

The Demand for Money During  
Hyperinflations Under Rational  
Expectations: II

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The Demand for Money During Hyperinflations  
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by

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## Introduction

This paper describes estimators of portfolio balance schedules of the form assumed by Phillip Cagan in his famous study of hyperinflation. Under suitable restrictions on the disturbance process, the estimators are statistically consistent under the circumstance that the public's expectations of inflation are rational in the sense of John F. Muth. Except in the special case where the money supply process obeys certain restrictions delineated in my earlier paper [ ], it will not be rational for the public to form its expectations using the adaptive expectations scheme assumed by Cagan. Under adaptive expectations, people ignore past rates of money creation in forecasting inflation, even though where Cagan's portfolio balance schedule prevails, past values of money can typically be used to forecast inflation under most monetary regimes. Interestingly enough, the estimators described here break down in the singular case in which Cagan's adaptive expectations mechanism is rational.

From a technical viewpoint, the model studied here is interesting because it provides an example of a case in which a key structural parameter is identified from restrictions that the model imposes on the systematic part of the vector autoregressive representation of the system. This circumstance is not common in econometrics, but does characterize a class of structures in which the public's expectations of future variables enter and in which the public's expectations are rational. It turns out that the vector autoregression fails to identify the slope parameter in the portfolio balance schedule in the special case that Cagan's adaptive expectations scheme is rational. That is what causes the estimators here to break down for that case, and causes resorting to estimators like those described in my earlier paper [ ].

The Model: Identification

I suppose that the portfolio balance schedule has the same form that Cagan assumed,

$$(1) \quad m_t - p_t = \alpha \pi_t + \varepsilon_t$$

where  $\varepsilon_t$  is a random variable with certain important properties to be specified shortly, and where  $m_t$  is the natural logarithm of the money supply,  $p_t$  is the natural log of the price level, and  $\pi_t$  is the public's current expectation of the rate of inflation next period, that is, the public's expectation at time  $t$  of  $p_{t+1} - p_t$ . I assume that the public's expectations are rational, which amounts to imposing<sup>1/</sup>

$$(2) \quad \pi_t = E_t(p_{t+1} - p_t)$$

or

$$\pi_t = E_t x_{t+1}$$

where  $x_{t+1} \equiv p_{t+1} - p_t$ , and where  $E_t x_{t+1} \equiv E[x_{t+1} | x_t, x_{t-1}, \dots, \mu_t, \mu_{t-1}, \dots]$ , where  $\mu_t \equiv m_t - m_{t-1}$ . Let  $\eta_t = \varepsilon_t - \varepsilon_{t-1}$ . Then substituting (2) into (1) and first differencing gives

$$(m_t - m_{t-1}) - (p_t - p_{t-1}) = \alpha(E_t x_{t+1} - E_{t-1} x_t) + \varepsilon_t - \varepsilon_{t-1}$$

or

$$(3) \quad \mu_t - x_t = \alpha(E_t x_{t+1} - E_{t-1} x_t) + \eta_t.$$

I now assume that  $\eta_t$  is serially independent with mean zero, that is

$$(4) \quad E_{t-1} \eta_t \equiv E[\eta_t | x_{t-1}, x_{t-2}, \dots, \mu_{t-1}, \mu_{t-2}, \dots] = 0.$$

(As indicated below, assumption (4) can be relaxed to permit  $\eta_t$  to follow a low order Markov process.) Assumption (4) implies that  $\eta_t$  is orthogonal to (i.e., uncorrelated with) lagged  $x$ 's and  $\mu$ 's, but permits  $\eta_t$  to be correlated with current and future values of  $x$  and  $\mu$ . Indeed, the sense of the model is that movements in  $\eta_t$ , which is the first difference of the disturbance in the demand function for money, cause responses in current and future values of the price level (and maybe the money supply too if the monetary authority is causing  $m$  to display feedback from  $p$ --as seemed to be the case in several hyperinflations).

Using assumption (4) and taking expectations in (3) conditional on information known at time  $(t-1)$  gives

$$(5) \quad E_{t-1}\mu_t - E_{t-1}x_t = \alpha(E_{t-1}x_{t+1} - E_{t-1}x_t)$$

where I am using the fact that  $E_{t-1}(E_t x_{t+1}) = E_{t-1}x_{t+1}$ . Furthermore, letting  $\theta$  be any subset of  $(x_{t-1}, x_{t-2}, \dots, \mu_{t-1}, \mu_{t-2}, \dots)$ , and taking expectations in (3) conditional on  $\theta$  gives

$$(6) \quad E(\mu_t - x_t) | \theta = \alpha E(x_{t+1} - x_t) | \theta,$$

since (4) implies that  $E\eta_t | \theta = 0$  for  $\theta$  any subset of  $(x_{t-1}, \dots, \mu_{t-1}, \dots)$ .

Now equation (5) is a (nonlinear) restriction on the systematic part of the vector autoregressive representation of the  $(x, \mu)$  process. It is a restriction which in general involves the parameter  $\alpha$  in a nontrivial way. Indeed, the restriction (5) or (6) in general identifies  $\alpha$ , and as will be seen, provides the basis for various possible methods of estimating  $\alpha$  consistently. Before proceeding to these methods, however, we take note of the fact that there is one singular case in

which  $\alpha$  is not identified by restriction (5). This is the case in which  $x_t$  is governed by the process

$$(7) \quad x_t = \frac{1-\lambda}{1-\lambda L} x_{t-1} + \xi_t$$

where  $E[\xi_t | x_{t-1}, x_{t-2}, \dots, \mu_{t-1}, \mu_{t-2}, \dots] = 0$ . Under (7), we have

$$E_{t-1} x_t = \frac{1-\lambda}{1-\lambda L} x_{t-1},$$

so that Cagan's assumption of adaptive expectations coincides with expectations being "rational." Furthermore, as Muth pointed out, under (7) we have

$$E_{t-1} x_{t+j} = E_{t-1} x_t = \frac{1-\lambda}{1-\lambda L} x_{t-1} \quad \text{for all } j \geq 1.$$

Consequently under (7), we have

$$E_{t-1} x_{t+1} - E_{t-1} x_t = 0$$

for all  $t$ . Under this circumstance, (5) becomes

$$(5') \quad E_{t-1} \mu_t - E_{t-1} x_t = 0,$$

which is a restriction on the systematic part of the vector autoregressive representation of the  $(x, \mu)$  process, but one that does not involve  $\alpha$ . Therefore, restriction (5') does not identify  $\alpha$ . This result is consistent with the findings of my previous paper [ ] in which I showed that under the circumstances in which Cagan's adaptive expectations mechanism is rational,  $\alpha$  is not identified even from knowledge of the entire vector autoregressive representation of the  $(x, \mu)$  process--which includes the systematic part and the covariance matrix of the innovations. That earlier finding implies the present result, since the present

result only considers the implications for identification of knowing a subset of the information assumed in the earlier paper. Throwing away information obviously cannot cure underidentification.

That a structural parameter  $\alpha$  is in general identified by restriction (5) on the systematic part of the vector autoregression is a special circumstance not encountered in usual macroeconomic applications, where identification of structural parameters almost always requires restrictions on the covariance matrix of innovations and on the matrix of contemporaneous structural coefficients. There are two special features of the present model that permit only the systematic part of the vector autoregression to identify the structural parameter  $\alpha$ . The first is the feature that in (1)  $m_t - p_t$  is posited to vary with the public's expectation of inflation one period forward. The second is that the public's expectations are assumed to be rational. Applications that share these two features regarding timing and rationality will often be associated with implied vector autoregressions, the systematic parts of which carry identifying restrictions.

#### Nonlinear Estimation of $\alpha$

To derive the restrictions on the vector autoregression implied by (5), assume quite generally that

$$(8) \quad E_t x_{t+1} = v(L)x_t + h(L)\mu_t$$

where  $v(L) = v_0 + v_1L + \dots$

$$h(L) = h_0 + h_1L + \dots$$

Substituting (8) into (3) gives

$$\mu_t - x_t = \alpha v(L)x_t + \alpha h(L)\mu_t - \alpha v(L)Lx_t - \alpha h(L)L\mu_t + \eta_t$$

or

$$[1 - \alpha h(L)(1-L)]\mu_t = [1 + \alpha v(L)(1-L)]x_t + \eta_t.$$

This can be written

$$\begin{aligned} (1 - \alpha h_0)\mu_t &= \alpha(h_1 - h_0)\mu_{t-1} + \alpha(h_2 - h_1)\mu_{t-2} + \dots \\ &+ (1 + \alpha v_0)x_t + \alpha(v_1 - v_0)x_{t-1} + \alpha(v_2 - v_1)x_{t-2} + \dots + \eta_t \end{aligned}$$

Substituting  $E_{t-1}\eta_t = 0$  and  $E_{t-1}x_t = v(L)x_{t-1} + h(L)x_{t-1}$  gives

$$\begin{aligned} (1 - \alpha h_0)E_{t-1}\mu_t &= \alpha(h_1 - h_0)\mu_{t-1} + \alpha(h_2 - h_1)\mu_{t-2} + \dots \\ &+ \alpha(v_1 - v_0)x_{t-1} + \alpha(v_2 - v_1)x_{t-2} + \dots \\ &+ (1 + \alpha v_0)(v(L)x_{t-1} + h(L)\mu_{t-1}) \\ &= (\alpha(v_1 - v_0) + (1 + \alpha v_0)v_0)x_{t-1} \\ &+ (\alpha(v_2 - v_1) + (1 + \alpha v_0)v_1)x_{t-2} + \dots \\ &+ (\alpha(h_1 - h_0) + (1 + \alpha v_0)h_0)\mu_{t-1} + \\ &+ (\alpha(h_2 - h_1) + (1 + \alpha v_0)h_1)\mu_{t-2} + \dots \end{aligned}$$



This can be written as

$$(9) \quad (1-\alpha h_0)E_{t-1}\mu_t = \{-L^{-1}\alpha v_0 + L^{-1}\alpha(1-L)v(L) + (1+\alpha v_0)v(L)\}x_{t-1} \\ + \{-L^{-1}\alpha h_0 + L^{-1}\alpha(1-L)h(L) + (1+\alpha v_0)h(L)\}\mu_{t-1}.$$

Equations (8) and (9) embody the restrictions across  $E_{t-1}x_t$  and  $E_{t-1}\mu_t$  implied by equation (5).

A way to estimate  $\alpha$  can now be advanced. Write

$$x_t = E_{t-1}x_t + a_{xt}$$

$$\mu_t = E_{t-1}\mu_t + a_{\mu t}$$

where  $a_{xt}$  is the innovation in  $x$  (the part of  $x$  that can't be predicted using lagged  $x$ 's and  $\mu$ 's) and  $a_{\mu t}$  is the innovation in  $\mu$ . Where  $E_{t-1}x_t$  and  $E_{t-1}\mu_t$  are given by (8) and (9), respectively, we have

$$(8') \quad x_t = h(L)x_{t-1} + v(L)\mu_{t-1} + a_{xt}$$

$$(9') \quad \mu_t = \frac{1}{1-\alpha h_0} \{-L^{-1}\alpha v_0 + L^{-1}\alpha(1-L)v(L) + (1+\alpha v_0)v(L)\}x_{t-1} \\ + \frac{1}{1-\alpha h_0} \{-L^{-1}\alpha h_0 + L^{-1}\alpha(1-L)h(L) + (1+\alpha v_0)h(L)\}\mu_{t-1} + a_{\mu t}.$$

On the assumption that  $(a_{xt}, a_{\mu t})$  has a bivariate normal distribution, maximum likelihood estimates of the  $h$ 's,  $v$ 's and  $\alpha$  under restriction

(5) can be obtained by minimizing

$$(10) \quad \left| \begin{array}{cc} \sum \hat{a}_{xt}^2 & \sum \hat{a}_{xt} \hat{a}_{\mu t} \\ \sum \hat{a}_{xt} \hat{a}_{\mu t} & \sum \hat{a}_{\mu t}^2 \end{array} \right|$$

subject to (8') and (9').

While this procedure is more efficient asymptotically than the procedures to be implemented below, its drawback is that the restrictions across (8') and (9') are highly nonlinear, so that minimizing (10) under (8') and (9') is computationally very difficult.

A computationally feasible variant of the above procedure can be obtained by using (6) and adopting a convenient parameterization of the  $(x, \mu)$  process. Define  $y_t \equiv \mu_t - x_t$ , and consider the restricted moving average autoregressive representation

$$\begin{bmatrix} y_t \\ x_{t+1} - x_t \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} y_{t-2} \\ x_{t-2} \end{bmatrix} \\ + \begin{bmatrix} a_{xt} \\ a_{\mu t} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a_{xt-1} \\ a_{\mu t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} a_{xt-2} \\ a_{\mu t-2} \end{bmatrix}$$

where  $E_{t-1} a_{xt} = E_{t-1} a_{\mu t} = 0$ . Under (11) we have

$$E_{t-1} y_t = B_{11} y_{t-1} + B_{12} x_{t-1} + \gamma_{11} a_{xt-1} + \gamma_{12} a_{\mu t-1}$$

$$E_{t-1} (x_{t+1} - x_t) = B_{21} y_{t-1} + B_{22} x_{t-1} + \gamma_{21} a_{xt-1} + \gamma_{22} a_{\mu t-1} .$$

In the above equations, restriction (6) implies

$$(12) \quad B_{11} = \alpha B_{21}, \quad \gamma_{11} = \alpha \gamma_{21}$$

$$B_{12} = \alpha B_{22}, \quad \gamma_{12} = \alpha \gamma_{22}$$

For fixed  $\alpha$ , maximum likelihood estimates of the B's and  $\gamma$ 's can be obtained by minimizing (10) under (11) and (12). The algorithm of Wilson [ ], for example, could be used to do this. Searching over  $\alpha$  for the value that minimizes the determinant (10) under (11) and (12) would then give maximum likelihood estimates under the restrictions (11) and (12).

A Two-Step Procedure

Equation (6) suggests the following two-step estimator of  $\alpha$ , which is statistically consistent. First, where  $\theta$  is any subset of information available at  $t-1$ --say some subset of lagged  $\mu$ 's and  $x$ 's, a constant, and a trend--regress  $x_{t+1}-x_t$  on  $\theta$  in the sample period and call the systematic part  $\hat{E}_{t-1}(x_{t+1}-x_t)$ . In the second step, compute the regression of  $\mu_t-x_t$  against  $\hat{E}_{t-1}(x_{t+1}-x_t)$ . (In practice, a constant and trend are also included in this regression.) Adopt the decompositions

$$E_t x_{t+1} - E_{t-1} x_t = (E_{t-1} x_{t+1} - E_{t-1} x_t) + \Psi_t \quad (E_{t-1} \Psi_t = 0)$$

$$E_{t-1} x_{t+1} - E_{t-1} x_t \equiv \hat{E}_{t-1}(x_{t+1}-x_t) + \xi_t \quad (E \xi_t | \theta = 0).$$

Substituting these two equalities into (5) gives

$$(13) \quad \mu_t - x_t = \alpha \hat{E}_{t-1}(x_{t+1}-x_t) + \eta_t + \alpha \Psi_t + \alpha \xi_t,$$

where  $\Psi_t$ , being in the nature of an innovation, obeys  $E[\Psi_t | x_{t-1}, \dots, \mu_{t-1}, \dots] = 0$ , and where by construction  $E \xi_t | \theta = 0$ . Then least squares estimation of the above equation gives a consistent estimate of  $\alpha$ , since the composite disturbance  $\eta_t + \alpha \Psi_t + \alpha \xi_t$  is orthogonal to the regressors. Notice that even though  $\Psi_t$  and  $\eta_t$  are serially uncorrelated,  $\xi_t$  is in general serially correlated. This means that the composite disturbance is in general serially correlated. To see this, use (13) to write the once lagged composite disturbance as

$$\eta_{t-1} + \alpha \Psi_{t-1} + \alpha \xi_{t-1} = \mu_{t-1} - x_{t-1} - \alpha \hat{E}_{t-2}(x_t - x_{t-1}).$$

Then we have that

$$\begin{aligned} & E\{(\eta_t + \alpha\Psi_t + \alpha\xi_t)(\eta_{t-1} + \alpha\Psi_{t-1} + \alpha\xi_{t-1})\} \\ &= E\{(\eta_t + \alpha\Psi_t + \alpha\xi_t)(\mu_{t-1} - x_{t-1} - \alpha\hat{E}_{t-2}(x_t - x_{t-1}))\}. \end{aligned}$$

Since  $E[\eta_t | \mu_{t-1}, \dots, x_{t-1}, \dots] = E[\Psi_t | \mu_{t-1}, \dots, x_{t-1}, \dots] = 0$ , the covariance above equals

$$\alpha E[\xi_t \{\mu_{t-1} - x_{t-1} - \alpha\hat{E}_{t-2}(x_t - x_{t-1})\}].$$

Now  $\xi_t$  is by construction orthogonal to  $\theta$ , which does not necessarily span the space in which  $(\mu_{t-1} - x_{t-1} - \alpha\hat{E}_{t-2}(x_t - x_{t-1}))$  lies. Therefore, there is no assurance that the above covariance is zero. The serial correlation in the composite disturbance will be smaller the smaller is the variance of  $\xi_t$  relative to the variances of  $\Psi_t$  and  $\eta_t$ . To the extent that for forecasting  $x$  and  $\mu$  the information set  $\theta$  well approximates the entire information set  $[x_{t-1}, \dots, \mu_{t-1}, \dots]$ , the serial correlation in the composite disturbance will be small.

It is tempting to use the serial correlation parameter of the estimated composite residuals from the two-step estimator as a diagnostic test of the assumption that  $\eta_t$  is serial uncorrelated. The preceding observations indicate that this can be done only with some care.

Table 1 records the results of applying the two-step estimator to six of the hyperinflations studied by Cagan for the same periods studied by Sargent [ ], and where  $\theta = \{x_{t-1}, x_{t-2}\}$ . Table 2 reports the results obtained by extending  $\theta$  in steps from  $\{x_{t-1}\}$  to  $\{x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}\}$ . Each of the estimators reported is consistent on

the null hypothesis. To say the least, we do not recover a statistically significant estimate of  $\alpha$ . Compared with Cagan's estimates of  $\alpha$ , which ranged from -2.30 to -8.55, the estimates in Table 2 are small in absolute value. Furthermore, they are as often positive as they are negative, and are uniformly of low statistical significance.

The disappointing nature of these results naturally leads one to question whether the condition is met that  $E_{t-1}\{\mu_t - x_t\} \neq 0$ , which is necessary for the estimators to work and which breaks down where Cagan's adaptive expectation scheme is rational. Table 3 contains F-statistics pertinent for testing the null hypothesis  $E\{(\mu_t - x_t) | \mu_{t-1}, \mu_{t-2}, \mu_{t-3}, x_{t-1}, x_{t-2}, x_{t-3}, t\} = 0$ . Only for the German data are we able to reject the null hypothesis at the .95 confidence level. This suggests that, except perhaps for Germany, the preconditions are probably not met that must be for the estimators here to be able to estimate  $\alpha$ . The parameter  $\alpha$  is only identified by the present procedures to the extent that  $E_{t-1}\{\mu_t - x_t\} \neq 0$ .

Correcting for Serial Correlation

The techniques above all require the assumption (4) that  $\eta_t$  is serially independent. The techniques can be modified to incorporate the assumption that  $\eta_t$  follows a low-order Markov process. To illustrate, I will suppose that

$$\eta_t = \rho\eta_{t-1} + \xi_t$$

where  $E[\xi_t | x_{t-1}, x_{t-2}, \dots, \mu_{t-1}, \mu_{t-2}, \dots] = 0$ .

Quasidifferencing (3) then yields

$$\begin{aligned} \mu_t - \rho\mu_{t-1} - x_t + \rho x_{t-1} &= \alpha(E_t x_{t+1} - E_{t-1} x_t) \\ &- \rho\alpha(E_{t-1} x_t - E_{t-2} x_{t-1}) + \xi_t. \end{aligned}$$

Taking expectations conditional first on information available at time  $t-1$ , then on information available at time  $t-2$  gives

$$\begin{aligned} (5a) \quad E_{t-1}(\mu_t - \rho\mu_{t-1}) - E_{t-1}(x_t - \rho x_{t-1}) &= \alpha(E_{t-1} x_{t+1} - E_{t-1} x_t) \\ &- \alpha\rho(E_{t-1} x_t - E_{t-2} x_{t-1}). \end{aligned}$$

$$\begin{aligned} (5b) \quad E_{t-2}(\mu_t - \rho\mu_{t-1}) - E_{t-2}(x_t - \rho x_{t-1}) &= \alpha(E_{t-2} x_{t+1} - E_{t-2} x_t) \\ &- \alpha\rho(E_{t-2} x_t - E_{t-2} x_{t-1}). \end{aligned}$$

From equation (5a), a two-step estimator along the following lines is indicated. First get estimates of  $E_{t-1}(x_{t+1} - x_t)$  and  $E_{t-1}(x_t - E_{t-2} x_{t-1})$

by replacing mathematical expectations with the corresponding values from linear least squares regressions. Second, estimate (5a) by (nonlinear) least squares, say a la Hildreth and Lu.

It is of some interest that if  $\rho \neq 0$ , then this two-step estimator recovers  $\alpha$  even under the situation in which

$$E_{t-1}x_t = \frac{1-\lambda}{1-\lambda L} x_{t-1},$$

so that Cagan's adaptive expectations scheme is rational. Under the above scheme, we have

$$E_{t-1}x_{t+1} - E_{t-1}x_t = 0$$

for all  $t$ , but also

$$E_{t-1}x_t - E_{t-2}x_{t-1} = \frac{1-\lambda}{1-\lambda L} (x_{t-1} - x_{t-2}),$$

so that (5a) becomes

$$E_{t-1}(\mu_t - \rho \mu_{t-1}) - E_{t-1}(x_t - \alpha x_{t-1}) = -\alpha \rho \left( \frac{1-\lambda}{1-\lambda L} \right) (x_{t-1} - x_{t-2}),$$

in which both  $\alpha$  and  $\rho$  are identified. Notice that identification of  $\alpha$  is lost if  $\rho=0$ , which is our previous result.



### Conclusions

The discouraging results from implementing the estimators described here were to an extent foreshadowed by the overfitting tests implemented by Sargent [ ]. Those tests indicated that the model formed by assuming that the money supply process was such as to make Cagan's adaptive expectations scheme rational is not drastically inconsistent with the data, and for three of the countries cannot even be rejected at conventional confidence levels relative to several more general stochastic models. The estimators in this paper break down in precisely those circumstances in which adaptive expectations coincide with rational expectations. Consequently, evidence that the model formed by assuming that adaptive expectations are rational is approximately adequate does not speak well for the prospects of implementing the estimators described here.

Nevertheless, from a technical point of view the estimators described here are of interest in a broader range of applications than we have studied here. Versions of the procedures described here are appropriate ones in a host of macroeconomic applications where expectations of rational agents are important explanatory variables, and where simultaneity is a problem. Examples include analysis of the permanent income consumption schedule and the labor supply schedule.

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## Footnotes

1/ The reader can regard  $E[z_{t+1} | x_t, \dots]$  either as the mathematical expectation of  $z_{t+1}$  conditioned on the indicated variables, or as the projection of  $z_{t+1}$  against the space spanned by the indicated variables. Either way, the argument goes through. The projection interpretation is the relevant one from a practical viewpoint, since it rationalizes the linear regressions to be computed below.

Table 1  
Two-Step  
Estimates of  $\alpha$   
 $\mu_t - x_t$  regressed on  $\hat{E}_{t-1}(x_{t+1} - x_t)$ , constant and trend.

	$\hat{\alpha}$	$\bar{R}^2$	d.w.	D.F.
Austria Feb'21-Aug'22	.220 (.354)	-.102	1.989	12
Greece Feb'43-Aug'44	.480 (.176)	.360	1.739	12
Hungary I Aug'22-Feb'24	-.124 (.401)	-.125	1.465	12
Russia Feb'22-Jan'24	.080 (.621)	.183	1.652	17
Poland May'22-Nov'23	-.148 (.769)	-.127	1.782	12
Germany Oct'20-July'23	.229 (.405)	-.051	1.598	27

Note  $\hat{E}_{t-1}(x_{t+1} - x_t)$  formed by regressing  $x_{t+1} - x_t$  on  $x_{t-1}$ ,  $x_{t-2}$ , constant and trend.

Table 2

$$\mu_{t+1} - x_{t+1} = \alpha(E_t x_{t+2} - E_t x_{t+1}) + \beta_1 t + \beta_0$$

<u>Country</u>	<u>Estimates</u>	<u>For Forecasts Based on No. of Lags</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	
AUSTRIA	.397 (.960)	.220 (.354)	.274 (.316)	.187 (.366)	
GREECE	.466 (.187)	.480 (.176)	.433 (.161)	.476 (.098)	
HUNGARY I	-.137 (.387)	-.124 (.401)	-.100 (.401)	-.245 (.436)	
RUSSIA	1.211 (.731)	.080 (.621)	-.330 (.725)	-.116 (.879)	
POLAND	-.566 (2.477)	-.148 (.769)	.255 (.500)	.231 (.267)	
GERMANY	.143 (.441)	.299 (.405)	.220 (.410)	.211 (.215)	

Standard errors are in parentheses.

Table 3

$$\text{Regression: } \mu_t - x_t = \sum_{i=1}^3 \phi_i \mu_{t-i} + \sum_{k=1}^3 \psi_k x_{t-k} + \beta_1 t + \beta_0$$

F-Statistics for Testing Hypothesis (1)  $\phi_i=0, i=1,2,3$  or (2)  $\psi_k=0, k=1,2,3$

<u>Country</u>	<u>1</u>	<u>2</u>	<u>3</u>
AUSTRIA	2.790 (3,6)	.400 (3,6)	1.41 (7,6)
GREECE	1.114 (3,6)	.248 (3,6)	1.15 (7,6)
HUNGARY I	1.421 (3,6)	1.222 (3,6)	2.25 (7,6)
RUSSIA	.341 (3,11)	1.333 (3,11)	1.87 (7,11)
POLAND	.948 (3,6)	.733 (3,6)	(1.29) (7,6)
GERMANY	6,830** (3,21)	1.906 (3,21)	3.637* (7,21)

Numbers in parentheses are degrees of freedom for F-statistics

Col. 1 Hypothesis:  $\phi_1 = \phi_2 = \phi_3 = 0$

Col. 2 Hypothesis:  $\psi_1 = \psi_2 = \psi_3 = 0$

Col. 3 Hypothesis: all coefficients = 0

\* Significant @ 5% level

\*\* Significant @ 1% level