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THE DESIGN OF A CONSUMPTION TAX UNDER CAPITAL RISK

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Abstract

This paper focuses on the design of a consumption tax in a world of capital risk. The certainty literature discusses two standard options, namely the *cash flow method* and the *pre-payment method* (i.e., the wage tax), and finds the two approaches to be equivalent. Models that consider capital risk (via asset choice) reach different conclusions. This discrepancy arises in part due to a different choice of the social discount rate. In light of the failure of the discount rate argument to resolve the issue at hand, we explore the market *certainty equivalence* of risky government revenue. We let revenue risks stay in the private sector, and examine the market value of the feasible transfer (e.g., in the form of a public good) back to households. We reach *three* broad conclusions. First, we find that if the state returns to each household its own tax revenue risks, equivalence will be re-established as in certainty models. Next, we show that if the state engages in intergenerational risk sharing (e.g., through a system of stochastic tax-transfers), the wage tax cannot be construed to be a valid pre-payment alternative to the cash flow or a modified wage taxation system. Efficient risk allocation across generations under a cash-flow tax (or, one that includes future captial gains as well as wages in the tax base) leads to a Pareto improvement over the simple wage tax. Finally, a major policy implication follows; in order to be practicable, a consumption tax would have to be implemented via registered savings accounts much in the fashion of the Canadian RRSP program rather than through the pre-payment route.

Keywords: Cash Flow Tax, Risky State Revenue, Intergenerational Risk Sharing

JEL Classification: H30, H43, H63

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The Design of a Consumption Tax under Capital Risk

Syed M. Ahsan and Peter Tsigaris[1]

1 Introduction

The principal aim of the present paper is to evaluate alternative methods of implementing a consumption tax in a savings-portfolio choice model. Proposals for implementing a consumption tax can be classified into two categories. The first, the cash flow method, allows a tax deduction for the entire amount saved, while the proceeds (principal and return) become taxable *only* upon withdrawal. Below we refer to this tax as a *cash flow tax* (CFT). The second method is a modification of the income tax system, where the return on savings is exempted from taxation altogether. In other words, individuals *pre-pay* tax on the total wage income (inclusive of inheritances, if any) such that savings occur out of net-of-tax resources. The latter, a simple wage tax (WT), is also known as the *pre-payment tax* (PPT). Focusing on proportional taxes, examination of lifetime budget sets in models *without* asset choice reveals that the CFT and WT (or, CFT and PPT) have identical effects on the household budget. And upon simple rules for setting taxes, the state would also obtain an identical present value of revenue under each regime. Since the budget set is identical, consumption allocation as well as the maximised level of lifetime utility must also remain unchanged between the two regimes. These observations have been taken to mean that the CFT and WT were *equivalent,* even though some economists would prefer that the government collect identical revenue in each period rather than an equal present value of revenue from each cohort.[2]

Does such equivalence continue to hold between the two taxes once asset choice (say, between safe

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[1] The authors are at Concordia University, Montreal, Quebec and Copenhagen Business School, Denmark, and at the University College, Cariboo, British Columbia, respectively. Ahsan would like to acknowledge the kind hospitality at the Central European University in Prague, Center for Economic Studies of the University of Munich, the University of Bonn, and at the Economic Policy Research Unit of the Copenhagen Business School on visits during the writing of the paper. The authors would like to thank Christian Ghiglino, Hans-Werner Sinn, seminar participants at the Universities of Bergen, Bonn, Dortmund, Heidelberg, Munich, Oslo, Purdue, California at Riverside, the 1995 IIPF conference in Lisbon, and especially, the referees for constructive comments and suggestions.

[2] In a life-cycle setting, note that the timing of tax payments would vary between consumption and wage taxes, implying that private savings (occurring in period one) would differ between the two. However, defining *total* savings to include government savings (i.e., tax revenue) as well, clearly the former is of an identical magnitude in both cases, at least in the perfect capital market case. Summers, in particular, notes the failure of the two taxes in yielding an identical amount of revenue in *each period.* Difficulties arise in a general equilibrium setting when the age structure of population is taken into account. Since the age-savings pattern would differ between the two taxes, wage taxation would significantly reduce capital intensity vis-a-vis a cash flow tax. See Atkinson-Stiglitz (1980, 70-71), Kotlikoff (1993) and Summers (1981) for a discussion.

and risky) is allowed? Views differ. There are economists who argue, much as in one-asset models, that all capital income should be exempted from income taxation in order to make it equivalent to a consumption tax [e.g., Zodrow (1995)], and hence conclude that a wage tax is indeed a valid pre-payment alternative even under risk. We show that the preceding result requires that one use a particular discount rate (namely, the expected *weighted* average return on the portfolio) in order to value future tax revenue. An alternative is to modify the pre-payment tax by adding in the excess return (or, *capital gains)* to the base. Thus one exempts the imputed risk free return on the entire portfolio in order to obtain the equivalence between CFT and PPT. The resulting tax has been baptised the "modified wage tax" (MWT) (Ahsan 1989, 1990). The latter proposition is valid when one uses the riskless rate as the discount rate. In effect, the argument put forward by Ahsan (1990) requires that capital gains or losses receive the same treatment under both cash flow and pre-payment versions; they are subject to tax.

The two results cited above have each relied on an *exogenous* discount rate, and little motivation has been provided in support of the particular choice. This paper attempts to ameliorate this sense of ambivalence. We start our analysis of tax equivalence by describing a behavioural model of risk disposition by the government. We consider two alternative mechanisms. In the first, the government returns lump sum to each household the lifetime revenue as taken from the same household. Taxpayers value the stochastic transfer by evaluating the latter's *market certainty equivalent*. The market value is computed by invoking the *capital asset pricing model* (CAPM). We find that only individuals who hold fully diversified *market portfolios* would remain indifferent between a consumption tax and a wage tax.[3]

We then turn to a second approach of risk disposal. We consider an environment where investors only bear the net-of-tax fraction of the risk; the balance is passed on to future, as yet unborn, generations through the intermediation of the government. In exchange, the current generation also shoulders a share of the risky tax revenue collected from past generations of taxpayers. Thus each generation receives as transfer a composite portfolio of current and past lotteries. The possibility of the state's ability to pre-commit a generation to engage in such risk sharing with future generations has already been recognised in the public economics literature (Gordon and Varian, 1988). We show that to the

[3] In valuing risky corporate tax revenue, Bulow and Summers (1984) and Hamilton (1987) focused on the excess return, and argued that the latter's market value was zero. The conclusion was a direct consequence of their belief in complete markets. However, the evidence cited by Friend and Blume (1975) and Gordon (1985) indicates that typically households do not diversify fully, not to speak of non-tradable assets such as human wealth. Also note that the US social security to date has not invested in equity, thus preventing households from holding such assets even indirectly.

extent there is *some* revenue sharing across generations, the household utility is higher under the consumption tax (or the MWT) than under the simple wage tax à la Zodrow. We again use the CAPM to evaluate the certainty equivalent of the public transfer.

One great advantage of using the concept of *market certainty equivalent* of risky revenue is that the social discount rate implicit in the certainty equivalent expression gets determined in an *endogenous* fashion. In particular, when analysing the intergenerational risk sharing behaviour, we find that if the number of generations were many, the social discount rate converges to the risk-free rate. Evidently, the *systematic* risk facing a given generation becomes idiosyncratic when it is shared with many future generations.

We therefore reach *three* broad conclusions. First, we show that the simple wage tax can be construed as a valid pre-payment alternative to the cash-flow tax only under extreme situations, namely where existing market institutions permit households to acquire portfolios consisting of all tradable assets. Or, alternatively, where there is no possibility for the state to engage in inter-generational risk sharing. To the extent that these premises are invalid, as evidence would suggest, household welfare would be increased by implementing a system of CFT than a simple wage tax due to additional risk shifting via the budget process. Indeed, the modified wage tax (MWT) referred to earlier does turn out to be equivalent to the CFT in the event the *current* revenue risks are shared with many future generations. Secondly, we find that a consumption tax would typically lead to a Pareto improvement in the risk allocation vis-à-vis the simple wage tax. Finally, we argue that on pragmatic grounds, it is perhaps easier to implement the CFT than a MWT since the latter would require that one track the time path of the relative return on household assets. The issues are similar to those involved in implementing a retrospective capital gains tax (Auerbach, 1991).

The rest of the paper proceeds as follows. Section 2 examines the notion of *equivalent taxes* in terms of household choice under alternative methods of implementing a consumption tax. Section 3 reviews the discount rate methodology, which has been used by previous writers to evaluate risky tax revenue. In section 4, we develop measures of certainty equivalent of risky government revenue by appealing to the capital asset pricing literature from two distinct perspectives, namely completeness of household portfolios as well as that of intergenerational risk shifting. In section 5, we briefly review the implications of our analysis for equity considerations. Section 6 offers some concluding remarks.

2. Household Budget Equivalence

This section introduces the household choice framework, and explores the meaning of equivalent budget sets from a household perspective. The model is the familiar *two-period* life cycle model, with a choice between a safe and a risky asset. No discussion of the preference structure of households is warranted at this stage.[4] The period by period pre-tax budget constraints facing the household born at time-t are given below.[5]

$$C_{1t} = Y_{1t} - (a_{1t} + m_{1t});$$
$$C_{2t} = a_{1t}(1+x_t) + m_{1t}(1+r),$$

where, the notation is as follows: consumption in each period is denoted by C_{it}, *(i= 1,2)*, and *Y* is first period income earned by the household by supplying a fixed amount of labour, while total saving, *S*, takes place via acquisition of assets.[6] The amount invested in the risky asset is given by *a*, and that in the safe form by *m*. The rates of return on investment are denoted by *x* and *r*, respectively, for the risky and the safe asset, with the former taken to be a random variable. When interpreted as financial assets, the institution of limited liability would require that *x* remain in the interval $[-1,\infty)$.

The lifetime budget constraint of the generation-t investor, in the absence of taxation, is therefore given by:

$$C_{1t} + \frac{C_{2t}}{(1+r)} = \left(Y_{1t} + \frac{a_{1t}\ z_t}{(1+r)} \right), \qquad (1)$$

where z_t (defined to equal $[x_t - r]$) is the excess return on the risky asset over the riskless. The latter quantity may be seen as the result of a change in the risky asset price over the holding period, and consequently, we often refer to the excess return as the *capital gain*. Evidently, the present discounted value of the consumption stream would be equal to the sum of endowment income and the present value of the capital gain.

Taxation. A proportional cash flow consumption tax (CFT), falls both on current (certain) and the second period (uncertain) consumption. With full loss-offset provisions, this tax therefore affects the budget constraint as follows:

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[4] In much of this paper we do not need to restrict household preferences except that they be consistent with the postulates of the capital asset-pricing model, and the latter are very general indeed.

[5] The subscript refers to the time of birth. Thus x_t denotes the random return earned by the t-investor when old.

[6] Assets may be either financial or real. In the latter event, we require that technologies be linear in each, and that one of the two marginal products be stochastic.

(2) $$C_{1t} + \frac{C_{2t}}{(1+r)} = \frac{1}{(1+\tau_c)} \left(Y_{1t} + \frac{a_{1t}\ z_t}{(1+r)} \right).$$

The wage tax (WT), on the other hand, falls only on the endowment (i.e., wage income), which is certain.[7] Thus, in the absence of inheritances, the wage tax constraint is:

(3) $$C_{1t} + \frac{C_{2t}}{(1+r)} = (1 - \tau_w)\ Y_{1t} + \frac{a_{1t}\ z_t}{(1+r)}.$$

Ex-Ante and Ex-Post Equivalence. By ex-ante equivalence of two taxes, we require that the household budget set, and hence expected utility, be the same between the two tax policies for any plausible decision rule. By contrast the concept of *ex-post* equivalence may be characterised as a fortuitous event where for a given decision rule, expected utility actually realised by a household happen to be the same for a pair (set) of tax polices in a given state of nature. In other words, the latter concept allows for the possibility of individuals to achieve an identical indirect utility for a given state of nature even where the budget sets were not identical *ex ante*. Valuation of government revenue would also have to be identical (again, *ex-ante or ex-post*, as appropriate) for any two taxes that are presumed equivalent.[8]

Ex-Ante Equivalence between Taxes. Given asset choice, note that while utility depends on the $\{C_1, C_2\}$-distribution, the right hand side (rhs) of the lifetime budget equations as presented above do not uniquely determine the feasible choice set for a given tax regime. The risk-taking variable would generally be a function of the tax regime. Examining equations (2) and (3), it is clear that *so long as both the government and households use the risk-free rate to discount future quantities, it would appear that WT and CFT budget sets are not ex-ante equivalent*. Taxation of the capital gain term would affect the asset choice in the latter event, an element absent from the wage tax world.

What if we include the capital gains component in the pre-payment tax base (call it a "modified wage tax" (MWT)? For a closer examination, let us write down the MWT budget constraint:

(4) $$C_{1t} + \frac{C_{2t}}{(1+r)} = (1 - \tau_d)\left[Y_{1t} + \frac{a_{1t}\ z_t}{(1+r)} \right].$$

---

[7] Note that an endowment uncertainty would get eliminated through aggregation if this risk were assumed to be of the idiosyncratic variety (Eaton and Rosen, 1980).

[8] Christiansen (1993), among those offering alternative equivalence criteria, lets the government collect an identical amount of revenue in each state of nature for all tax alternatives. Kaplow (1994), on the other hand, allows the state to actively participate in risk markets much like private agents, and consequently he has more rigid requirements of policies to be deemed equivalent. For example, one condition is that the total demand for each asset (combining private and public) remains the same under each policy.

Comparing CFT and MWT, it is apparent that neither tax distorts the choice between consuming now and later, but they each distort the choice of the portfolio mix. Clearly the two choice sets (i.e., equations (2) and (4)) over $\{C_1, C_2\}$ would be identical if we set $\tau_d = [\tau_c(1+\tau_c)^{-1}]$, *and* if individual behaviour is such that $\{a_{1t}(\tau_d)\}$ were to equal $\{a_{1t}(\tau_c)\}$.[9] It may be verified that along a constant expected utility contour (for any quasi-concave function), the latter condition on asset demand does indeed hold. This may be seen as follows. Risky asset choice functions turn out as follows:

$$(5) \qquad a_{1t}(\tau_c) = (1+\tau_c)a(0), \quad and, \quad a_{1t}(\tau_d) = \{(1-\tau_d)^{-1}\}\, a(0),$$

where $a(0)$ is the pre-tax optimal risky asset demanded by the investor.[10] Thus the tax rate condition specified above, namely $\tau_d = \{\tau_c/(1+\tau_c)\}$, and hereafter labelled *tax rate symmetry*, is necessary for the *two* risk-taking functions given by equation (5) to equal each other. However this is along an expected utility contour. Each of these taxes would also induce income effects. It may be further checked that the income effects are such as to require additional compensation of $\{\tau_c Y\}$ and $\{\tau_d/(1-\tau_d)Y\}$, respectively in the two tax situations.[11] Again, the tax rate symmetry is necessary for the income compensation to equal each other. Finally, from the budget equations (2) and (4) and the tax rate symmetry condition, it also becomes clear that the latter is also sufficient for the two budget sets to be identical. *Hence we may conclude that the CFT and MWT are ex ante equivalent if and only if the tax rate symmetry condition applies.*

The ex-ante budget equivalence clearly implies that the government too expects to collect an identical amount of revenue in the two cases for any feasible decision rule. Substituting from the household budget constraints, we have the following expressions for government revenue, respectively, for CFT and MWT:

$$(2a) \qquad R_c = \frac{\tau_c}{(1+\tau\ )}\left[\ Y_{1t} + \frac{a_{1t}(\tau_c)z_t}{(1+r)}\ \right];$$

$$(4a) \qquad R_d = \tau_d\,[\ Y_{1t} + \{a_{1t}(\tau_d)z_t\}/(1+r)]$$

---

[9] Since the optimal choice of assets would, in principle, depend on the relevant tax regime, here we have written them such as to allow easy identification

[10] Derivation is simple. For the MWT, one finds that along the constant expected utility contour, $(da/d\tau_d) = \{a(\tau_d)/(1-\tau_d)\}$. Upon integration, $a(\tau_d) = \{k/(1-\tau_d)\}$, where k, the constant of integration, may be seen to equal $a(0)$, the pre-tax value. Similarly for the CFT, one finds that $(da/d\tau_c) = \{a(\tau_c)/(1+\tau_c)\}$, and hence (5).

[11] For expected utility to remain constant, the $(dY/d\tau_c)$ derivative in the consumption tax case is $\{Y/(1+\tau_c)\}$. Upon integration and subtracting the initial endowment, the Hicks compensated term becomes $(\tau_c Y)$. Similarly for the MWT, utility compensation calls $(dY/d\tau_d)$ of $\{Y/(1-\tau_d)\}$, and hence a total income of $\{Y/(1$

The preceding discussion establishes that the given the optimal risky asset demand induced by the two taxes, the revenue constraints would be identical if and only if the tax rate symmetry holds. Indeed, this was the pre-payment method put forward by Ahsan (1989, 1990), and has been further discussed by Davies (1994). Note that the present result relating CFT and MWT in the asset choice model would seem to be exactly parallel to the well-known equivalent relationship between the simple wage tax and CFT that obtains in a model without uncertainty. As before, while the household utilities are affected the same way, and the government gets to keep the same expected value of revenue, the time path of revenue would differ. To summarise the above discussion, let us put together the main conclusions as follows.

> ***Proposition 2.*** *(a) The cash-flow tax (CFT) and the simple wage tax (WT) cannot be equivalent in the ex-ante sense so long as both households and the government use the risk-free rate as the discount rate.*
>
> *(b) The cash-flow tax and the modified wage tax (MWT) are ex-ante equivalent if and only if tax rates are symmetric, i.e.,* $\tau_d = (\tau_c/(1+\tau_c))$.

3. Social Valuation of Revenue: The Discount Rate Approach

There are two broad manners by which one may account for the riskiness of tax revenue. One can use a discount rate that is higher than the risk free rate.[12] Alternatively, we may use the certainty equivalent of the tax revenue, an approach that we adopt later in the paper. While both of these methods should ideally lead to equivalent results, a comparison of the two approaches is deferred to the next section.

Valuation of the CFT Revenue. The present value of the tax revenue from the cash flow consumption tax may be written as follows:

$$(6) \qquad E(R_c) = t_c \left[C_{1t} + \frac{E(C_{2t})}{(1+\rho)} \right],$$

where ρ denotes the public discount rate. Substituting the budget constraint (2) of the t-household we get:

$-\tau_d)\}$. Thus the incremental Hicks compensation is $\{\tau_d/(1-\tau_d)Y\}$.

[12] The only discussion on the proper discount rate for the evaluation of risky tax revenue comes from Hamilton (1987) who refers to the literature of risky public investment [e.g., Arrow and Lind (1970), Hirshleifer (1966), Mayshar (1977) and Sandmo (1972)]. Arrow and Lind (1970) argued that the appropriate social discount rate for risky public investment is the *risk free rate*. They rely on the state taking on projects with uncorrelated returns, and where each project is small in size for the generation of current taxpayers. Hirshleifer (1966), on the other hand, posited perfect capital markets and derived that government should use discount rates similar to that used in private sector for projects with comparable risk characteristics. Note that the Arrow-Lind argument was independent of any assumption on the completeness of markets.

(7) $$E(R_c) = \{\frac{\tau_c}{(1+\tau_c)}\} Y_{1t} \, [\, 1 - \frac{S_{1t}\ (\tau_c)}{Y_{1t}} (\, 1 - \frac{1+r+\beta_{1t}(\tau_c)\, E(z)}{(1\ +\ \rho)})] ,$$

where $\beta_{1t}(\tau_c)$, a measure of proportional risk taking, is defined to be the share of the risky asset in total savings, i.e., $\{a(\tau_c)/S(\tau_c)\}$.

Evidently, the value of CFT revenue to equal that under WT, we must have:

(8) $$\tau_w^* = \frac{\tau_c}{(1+\tau_c)} [1 - \frac{S_{1t}(\tau_c)}{Y_{1t}} (\, 1 - \frac{1+r+\beta_{1t}(\tau_c)E(z)}{(1\ +\ \rho)})\,] ,$$

where τ_w^* is the required wage tax rate. Clearly, setting symmetric tax rates, i.e., $\{\tau_w = \tau_c(1+\tau_c)^{-1}\}$, will not in general equalise revenue; the valuation of the risky tax revenue depends on the choice of the discount rate. Interestingly, if the weighted average market return, $[r + \beta_{1t}(\tau_c)E(z)]$, is used as the public discount rate, both methods yield an identical present value tax revenue. On the other hand, were the discount rate to be lower than the weighted average, the cash flow consumption tax would raise more revenue than the wage tax in expected present value terms if tax rates were symmetric. Thus a higher wage tax would be needed in order to obtain the same amount of revenue as the cash flow consumption tax.[13]

Indeed several authors have advocated the use of weighted average discount rate, though without much of a motivation.[14] One pitfall of using the weighted average as the discount rate is that the weight, namely the proportion of saving allocated to the risky asset, is an *endogenous* variable depending on the tax regime as well as the tax rate itself, and one that would not be known *a-priori*. Hence the tax equivalence result that follows is very much of the *ex-post* variety. Another awkward feature of the scheme is that it would require the public authorities to possess information that may be costly to obtain.

We note that the possible *ex-post* equivalence between WT and CFT contrasts with the result between CFT and MWT discussed in Section 2 above. Not only the previous result holds *ex-ante*, it does so for any feasible choice of the discount rate. Examining equations (2a) and (4a), it is further clear that the

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[13] Zodrow (1995) argued that even if the government uses the safe rate of return to discount the risky tax revenue, Ahsan's "modified wage tax" is not the only method to design a wage tax system that is equivalent in present value terms to the cash flow tax. Zodrow insists that the pre-payment approach could still be used by increasing the wage tax rate until the revenue requirement is met. Elsewhere Tsigaris (1994) has shown that if the social discount rate were to be lower than the weighted average rate, individuals would then prefer the modified wage tax, and not a wage tax set higher than $\{\tau_c/(1+\tau_c)\}$.

[14] See Hamilton (1987) and Zodrow (1995).

discount rate may well differ between the private and the public sector.

Valuation of the MWT Revenue. We have just seen that wage tax revenue would be equivalent to cash flow consumption tax revenue were the weighted average market return on assets to be used as the social discount rate. Question arises if one may use the discount rate argument to obtain a possible equivalence between a WT and a MWT. The answer is generally no! Denoting the social discount rate by an arbitrary parameter φ, the present value of MWT revenue is:

$$(9) \qquad E(R_{dt}) = \tau_d \left[Y_{1t} + \frac{a_{1t}(\tau_d)\, E(z)}{(1 + \varphi)} \right].$$

Hence it is clear that, for any *finite* value of φ, we do not have *ex-ante* equivalence. Insofar as *ex-post* considerations are concerned, one can always compute wage tax rates that satisfy the revenue constraints, but these do not appear to be interpretable, and hence are devoid of policy implications.[15]

The preceding analysis shows that the issue of equivalent revenue between tax regimes cannot be settled through the public discount rate approach. Different hypotheses on the magnitude of the rate yield different conclusions. This is especially troublesome given the ambivalence in the literature on the criteria for choosing the social discount rate. Problem with setting the rate arbitrarily is that mutual consistency between the chosen rate and the set of behavioural and market parameters becomes tenuous. In the next section we follow a different methodology, namely the *certainty equivalence approach*, which we argue, alleviates these pitfalls. Before doing so, let us conclude this section by collecting the main points in the form of a proposition.

> ***Proposition 3*** *(a) Should the social discount rate equal the weighted average rate of return on assets, then a cash flow consumption tax would be equivalent to a wage tax, albeit in the ex-post sense. If the discount rate is lower, a larger amount of revenue (in present value terms) can be raised with a cash flow consumption tax than a wage tax under symmetric tax rates (i.e.,* $\tau_w = \{\tau_c(1+\tau_c)^{-1}\}$*).*
>
> *(b) By contrast, the ex-ante equivalence, given tax rate symmetry, between a CFT and MWT, holds for any plausible discount rate. Moreover, the discount rate may well differ between the private and the public sector.*

4. The Certainty Equivalence Approach to Valuing Revenue

It is useful at this stage to dwell on the budget process a bit further. We hypothesise that the state uses the tax revenue to provide a public consumption good in period 2, i.e., to the old generation. The utility

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[15] Indeed a tax rate, τ_w^* set equal to $(\tau_d\, [1+ \{\alpha\, (\tau_d) E(z)/(1+ \varphi)\}])$, where $\alpha(\tau_d)$ is the share of the endowment allocated to the risky asset, would do.

of the transfer enters additively in household preferences. We analyse two alternative budget rules. In the first, the transfer to a generation is financed out of the revenue collected from that generation alone. Next we allow the state to engage in intergenerational risk sharing. The central question we pose is as follows. *What determines the market value of this transfer, and the mechanism for financing the outlay?*

Presently we adopt the *certainty equivalence* (CE) approach to valuing risky revenue.[16] The certainty equivalent, namely the expected value less the risk premium, then becomes the effective budget constraint for the government. We argue that this approach is preferable to arbitrarily setting the risk discount rate for at least two reasons. First, as we shall see below, the CE quantity yields the implicit discount rate. In other words, the present approach endogenises the choice of the discount rate. Secondly, since the *risk premium* associated with the CE would incorporate the underlying behavioural and institutional assumptions, we gain insights into behaviour consistent with a particular discounting methodology.

4.1 Pricing of Within Generation Risks: MWT Revenue. In what follows, we propose to use asset-pricing models to determine the risk premium implicit in the tax revenue flows. Investors evaluate the certainty equivalent of the risky transfer using the standard *capital asset pricing model* (CAPM) due to Lintner (1965), Mossin (1966), Sharpe (1964) and others. In this interpretation of CAPM, one needs to evaluate the covariance of the tax revenue flows with the behaviour of the *market portfolio*.[17] Were government revenue to embody *idiosyncratic risks* only, the covariance, and hence, the risk premium would be zero. This takes us back to the Arrow-Lind world, where the risk-free return (RFR, here denoted by *r*) becomes the correct public discount rate. In contrast, to the extent individual capital risks are mutually correlated, taxation of such returns does not achieve further risk pooling, and there arises *systematic risk*. Thus the present procedure determines the risk premium on government debt by the same rigours that apply to corporate cash flows.

Generally one would write CE as follows:

$$\text{CE}(R_{dt}) = E(R_{dt}) - \lambda \operatorname{cov}(R_{dt}, r_{mt}); \quad \lambda = \frac{E(r_m) - r}{\sigma_m^2}, \tag{9}$$

[16] Richter and Wiegard (1993) also use functions describing value of tax revenue that supposedly reflect consumer valuation. They do not spell out how the valuations is done, however,

[17] Strictly speaking, the corporate finance literature defines the market portfolio as the "market value weighted portfolio of all existing securities" (Ross, Westerfield and Jaffe, 1993, p301). In practice, however,

where, R_{dt} is the present value of the lifetime tax liability of the t-generation and r_{mt} is the return on the market portfolio in the second period, while $E(r_m)$ and σ_m^2, respectively, are the expected return and variance of the market portfolio. The market price of risk (i.e., the expected excess return on the market portfolio per unit of risk) is measured by λ, while the standardised covariance is the market *beta*. The tax revenue, R_{dt}, given by equation (4a) above, when substituted in (9), yields:[18]

$$(10) \qquad CE(R_{dt}) = \tau_d Y + \tau_d \{\frac{a_{1t}(\tau_d)}{(1+r)}\}[E(x_t) - (r + \lambda \operatorname{cov}(x_t, r_{mt})] .$$

According to the postulates of standard CAPM, investors hold diversified portfolios, possess homogeneous expectations, and are able to borrow and lend at the RFR. Consequently, in market equilibrium, the expected excess return, E(z) would equal $\lambda cov(x_t, r_{mt})$. In other words, the terms within square brackets in (10) become zero; in equilibrium all financial assets will be priced for the systematic risk embedded in them. Thus the risk premium on the MWT revenue exactly equals the expected revenue from the capital gain. Under these circumstances, the market value of the modified wage tax revenue equals that under the wage tax.[19]

Essentially the point is that if an asset is tradable in a well functioning capital market, the government cannot bear risk at a lower cost than the private sector if it returns the same risks back to households. It is plausible, and indeed there is strong evidence to suggest so, that the conditions of CAPM may fail to hold. Many individuals do not hold diversified portfolios. Moreover, human capital risks are typically non-traded, and the latter (via taxation of realised values of payroll, wages, and entrepreneurial income) is possibly the largest source of government revenue in many cases.[20] It would be of interest to pursue the consequence incomplete portfolios for tax design. Presently, however, we follow a different route in moving the argument forward. In the next section, we show that a policy of redistributing the risky revenue to the same generation that experiences the risk is inefficient. The optimal policy would be to spread the revenue, and hence risks, across generations.

To conclude this section, we state the principal result.

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a broad based index such as the S&P 500 is often used as a good proxy.

[18] The procedure we follow requires that the interest rate used to obtain present values be set equal to the risk free rate since the certainty equivalent calculations take riskiness explicitly into account.

[19] Hamilton (1987) and Zodrow (1995) may see this result as a rehabilitation of the advocacy for using the weighted average return on assets as the correct discount rate for risky tax revenue.

[20] Merton (1983) has argued that non-tradability of human capital, a significant part of national wealth, renders tradable "market" portfolio of financial assets *incomplete.*

Proposition 4.1.1 *If the government re-distributes to the same generation its own risky tax revenue, and the latter were valued in security market equilibrium, a modified wage tax or a cash-flow tax would become equivalent to the simple wage tax.*[21]

4.2 Intergenerational Risk Sharing. This section puts forward another interpretation of the certainty equivalent tax revenue based on the idea of intergenerational risk sharing. The latter has been formalised, among other, by Gordon and Varian (1988), hereafter G-V. They have argued that the government's ability to pre-commit unborn generations suggests that it can reallocate risk across generations cheaper than private markets through a stochastic debt management policy.[22] Thus when there is an unlucky (lucky) outcome, the latter's distribution being assumed independent through time, the government can create (retire) public debt. The burden of debt can be passed on to all future generations even though it was incurred by the current unlucky generation. It is intuitive that efficient allocation would call for each future generation to bear a small share in the risks of today's lottery. Below we extend the above analysis to the case of a *stochastic tax transfer policy* whereby all future generations share the risky capital gains revenue attributed to the current generation of tax payers.

We find that even if households held the market portfolio, and thus were exposed to "market" or "social" risks, sharing of uncertain returns still reduce the current "market" risks and provide insurance. Recall that the standard Domar-Musgrave Phenomenon (DMP) obtains where agents are able to share the risk with a third party (e.g., the government) in the ratio $\{(1-\tau): \tau\}$ via loss-offset provisions. Hitherto in the literature, whenever the latter is assumed, the government is supposed to bear the risk in an unexplained way without any further consequence on private behaviour. While we continue to assume that the collective risks remain in the system, through generational risk sharing analysed here, agents are indeed able to pass on the τ-*fraction* of the risky income to future generations in a cost-less fashion. *Thus the present interpretation of sharing revenue risk through generations may indeed be thought of as a theory of loss-offset behaviour.* We shall see that as the number of risk sharers increases, the cost of this collective risk becomes negligible. In other words, such a scheme would allow households to enjoy a higher expected steady state consumption (made possible by greater risk taking), but at a lower unit risk. In other words, market risk facing a generation becomes idiosyncratic when pooled with the *independent* lotteries of many later generations.[23]

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[21] While the text deals with MWT, the arguments apply to the cash flow consumption tax case as well.

[22] Richter has argued that the availability of an asset like land may limit the scope of intergenerational risk shifting through government intervention such as social security (1993). However, in his model, utility of a generation is evaluated in *historic* time, i.e., at birth. In contrast, like Gordon-Varian, we let this calculation be done at the time of policy enactment, hence in *non-historic* time.

[23] It is of interest to note the formal similarity of the present argument with the much earlier contribution by

We note that unlike CFT or MWT, the wage tax has no risk sharing elements since it falls on the endowment of a household, which if stochastic, is purely idiosyncratic in nature. Nor do we allow for the tax rate to depend on the realisation of the risky event.[24] Consequently, there is no room for risk sharing in the WT case regardless of the coercive powers of the state. We also abstract from modelling debt.

In order to explore the risk sharing argument, we suppose that all generations (*n* in number) share in the current "market" risk via a stochastic tax–transfer policy. The t-generation retains (1-τ)-fraction of its own risk through the tax schedule, and thus would share the remainder (τ) with the yet unborn (indeed the next *n-1*) generations. In exchange it is entitled to share in the past revenues. The *ex-ante* random transfer that will be evaluated by a steady state generation born at time t is given by:[25]

$$(11) \qquad A_{dt} = \sum_{i=0}^{n-1} \gamma_i R_{d(t-i)}, \sum_{i=0}^{n-1} \gamma_i = 1$$

where γ_i is the weight of each lottery that is allocated to the generation born i-periods earlier, and $R_{d(t-i)}$ is the tax revenue from the MWT tax revenue realised by the (t-i) generation. Consequently, the t-generation would value the resulting public provisions it receives by the certainty equivalent value of the stochastic stream A_{dt}, whose expected value is indeed R_{dt} since the risks are identical through time.[26]

We note that the present scheme does allow for no risk sharing as a possible outcome, which would entail setting $\gamma_0 = 1$, and $\gamma_j = 1$, j = 1,..,n-1. In the latter scenario, we would then take us back to the model encountered in the preceding section (4.1), and to the same conclusion. However, below we show that these weights are not optimal. To further illustrate how the government may spread risk across generations, let us consider an example where only *three* generations share their financial risks as stipulated above.

Arrow and Lind (1970). In the latter framework, the state undertook many uncorrelated public projects at a point in time, while we consider the state pooling uncorrelated risky revenue of many generations.

[24] Indeed a graduated tax schedule (even a piece-wise linear) may be construed as a state contingent tax function. However, even with a simple proportional rate, there is some risk sharing, though clearly more is possible within a progressive schedule so long as the incentive effects are minimal. Indeed Barro (1979) has argued that the government would prefer a constant marginal rate in view of the efficiency costs.

[25] By a steady state generation we mean one that shares in the full complement of *(n-1)* previous risks. The calculation facing earlier (i.e., non-steady state) generations would be a little different since there are fewer parties to the exchange.

[26] We assume that the economy is operating at the golden rule of capital accumulation, and hence the interest

Table 1: Three Generations Sharing Systematic Risk Through the Tax System

Time of tax/transfer policy enactment t = 0	t=0	t=1	t=2	t=3	t=4
The allocation of the tax payment by the old at t=0	$\gamma_0 R_{d0}$	$\gamma_1 R_{d0}$	$\gamma_2 R_{d0}$		
The allocation of the tax payment by the old at t=1		$\gamma_0 R_{d1}$	$\gamma_1 R_{d1}$	$\gamma_2 R_{d1}$	
The allocation of the tax payment by the old at t=2			$\gamma_0 R_{d2}$	$\gamma_1 R_{d2}$	$\gamma_2 R_{d2}$
The allocation of the tax payment by the old at t=3				$\gamma_0 R_{d3}$	$\gamma_1 R_{d3}$
The allocation of the tax payment by the old at t=4					$\gamma_0 R_{d4}$

Each row illustrates the allocation of a given old generation's stochastic tax payment through time to yet unborn generations. For example the currently old (second row) receives only a fraction of their own tax liability: $\gamma_0 R_{d0}$. The rest of the tax payment is transferred into the future, to time periods t = 1 and 2. Each horizontal row therefore adds up to the gross tax payment by the relevant generation. Clearly in the above example the steady state generations are all who become old at t=2 and later (i.e., born at t=1 and after). To focus on the young generation born at time t=1, who stands to receive $\gamma_0 R_{d2}$ of their own tax revenue risks when old (at time t=2), and also $(\gamma_1 R_{d1}+\gamma_2 R_{d0})$ from the preceding two lotteries. Thus each generation gets as a stochastic transfer the sum of each *column* (as in equation 11). As stated above, society evaluates this composite risk behind the Rawlsian *veil of ignorance*, i.e., at the time of the enactment of the tax/transfer policy.

Since the distribution of R_{dt} is identical through time, the transfer scheme proposed above does not affect the expected value of the composite lottery at all. However from (11) it is evident that higher moments of A_{dt} are all lower that those corresponding to R_{dt}. Thus steady state expected utility would be increased under the proposed transfer scheme.[27] For concreteness, however, we focus on the second moment, and let the social planner choose the appropriate weights γ_i for the steady state generation by minimising the variance of the public provisions:

$$(12) \qquad \underset{\gamma_i}{\text{Min}} \quad \text{Var}\left(\sum_{i=0}^{n-1} \gamma_i R_{dt-i}\right) + \xi\left(1 - \sum_{i=0}^{n-1} \gamma_i\right)$$

It is easily seen that the weights which minimise the variance of the public transfer is such that γ_i = (1/n) under the present risk assumptions.[28] It is thus optimal for each generation to equally share all

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rate is equal to the growth rate of population. We leave other steady state possibilities for future work.

[27] In the appendix, we discuss the non-steady state case, and establish that the proposed scheme is indeed Pareto-efficient.

[28] The variance is simply: $Var\ \{A_{dt}\} = \{(\tau/(1+r)^2\ \{a(\tau)\}^2(Var\ (z))\Sigma_i\ (\gamma_i)^2\}$, all i. Now setting up the Lagrangean in (12) above, we immediately obtain that all weights are identical, and hence equal to (1/n).

risks.

Now we turn to the issue of the appropriate public discount rate relevant for evaluating risky tax revenue flows. As before, we let the capital market value the transfer that the steady state generation is entitled to:

$$(13) \qquad E(A_{dt}) = E\left[\sum_{i=0}^{n-1} \gamma_i R_{dt-i}\right] = E(R_{dt}).$$

Using CAPM, we have the following certainty equivalent:

$$(14) \qquad CE(R_{dt}) = E\left[\sum_{i=0}^{n-1} \gamma_i R_{dt-i}\right] - \lambda \operatorname{cov}\left[\sum_{i=0}^{n-1} \gamma_i R_{dt-i},\ r_{mt}\right].$$

Expanding terms and using the properties of covariance we get:

$$(15) \qquad CE(R_{dt}) = E(R_{dt}) - \lambda \operatorname{cov}\left[\gamma_0 R_{dt} + \gamma_1 R_{dt-1} + \gamma_2 R_{dt-2} + \ldots,\ r_{mt}\right].$$

Given that the market or social risks across generations are idiosyncratic, i.e.,

$$(16) \qquad \operatorname{cov}\left[\gamma_i R_{dt-i},\ r_{mt}\right] = 0, \quad \forall\ i \neq 0,$$

the covariance of R_{dt} is the only relevant term in the evaluation of the risky tax revenue. Hence:

$$(17) \qquad CE(R_{dt}) = E(R_{dt}) - \lambda \operatorname{cov}(\gamma_0 R_{dt},\ r_{mt}).$$

Even assuming that the household held the market portfolio, the valuation is given by (18). Thus as long as there is *some* intergenerational risk sharing, (i.e., $\gamma_0 < 1$), the certainty equivalent of the transfer will be higher relative to the value in the absence of risk sharing.

$$(18) \qquad CE(R_{dt}) = \tau_d \left[Y_{1t} + (1 - \gamma_0) \frac{a_{1t}(\tau_d)\, E(z)}{(1+r)} \right].$$

The public discount rate for the evaluation of risky tax revenue (denote it by φ) implicit in the present derivation is given by:[29]

[29] This is obtained by setting the present value of the risky tax revenue using the certainty equivalent method equal to the present value of the tax revenue using the discounting method, and solving for the required rate of return.

(19) $\varphi = [\{(1+r)/(1-\gamma_0)\} - 1]$.

Setting $\gamma_0 = 1/n$, it follows that as the number of generations sharing risks increases the discount rate approaches the risk free rate. Revenue risks become idiosyncratic, and hence the certainty equivalent of the modified wage tax revenue is given by (20).[30]

$$(20) \quad CE(R_{dt}) = E(R_{dt}) = \tau_d \left[Y_{1t} + \frac{a_{1t} (\tau_d) E(z)}{(1+r)} \right].$$

Thus we see that capital asset pricing model fails to set a zero market value to the stochastic transfer because such a security is not traded in the financial market. Gordon and Varian state why the private market fails to share risks across generations:

> Later generations cannot participate in the securities market for lotteries, which occur before they are born because they are not alive ex ante to buy shares in these lotteries. An agent cannot profitably buy shares on behalf of later generations since there would be no legal mechanism to force these later generations to accept any losses, implying that the agent would have no incentive to pass on any gains. ..The Government, however, could well have the power to commit later generations to share in the outcome of earlier lotteries. If so there is a potential for Pareto-improving policies which share the risks between generations. (p186)

A family can also engage in a similar risk sharing arrangement, as has also been noted by G-V, and possibly eliminate the need for government intervention. One may indeed interpret the above result as an optimal family behaviour. To observe this suppose that there is only a wage tax and hence no tax on stochastic incomes. Will it be optimal for the family to engage in the following scheme ex ante? The young generation accepts a transfer from the parents, equalling half of latters'retirement fund, which is then combined with its own savings and invested in the market portfolio. In the next period, the now old consumes half the accumulated value of the fund, and the other half is transferred to the next generation, and so on. Were the family to go for this plan, it would be indifferent between a wage tax and a consumption tax. However, a family may fail to establish such a risk sharing arrangement because of the non-existence of a legal mechanism to allow the acceptance of past losses, even though such a scheme would be *ex ante* Pareto improving if implemented.

To conclude this section, we may summarise the main findings as follows.

Proposition 4.2.1: *Even if households held the market portfolio, intergenerational risk sharing leads to a positive market valuation of the risky tax revenue.*

Proposition 4.2.2: *The public discount rate becomes finite, and monotonically decreases*

[30] The appendix shows the derivation for the cash flow consumption tax case.

as the number of intergenerational risk sharers increases.

Proposition 4.2.3: *The social discount rate would equal the risk free rate when all future unborn generations bear a small share of the "market" risks of today's lottery.*[31]

5. Distributional Aspects of Tax Base Choice

While the present paper has focused on the efficiency aspects of risk allocation via taxation, we would like to point out that MWT and CFT would each be consistent with the requirement of both *horizontal* and *vertical* equity. The wage tax, however, would not. Horizontal equity is ensured by the fact that under MWT or CFT, households who are identical *ex-ante* also fare identically *ex-post*; while with WT there is the problem of ex-post equity. If a 'financial crisis' were to occur, the "unlucky investors will have prepaid a tax on expected returns and will then obtain no deduction for the losses they incur. By contrast, if the 'favourable' outcome would occur, 'lucky' investors might become very rich and owe no additional tax liability on future consumption of their wealth", thus creating *ex-post* "horizontal inequities" (US Treasury, 1977). With a wage tax, large fluctuations in the old generation's financial wealth would be observed.

Vertical equity may also be guaranteed by replacing the proportional consumption tax (either CFT or MWT) by a linearly progressive schedule. It may be verified that the analysis presented above would also go through in the latter case. The difficulties regarding ex-post equity under a wage tax get exacerbated in a world of unequal initial endowment. Those with the larger endowment may turn out to be plungers, and depending on the state of nature, may lose pretty much everything. Hence ranking of post-tax realised lifetime wealth would be reversed from the pre-tax distribution.[32]

Finally, we reiterate the result that in a regime of intergenerational risk sharing, the tax-transfer possibilities obtainable under the CFT or MWT offer a Pareto improvement over the status quo world of a simple wage tax.

6. Conclusions

This paper has examined the valuation of risky revenue from two perspectives, namely the use of a risk-adjusted discount rate and that of the market certainty equivalence. The discussion takes place in the context of the possible substitution of a consumption tax for the income tax. Armed with the conventional belief that a wage tax is formally equivalent to a consumption tax, the advocates of

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[31] Intergenerationalrisk sharing would not hold in a model where the household has an infinite horizon as in Hamilton (1987). In fact, Hamilton's model is akin to a world where households exhibit altruism towards future generations.

[32] See Ahsan (1990) and Davies (1994) for further elaboration on these and related matters.

consumption tax have cited the ease of administration and compliance for recommending the pre-payment method (namely the simple wage tax) as the preferred reform alternative rather than the cash-flow tax. The preceding literature has not explicitly dwelt on the implications of the naïve equivalence for the social discount rate. This paper takes the position that the presumed equivalence between CFT and WT rests on extreme hypotheses on individual and government behaviour. One is that households hold the "market portfolio", while the other posits that the government is unable to share the aggregate risk facing a generation with future ones. On both of these presumptions, there is strong evidence to the contrary, and hence we argue that a wage tax would not be equivalent to a consumption tax under uncertainty. Inclusion of the capital gains term in the wage tax base would typically be necessary to secure equivalence.

We go on to establish that if the risks of the cash flow consumption tax revenue were shared equally with all generations, the cash flow consumption tax (or the modified wage tax) would be preferred by tax payers relative to the wage tax. Should the government use tax revenue to finance lump-sum transfers, households would be better off under a MWT compared to a simple wage tax.[33]

For the reform debate, the practical implication of our analysis is that the consumption tax ought to be implemented via registered savings accounts since no convenient pre-payment alternatives are to be had. It is interesting to observe that the Canadian personal income tax, which is perhaps closer to any real world consumption tax system, indeed relies on the savings account route (the *registered retirement saving plans*, RRSPs). We also note that, while the scope is perhaps limited, the US scheme of individual retirement accounts (the IRAs and related innovations) work along similar principles.

Finally we note that it would be of interest to examine the welfare gains of a cash flow consumption tax vis-à-vis the wage tax using a framework of differential incidence. If the gains are negligible then the insurance elements of a cash flow consumption tax may not outweigh the added administrative complexity of the tax system. Indirectly the results of such an analysis could be used to debate whether the intergenerational risk sharing argument is indeed policy relevant.

[33] Kaplow (1994) lets the government itself deal in risky securities, and finds that under suitable strategies, the risk position of the state may fully offset the tax-induced change in the risk characteristics of revenue obtained from the private sector. Consequently consumption and wage taxes become equivalent: "a tax on excess returns is equivalent to no tax at all." (p793). Such reasoning is curious for a number of reasons. For one thing, one does not observe the state to be engaging in portfolio activities required by his analysis. Even if one were to allow risk trading by the state, we note that the range of feasible (public) investment strategies may not match the risk characteristics of private behaviour.

APPENDIX

1. Valuing CFT Revenue under Intergenerational Risk Sharing. If we assume that all generations (n in number) share in the current "market" risk via stochastic tax-transfers, the relevant tax revenue is the average cash flow consumption tax revenue across all generations. Using CAPM to evaluate this stream of revenue, the certainty equivalent becomes:

$$CE(R_{ct}) = E\left[\sum_{i=0}^{n-1} \frac{R_{ct-i}}{n}\right] - \lambda \operatorname{cov}\left(\frac{\sum_{i=0}^{n-1} R_{ct-i}}{n},\ r_{mt}\right)$$

Here we let the weight of each generation in the average be equal (which has been established in the text). Expanding the terms and using the properties of covariance we get:

$$CE(R_{ct}) = E(R_{ct}) - \left(\frac{\lambda}{n}\right) \operatorname{cov}\left[R_{ct} + R_{ct-1} + R_{ct-2} + \ldots,\ r_{mt} \right]$$

In keeping with the hypothesis that aggregate risks across generations are idiosyncratic, we have:

$$\operatorname{cov}\left[R_{ct-i},\ r_{mt} \right] = 0 \qquad \forall\ i \neq 0$$

Substituting the cash flow tax revenue into the above expression, and assuming that the household holds the market portfolio, yields:

$$R_{ct} = \frac{\tau_c}{(1+\tau_c)}\left(Y_t + \left(1-\frac{1}{n}\right)\frac{a_t(\tau)E(z)}{(1+r)}\right)$$

Proceeding analogous to the text, we find the implied public discount rate to be:

$$\rho = (1+r)\left[\frac{1+r+\beta(\tau)E(z)}{1+r+\left(1-\frac{1}{n}\right)\beta(\tau)E(z)}\right]-1$$

As before, as n approaches infinity the social discount rate approaches the risk free rate. By contrast, the discount rate converges to the weighted average rate if n is set at unity.

2. Pareto Efficiency under Intergenerational Risk Sharing. In order to discuss Pareto efficiency, we note that consumption when young is unaffected by the tax polices in review (namely, CFT, WT and MW), or to be exact, affected identically by the income effect. What matters then is the level of future consumption. We also require a benchmark. Here we have two choices. One is where there is no government, and alternatively we may return to the model described in section 4.1 above, namely where the government was unable to improve on private risk allocation. Clearly in both these reference scenarios, future consumption is:

$$(1) \qquad C_{2t} = \{(Y - C_{1t})(1+r) + Z_t(0)\}, \text{ where } Z_t(0) \equiv \{a_t(0)z\},$$

the subscript-t denotes any steady state generation, and $a(0)$ corresponds to the pre-tax, (i.e., $\tau = 0$) values. Since both consumption by the young and wage income are non-stochastic, we focus on the discretionary part, and call it C_{zt}. The mean and variance of C_{zt} would be:

$$(2) \qquad E\{C_{zt}\} = E\{Z(0)\} = b > 0, \qquad \text{and,} \quad Var\{C_{zt}\} = Var\{Z(0)\} = s > 0.$$

With the transfer scheme proposed in the paper, discretionary consumption for members of the steady state t-generation is augmented to:

$$(3) \qquad C_{zg} = (1-\tau)\, Z_t(\tau) + \Sigma_i \gamma_i R_{(t-i)},\ \Sigma_i \gamma_i = 1,\ i = 0, 1, \ldots, n-1, \text{ and,}$$

$$(4) \qquad Z_i(\tau) = \{a_i(\tau)z\}, \text{ and, } \ R_i = \tau\, Z_i(\tau), \ \text{all } i.$$

As derived in section 2 of the text, we have

$$(5) \qquad a_i(\tau) = \{a_i(0)/(1-\tau)\}, \quad \text{and, hence, } Z_i(\tau) = Z_i(0)/(1-\tau), \ \text{all } i.$$

Therefore, in view of (5) we may, utilising (1), restate (3):

$$(3a) \qquad C_{zg} = Z_t(0) + \Sigma_i (\tau/(1-\tau)\gamma_i Z_i(0) = Z_t(0) + C_G, \text{ all } i.$$

With separable preferences,[34] one needs to evaluate the utility of the transfer, i.e., the C_G -term in (3a). Evidently,

$$(3b) \qquad E\{C_G\} = E\{Z(0)\}[\tau/(1-\tau)] = \{b\tau/(1-\tau)\} > 0.$$

Given the optimal value of $\gamma = (1/n)$, and that risks are *independent*, we also have from (3a):

$$(3c) \qquad Var\{C_G\} = (s)\, \{[\tau/(1-\tau)]^2 (1/n)\}.$$

Thus as the number of risk sharers increases, the variance tends to zero. Thus members of steady state generations are strictly better off. Out of steady state, the weakest conceivable position is that of the first generation that participates at the point of policy implementation. It is able to shift forward its own risk, and hence DMP obtains, but it does not share in any past lotteries. Their consumption is given by only the first term in (3a), and hence conditions (1) and (2) again apply. Thus they are no worse off than in the status quo. Similarly one can show that the intermediate non-steady state people are also strictly better off.

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[34] It is trivial to extend the analysis to the case of more general preferences.

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