THE DEMAND FOR INSURANCE AND PROTECTION:
THE CASE OF IRREPLACEABLE COMMODITIES*

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Insurance and protection against various kinds of losses are both valuable activities provided to a large and perhaps increasing extent by the public sector.

If these activities are to be organized at an appropriate level of intensity, it is necessary to have a conceptual understanding of their value to the individual. While I. Ehrlich and G. Becker[1] have provided a theoretical development of individual demands for insurance and self-protection (and the interactions between these two activities) for the case of commodities that are valued appropriately in the market place, a similar theory is lacking for the large class of commodities that are essentially unique or irreplaceable (commodities for which there are no perfect market substitutes) such as family snapshots, the family pet, good health, the life of a beloved spouse or child, etc.

In this paper we present a new theoretical characterization of such commodities and develop some results concerning the demand for insurance and the value of increases in the level of protection for such commodities. Replaceable commodities are shown to be a special case of the more general theory.

Some of our more interesting results are as follows:

1. A rational individual, risk-averse with respect to lotteries on wealth, will typically not fully insure an irreplaceable commodity and may even choose to bet *against* losing it. The conventional explanation of such risk-taking behavior in the state-preference approach depends upon state preferences for wealth being asymmetrical. (See, for example, discussions in J. Hirshleifer[2] and R. Zeckhauser.[3]) For example, a dollar received contingent upon the state "individual lives" is to be viewed as a different commodity than a dollar contingent upon

1. "Market Insurance, Self-Insurance, and Self-Protection," *Journal of Political Economy*, LXXX (July/Aug. 1972), 623–48.
2. "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," this *Journal*, LXXIX (Nov. 1965), 509–36.
3. "Coverage for Catastrophic Illness," *Public Policy*, XXI (Spring 1973), 149–72.

"individual dies." We provide a complementary interpretation by focusing upon the value that the individual places upon the commodity whose loss distinguishes the two states (e.g., the person's life) and the manner in which this value changes with his wealth. The rational insurance decision is shown to depend in a simple way on the wealth elasticity of the individual's personal valuation of the commodity.

2. In assessing the benefit of an increase in public protection activity, the correct value of a commodity is bracketed by the amount of money the owner would pay to avoid its loss and the amount of money required to fully compensate him for its loss, assuming that there is no bar to other forms of contingent payments being made. One application of this result is to clarify the appropriate benefit measure for public investments, which have the effect of changing the rates of serious injury or death.[4]

IRREPLACEABLE COMMODITIES

The individual's demand for insurance coverage or protection of an asset depends upon his *personal* valuation of the asset; its *market* (sale) value is relevant only insofar as it influences his personal valuation. We expect market and personal valuations to coincide for assets that are usually perceived to have perfect substitutes readily available in the market (e.g., General Motors stock certificates), but these values may diverge widely for an asset that is perceived by the owner as having unique attributes. Indeed, there are no markets for some continuing sources of utility, such as good health, the life of a friend, or freedom of speech. Although the individual may be able to assess the monetary value of such "assets" (and may indeed be faced with decisions that in effect require such assessments), his personal valuation is not tied to any market price. Since in general the individual's personal valuation of unique or irreplaceable assets will change with changes in his wealth, this possibility should be incorporated into a general theory of behavior under risk.

Consider the value its owner places on a commodity θ in the context of calculating his demand for insurance or protection. This value is equivalently defined as the owner's minimum selling price;

4. See E. J. Mishan, "Evaluation of Life and Limb: A Theoretical Approach," *Journal of Political Economy*, LXXIX (July/Aug. 1971), 253–71; M. Jones-Lee, "The Value of Changes in the Probability of Death or Injury," *Journal of Political Economy*, LXXII (July/Aug. 1974), 835–49; and B. C. Conley, "The Value of Human Life in the Demand for Safety," *American Economic Review*, LXVI (March 1976), 45–55. Conley's model is closest in its basic approach to ours; see P. J. Cook, "The Earnings Approach to Life Valuation: A Reply to Conley," Duke University, March, 1976.

the rate at which he is willing to exchange θ for "all other goods;" or the minimum payment necessary to fully compensate him in the event that he loses θ. We denote this value by C. C is under some circumstances bounded by market prices: Assuming that there are no transactions costs, we see that C is not less than the market price of commodities, which potential buyers view as equivalent to θ, since otherwise it would pay the owner to sell θ. Furthermore, C is not greater than the market price of any commodity that the owner perceives as equivalent to θ.[5] When these lower and upper bounds are equal, C is precisely determined by market prices and invariant with respect to the owner's wealth—this is the conventional case in economic theory of "homogeneous" product and a "frictionless" market. More commonly, perhaps, transactions and information costs introduce a wedge between these lower and upper bounds; it is also possible (and most relevant to our analysis) that no market exists for θ or that the owner perceives no other commodity as equivalent to θ. In any of these cases C is determined by the owner's tastes and may in general change with changes in his wealth.

In the analysis that follows, we shall denote a commodity θ as "irreplaceable" if (in the owner's view) equivalent commodities are not available in the market or if C is less than the price of an equivalent commodity for at least some levels of the owner's wealth.[6] Oth-

5. "Equivalence" is necessarily a matter of the individual's tastes and perceptions, since no commodity has an exact physical duplicate; each snowflake, sugar crystal, and Chevrolet is essentially unique. However, differences between commodities will not influence an individual's economic behavior if he perceives whatever differences that do exist as unimportant. Formally, we define two commodities as equivalent if the individual in question is indifferent between any two commodity bundles that differ only in which of the two commodities is included.

6. This definition would include the "irreplaceable assets" discussed in the "option value" literature. See most recently C. Henry, "Option Values in the Economics of Irreplaceable Assets," *Review of Economic Studies, Symposium,* (1974), 89–104.

When the market is not homogeneous and frictionless, the availability of an equivalent commodity does not guarantee that the individual will feel that replacement of θ with the equivalent commodity is the least-cost method of making himself "whole" following the loss of θ. Some other commodity (or even some extensive change in his asset holdings) may accomplish this purpose at less cost. The cost of restoring himself following the loss of θ may then change with changes in his wealth simply because the least cost combination of commodities required to restore him to his pre-loss utility level may depend on his other asset holdings.

For example, consider a woman who inherits a diamond brooch that she could sell for $200, but that retails for $500 (i.e., she could buy another brooch she perceives as equivalent for that amount). Her personal valuation of the brooch will most likely lie somewhere strictly between these two numbers, implying that if the brooch were stolen she would not buy another one even if she received $500 compensation for the loss. Thus, although the brooch is replaceable in one sense, she would not actually choose to replace it. We have limited the definition of replaceability to those commodities that both *can and would* be replaced when their loss is fully compensated. Perhaps the most interesting applications of the theory, however, are to commodities for which it is highly unlikely that any equivalent commodities are available (e.g., commodities whose value depends largely on sentiment).

erwise, θ will be denoted "replaceable." Note that C is necessarily fixed by a market price if θ is replaceable, but may vary with the owner's wealth if it is irreplaceable. The remainder of this section presents a somewhat more formal discussion of this wealth effect, as a prelude to next section's discussion of the demand for insurance.

Suppose that an individual faces two states of the world: in state b the commodity in question is kept; in state a it is lost. We assume that the individual's preferences can be represented by the von Neumann-Morgenstern utility function

$$U(W, \theta),$$

where W represents a composite commodity involving all goods other than the commodity in question and is measured in dollars, and where θ (an indicator for the given commodity) equals zero in state a and one in state b. For simplicity we define

$$U_a(W) \equiv U(W, 0)$$

$$U_b(W) \equiv U(W, 1)$$

and assume that for all $W \geqq 0$

$$U_a(W) < U_b(W)$$

and

$$U_i''(W) < 0 < U_i'(W) \ i = a, b.$$

How much is the commodity worth to the individual? As previously discussed, one measure is the minimum compensation (selling price) that would induce the individual to accept a certainty of state a in exchange for a certainty of state b. This compensation $C(W)$ is defined by

$$U_a(W + C(W)) = U_b(W),$$

provided that such a $C(W)$ exists and by $C(W) = \infty$ otherwise.[7]

Alternatively, the value of the commodity could be expressed as the maximum amount the individual would be willing to pay to exchange a certainty of state a for a certainty of state b. This *ransom* $R(W)$ is defined by

$$U_a(W) = U_b(W - R(W)),$$

7. As Mishan, *op. cit.*, p. 693, footnote, points out, $C(W)$ may be finite for small values of W and infinite for others where the loss of life is involved: "if a man and his family were so destitute and their prospects so hopeless that one or more members were likely to die of starvation, or at least suffer from acute deprivation, then the man might well be persuaded to sacrifice himself for the sake of his family. But without dependents or close and needy friends, the inducement to sacrifice himself for others is not strong."
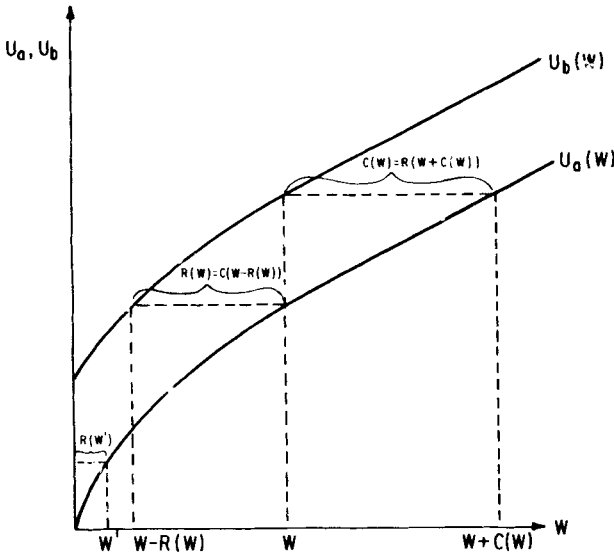
FIGURE I

provided that such an $R(W)$ exists and by $R(W) = W$ otherwise.[8]

Of course compensation and ransom differ by only a wealth effect, since

(1) $$C(W - R(W)) = R(W)$$

(2) $$C(W) = R(W + C(W))$$

for all W such that the defining equalities hold. There is thus no loss of generality in focusing the analysis upon the ransom value of the commodity. These relationships are illustrated in Figure I.

Notice that the derivative of $R(W)$ is given by

(3) $$R'(W) = 1 - \frac{U_a'(W)}{U_b'[W - R(W)]}.$$

The irreplaceable commodity can be classified as normal or inferior, respectively, according to whether $R'(W)$ is positive or negative.[9] Thus,

(4) $$U_a'(W) < U_b'(W - R(W))$$

8. The fact that $U_b(W) > U_a(W)$ and $U_a'(W)$ and $U_b'(W) > 0$ implies that $R(W)$ is uniquely defined and positive for all $W > 0$.

9. This terminology conforms to conventional usage: if a commodity is normal in the sense that an increase in wealth, *ceteris paribus*, entails an increase in the consumption of the commodity, then it is also necessarily true that the maximum amount an individual would pay for a *given amount of the commodity* increases with wealth. It is our impression that most irreplaceable commodities are normal for most owners. In fact, we have not been able to think of clear examples of an inferior commodity.

if the irreplaceable commodity is normal, and

(5) $$U_a'(W) > U_b'(W - R(W))$$

if it is inferior. Moreover, given this ransom concept, a replaceable commodity can be viewed as a special case of an irreplaceable commodity in which the wealth effect $R'(W)$ is zero and thus for which

(6) $$U_a'(W) = U_b'(W - R(W)).$$

Last, notice that[10]

$$C(W) \gtreqless R(W) \quad as \quad R'(W) \lesseqgtr 0.$$

INSURING IRREPLACEABLE COMMODITIES

In what way will the risk-averse individual's demand for insurance coverage of an irreplaceable commodity differ from his demand for insurance coverage of a replaceable commodity? A defining characteristic of a risk-averse individual is that he will insure fully against the loss of a replaceable commodity if he is able to buy any actuarially fair policy (i.e., he will exchange any risky portfolio of replaceable assets for a riskless portfolio of the same expected value if given the opportunity): surprisingly, he will buy less than full coverage for a normal irreplaceable commodity if he can buy insurance at actuarially fair rates.

With "fair" transfers of wealth between states available, the individual's budget constraint is

(7) $$W \equiv p\overline{W}_a + (1 - p)\overline{W}_b = pW_a + (1 - p)W_b,$$

where p is the probability of state a, $\overline{W}_i$ is endowed wealth in state $i, i = a, b$, and W_i is the financial claim contingent upon state i to be purchased, $i = a, b$.[11] The utility maximization problem[12] is then

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10. This follows from the fact, for example, that if $R'(W) > 0$, then $R(W) < R[W + C(W)] = C(W)$.

11. Note that endowed wealth is not assumed to be the same in both states. For example, if the commodity in question were the individual's right arm, $\overline{W}_b$ would be greater than $\overline{W}_a$ by an amount equal to the loss in earning potential associated with the loss of the arm.

12. A rigorous justification for an expected utility approach in a state-dependent utility framework is presented in P. Fishburn, "On the Foundations of Decision Making Under Uncertainty," in M. Balch, D. McFadden, and S. Wu, eds., *Essays on Economic Behavior Under Uncertainty* (Amsterdam: North Holland, 1974), pp. 25–44. We do not require that the loss of θ be the only risk faced by the individual. To see this, let Y be a random variable representing fluctuations in wealth from sources other than the risk involved in the loss of θ, where $E_Y Y$, the expectation over the distribution of the random variable Y, is zero. Then we may replace $U_a(W)$ with $\overline{U}_a(W) \equiv E_Y U_a(W + Y)$ and $U_b(W)$ with $\overline{U}_b(W) \equiv E_Y U_b(W + Y)$. These new expressions have the same concavity properties assumed for the original expressions. We are indebted to John Marshall for this observation.

$$\max_{W_a, W_b} pU_a(W_a) + (1 - p)U_b(W_b),$$

subject to equation (7). It is necessary and (given our assumptions) sufficient for W_a^*, $W_b^* > 0$ to be a solution that

$$(8) \qquad U_a'(W_a^*) = U_b'(W_b^*).$$

The solutions W_a^* and $W_b^* = (W - pW_a^*)/(1 - p)$ are unique and have a close correspondence to the value of $R'(W)$. A number of possibilities exist. If $R'(W_a^*) = 0$, then equations (6) and (8) imply that $W_b^* = W_a^* - R(W_a^*)$ or that

$$(9) \qquad W_a^* - W_b^* = R(W_a^*).$$

The individual has insured himself fully ($U_a(W_a^*) = U_b(W_b^*)$) and is indifferent as to which state of the world occurs. This, of course, is the case corresponding to a replaceable commodity.

If the irreplaceable commodity is normal (if $R'(W_a^*)$ is positive), then equations (4) and (8) imply that $W_b^* > W_a^* - R(W_a^*)$ or that

$$(10) \qquad W_a^* - W_b^* < R(W_a^*).$$

In this case the individual stops short of full insurance ($U_a(W_a^*) < U_b(W_b^*)$) and prefers the occurrence of state b to state a. If the wealth effect is sufficiently large, and in particular if

$$R'(W_a^*) = 1 - U_b'(W_a^*)/U_b'(W_a^* - R(W_a^*)),$$

then $U_a'(W_a^*) = U_b'(W_a^*)$, and the individual will purchase insurance against only the financial loss associated with the loss of the commodity ($W_a^* = W_b^*$).[13] Still larger values of $R'(W)$ would be associated with less than complete insurance against even the financial loss; he may buy no insurance whatever, or even bet on the occurrence of state b (e.g., with the purchase of an annuity rather than the life insurance).[14] Examples illustrating these possibilities are examined in the Appendix.

Last, if the irreplaceable commodity is inferior, then equations (5) and (8) imply that $W_b^* < W_a^* - R(W_a^*)$ or that

13. For example, a household would demand life insurance on each household member equal to the net financial contribution of that person to the household (plus funeral expenses). The demand for insurance on a dependent child would be negative.

14. A fan's decision to bet for or against his favorite sports team may thus depend on how much the team's winning is worth to him and how his value changes with his wealth; his decision does not necessarily reflect his assessment of the odds. This possibility suggests that two risk-averse people who agree on the probability of an event may find it profitable to bet with each other as long as at least one of them cares about the outcome itself. Ordinarily, a fan would be expected to bet against his own team, but he will bet for his team if the wealth effect is large enough.
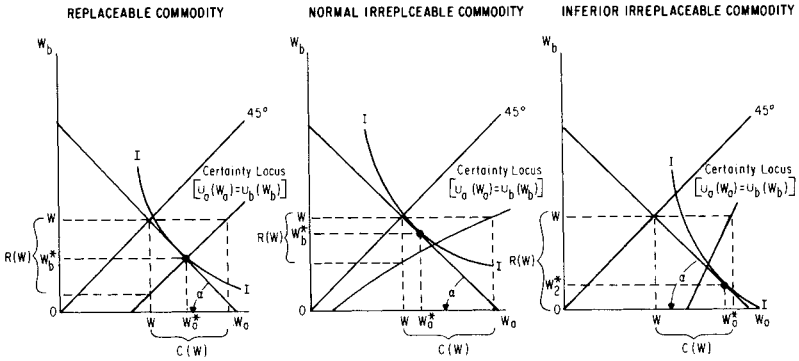
REPLACEABLE COMMODITY   NORMAL IRREPLCEABLE COMMODITY   INFERIOR IRREPLACEABLE COMMODITY

FIGURE IIa
Replaceable Commodity

FIGURE IIb
Normal Irreplaceable
Commodity

FIGURE IIc
Inferior Irreplaceable
Commodity

(11) $W_a^* - W_b^* > R(W_a^*).$

Here the individual overinsures $(U_a(W_a^*) > U_b(W_b^*))$ attaining a position in which state a is the preferred outcome.

These possibilities are illustrated in Figures IIa through IIc. In all cases the slope of the budget line is given by

$$\tan \alpha = p/(1 - p),$$

and the indifference curve corresponding to the best insurance purchase is labeled I. The certainty locus represents arrangements of contingent claims under which the individual would be indifferent as to which state of the world occurs and which therefore may be regarded as riskless. Since the horizontal distance from the 45° line to the certainty locus corresponds to R, this distance will increase if $R'(W)$ is positive and fall if $R'(W)$ is negative. The tangency of an indifference curve to the fair bet budget line, which necessarily characterizes the optimal insurance purchase, must occur to the left of the certainty locus if the irreplaceable commodity is normal, and to the right if it is inferior.

Before making behavioral predictions from this theoretical analysis, it should be emphasized that it is directly applicable only to the individual's *demand* for insurance when the probability of loss p is determined exogenously and can be costlessly monitored by all parties. Actual insurance transactions are of course influenced by the insurer's perception of the moral hazard created by certain contractual arrangements. For this reason, and because (contrary to our assumption of a "fair" price) insurance is usually available only with

a positive "loading," the actual quantity of insurance purchased on the private market for an irreplaceable commodity will tend to be *less* than the quantity demanded under our assumptions.[15] A normative implication of our analysis follows from the reasonable assumption that life, good health, the absence of pain and suffering, etc. are "normal" irreplaceable commodities: the goal of full compensation to victims of violent crime or accidents that result in injury or death is not compatible with economic efficiency (since the settlements will be "too high"). This conclusion is strengthened by the fact that administering programs for criminal victim compensation, workmen's compensation, and other tort settlements is costly (i.e., there is a large positive loading on these types of "insurance").

THE VALUE OF COLLECTIVE PROTECTION

The individual probability of loss in many cases is influenced by public activities such as law enforcement, highway design, and medical research. Investments in such areas produce a good (reductions in the probability of loss for each of a number of individuals) that, from an efficiency point of view, should be valued at an amount equal to the sum of the resulting benefits accruing to individuals.

E. Mishan points out that the appropriate individual benefit measure in such cases is the "compensating variation" in wealth: the reduction in the individual's wealth, which, when coupled with a reduction in the probability of loss, leaves him at the same (expected) utility level.[16]

Mishan does not mention what we are to assume about the existence and nature of contingency markets in calculating this benefit measure, although this issue is clearly salient; note Zeckhauser's example of the community that, because it lacks fire insurance, finds it worthwhile to rent a fire engine for $12,000 a year to prevent one $10,000 fire a year.[17] (If the community could organize an insurance pool, the benefit of the fire engine would be only $10,000.) We think it theoretically appropriate and useful to identify the *pure protection benefit* of a proposed public investment as its value when fair transfers of wealth between states are possible. An investment may have some

15. For example, if the premium for a payment of I dollars contingent on state a is βI, then it can be shown that the demand for insurance falls as β increases. It should be noted that the presumption that β is typically greater than $p/(1 - p)$ (the fair rate) is not necessarily correct, since p is the individual's *perception* of the probability of loss and may be an exaggeration of the actuarial probability.
16. *Op. cit.*, pp. 691–95.
17. "Resource Allocation with Probabilistic Individual Preferences," *American Economic Review*, LIX (May 1969), 546–52.

additional benefit (as in Zeckhauser's example) if the project is un-
dertaken in the context of imperfect contingency markets; this type
of benefit results from changes with respect to the efficiency of the
distribution of wealth among states of the world, and is conceptually
distinct from changes in the expected value of wealth.[18] The pure
protection benefit of a reduction in p is then defined as the maximum
expected payment made by the individual, which, when coupled with
the reduction in p, leaves the individual's expected utility unchanged.
This benefit measure is the same whether (1) the individual is viewed
as contracting to make *identical* payments in states a and b and then
adjusting his insurance coverage appropriately; or (2) the individual
is viewed as contracting for payments in the two states that are chosen
so as to leave him in equilibrium. (In the latter case the payments will
in general be different, and one "payment" may even be negative.)
We employ the latter definition in the analysis that follows.

Suppose that an individual is initially endowed with expected
wealth $\overline{W}$ and a probability of loss $\overline{p}$. This endowment is one point
on the indifference curve depicting the trade-off between expected
wealth $W(p)$ and the probability of loss, where

$$(12) \qquad W(p) \equiv p W_a^*(p) + (1 - p) W_b^*(p),$$

such that

$$(13) \qquad U_a'(W_a^*(p)) = U_b'(W_b^*(p)),$$

and

$$(14) \quad p U_a(W_a^*(p)) + (1 - p) U_b(W_b^*(p))$$
$$= \text{expected utility of the endowment.}$$

Equation (12) defines expected cost of that bundle $(W_a^*(p), W_b^*(p))$,
which would be purchased at fair odds (equation (13)) and which
yields the same expected utility as the endowment bundle (equation
(14)).

18. The value of any public investment in protection can be analyzed as the al-
gebraic sum of (1) the value of moving from the initial wealth distribution to an efficient
risk distribution of wealth; (2) the pure protection benefit of the investment; and (3)
the cost of financing the investment inefficiently if the postinvestment distribution
of wealth is inefficient. This trichotomy provides a useful framework in which to
evaluate the financing scheme associated with the project, as distinct from the activity
that produces greater protection. Such an analysis would perhaps facilitate a quest
for more efficient financing mechanisms. It should be pointed out that this analysis
implicitly assumes that the law of large numbers applies to the losses we are dis-
cussing—there is a large group of potential insurers whose aggregate wealth is invariant
over states of the world. Some such assumptions are necessary to guarantee even the
theoretical possibility of actuarially fair insurance. See J. M. Marshall, "Insurance
Theory: Reserves Versus Mutuality," *Western Economic Journal*, XII (Dec. 1974),
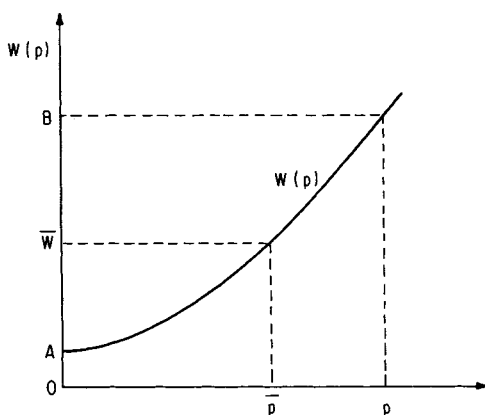476–92.

FIGURE III

The indifference curve $W(p)$ is illustrated in Figure III.[19] The shape of this curve is dictated by the conditions (derived below) that $0 \leq W'(p)$, $W''(p)$. The slope of the indifference curve at $(\overline{p}, \overline{W})$ can be regarded as the *value of the commodity* for purposes of calculating the value of small reductions in p. That is, if we define

$$V \equiv W'(\overline{p}),$$

then the value of a 0.01 reduction in the probability of loss is approximately 0.01 V.[20]

Differentiation of (12), (13), and (14) with respect to p yields (after simplifying)

(15) $\qquad V = W_a^* - W_b^* + (U_b(W_b^*) - U_a(W_a^*))/U_b'(W_b^*).$

In the special (and conventional) case that the commodity is replaceable, $R'(W)$ equals zero, $U_a(W_a^*) = U_b(W_b^*)$, and

$$V = W_a^* - W_b^*.$$

19. Figure III can be related to the results of previous sections. If the endowment wealth were OB, for example, then AB would be the ransom value of the commodity. Alternatively, if endowment wealth were OA, then AB would give the compensation value of the commodity. For the case in which the endowment is given by $(\overline{p}, \overline{W})$, the fact that the indifference curve has a positive vertical intercept means that there is a positive certain prospect of wealth in state b that the individual would consider equivalent to his uncertain endowment. Alternatively, the individual would not bankrupt himself to buy a certainty of state b. See the related result in Jones-Lee, *op. cit.* The case in which $C = \infty$ corresponds to the indifference curve being asymptotic to the vertical line at $p = 1$.

20. Conversely, V can be approximated as one hundred times the amount the individual would be willing to pay for a 0.01 reduction in the probability of state a.

Here the price appropriate for calculating the value of probability reductions can be inferred directly from knowledge of the amount of insurance that the individual would buy at fair odds.[21] For the case in which the commodity is irreplaceable and normal, the amount of insurance that would be purchased at fair odds, $W_a^* - W_b^*$ understates the correct V, since $(U_b(W_b^*) - U_a(W_a^*))/U_b'(W_b^*)$ is positive in such circumstances.[22]

In general, V is bounded above and below by two other measures of the value of the irreplaceable commodity—R and C. This observation is demonstrated for the normal case $(U_a(W_a^*) < U_b(W_b^*))$ as follows: If $C(W_b^*)$ is finite,

$$U_b(W_b^*) = U_a(W_b^* + C(W_b^*))$$

$$< U_a(W_a^*) + (W_b^* + C(W_b^*) - W_a^*)U_a'(W_a^*)$$

from the definition of $C(W_b^*)$ and concavity of $U_a(W_a^*)$. Substituting in (15) and (13) yields

$$V < C(W_b^*),$$

which holds trivially for $C(W_b^*) = \infty$ as well. Similarly,

$$U_a(W_a^*) = U_b(W_a^* - R(W_a^*))$$

$$< U_b(W_b^*) + (W_a^* - R(W_a^*) - W_b^*)U_b'(W_b^*),$$

together with (15) and (13) gives

$$V > R(W_a^*).$$

21. Notice that $W_a^* - W$ represents the market value or price of a replaceable commodity that is subject to a probability loss p, provided that a fair insurance market exists. (I.e., $W_a^* - W$ equals the maximum amount an individual with wealth W and a certainty of state a would pay for a lottery involving a probability p of state a, probability $(1 - p)$ of state b, and an ability to make fair bets.) This price is, of course, "discounted" by the probability of loss and is less than the ransom value of the same commodity $R(W) = W_a^* - W_b^*$, which represents the purchase price of a replaceable commodity that is subject to a zero probability of loss. It is this ransom value or undiscounted price *and not the market price* that is appropriate for valuing probability reductions.

22. Since R and C are nonnegative, this establishes the fact that $W'(p) = V > 0$. $W''(p) \geq 0$ is obtained by differentiation of (15) with respect to p and simplification:

$$W''(p) = - [U_b(W_b^*) - U_a(W_a^*)]U_b''(W_b^*)W_b^{*'}(p)/U_b'(W_b^*)^2.$$

or

$$\text{sign } W''(p) = \text{sign } [U_b(W_b^*) - U_a(W_a^*)] \cdot \text{sign } [W_b^{*'}(p)].$$

But from (13) and (14) sign $[W_b^{*'}(p)] = \text{sign } [W_a^{*'}(p)] = \text{sign } [U_b(W_b^*) - U_a(W_a^*)]$.

Thus, sign $W''(p) = \text{sign } [U_b(W_b^*) - U_a(W_a^*)]^2 > 0$.

Combining results for the case of normal irreplaceable commodity, we have[23]

(16) $$W_a^* - W_b^* < R(W_a^*) < V < C(W_b^*).$$

Equation (10) suggests that in general it is necessary for calculating the value of collective protection that one have knowledge not only of the amount of insurance the individual would purchase at fair odds but also of the ransom function of the individual $R(W)$.[24]

    While a complete analysis of the issue would require introducing considerations that are well beyond the scope of this paper, it is interesting to note that our conclusion that $V < C(W_b^*)$ adds a second dimension to our argument that full compensation is an inefficient policy for tort settlements that involve irreplaceable commodities. Tort law provides an incentive for private firms and individuals to invest in reducing the probability of becoming responsible for an injury; if courts typically award "full compensation" settlements, then this incentive is "too strong" in the sense that induced investments in safety will be larger than the efficient level.

APPENDIX

    We present here examples of utility functions for states a and b for which the irreplaceable commodity is normal. In all cases $b > a > 0$. Example 1 (linear utility functions) has the property that, given fair odds, the individual would bet everything on state b. In this case $R(W_a^*) = V = 0$; the value of the irreplaceable commodity for purposes of calculating the value of reductions in the probability of loss is zero despite the fact that the individual who has placed the appropriate fair bet still prefers the occurrence of state b to that of state a. The less advantageous terms at which bets could be placed following a probability reduction has a cost to the individual equal to the benefit of the probability reduction.

    In example 2 the individual purchases insurance against the loss of the irreplaceable commodity for values of $W > [1/(b - a)] \ln b/a$.

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23. Should the utility functions be concave rather than strictly concave, i.e., $U_i'(W) \leqq 0$, $i = a, b$, then the strict inequalities in this expression would be replaced with weak inequalities. For the case of a replaceable commodity, $U_a(W_a^*) = U_b(W_b^*)$, and $W_a^* - W_b^* = R(W_a^*) = V = C(W_b^*)$. The case of an inferior irreplaceable commodity is characterized by $U_a(W_a^*) > U_b(W_b^*)$ and $W_a^* - W_b^* > R(W_a^*) > V > C(W_b^*)$. As in the normal case these inequalities would be weakened if concavity rather than strict concavity were assumed.

24. From a knowledge of $R(W)$ one can obtain $Z(W) \equiv W - R(W)$ and thus $C(Z)$, since by equation (1), $C(W - R(W)) = R(W)$, provided that the defining equality holds. Moreover, since $Z'(W) > 0$ if $0 < R'(W) < 1$, one can solve $Z(\hat{W}) = W_b$ uniquely for $\hat{W}$ and thus obtain $C(W_b) = R(\hat{W})$. In this case knowledge of $R(W)$ is sufficient to bracket V.

APPENDIX

Example	$U_a(W)$	$U_b(W)$	$R(W)$	$C(W)$	Fair bet solutions
1	aW	bW	$(b-a)W/b$	$(b-a)W/a$	$W_a^* = 0$ $W_b^* = W/(1-p)$
2	$-\exp(-aW)$	$-\exp(-bW)$	$(b-a)W/b$	$(b-a)W/a$	$W_b^* = a/b \; W_a^* +$ $\ln \; b/a$
3	$\ln \; aW$	$\ln \; bW$	$(b-a)W/b$	$(b-a)W/a$	$W_b^* = W_a^*$
4	$a \ln W$	$b \ln W$	$W-W^{a/b}$	$W^{a/b} - W$	$W_b^* = b/a \; W_a^*$

In example 3 he purchases no insurance against the loss of the irreplaceable commodity, and in example 4 he bets on the occurrence of state b. In any of these cases, state a may entail a pure financial loss in addition to the loss of the irreplaceable commodity; in this case we can view him as initially buying full insurance against the financial loss and then modifying his risk position according to the rules specified above.

Analytic solutions for V can be calculated for examples 2–4, but they are complex and not particularly enlightening.

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