

The Demographic Transition and the Sexual Division of Labor*

Abstract

There have been significant increases in the labor supply of women in the last decades, both in developed and developing countries. Traditional views of this process interpret it as being caused by reduced fertility and increased wages for women in the labor market. This paper suggests that all these changes can be understood as part of a single process of demographic transition, triggered by reductions in mortality. Mortality reductions have important effects on the incentives of individuals to invest in human capital and to have children. We develop a model where households composed by two members (male and female) decide on their allocation of time, number and quality of children, and human capital investments. The paper shows that gains in adult longevity can lead to reductions in fertility, increases in investments in market human capital, increases in female labor force participation, and reductions in the gender wage-gap.

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1 Introduction

This paper develops a model where households composed by two members (female and male) decide on their allocation of time, number of children, and human capital investments. We investigate the effects of changes in mortality on fertility, investments in human capital, female labor force participation, the gender wage-gap, and the quality of children. The paper shows that, as long as women are marginally more productive at child raising, gains in adult longevity lead to reductions in fertility, increases in female labor force participation, and reductions in the wage differential between men and women. The effect on the quality of children is ambiguous, depending on the characteristics of the household and the nature of the technology available for investments in children. But even when reduced mortality reduces the quality of children, it does so only in a very narrow interval. Successive gains in longevity are eventually accompanied by increased investments in children. The model can, therefore, generate either a positive or a non-monotonic relationship between labor supply of women and the quality of children.

There have been profound changes in the labor supply of women in the last decades, both in developed and developing countries. In the United States, female participation in the paid labor force changed drastically in the course of the last century. In 1880, only 17% of all American women at working ages participated in the labor market. By 2000, this number had risen to more than 60%. Together with this change in the role of women, individuals and families were changing in other important ways. Figure 1 shows the evolution of life expectancy, fertility and female labor force participation for the United States between 1880 and 2000. Throughout the 20th century, life expectancy at birth rose by 30 years, from 47 to 77. The total fertility rate dropped from above 4 to 2.1.¹ These changes in life expectancy and fertility reflect trends that were observed at least since the beginning of the 19th century, when the total fertility rate was above 7 points and life expectancy at birth was below 40 years.

Over this same period, similar trends were observed in other developed countries. Figure 2 illustrates the case of Great Britain, where fertility dropped from 4.9 in 1851 to 1.7 in 2001, while life expectancy at birth increased from 40 to 77 years. Also similarly, after some delay, female labor force participation started rising, from 35% in 1900 to 53% in 2000.

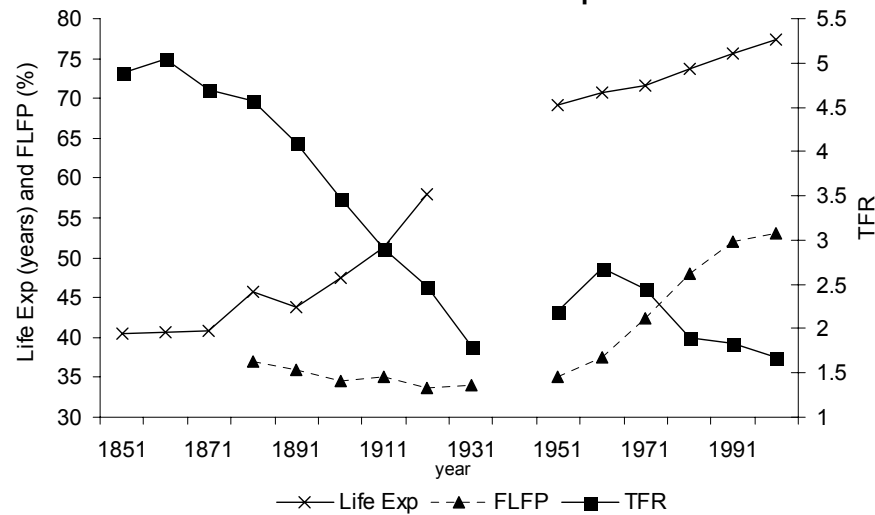
¹ The temporary increase in fertility in the post-war period is the well-known “baby-boom” phenomenon. Our focus here is on the long term trend of fertility decline, which was only temporarily interrupted by the increased fertility rate of the 1950’s and 1960’s.

Figure 1: United States - Life Expectancy, Fertility and Female Labor Force Participation



—x— Life Exp —▲— FLFP —■— TFR
 Note: Total fertility rate prior to 1920 is for the black and white populations only.
 Source: Costa (2000), National Center for Health Statistics (2003), and US Census Bureau (1975).

Figure 2: Great Britain - Life Expectancy, Fertility and Female Labor Force Participation

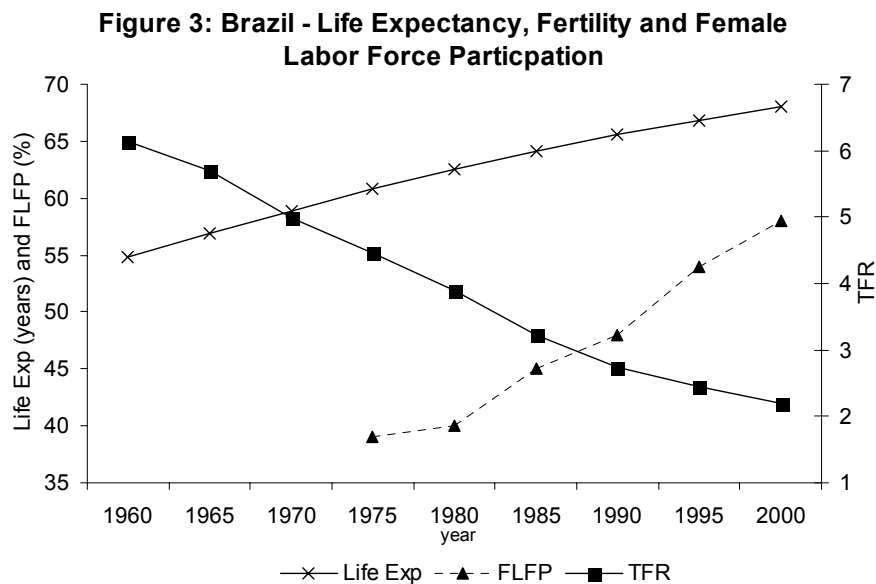


—x— Life Exp —▲— FLFP —■— TFR
 Source: Wrigley et al (1997), Keyfitz and Fieger (1968), Costa (2000) and World Bank (2003).

The reductions in mortality and fertility depicted above are part of the process of demographic transition, which spread through most of the world in the post-war period. The empirical pattern characterizing the transition has been widely documented and discussed in the demographic literature. It is usually understood as being marked by an initial reduction in child mortality, followed

by reductions in adult mortality and fertility (see, e.g., Heer and Smith, 1968, Cassen, 1978, Kirk, 1996, and Mason, 1997). Maybe less acknowledged is the fact that changes in female labor force participation have also reached several of the “latecomers” of the demographic transition. Though data in these cases are more recent and sparse, it is already possible to notice some trends.

Figure 3 illustrates the demographic changes experienced by Brazil in the period between 1960 and 2000. In this forty-year interval, Brazil went through radical demographic changes: life expectancy at birth rose from 55 to 68 years, while the total fertility rate declined from 6 to 2.2 points. Though data on female labor force participation is not available before the mid 1970’s, the pattern of the series suggests that modest increases were being observed before 1975. But it is really only after 1980 that the increase in female labor force participation gained momentum. In the twenty five years between 1975 and 2000, women’s labor market participation in Brazil increased from 39% to 58%.²



Source: Soares and Izaki (2002) and World Bank (2003)

Also, in the three experiences discussed in the figures – as well as in virtually every other documented history of increased female labor force participation – the wage differential between men and women has been shrinking. This is a quite general phenomenon, irrespective of cultural tradition or level of economic development (see Blau and Kahn, 2000). We illustrate this point in Table 1, with data for the three countries discussed before. Among the cases portrayed in Table

² The World Bank’s World Development Indicators show that, between 1960 and 2000, the fraction of the labor force composed by women increased substantially in almost every single country that experienced significant reductions in fertility.

1, the highest degree of convergence is observed in Brazil, where the wage-gap was reduced by 15 percentage points in the period between 1979 and 1998.

Table 1: Female/Male Earnings Ratio

Country	1979	1989	1998	Change btwn 1979 and 1998
Brazil	0.508	0.541	0.661	0.153
Great Britain	0.626	0.667	0.749	0.123
United States	0.625	0.706	0.763	0.138

Source: Blau and Kahn (2000) and Simão et al (2001).

Notes: For the United States, the 1998 number corresponds to 1996. Median earnings for Great Britain and the US, and mean earnings for Brazil

The implications of the increase in women’s labor force participation encompass issues such as the bargaining power of husband and wife within the household, the enhanced role of women in modern society, and the availability of parents to invest in their children. On the one hand, this change has been linked to cultural transformations within society, which led to a change in the role of women in the household and in the labor market. On the other hand, economists have linked it to rising wages and falling fertility. Though these closer explanations are relevant, they do not identify the ultimate determinants of the observed trends. What is the underlying reason behind society’s change of attitude toward women? Why did fertility decline and, even with rising labor supply, why did women’s wages increase? Also less understood is the theoretical mechanism linking the reductions in mortality to reductions in fertility, increases in female labor force attachment, and narrowing of the gender wage-gap. Nevertheless, the evidence suggests that increased female labor force participation follows a quite general pattern, which, therefore, cannot be explained simply by changes that are particular to one specific country. It must be part of a broader process of social transformation, which seems to be directly associated with the demographic transition. Though the empirical pattern of the transition has been thoroughly documented in the most diverse regions of the world, its relation to the subsequent increase in female labor force participation – which is commonly regarded as an almost separate phenomenon – is still much less understood.

This paper suggests a single explanation for all these changes, based on the impact of reductions in mortality on household decisions regarding fertility and investments in human capital. Particularly, we develop a model where gains in adult longevity reduce fertility and increase the returns to investments in market-oriented human capital. Since women are initially responsible for child raising, reduced demand for household production leads to an increase in investments in human capital and labor supply that is more than proportional to the increase in longevity

and larger than the one observed for men. The differential change across genders translates into increased labor force participation of women and narrowing of the gender wage-gap.

In addition, and contrary to the superficial intuition, the model does not generate increases in female labor force participation or narrowing of the gender wage-gap as results of reduced child mortality. Reductions in child mortality do not increase the returns to market attachment directly, but they do increase the returns to investments in children. In our setup, when women initially allocate part of their time to household production, the increased return from investments in children is large enough to guarantee that female labor force participation and investments in market human capital do not rise. Therefore, increased female labor force participation is directly linked to changes in adult longevity, which is in line with the timing of events observed during the demographic transition (more on this in Section 6).

This simple theory explains the rising presence of women in the labor market without resorting to exogenous changes in culture, habits or preferences. The changing role of women arises as a natural consequence of the same process that led to reductions in the size of families and to higher investments in human capital. We argue that exogenous reductions in adult mortality – driven by technological progress in medical and biological sciences³ – have two fundamental effects: they reduce the returns from large families and increase the returns from investments in market-oriented human capital. With higher longevity, there are higher returns to specialization, which then increase investments in human capital and the cost of time. A higher cost of time increases the cost of children, which tends to reduce fertility. In addition, lower mortality reduces the gains from higher fertility, reinforcing the previous effect. This setup suggests a unified explanation for all the social changes discussed above, and also sheds light on recent debates regarding their implications for future generations.

Most notably, increased female labor force participation has raised concerns about the possibility of negative impacts on the quality children, due to reduced presence of parents in the household. This issue has been the topic of a series of recent presidential addresses to the Population Association of America (see, for example, Preston, 1984, Bianchi, 2000, and McLanahan, 2004), and is still subject of debate (e.g. Sayer et al, 2004, Gauthier et al, 2004, and James-Burdumy, 2005). The model proposed here allows us to identify under what conditions increased participation of women in the market will take place at the expense of child quality, and under

³ Our theory has exogenous reductions in mortality as the main driving force behind all other demographic changes mentioned before. Though there are several dimensions on which individuals can invest in their own health, our focus here is on changes in mortality brought about by technological advances or diffusion of previously existing technologies. An extensive literature has pointed to the fact that recent changes in mortality have been largely unrelated to income and living conditions, and are to a great extent exogenous to individuals and countries. This issue is discussed in Preston (1975, 1980), Becker et al (2005) and Soares (2005), and will not be explored in further detail here.

what conditions it will occur along with increased quality of children. This is done in a setup that encompasses several social changes usually treated as different phenomena: the classic demographic transition (understood as reductions in mortality followed by reductions in fertility), the increased female labor force participation, the narrowing wage-gap, and the changing pattern of parental investments in children.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on the demographic transition and the recent changes in female labor force participation. Section 3 presents the basic framework of the model. Section 4 discusses the effect of adult longevity gains. Section 5 analyzes the impact of child mortality reductions. Section 6 discusses the pattern of historical changes generated by the model. Finally, section 7 concludes the paper.

2 Related Literature

There are numerous empirical papers on female labor force participation, mainly within the literature on the determinants of labor supply. Typically, these papers concentrate on the analysis of the proximate determinants of female labor supply. For example, the first generation of models of labor supply of women took lifetime wage profiles or fertility decisions as given, and analyzed their impact on labor supply decisions (see, e.g., Mincer, 1962, Gronau, 1973, or the survey in Heckman, 1978). More recent papers acknowledge the joint household decision regarding number of children, investments in human capital and labor supply of women. This literature usually tries to instrument for one of these dimensions of choice, and analyze its impact on the other dimensions (Hotz and Miller, 1988, Rosenzweig and Schultz, 1985, and Angrist and Evans, 1998). Overall, this line of research is mainly focused on explaining the determinants of labor supply at a point in time, given the skills accumulated by women and the wages they face on the market. The objective is not to explain the joint historical evolution of these variables or the long-term pattern of change.

From a theoretical perspective, few papers have focused on the structural determinants of the changing labor market participation of women and the narrowing of the gender wage-gap. In this respect, Jones et al (2003) is the paper closest in spirit to the labor supply literature. These authors calibrate a general equilibrium model of household decisions regarding labor supply and human capital accumulation, and argue that the narrowing wage-gap alone is enough to explain a large part of the recent changes in female labor force participation. In a similar vein, Olivetti (2001) claims that there are specific changes in the life-cycle profile of female labor force participation that cannot be accounted for by the narrowing wage-gap alone. She then argues that technologically induced changes in the return to one specific productive attribute – labor market experience –

can account for these patterns. Both these papers take the time path of wages – or returns to productive attributes – as given, and try to understand the changes in female labor force participation from that. As argued before, our goal is to go one step further, and also understand what determines the changes in the market wages faced by women.

With a different perspective, Greenwood et al (2004) suggest that a major force determining the liberation of women from household chores was the technological revolution in household production induced by the birth of electricity. The introduction of electricity in the household – followed by a series of new durable goods – liberated women from household production and allowed increased labor force participation, without implying a reduction in the consumption of home produced goods. Goldin and Katz (2002), on the other hand, suggest that a technological innovation of a different nature was the main factor determining the changes in women’s professional choices. They argue that the introduction of the oral contraceptive in 1960 was the driving force behind the changes in women’s career and marriage decisions in the US.

Though the mechanisms described in the last two paragraphs are important components of the changes observed in women’s attachment to the labor market, they leave out key transformations that took place at the core of the family unit. It is difficult to conceive that the change in female labor force participation is not intrinsically related to the substantial reduction in family size that characterizes the last stage of the demographic transition. Reduced wage-gap, increased returns to market experience, and the availability of new household technologies are factors that certainly allowed and induced women’s increased participation in the labor market at the observed pace. But the underlying factor that determined these changes – or the demand for the new technologies – must be linked to the determinant of the prior reductions in fertility. The fact that these changes were observed in various parts of the world, at different points in time, suggests that its ultimate determinants cannot be technological or institutional transformations particular to one specific country.

In this respect, the theory proposed here is closest in spirit to the work of Galor and Weil (1996). These authors analyze the process of increased participation of women in the labor market as a joint consequence of the reduced demand for children induced by economic growth. In their model, women are assumed to have comparative advantage in mental labor, as opposed to physical labor. In addition, capital is assumed to be complementary to mental labor, so that growth induced by capital accumulation tends to increase the relative return to mental labor and, therefore, the relative wage of women. In a situation where the capital stock is small, and the return to physical capital relatively high, women specialize in raising children. But capital accumulation increases the relative wage of women, therefore increasing the opportunity cost of children, reducing fertility,

and raising female labor force participation.

We share the same basic goal of Galor and Weil (1996), but key aspects of our theory differ from theirs, and we also extend the analysis in new directions. First, we focus on the reductions in mortality that characterize the onset of demographic transition as the only reason behind all the changes observed thereafter, while Galor and Weil (1996) do not incorporate mortality changes as part of the transition. Second, the only difference between men and women in our model is that women are more productive at raising children (childbearing, breast-feeding, etc.), while they rely on differential productivity of men and women in physical and mental labor. And third, we explore the consequences of these changes for parental investments in children and, ultimately, for the quality of children, while they do not incorporate investments in children in their model.

Experiences of demographic transition in the vast majority of cases have been characterized by initial reductions in mortality that are, after some delay, followed by reductions in fertility (see, e.g., Heer and Smith, 1968, Cassen, 1978, Kirk, 1996, and Mason, 1997). This has taken place in different areas of the world at very different development levels, so that to link the changes exclusively to economic growth does not seem fully satisfactory. This paper follows the literature that stresses the interaction between fertility and investments in human capital as one of the distinguishing features of modern economies, and as the main determinant of the differential behavior of population before and after the demographic transition (as Becker et al, 1990). As Meltzer (1992) and Soares (2005), it explores the particular way in which health affects this interaction in order to understand the long term determinants of social changes.

We propose a theory that places reductions in mortality at the center stage in explaining the long-term behavior of society. In addition, we start from a model where women's specialization in household production arises for a very natural reason: women are relatively more productive at raising children. This can be interpreted as reflecting the simple fact that women are the ones that actually bear children and are capable of breast-feeding. Our main question then is the following. When the only difference between men and women is that women are relatively more productive at raising children, can reductions in mortality generate: (i) reduced fertility, (ii) increased investments in market-oriented human capital, (iii) increased female labor force participation, and (iv) a narrowing of the gender wage-gap?

Finally, there are important implications of the increased labor force participation of women that have not yet been addressed in the theoretical literature. A large body of empirical work has tried to evaluate whether increased labor force participation of women has come at the expense of parental investments in children (see, e.g., Gershuny and Robinson, 1988, Bianchi, 2000, Gauthier et al, 2004, and Sayer et al, 2004). By placing this question within a broader context, we can

understand the exact circumstances under which increased female labor force participation will come with reductions or increases in child quality.

3 The Model

Consider an economy inhabited by families that live for a deterministic amount of time. Each family is composed by a male and a female (denoted by subscripts m and f , respectively), who jointly decide on the allocation of time of each member towards investments in adult human capital, work, and raising children. As in Galor and Weil (1996), fertility is realized in terms of couples who grow up to be a household, so that we abstract from questions of matching and the formation of families. Each member of the family is endowed with a level of basic human capital determined from the previous generation's decision (both members receive the same level of basic human capital). This basic human capital determines the productivity of these individuals in acquiring market human capital and raising children. Market human capital, on its turn, determines the productivity of each unit of time allocated to labor supply, and goods produced in the market may be used for consumption or investment in children.

In such context, where we incorporate the effects of mortality, quality of children cannot be understood simply as productive human capital. Evolutionary considerations lead to a formulation where parents derive utility not only from the number of children and human capital of each child, but also from the child mortality rate and adult longevity faced by the children. In addition, these considerations imply that parents regard life expectancy and fertility in similar ways. This formulation requires the recognition that adult longevity can be seen as an additional dimension of offspring quality, justified by the fact that survival into adulthood affected evolutionary fitness in earlier hunter-gatherer populations (due to the necessity of providing to offspring until the early teenage years). It also implies the existence of a biological trade-off between quantity and quality of offspring (for a detailed discussion, see Robson and Kaplan, 2003, Robson, 2004, or Soares, 2005). In this case, natural selection would impose a trade-off between life expectancy and number of offspring (fertility) that, if recognized by preferences, would imply a dominant evolutionary strategy. This is the logic underlying a recent model developed by Robson (2004, section 3), in which preferences that aim at maximizing the product of life expectancy (quality) and fertility (quantity) arise as evolutionarily dominant in the long run. In his model, the interaction between life expectancy and fertility is relevant for purely biological reasons. But their interaction in the induced preferences is exactly the same that we assume here: exogenous increases in life expectancy are perceived as being similar to increases in fertility. This is a natural assumption when parents are concerned not only with the survival of their children, but with the continuing

survival of their lineage. In this case, families should be concerned about whether their children would live long enough to have and raise their own offspring, therefore guaranteeing the long term survival of descendants.

For these reasons, we adopt a simplified version of the formulation proposed in Soares (2005). Households derive utility from consumption in each period of life $(c(t)^\sigma/\sigma$, over T years of life) and from children. We assume that parents derive utility from the basic human capital of children (h_c^α/α) , and that this utility is affected by the number of children (n), the child mortality rate (β), and the lifetime that each child will enjoy as an adult (T). We assume that the discount factor applied to basic human capital is a concave and increasing function $\rho(\cdot)$ of the total expected lifetime of the children $(nT(1-\beta))$, or the total of “child-years.”⁴ With this formulation, the household utility function is⁵

$$\int_0^T \exp(-\theta t) \frac{c(t)^\sigma}{\sigma} dt + \rho(nT(1-\beta)) \frac{h_c^\alpha}{\alpha} \quad (1)$$

where $c(t)$ is consumption at instant t , θ is the subjective discount factor and $0 < \alpha, \sigma < 1$. The first term is the utility that parents derive from consumption in each period of life, and the second term is the utility that they derive from their children.

Production of basic human capital for children (h_c) can use either one of two inputs. The first input (x_T) is produced with time invested by adult members of the household, while the second input (x_y) is a good purchased with income on the market (at a fixed price p). For simplicity, we assume that these two inputs are perfect substitutes in the production function of h_c :

$$h_c = ax_T + (1-a)x_y, \quad (2)$$

where a is a constant between zero and one. The household produced input (x_T) makes use of the time of parents according to the following production function:

$$x_T = (Bb_f + Cb_m)h_p, \quad (3)$$

⁴ Soares (2005) shows that the main results generated by this functional form are also present under a more general formulation for the $\rho(\cdot)$ function. Specifically, if the altruism function assumes the general form $\rho(n, T, \beta)$, and $\varepsilon(n, T, \beta) = \frac{\rho_n(n, T, \beta)n}{\rho(n, T, \beta)}$ denotes its elasticity in relation to n , the condition needed is $\text{sign}\{\varepsilon_n(n, T, \beta)\} = \text{sign}\{\varepsilon_T(n, T, \beta)\} = -\text{sign}\{\varepsilon_\beta(n, T, \beta)\}$, where the subscripts denote partial derivatives. We adopt the alternative formulation because it relates directly to the endogenous preference results from the evolutionary literature (Robson, 2004). In addition, it is intuitively more appealing and simpler.

⁵ We investigate the effect of technologically induced mortality changes, which are perceived as permanent by agents. This is why we do not distinguish between parent’s and children’s adult longevity in this formulation. As long as the changes are permanent, the long-run effects are the ones discussed here, even if they apply only from the children’s generation on. In this case, we could understand our exercise as comparing households two generations apart.

where h_p is the basic human capital of parents and B and C are constants. In this context, the idea that women are more productive at raising children can be translated into $B > C$, such that each unit of time invested in a child by a woman generates more of the input x_T than the same unit invested by a man. This hypothesis is maintained throughout the paper. It is the only intrinsic difference between men and women in our model.

Basic human capital of each parent is also used, together with time invested in adult education, to produce market (productive) human capital:

$$H_i = Ah_p e_i. \quad (4)$$

This is the human capital that is actually used to produce goods. The market human capital of each member determines the productivity of each unit of time used as labor. The total amount of market goods produced by the household is, therefore,

$$y = l_m H_m + l_f H_f, \quad (5)$$

where l_i indicates the labor supply of agent i . The distinction between basic and market human capital highlights the different types of human capital acquired at different points in the life cycle. Basic human capital (h) refers to basic skills (language, motor ability, etc.) and general knowledge accumulated during early stages of life, and while investments decisions are still taken by parents. Market human capital (H) refers to the accumulation of skills related to a specific occupation or profession, and when investment decisions are already taken by the individuals themselves. Since the model is unable to capture all the subtleties of human capital investments, we understand market human capital very broadly as referring to any type of human capital investment specific to a particular task, including college and graduate education, professional training, and the investment dimension of on-the-job training and learning-by-doing. The distinction between these two types of human capital turns out to be key in identifying the different effects of changes in child mortality and adult longevity.

The total amount of goods produced by the household is allocated between consumption and raising children. Borrowing from future generations and bequests are not allowed, so that the budget constraint is

$$y \geq \int_0^T \exp(-rt) c(t) dt + \exp(-r\tau) n p x_y, \quad (6)$$

where r is the interest rate, x_y is the goods investment in the basic human capital of each child,⁶ and all the children are assumed to be born in period τ .

⁶ The model could also incorporate a parameter f capturing a fixed cost of having children in terms of goods. But since the economy analyzed here displays long-run growth, this parameter would be asymptotically irrelevant and would not affect the long-run behavior of the economy.

Each adult member of the family also faces a time constraint. Her/his adult lifetime has to be allocated between studying, working and, possibly, raising children. The time constraint of agent i is

$$T = l_i + e_i + nb_i. \quad (7)$$

In order to simplify the problem and concentrate the analysis on the intergenerational behavior of the economy, we abstract from life-cycle considerations and set discount rates and interest rates to zero. In this context, once we substitute for y in the budget constraint and for h_c in the utility function, the problem of the household can be written in a simpler form as

$$\begin{aligned} \max V &= T \frac{c^\sigma}{\sigma} + \rho(n(1-\beta)T) \frac{[ax_T + (1-a)x_y]^\alpha}{\alpha} & (8) \\ \text{subject to } l_m Ah_p e_m + l_f Ah_p e_f &\geq Tc + pn x_y, \\ T &= l_i + e_i + nb_i, \text{ for } i = f, m, \text{ and} \\ x_T &= (Bb_f + Cb_m)h_p. \end{aligned}$$

In this framework, there are two forces working toward specialization. First, since women's time is relatively more productive in household activities, simple comparative advantage considerations would lead to women's partial specialization in household production. Second, the possibility of increasing market productivity through human capital investments generates increasing returns to the total amount of time allocated to market related activities (both investments in adult education and labor supply together).⁷ The presence of increasing returns to market related activities enhances the tendency toward specialization generated by any minor difference in comparative advantages, and exacerbates *ex-post* differences.

Therefore, there can be no equilibrium where both agents share their time between market activities and household production. Comparative advantage and investments in education generate an incentive toward specialization, and at least one agent will always be completely specialized in some activity. When agents spend some amount of time investing in children, there can be only three possible equilibria: (a) m specializes in the market and f works both in the market and in the household; (b) m works in the market and in the household and f specializes in the household; or (c) m specializes in the market and f specializes in the household. This has to be the case because if both agents share their time between market and household activities, the household

⁷ If \tilde{l}_i denotes the total amount of time allocated to market related activities by agent i ($\tilde{l}_i = e_i + l_i$), the optimal allocation of time between human capital investments and labor supply is $e_i = l_i = \tilde{l}_i/2$. Substituting both back into the production functions, this would imply a total production of $Ah_p \tilde{l}_i^2/4$ for agent i . So, there are increasing returns to the total amount of time \tilde{l}_i dedicated to market related activities. This is the type of return to specialization discussed in Becker (1985).

can always increase its total production by increasing the market time of m (there are increasing returns to the total amount of time dedicated to the market). In addition, since $B > C$, if only one agent works in the household, it must be f . It is also possible that both agents end up spending all of their time investing in market human capital and working, in which case investments in children are made using goods purchased in the market.

In what follows, we concentrate the discussion on the cases where women spend at least part of their time on the market, and men are completely specialized on market production. Since women labor force participation has been positive in modern economies for most of the recent past, this seems to be the most relevant equilibrium from an empirical perspective (the other equilibria, and the effects of mortality on the transition between different equilibria, are briefly described in the Appendix). Early stages of the process of economic development and of the movement from agriculture to industry may be better characterized as a movement of men from household activities to the market. This is a possibility that deserves further thought and discussion, but we do not deal with it here. Our main focus is on the increased female labor force participation that characterizes most industrial societies and also many less developed countries that have already experienced the demographic transition.

4 The Effect of Longevity Gains

From the first order conditions for an optimum (see Appendix), it is easy to show that, when the woman shares her time between market and non-market activities, the optimal allocation of time of m and f is characterized by

$$\begin{aligned} e_m &= l_m = \frac{T}{2}, \quad b_m = 0, \\ \text{and } e_f &= l_f = \frac{T - nb_f}{2}. \end{aligned}$$

Using the first order conditions for n , b_f , and x_y (see Appendix):

$$\frac{\rho' n T (1 - \beta)}{\rho} = \alpha \tag{9}$$

This expression determines the response of n to exogenous changes in longevity (T) and child mortality (β). Particularly:

$$\frac{dn}{dT} = -\frac{n}{T} < 0. \tag{10}$$

This is the same relationship found in Soares (2005), and it reflects the interaction between n and T inside the function ρ .

Since mother's time (b_f) and market goods (x_y) are substitutes in the production function for children's basic human capital (h_c), only the one with the higher relative return will be used.

Analyzing the first order conditions, one can see that there are two possible choices: one where the investment in basic human capital is done using the domestic technology ($b_f > 0$ and $x_y = 0$, or Equilibrium A), and another where this investment is done using goods purchased on the market ($b_f = 0$ and $x_y > 0$, or Equilibrium B). In other words, in Equilibrium A the woman shares her time between market and household activities, while in Equilibrium B both man and woman spend all their time on the market, and investments in children are undertaken with goods purchased on the market.

Longevity affects the investment in children's human capital through two channels. First, it affects the choice of the technology to be used in this investment (extensive margin). And second, it affects the total amount of investment undertaken, possibly through both income and substitution effects (intensive margin).

Equilibrium A

We first analyze the effect of T on the intensive margin, when women use part of their time to invest in children. In Equilibrium A, the fraction of time that the woman dedicates to the market (human capital investments plus labor supply) increases in T . The fraction of time spent in the household declines, but the net effect on the absolute value of b_f (investment in each child) is ambiguous because fertility is also reduced (see Appendix). The impact on consumption is positive. These effects are described by

$$\begin{aligned} \frac{d(e_f/T)}{dT} &= \frac{d(l_f/T)}{dT} > 0, \quad \frac{dc}{dT} > 0, \\ \text{and } \frac{d(nb_f/T)}{dT} &< 0, \quad \text{but } \frac{db_f}{dT} \geq 0. \end{aligned} \tag{11}$$

When women spend part of their time in the household, an increase in T increases overall incentives to invest in market human capital, because the time over which the returns can be enjoyed is longer. Therefore, the opportunity cost of time (and fertility) increases. In addition, the gain in longevity itself also reduces the benefits from larger families, and these two forces together determine a reduction in fertility. The result is that the total amount of time spent on market related activities increases more than proportionally with the gain in life-span, reducing the fraction of time allocated to the household. In the end, the fraction of f 's productive life allocated to labor supply (l_f/T) – the female labor force participation – rises, and the time invested in each child may respond positively or negatively, depending on the relative responses of fertility and the total time allocated to the household.

Understanding the market wage as the productivity of one unit of time allocated to labor

supply, we can define the wage rate as $w_i = H_i = Ah_p e_i = Ah_p \tilde{t}_i / 2$, where \tilde{t}_i denotes the total amount of agent i 's time allocated to the market. Since gains in longevity lead to more than proportional increases in the time women allocate to the market, while increases in men's time are just proportional to T , the gender wage-gap $(1 - w_f/w_m)$ is reduced as the process described in the previous paragraph takes place.

In relation to the quality of children, the response of investment per child (b_f) to changes in longevity, though indeterminate, follows a very specific pattern. For lower levels of female labor force participation (below 40% irrespective of the parameters of the model), increases in longevity bring both increases in female labor force participation and improvements in the quality of children. For intermediary values of b_f , and conditional on other parameters, investments in children may decrease as longevity and female labor force participation increase. But for sufficiently high levels of female labor force participation, irrespective of the parameters of the model, increases in longevity are again accompanied by increased attachment of women to the labor force and increased quality of children. Therefore, for a given set of parameters, the relation between female labor force participation and investments per child is either always positive or non-monotonic. When the relation is non-monotonic, it starts as positive for low levels of female labor force participation, turns into negative for intermediary levels, and becomes positive again at high levels. Even in these cases, the quality of children increases with longevity for the vast majority of initial levels of female labor force participation.

The claims from the previous paragraph are proven in the Appendix, but here we give the intuition behind these results. Three forces relevant to decisions regarding investments in children are at work when longevity increases. First, the increase in longevity itself relaxes the resources constraint and increases full lifetime income. Second, the reduction in fertility that accompanies the gain in longevity reduces the relative price of investments in children. And third, due to increasing returns to market related activities, longevity increases the opportunity cost of time used in the household. The first two forces work toward increased investments in children, while the third works against it. For low levels of female labor force participation, the effect of increasing returns is relatively weak, so the first two forces dominate. At the other extreme, for high levels of female labor force participation, consumption is very high and fertility is very small, so the income effect is strong and it takes relatively little time to increase the quality of children. Also in this case, the first two forces dominate. The only situation where the third force may dominate is the intermediary one, where increasing returns kick in strongly and, to take advantage of them, women shift their time abruptly toward the market. In any case, it is not generally guaranteed that this third force will be strong enough to overcome the first two and, when even it does, it is

only for a relatively short interval.

Equilibrium B

In the case of Equilibrium B (market investments in children), the expressions derived before fully describe the household's allocation of time and its response to longevity changes. Both adult members of the household allocate their total lifetime to market related activities, sharing it equally between investments in human capital and labor supply. Contrary to before, the effect of longevity gains on the quality of children is unambiguously positive ($dx_y/dT > 0$), but the effects on consumption are ambiguous ($dc/dT \gtrless 0$). In this case, changes come from the income effect and from the fact that increases in longevity reduce the returns from larger families. As fertility is reduced as a result of the latter, the shadow cost of investments in children goes down, so that parents end up investing more in each child (there are no time costs in this case, and this effect can be seen as a positive substitution effect towards the quality of children). Whether consumption increases or not depends on how strong the income and substitution effects are.

Summary of the Two Equilibria and the Extensive Margin Choice

In Equilibrium A, longevity gains lead to increased investments in human capital, increased labor supply by women, narrowing of the gender wage-gap, and less time allocated to the household. The final effect on child quality depends on the initial position of the household, but there is a tendency for investments in children to increase with gains in longevity. Investments per child may be reduced by increases in longevity only for an intermediate and relatively short interval of values of female labor force participation. In Equilibrium B, on the other hand, women spend all their time on market related activities and there is no wage differential between genders. In this situation, increases in longevity lead to reductions in fertility and increases in the quality of children. Though Equilibrium B may seem somewhat extreme (mother's time entirely allocated to the market), qualitative results would be identical if we imposed an additional constraint requiring a minimal amount of mother's time to be allocated to the children. The only difference in this case would be the absence of complete wage convergence between men and women.

Finally, the extensive margin choice of the technology used to invest in children is also affected by longevity. For a large enough T , returns to market human capital are so high that both members of the household spend their entire time on market related activities, and make their investments in children through the market. For lower levels of T , this may not be the case, and women may share their adult lifetime between market activities and raising children. The intuition for this is clear: for lower T , the returns to market human capital and the cost of time are lower, the family is poorer, and, therefore, it is cheaper to spend time investing in children

instead of buying this investment in the market. As longevity increases, returns to human capital (and family income) rise, so that women increase their level of education and the opportunity cost of time. When this change is large enough, it becomes cheaper for the family to make its investments in children through the market, and allocate all the available time of its members to investments in adult education and labor supply.

Long-Run Growth

Differences in investments in children in this model translate into long-run differences in growth rates, since basic human capital determines the productivity of later investments in market specific human capital. In steady-state, the growth rate between generations is determined by the evolution of basic human capital between parents and children.⁸ Changes in women's labor force participation may affect the accumulation of basic human capital in different ways, depending on the equilibrium that characterizes the economy. In the case of Equilibrium A, the effect is ambiguous, but tends to be positive. This comes directly from the fact that longevity has ambiguous effects on the quality of children, which nevertheless are positive for relatively low and high levels of female labor force participation. In terms of growth rates,

$$1 + g = \frac{h_c}{h_p} = aBb_f, \text{ and} \tag{12}$$

$$\frac{d(1 + g)}{dT} = aB \frac{db_f}{dT} \geq 0.$$

In Equilibrium B, all the investment in children is purchased through the market. Also in this situation, increases in longevity reduce fertility, relax the budget constraint, and increase investments in children:

$$1 + g = \frac{h_c}{h_p} = (1 - a) \frac{x_y}{h_p}, \text{ and} \tag{13}$$

$$\frac{d(1 + g)}{dT} = \frac{(1 - a)}{h_p} \frac{dx_y}{dT} > 0.$$

It is therefore possible to observe an intermediary period when the growth rate of the economy is reduced as women increase their attachment to the market. This would be due to reduced investments in children during this transition period. But eventually, the quality of children and the growth rate start rising again as women intensify their attachment to the market. This

⁸ As shown in Soares (2005), a steady-state only exists in this type of economy when $\alpha = \sigma$. We implicitly make this assumption whenever talking about steady-states. To keep notation to a minimum, we are not indexing by generations, and are distinguishing parent's and children's basic human capital by the subscripts p and c . These are obviously related across generations. If we let i index different generations, $h_{p,i+1} \equiv h_{c,i}$. So the growth rate of basic human capital from one generation to the next is $h_{c,i}/h_{p,i} = h_{p,i+1}/h_{p,i} = h_{c,i}/h_{c,i-1}$.

tendency is reinforced after women enter fully into the job market, and investments in children are bought outside of the household. From this point on, the cost of time becomes irrelevant in determining the quality of children and the long-run growth of the economy.

5 The Effect of Child Mortality Reductions

The impact of child mortality changes is much simpler in nature than that of adult longevity. The effects of child mortality on fertility can be seen, as before, from the expression $\rho'n(1-\beta)T/\rho = \alpha$. This relation implies that reductions in child mortality will be accompanied by reductions in fertility:

$$\frac{dn}{d\beta} = \frac{n}{(1-\beta)} > 0. \quad (14)$$

This is the only direct effect of child mortality in the model, and all other changes follow from how fertility affects other dimensions of the household decision. In Equilibrium A (woman sharing her time between market and non-market activities), the household allocation of time between domestic and market related activities is also affected by the change in fertility. The reductions in fertility and child mortality imply a reduction in the shadow price of investments in children (or an increase in the rate of return to these investments). In the presence of increasing returns to market activities, the marginal cost of market goods in terms of time is decreasing in total consumption (or, alternatively, the marginal productivity of goods is increasing in the total amount of time allocated to the market). This non-linear time cost tends to magnify responses to price changes. For this reason, the effect of the reduction in the shadow price of child quality represented by the fertility decline is strong enough to increase the total amount of time allocated to children (see Appendix). This is guaranteed to happen just because there are increasing returns to market activities. Otherwise, the reduction in fertility would generate the typical price response of a normal good, where consumption (b_f or h_c) necessarily increases, but total expenditures (nb_f) may not. Notice that, since there is a reduction in the total amount of time allocated to the market, female educational attainment and labor force participation are actually reduced by reductions in child mortality, and the wage differential between men and women rises.

In Equilibrium B (investments in children through the market), the reduction in child mortality and fertility is reflected on higher investments in children, as the simple response of a normal good to price changes. But the choice between the two equilibria is also affected by child mortality. In order to be optimum for the household to allocate part of the woman's time to the market, it must be the case that $p\frac{a}{(1-a)}\frac{B}{A} > e_f$. Since the reduction in child mortality tends to reduce female investments in market human capital, reductions in child mortality tend to move the economy away from Equilibrium B and toward Equilibrium A.

In both equilibria, increased investments in children lead to higher growth rates and, in the long run, higher consumption (notice that this may take place at the expense of reductions in present consumption). But, most importantly, reductions in child mortality cannot generate increased female labor force participation nor a narrowing gender wage-gap. In fact, the model generates exactly the opposite result when women spend part of their time on domestic activities: reductions in child mortality lead to reduced participation of women in the labor market and to a widening wage differential between men and women. This result highlights the key position occupied by adult longevity in our theory. Specifically, reductions in fertility are not enough to generate the typical changes in women’s participation in the labor force. Increased return to market specific human capital is an additional feature that is essential in explaining the observed trends.

6 Predictions the Model and Related Evidence

In the theory developed here, the different productivity of men and women at raising children generates a tendency for specialization of women in household activities. But increases in adult longevity change the return to time invested in the market and affect the opportunity cost of children. When women share their time between the household and the market, increases in longevity reduce fertility and increase women’s investments in human capital and labor market participation. In this case, there is actually an increase in the fraction of total female time devoted to market related activities. Therefore, an increase in longevity leads to a more than proportional increase in female labor supply and investments in human capital. Since longevity leads to a proportional increase in the time men allocate to the market, increases in longevity lead to increased female labor force participation (l_f/T) and reductions in the gender wage-gap ($1 - w_f/w_m$).

This remains true as long as women spend some amount of time in the household. When longevity is high enough so that the economy moves to Equilibrium B (men and women allocating all their time to the market), the gender-wage gap disappears, and so do all the observable differences between men and women in terms of economic outcomes. A less extreme version of this result would be obtained if we imposed the additional constraint that a minimal amount of woman’s time is required to conceive a child (pregnancy). In this case, the gender-wage gap would decrease monotonically until Equilibrium B were reached, when it would then reach its minimum value (determined by the equilibrium level of fertility). Therefore, the model implies that if there is a minimal requirement in terms of women’s time in order to conceive a child, there will never be a complete elimination of the gender wage-gap.

The effect of these changes on the quality of children, on the other hand, is less clear cut.

For lower levels of female labor force participation, increases in longevity improve the quality of children. For intermediary values of female labor force participation, investments in children may decrease in a certain interval as longevity increases, but for sufficiently high labor supply of women, increases in longevity lead again to increased quality of children. When longevity is high enough so that women allocate all their available time to labor supply and investments in adult human capital, the positive effect on the quality of children is reinforced by the fact that investments in children are purchased on the market, at a fixed price.

The model can, therefore, generate either a positive or a non-monotonic relationship between women's participation in the labor market and child quality. For certain levels of longevity and income, increases in female labor force participation may reduce child quality, while the opposite certainly happens for sufficiently low or high levels. When comparing these results with the available demographic evidence on the allocation of time within families, it should be kept in mind that there is one important dimension of the family's problem that is absent from the model. This is the one related to the consumption value of leisure and time with children. Both of these are thought to be luxury goods, in which case they should increase with gains in full lifetime income. We chose to keep these dimensions out of the model to keep things simple, and make our main points as clear as possible. In addition, typical data on allocation of time distinguishes between childcare activities – which involve direct personal contact with the child – and general housework. In our model, all activities related to raising a child, including those that do not involve direct contact with the child (cooking meals, doing laundry, cleaning the room, etc.), are incorporated in the variable b_i , and therefore should be counted as childcare activities.

Keeping these limitations of matching the model to the time-use data in mind, some recent demographic evidence seems to support the type of non-monotonic relationship potentially generated by our theory. Gauthier et al (2004), for example, present evidence for the US, UK and Canada indicating that average time spent on child-care activities was reduced up to the end of the 1970's, after when it is started rising again. They also show that since 1960 average time spent on housework by mothers was reduced by more than 30% (this reduction was partly compensated by increased housework by men, but the compensation accounts for less than one-third of the reduction in women's time). Overall, time spent by women in housework and childcare together was reduced by roughly 1.2 hours a day. The differential behavior in the allocation of time across activities that involve direct contact with the child and activities that do not probably reflects different degrees of substitutability between parents' time and market goods along these dimensions, as well as different prices of the market substitutes available.

Sayer et al (2004) also find the same non-monotonic trend in terms of time allocated to direct

childcare by mothers, with reductions observed between 1965 and 1975, and increases afterwards. In addition, they also find evidence that activities involving a direct and deeper interaction with children have increased in relative importance. In particular, they show that the allocation of time to different types of childcare activities shifted toward “human capital-intensive” activities and away from more basic activities (“teaching and playing” increases in relative importance when compared to “primary and daily care”). Finally, they also show that reductions in fertility have reduced the total amount of time allocated to childcare.⁹

A different strand of literature whose results are also relevant to our theory refers to the direct impact of maternal labor force participation on outcome measures of child development. Typically, papers in this literature try to estimate the effect of mother’s employment and hours worked on a child’s educational attainment, achievement test performance, teacher’s rating, and behavior. The results are largely mixed, with negative, positive, and quantitatively irrelevant effects all being common (see discussion and summary of this literature in James-Burdumy, 2005). In a recent study, James-Burdumy (2005) addresses some of the problems common to the previous papers – absence of control for family specific effects and instruments for mother’s labor force participation – and finds only modest effects of mother’s labor supply on child performance. Specifically, the author finds that math performance at age 9 measured by the Peabody Individual Achievement Test (PIAT) is negatively affected by maternal hours and weeks of work in the first year of the child’s life, while reading is negatively affected only by weeks worked. On the other hand, weeks worked in the third year of the child’s life are found to increase the PIAT math scores at age 9. Nevertheless, these statistically significant results are quantitatively very small, and even extreme changes in mother’s labor force participation can only explain a tiny fraction of the variation in test performance. So rather than settling the debate, the results from James-Burdumy (2005) add to the controversy. Our theoretical predictions suggest that contradictory results in studies looking at different groups of the population, or weak quantitative results in studies looking at broad groups, might be exactly what should be expected, given the possibility of non-monotonicity in the relation between female labor force participation and quality of children. A test of this specific possibility should include non-linear terms in the interaction between mother’s labor force participation and, for example, education or income, in regressions like the ones ran by James-Burdumy (2005).

Finally, our model cannot generate a link between reductions in child mortality and increases in female labor force participation (or reductions in the gender wage-gap). This prediction is in

⁹ McLanahan (2004) argues that increased female labor force participation in the US was accompanied by differential trends in terms of child quality for poor and wealthy families. But her focus is on issues related to family structure and stability, and their impact on child development. These dimensions are not captured by the theory proposed in this paper, so her evidence, though consistent with our results, cannot be directly used to support them.

line with historical evidence, since changes in female attachment to the labor force only increase significantly at later stages of the demographic transition, when the main child mortality reductions have already been observed and changes in adult longevity become relatively more important. This would explain the lag between the initial reduction in life expectancy during the onset of the demographic transition and the later increase in female labor force participation, as Figures 1, 2, and 3 illustrate. Indeed, the main issue that made it difficult to relate female labor force participation to the demographic transition is precisely the fact that increased labor supply of women only starts considerably after significant reductions in mortality and fertility are observed. Our theory overcomes this problem by showing that reductions in fertility are not enough to generate a movement of women out of household work. The latter is only guaranteed when reduced fertility is accompanied by increased adult longevity, which does not happen until later stages of the demographic transition.

7 Concluding Remarks

This paper proposes a theory where, ultimately, longevity gains are solely responsible for the increased participation of women in the labor market and the narrowing gender wage-gap. Though the direct link between these two phenomena may seem obscure, the intuition becomes clear once other dimensions of the demographic transition are brought into the analysis. Increased longevity increases the returns to investments in market oriented human capital. Higher returns to investment in human capital increase the cost of time and shift the quantity-quality trade-off towards fewer and better educated children. In addition, lower mortality reduces the returns from large families, reducing fertility also directly as an independent force. With higher returns to human capital and fewer children, women increase their investments in human capital and their attachment to the market. Since women are initially specialized in the household sector, this takes place at a rate more than proportional to the one observed for men, so that the gender wage-gap is reduced. These changes also have important consequences for investments in children, and may generate a non-monotonic relationship between female labor force participation and child quality. For intermediary levels of women's labor supply, further increases in the female attachment to the labor market may take place at the expense of investments in offspring, therefore reducing the quality of children. Nevertheless, even in these cases, sufficiently large gains in longevity eventually lead to increased investments in children. This framework explains several social changes observed in the course of the last century as being caused ultimately by reduction in adult mortality.

A Appendix

A.1 First Order Conditions

Let Ψ , λ_f , λ_m , and Π denote sequentially the multipliers for the constraints in the maximization of equation 8. Substituting for h_c and x_T in the objective function, the first order conditions for c , n , e_m , l_m , b_m , e_f , l_f , b_f and x_y are given by, respectively:

$$\begin{aligned}
Tc^{\sigma-1} &= \Psi T, \\
\rho' T(1-\beta) \frac{h_c^\alpha}{\alpha} &= \Psi p x_y + \lambda_f b_f, \\
A h_p l_m \Psi &= \lambda_m, \\
A h_p e_m \Psi &= \lambda_m, \\
\rho h_c^{\alpha-1} a C h_p &< \lambda_m n, = \text{if } b_m > 0 \\
A h_p l_f \Psi &= \lambda_f, \\
A h_p e_f \Psi &= \lambda_f, \\
\rho h_c^{\alpha-1} a B h_p &< \lambda_f n, = \text{if } b_f > 0 \text{ and} \\
\rho h_c^{\alpha-1} (1-a) &< \Psi p n, = \text{if } x_y > 0.
\end{aligned}$$

A.2 The Choice of the Technology for Investments in Children

When investments make use of the mother's time, $b_f > 0$ and we have that $\rho' T(1-\beta) \frac{h_c^\alpha}{\alpha} = \frac{\rho h_c^{\alpha-1} a B h_p}{n} b_f$. And when investments make use of income, $x_y > 0$ so that we can write $\rho' T(1-\beta) \frac{h_c^\alpha}{\alpha} = \frac{\rho h_c^{\alpha-1} (1-a)}{n} x_y$.

Proposition 1 *There is a T^* such that, in an equilibrium with growth, for every $T < T^*$ investments in children are done domestically (using the mother's time), and for every $T \geq T^*$, investments are done with goods purchased on the market.*

Proof. The rate of return to investments using the domestic technology (marginal productivity divided by opportunity cost) is given by $RRb_f = \frac{\rho h_c^{\alpha-1} a B h_p}{\lambda_f n}$. The rate of return to investments using income is $RRx_y = \frac{\rho h_c^{\alpha-1} (1-a)}{\Psi p n}$. The household will choose to use time whenever $RRb_f \geq RRx_y$, or $\frac{a B h_p}{\lambda_f} > \frac{(1-a)}{\Psi p}$. Substituting for λ_f , this inequality can be rewritten as $e < p \frac{a}{(1-a)} \frac{B}{A}$. For T sufficiently low (in particular, for $T < p \frac{a}{(1-a)} \frac{B}{A}$) $RRb_f > RRx_y$ and investments in children are done domestically. In addition, since e increases at least proportionately with T , there is a T large enough so that $e \geq p \frac{a}{(1-a)} \frac{B}{A}$. In this case, $RRb_f \leq RRx_y$ and investments in children are done through the market. ■

A.3 The Equilibrium with Time Investments in Children

When investments in children make use of the domestic technology, it is convenient to rewrite the problem in terms of the shares of total lifetime (T) dedicated to each different activity. Define the new variables $e_i^* = e_i/T$, $l_i^* = l_i/T$ and $d_i = nb_i/T$ as the shares of total lifetime allocated to, respectively, investments in adult human capital, labor supply, and domestic activities (raising children). Incorporating $x_y = 0$, the original problem can be rewritten as

$$\max_{e_i^*, l_i^*, d_i, n, c} T \frac{c^\sigma}{\sigma} + \rho[(1 - \beta)Tn] \frac{h_c^\alpha}{\alpha}$$

subject to

$$\begin{aligned} (Ah_p e_m^* l_m^* + Ah_p e_f^* l_f^*) T^2 &\geq Tc, \\ 1 &\geq e_i^* + l_i^* + d_i, \text{ with } i = m, f, \\ h_c &= \frac{T}{n} a (Bd_f + Cd_m) h_p, \text{ and} \\ e_i^*, l_i^*, d_i &\geq 0. \end{aligned}$$

Let λ be the multiplier on the first constraint, and λ_i the multiplier on the second constraint for agent i . In the equilibrium where men specialize in market activities and women share their time between market and domestic activities, first order conditions can be written in terms of the new variables as

$$\begin{aligned} c^{-\sigma} &= \lambda, \\ \rho'(1 - \beta)T \frac{h_c^\alpha}{\alpha} &= \rho h_c^{\alpha-1} \frac{T}{n^2} a (Bd_f + Cd_m) h_p, \\ \lambda T^2 Ah_p l_m^* &= \lambda_m, \\ \lambda T^2 Ah_p e_m^* &= \lambda_m, \\ \rho h_c^{\alpha-1} \frac{T}{n} a C h_p &< \lambda_m, \\ \lambda T^2 Ah_p l_f^* &= \lambda_f, \\ \lambda T^2 Ah_p e_f^* &= \lambda_f, \text{ and} \\ \rho h_c^{\alpha-1} \frac{T}{n} a B h_p &= \lambda_f. \end{aligned}$$

Therefore, $e_m^* = l_m^* = 1/2$, $d_m = 0$, and $l_f^* = e_f^*$. For each individual, the time spent with investments in adult human capital and labor supply is always the same. Therefore, to save on notation, we write t_i as the proportion of total lifetime allocated to market related activities (investments in human capital plus labor supply), so that $l_i^* = e_i^* = t_i/2$ ($t_i = \tilde{t}_i/T$, where \tilde{t}_i was defined before as the total amount of agent i 's time allocated to the market).

Rewriting the problem after substituting for the budget constraint directly into the utility function, and considering the equilibrium where $t_m = 1$, and $d_f, t_f \in [0, 1]$:

$$\max_{t_f, d_f, n} T^{1+\sigma} \left(\frac{Ah_p}{4} \right)^\sigma [1 + t_f^2]^\sigma / \sigma + \rho[(1 - \beta)Tn] \frac{h_c^\alpha}{\alpha}$$

subject to

$$\begin{aligned} 1 &\geq t_f + d_f, \text{ and} \\ h_c &= \frac{T}{n} aBh_p d_f, \end{aligned}$$

and to the additional constraint that $t_f, d_f \geq 0$. In this form, first order conditions become

$$\begin{aligned} (1 - \beta)T\rho' \frac{h_c^\alpha}{\alpha} &= \rho h_c^{\alpha-1} \frac{T}{n^2} aBh_p d_f, \\ T^{1+\sigma} \left(\frac{Ah_p}{4} \right)^\sigma [1 + t_f^2]^{\sigma-1} 2t_f &= \lambda_f, \text{ and} \\ \rho h_c^{\alpha-1} \frac{T}{n} aBh_p &= \lambda_f. \end{aligned}$$

The objective function is concave on d_f and convex on t_f , so we cannot, in principle, guarantee unicity of the internal solution nor trust on the Hessian to verify that a point of maximum is reached. The issue in question and the optimal solution to the problem can be better understood with the help of a figure. Define

$$\begin{aligned} f(t) &= T^{1+\sigma} \left(\frac{Ah_p}{4} \right)^\sigma [1 + t_f^2]^\sigma / \sigma, \text{ and} \\ g(d) &= \rho[(1 - \beta)Tn] \frac{\left(\frac{T}{n} aBh_p d_f \right)^\alpha}{\alpha}. \end{aligned}$$

Conditional on being on this equilibrium, and given the choice on the number of children (n), the optimal allocation of the woman's time is the solution to the following problem.

$$\max_{t_f, d_f} f(t_f) + g(d_f)$$

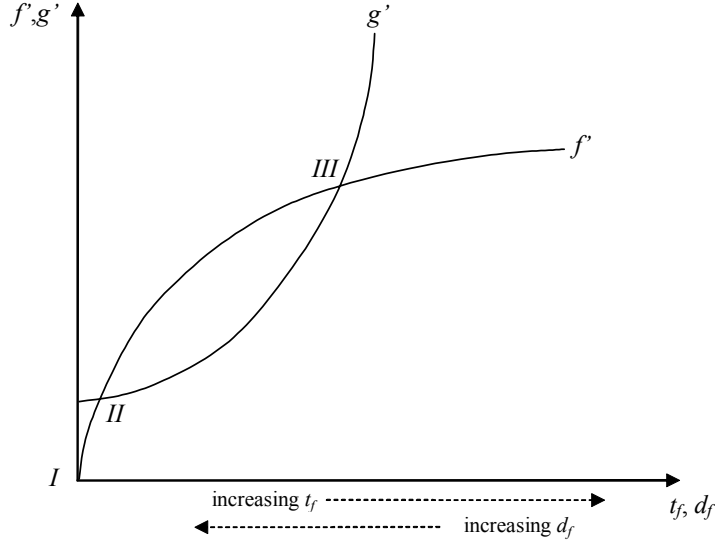
subject to $t_f + d_f = 1$ and with $t_f, d_f > 0$.

The optimum is characterized by $f'(t_f) = g'(d_f)$, where these derivatives are given by the left hand side of the last two first order conditions above. One can show that

$$\begin{aligned} f''(t_f) &= 2T^{1+\sigma} \left(\frac{Ah_p}{4} \right)^\sigma [1 + t_f^2]^{\sigma-2} [1 + t_f^2 + 2(\sigma - 1)t_f^2] > 0, \\ g''(d_f) &= (\alpha - 1) \rho[(1 - \beta)Tn] \left(\frac{T}{n} aBh_p \right)^\alpha d_f^{\alpha-2} < 0, \text{ and} \\ f'''(t_f) &= 2T^{1+\sigma} \left(\frac{Ah_p}{4} \right)^\sigma [1 + t_f^2]^{\sigma-3} [6(\sigma - 1)t_f + 2(\sigma - 1)^2 t_f^2 + 2\sigma t_f^3 (\sigma - 1)] < 0, \end{aligned}$$

where the inequality comes from the fact that $t_f > t_f^3$. In addition, these functions are characterized by the following properties: $g'''(d_f) > 0$, $\lim_{x \rightarrow 0} g'(d_f) = \infty$, $\lim_{x \rightarrow 1} g'(d_f) = \text{constant} > 0$, $\lim_{t \rightarrow 0} f'(t_f) = 0$, and $\lim_{t \rightarrow 1} f'(t_f) = \text{constant} > 0$. So we can plot the functions f' and g' against the fraction of time allocated to market and non-market activities (see Figure A.1).

Figure A.1: Characterization of First Order Conditions



In the figure, we assume that the two curves intersect. If they did not, the origin would be the optimal choice. Points *II* and *III* are the ones that satisfy the first order conditions.

Proposition 2 *Point III is preferable to point II and, therefore, is the solution to the household problem.*

Proof. Starting from point *II* and moving to the right, t_f increases while d_f is reduced. The gains in terms of utility are given by f' , while the losses are given by g' . Since $f' > g'$ in (*II*, *III*), *III* is preferable to *II*. ■

Points *I* and *III* are not comparable on a strictly graphical basis. More rigorously, their ordering would depend on the value of the integral $\int_0^c (f' - g') dt$. Point *I* corresponds to the equilibrium where there is total specialization within the household (man on the market and woman in the household). For lower values of T , the integral above is negative, and the optimal choice is point *I*. For a higher T , the value of the integral increases and eventually becomes positive. This is what characterizes the first movement of women into the labor market.

Generally, the comparative statics of the problem can be analyzed from the impact of changes in T on the curves f' and g' . As T increases, f' moves vertically at a rate $(1 + \sigma)$, and g' moves

vertically at a rate 2α .¹⁰ In order for a steady state to exist in this economy, we must have $\alpha = \sigma$ (see Soares, 2005). In this case, f' shifts at a faster rate and, therefore, point *III* moves to the right, corresponding to a higher t_f and a lower d_f .

A.4 The Quality of Children in the Equilibrium with Time Investments

With the first order conditions and after some tedious algebra, it can be shown that

$$\frac{db_f}{dT} = \frac{D}{\left(2D + (1 - \sigma) \left[(1 - d_f) + (1 - d_f)^3 \right] \right)} \frac{b_f}{T},$$

where $D = -(1 - \alpha)(1 - d_f) - (1 - \alpha)(1 - d_f)^3 + d_f + d_f(1 - d_f)^2 - 2(1 - \sigma)(1 - d_f)^2 d_f$ and $d_f = nb_f/T$. This implies that $db_f/dT < 0$ if and only if

$$-\frac{(1 - \sigma) \left[(1 - d_f) + (1 - d_f)^3 \right]}{2} < D < 0.$$

Expanding the polynomials on D and incorporating the condition for existence of steady-state ($\alpha = \sigma$), this inequality can be rewritten as

$$-\frac{(1 - \sigma) \left[(1 - d_f) + (1 - d_f)^3 \right]}{2} < -(1 - \sigma)2 + (2 - \sigma)2d_f - (1 + \sigma)d_f^2 + \sigma d_f^3 < 0.$$

The effect of the change in longevity on the quality of children, therefore, depends on the equilibrium fraction of time allocated to the household (d_f). In addition, the set of possible values of d_f compatible with Equilibrium A being optimum depends on the other parameters of the model. From the extensive margin choice between the two technologies, the maximum level of longevity compatible with Equilibrium A is determined implicitly from

$$e_f = \frac{T(1 - d_f)}{2} = p \frac{a}{1 - a} \frac{B}{A}.$$

Since the left hand side is constant and $T(1 - d_f)$ increases more than linearly with T , we know that there is a T high enough so that this expression is satisfied (even though it may be already with $d_f = 0$). This is the level of longevity at which the family stops using time to invest in the children and starts using market goods. Call this level of longevity and the associated fraction of time spent in the household \hat{T} and \hat{d}_f , respectively. Notice that the expression above implies that any value of d_f between 0 and 1 is always compatible with Equilibrium A being optimum for some set of parameters. Particularly, changes in p do not affect the intensive margin choice of d_f in Equilibrium A, but do affect the extensive margin choice between Equilibria A and B. So, for

¹⁰ This result uses the fact that, in equilibrium, $(1 - \beta)nT$ is constant.

any given values of the other parameters, there is always a p such that virtually all values of d_f (corresponding to changing T 's) are compatible with the optimality of Equilibrium A.

In principle, this means that d_f can really take any value between 0 and 1 for any value of σ , depending on the set of parameters. So, in order to check when the inequalities above are satisfied, we have to check whether they hold for all the values of d_f and σ between 0 and 1. Based on the inequalities above, the table below shows whether db_f/dT is negative or positive for a grid of values of d_f and σ between 0 and 1.

Table A.1: Signal of db_f/dT for Different Values of d and σ

$d\sigma$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.05	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.10	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+
0.15	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	+	+	+
0.20	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	+	+	+	+
0.25	+	+	+	+	+	+	+	+	+	-	-	-	-	-	+	+	+	+	+
0.30	+	+	+	+	+	+	+	+	-	-	-	-	+	+	+	+	+	+	+
0.35	+	+	+	+	+	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.40	+	+	+	+	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+
0.45	+	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+
0.50	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.55	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.60	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.65	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.70	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.75	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.80	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.85	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.90	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.95	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

It is clear from the table that, in terms of this grid, db_f/dT is positive in the vast majority of cases (or, generally, it is positive in the vast majority of the area defined when d_f and σ vary between 0 and 1). Nevertheless, this result does not mean that db_f/dT is positive for all the empirically relevant cases.

But it does mean that when female labor force participation starts rising from low levels (typically well below 40%), child quality increases together with the increased attachment of women to the labor market. After this process takes place for some time, it is possible that further increases in female labor force participation are accompanied, in a certain interval, by reductions in the quality of children. Yet, even in this case, this would be no more than a temporary phenomenon. Further increases in longevity eventually bring back the positive association between increased female labor force participation and child quality. This happens either because, as d_f

is reduced within Equilibrium A, the range of negative values of db_f/dT is surpassed, or because the household eventually finds it optimum to move from Equilibrium A into Equilibrium B.

Therefore, the model generates either a positive or a non-monotonic relationship between female labor force participation and quality of children. In the case of the non-monotonic relation, increases in female labor force participation are initially accompanied by increases in child quality, then by reduced investments in children (for intermediary values of female labor force participation), and then again by increased quality of children. Section 4 discusses the intuition for these results.

A.5 The Equilibrium with Market Investments in Children

When investments in children's basic human capital is done through the market, increases in longevity reduce the opportunity cost of investments in children, because of a lowered fertility rate and an expanded budget constraint (higher educational attainment on the part of both members of the family)

From the first order conditions in this situation, we can obtain $c^{\sigma-1}pn = \rho h_c^{\alpha-1} (1 - a)$. Working with this expression and incorporating the condition for a steady-state ($\alpha = \sigma$), we obtain the following relation between the changes in c , n , and h_c .

$$\frac{dh_c}{h_c} = \frac{dc}{c} + \frac{dn}{n} \frac{1}{(\sigma - 1)}.$$

In addition, previous results related to educational attainment and fertility hold, so that we have $e_i = l_i = \frac{T}{2}$ for $i = m, f$, and $\frac{dn}{dT} < 0$.

When T increases, n falls and y increases more than proportionately ($y = l_m H_m + l_f H_f = Ah_p T^2/2$), so either c or x_y must increase (from the budget constraint, $Tc + fn + px_y = y$). If c increases, from the equation above, h_c must also increase, so x_y grows. If c falls, from the budget constraint, x_y increases, and so does h_c . In any case, increases in longevity lead to increases in investments in children.

A.6 Child Mortality and the Allocation of Time

Proposition 3 *A reduction in the child mortality rate increases the amount of time that the woman allocates to household activities.*

Proof. A reduction in the child mortality rate reduces the fertility rate (see derivation in the text). In Figure A.1 from the Appendix, it can be seen that changes in child mortality affect the function g . The reduction in fertility generated by the child mortality reduction shifts the the curve g' upwards, shifting the optimal point III to the left (increasing the amount of time that the woman allocates to the household). ■

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