lambda_1^2\beta^2} \tag{34}$$

$$\delta_2 = \frac{\lambda_1 - 1}{1-2\lambda_1\beta\cos\omega + \lambda_1^2\beta^2} \tag{35}$$

where

$$\lambda_1 = \frac{1+\beta a + a - \sqrt{(1+\beta a + a)^2 - 4\beta a^2}}{2\beta a}$$

Since λ_1 is a function of *a* and β, the δ_i are functions of *a*, β and ω.

2. Coefficients of sprectral regressions (Section 3.C.).

$$b_{RL}(\omega) = \frac{(1-\lambda_1)(1-\lambda_1\beta)\left[(1-\lambda_1)(1-\lambda_1\beta) + \lambda_1(1+\beta)(1-\cos\omega)\right]}{\left[(1-\lambda_1)^2 + 2\lambda_1(1-\cos\omega)\right]\left[(1-\lambda_1\beta)^2 + 2\lambda_1\beta(1-\cos\omega)\right]} \tag{36}$$

$$b_{\Delta KL}(\omega) = b_{RL}(\omega) - 1 \tag{37}$$

$$b_{KL}(\omega) = \frac{-\lambda_1\left[1+\lambda_1\beta^2 + \lambda_1^2\beta - \lambda_1\beta - (\lambda_1 + \beta + \lambda_1\beta + \lambda_1^2\beta^2)\cos\omega + 2\lambda_1\beta\cos^2\omega\right]}{\left[(1-\lambda_1)^2 + 2\lambda_1(1-\cos\omega)\right]\left[(1-\lambda_1\beta)^2 + 2\lambda_1\beta(1-\cos\omega)\right]} \tag{38}$$

The first two expressions satisfy $0 \le b_{RL}(\omega) \le 1$ and $-1 \le b_{\Delta KL}(\omega) \le 0$. The sign of the expression for $b_{KL}(\omega)$ is not obvious, but we have that:

$$b_{KL}(\omega) \le \frac{-\lambda_1(1-\lambda_1)(1-\beta)(1-\lambda_1\beta)}{\left[(1-\lambda_1)^2 + 2\lambda_1(1-\cos\omega)\right]\left[(1-\lambda_1\beta)^2 + 2\lambda_1\beta(1-\cos\omega)\right]} \le 0$$

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Table 1: Estimate of model of the U.S. real GDP gap, given by equations (33) in the text

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t$$
$$h_t = \gamma_0 \exp\left(\gamma_1 \left(\alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2}\right)\right)$$
$$\varepsilon_t \sim N(0, h_t) .$$

Q1 1966 to Q4 2000

Coefficient	Estimate	Standard Error	*t* statistic	*p* value
α_0	-.0004	.0007	-.58	.560
α_1	1.220	.078	15.66	.000
α_2	-.293	.083	-3.52	.000
γ_0	.0000635	.0000064	9.99	.000
γ_1	-6.64	5.20	-1.28	.202

Equation estimated using maximum likelihood.

Table 2: Theoretical and numerical coefficients of regressions of capital flows on L_t .

Parameter	Theoretical model			Empirical model
	a = 1	*a* = 5	*a* = 10	
1- λ_1	.617	.356	.267	—
$b_{RL}(\omega)$	.911	.672	.506	—
numerical b_{RL}	.911	.672	.506	.661
std. error	.001	.002	.002	.099
R^2	1.000	1.000	1.000	.761
$b_{\Delta KL}(\omega)$	-.089	-.328	-.494	—
numerical $b_{\Delta KL}$	-.089	-.328	-.494	-.359
std. error	.001	.002	.002	.095
R^2	.999	1.000	1.000	.506

Figure 1: Impulse responses, quarterly data
Initial shock of size $\sin\omega$ to L_t in quarter number 1

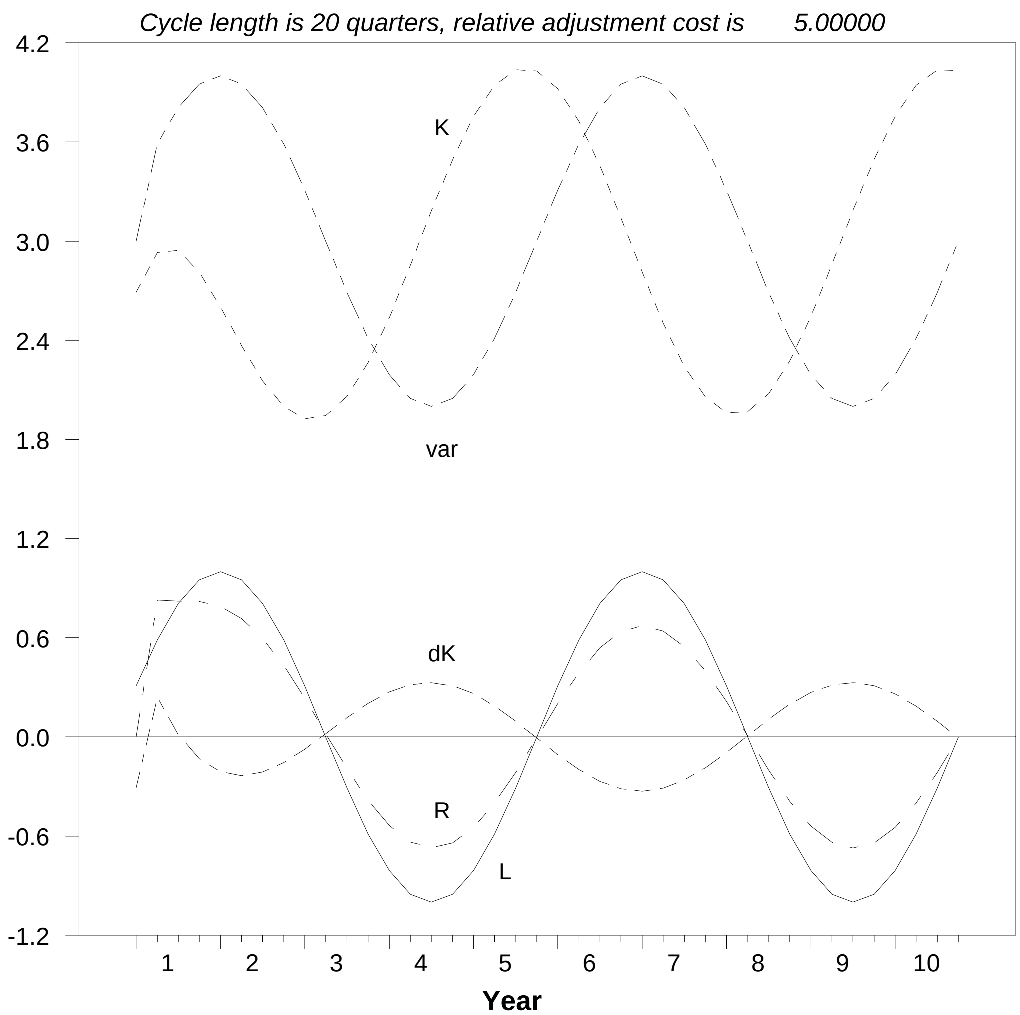

Figure 2: Impulse responses, quarterly data
Initial shock of size $\sin\omega$ to L_t in quarter number 1, $c_0 = -.05$

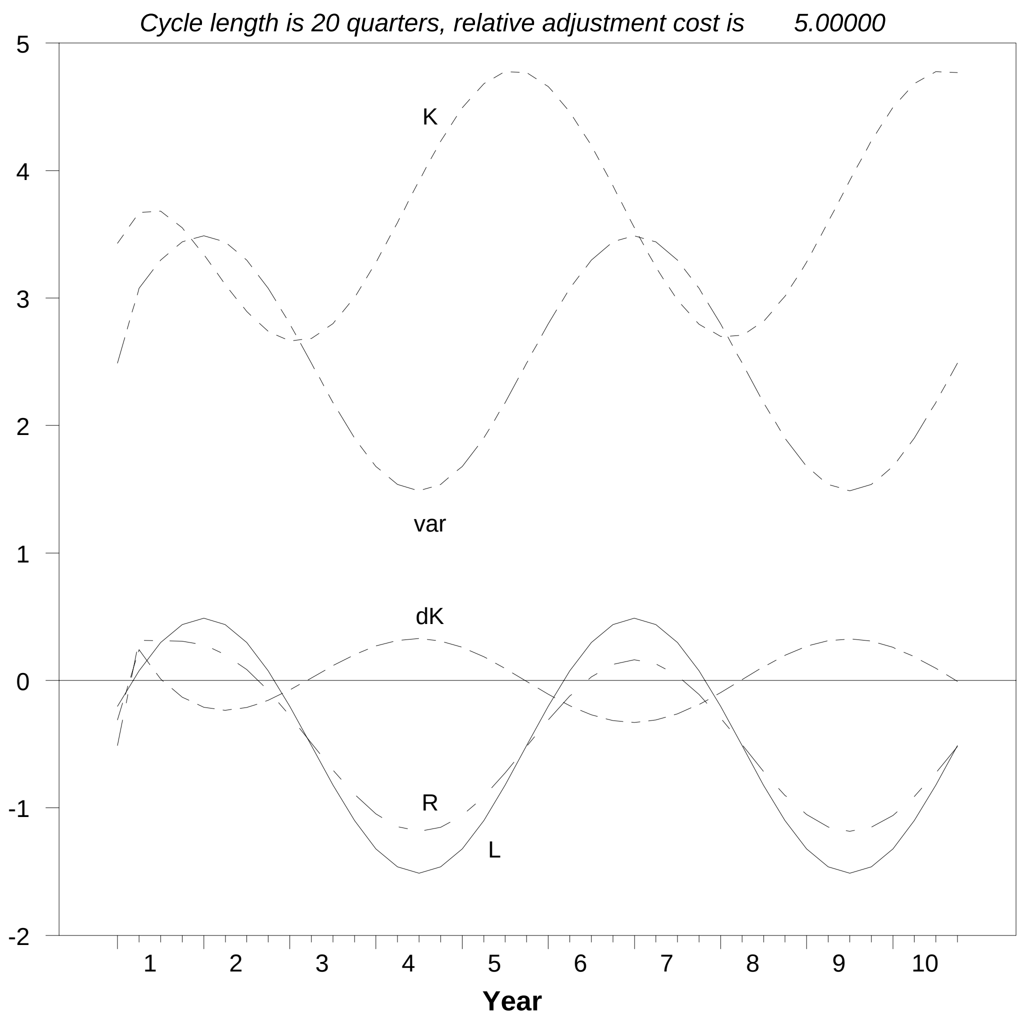

Figure 3: Time series of capital variables
All FDIC insured banks
Millions of 1999 dollars (GDP deflator)

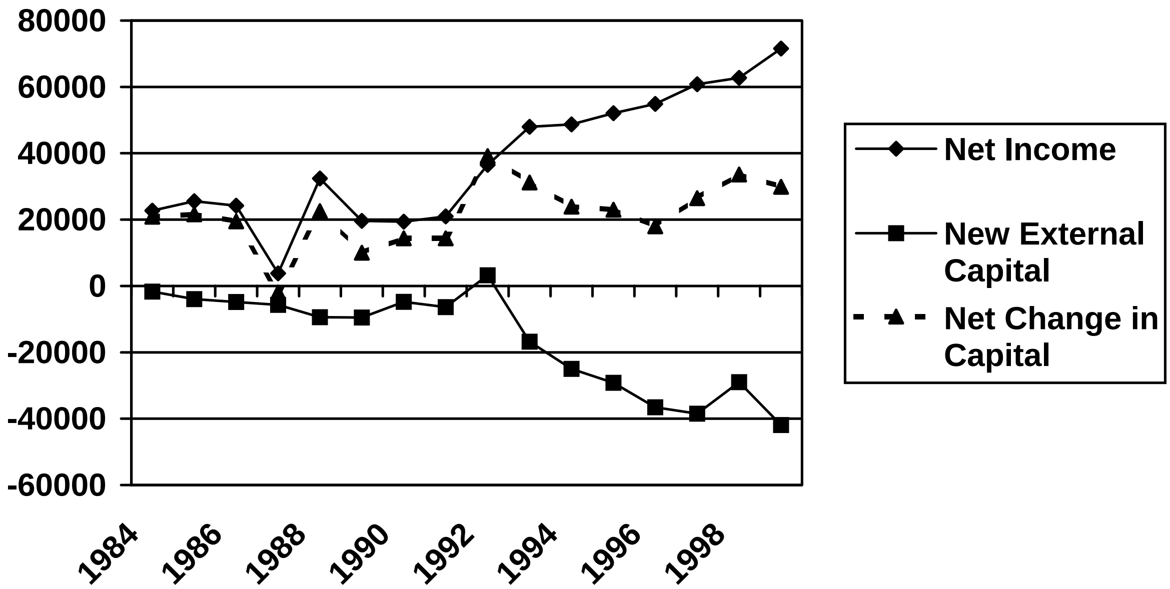

Figure 4: Net income, net external capital raised, and net change in capital
Millions of 1999 dollars (GDP deflator)