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# THE DESIGN OF HIGH-PRESSURE DISTRIBUTION SYSTEMS. 

By J. R. Beard, M.Sc., Associate Member.<br>(Paper first reccived 20 February, and in final form 9 October, 1915 ; read before The Institution 16 December, before the Western Local Section 13 December, and before the Manchester Local Section i4 December, igi5.)

## Introduction.

The general distribution of electrical energy to individual consumers at high pressures is of comparatively recent development, one of the first schemes in this country being that which was inaugurated in the year igor for the purpose of supplying power to shipyards on the Tyne and which has since grown into the extensive system that now covers the principal industrial districts of Northumberland, Durham, and North Yorkshire. High pressures had previously been used, but only for transmission purposes to enable the power stations to be situated outside the areas of supply in order to utilize special advantages such as waterfalls, cheap sites, plentiful condensing water, or easy delivery and storage of coal. In such cases the highpressure energy was transmitted to a point near the centre of the area of supply, where the whole of it was transformed to low pressure for distribution to the consumers. Such schemes, for example, were Dr. Ferranti's pioneer high-pressure transmission from Deptford to London, and various water-power schemes in America and on the Continent.
The advantages of supplying concentrated power loads, of say 100 kilowatts and over, direct from the power station at high pressure and transforming to the working voltage at individual sub-stations adjacent to the load centres, have since been generally realized, and the last decade has seen an enormous development both in the number and in the size of such high-pressure distribution systems. Side by side with the development of high-pressure distribution for power purposes, similar methods were found essential for the various railway electrification schemes, and also for the supply of power to the larger tramway systems which were spreading out over an area that it was impossible to supply at low pressure from a single power station. Up to the present the method of distribution for lighting has not been radically altered, although occasionally the existence of a high-pressure distribution system, primarily installed for other purposes, has resulted in bulk supplies
being given to outlying districts. It is, however, being more generally realized that only under exceptional circumstances is it possible to find near the centre of a large lighting load conditions which permit of economical generation, and consequently the original Deptford example is being more widely followed. In many such cases, owing to the ease with which a sub-station-unlike a power station-can be installed in almost any position, there is a growing tendency to split up the more extensive and cumbersome low-pressure networks into several sections, each fed by a separate sub-station. As the use of electricity for lighting and other domestic purposes increases, there is little doubt that these sections will gradually decrease in area, with a consequent increase in the number of high-pressure feeding points, so that in this, as in other branches of electricity supply, the distribution will eventually be at high pressure practically to the consumers' terminals.
Not only is the cost of distribution at high pressures considerably less than at low pressures, but also it is thus possible for a single power station to supply economically over an enormously greater area than that which was previously practicable, and in consequence by better diversity factor and larger and more efficient generating plant great reductions have been effected in the cost of electricity. The maximum economy from these advantages is still far from being obtained, and there is therefore every prospect that in the future the growth of high-pressure distribution systems will be equally as rapid as it has been in the past. The growth should, however, be in size and not in number, since in order to reap the full benefit of improved diversity factors and increased efficiency of generation it is essential that each system should supply the whole of the demands of the area which it serves, and also that the area served should be as large as possible.
The term "distribution system" is often used in a broad sense to indicate everything outside the power station, but in the present instance it will be restricted to mean the mains which convey the electrical energy from the power stations where it is generated to the sub-stations where it
is transformed to a form suitable for the consumers' use, together with the switchgear controlling them. The paper will further be limited to a consideration of 3 -phase alter-nating-current systems, as it is generally recognized that these are most suitable for high-pressure distribution.

## General Principles.

A well-designed distribution system is that which secures the following essential characteristics at a minimum total annual cost :-
(a) Safety in operation both as regards the operating staff and the public generally.
(b) Suitability of the supply for the purposes for which it is required.
(c) Freedom from interruption of supply.

Safety in operation is of primary importance, since with high pressures accidents are frequently fatal and, in this country at any rate, no commercial advantage is considered sufficient to justify unnecessary danger to human life.

The other two limitations have a commercial basis. In all its various uses electricity is a competitor with other forms of energy, many of them very firmly established, and if it is not supplied in suitable form, or if the supply is subject to interruption, the sale of it will be restricted. For example, the cost of power is such a small proportion of the total cost of running a works that material irregularity in the supply would quickly involve more loss than the whole cost of the power bill. In such circumstances a supply of electricity would not be a paying proposition if it could be obtained for nothing.

Failure of supply is usually caused by the breakdown of apparatus, and the primary precaution is therefore careful attention to the design, manufacture, and maintenance of the various parts of the system; but since no apparatus can be made absolutely immune from breakdown through external damage, the secondary precaution is to make arrangements so that the effects of a breakdown to any part of the system are localized as much as possible. In some cases such precautions may mean increased capital cost, but they undoubtedly result in a net economy if a broad view is taken. Fortunately, however, well-designed apparatus does not necessarily cost more than badly designed apparatus, and, as will be seen later, it is possible to cheapen the system by closer localization of breakdowns.

The chief items which make up the annual cost are the following, and it is the sum total of these which should be a minimum :-
(1) Interest on capital expenditure.
(2) Repairs and provision for depreciation.
(3) Switchgear attendance.
(4) Energy losses in the mains.

It may be useful to consider briefly these items in turn, and to note in what ways they are interdependent and how they are affected by the essential characteristics previously detailed.
(1) It is unnecessary to say anything as to the desirability of keeping down capital expenditure ; the chief danger is usually lest this point should receive too much attention at the expense of other items. There is, however a further way of reducing interest charges, namely, by a low rate of
interest, and this can only be secured by a permanent and steady business. This is a further argument for extending and consolidating the area of supply so that the revenue of the undertaking is less subject to trade fluctuations.
(2) A low cost of repairs goes hand in hand with low depreciation charges, since both depend on the permanence of apparatus and its freedom from breakdown. Money is doubly well spent on these points, since it not only reduces repairs and depreciation but provides the first precaution already postulated as necessary to ensure security of supply.
(3) Particularly in the case of static sub-stations, which have not an operating staff continually on shift, attendance is largely a matter of operating switches should a fault occur on some part of the system. Apart from cost, such attendance is often very difficult to arrange for, and there is therefore a strong reason for reducing it as much as possible. This can be done by closer localization of breakdowns so that the only part of the system which is affected is that in the immediate neighbourhood of the fault, thus at the same time taking the second precaution postulated as necessary to ensure security of supply. The cost of attendance is not only a question of amount but also of quality, and cheaper labour can be employed if apparatus is designed to prevent automatically the more serious mistakes in operation. Such design tends to increased safety in operation, which has been previously mentioned as the first essential characteristic of the system.
(4) Loss of energy in the mains can be reduced by increased capital expenditure, and it is therefore a question of the correct balance between the two. It can also be reduced by linking up the various parts of the system in order to utilize the diversity of their loads, a process only rendered possible by again using means for localizing the effects of breakdowns.

It is interesting to note that the essential characteristics of the system are not incompatible with designing it on the basis of minimum annual cost, since though they may in a few cases increase the capital expenditure this is discounted by savings in other ways.

The object of this paper is to endeavour to indicate on the basis of the preceding general principles the most suitable choice of apparatus and the means whereby that apparatus can be used to the greatest advantage. For this purpose the paper will be divided into the following sections:-
I. The important points in the choice of individual apparatus :-
(a) Mains.
(b) Switchgear.
II. The methods by which such apparatus may be most economically utilized, the consideration of which naturally falls into three successive stages. The basis of the investigations is :-
(a) The determination of the economical section of mains; that is to say, the correct balance between losses and capital expenditure.
From this it is possible to consider progressively :-
(b) The lay-out of the distribution system to give the maximum economy.
(c) The most suitable distribution voltage.

## $\mathrm{I}(a)$. Mains.

As it is desirable to avoid covering ground which has been recently covered by other papers read before the Institution * this section of the paper will be limited to a brief consideration of the conditions which determine whether underground or overhead mains are most suitable for particular cases. Both have reached a high degree of perfection, but owing to the fact that the conductors are exposed, overhead mains are necessarily more subject to breakdowns than underground mains, so that from the point of view of security of supply the latter are to be preferred, unless special circumstances such as liability to serious subsidence increase the risk of damage to cable. This point is, however, not so important as might be thought, since in spite of the numerous ways in which overhead mains can be damaged by snow, wind, lightning, malicious damage, and shortcircuits due to birds, flying straw, kite strings, etc., it is found in practice from extended experience that per mile of main the average number of serious breakdowns of overhead mains is only about double that on underground mains. Overhead mains are, however, more liable to cause temporary interruptions of supply by being automatically disconnected on transitory short-circuits which do not cause permanent damage. The advantage of overhead mains is their lower capital cost, more particularly at higher voltages; this is readily seen by glancing forward for a moment to Fig. 7, which shows comparative annual costs per mile for various voltages, allowing a slightly higher figure for the maintenance of overhead mains and including a reasonable figure for wayleaves. For low voltages and small sizes there is not much to choose on the score of cost, and underground mains are therefore preferable on account of their other advantages. As the voltage and size increase, however, a considerable saving is effected by the use of overhead mains; and such saving is often increased by the difficulty of finding direct routes for cables in the open country where roads are few and winding.
While it will be seen from the foregoing that the use of overhead lines in open country is frequently fully justified, they have further disadvantages which must always be borne in mind, viz. :-
(a) They usually require a special wayleave.
(b) The inductive drop is much greater than that of an equivalent cable.
(c) If overhead lines are run in parallel with cables neither can be operated at their maximum economy.
(d) They tend to lower the power factor of the system as a whole.
(a) The objection to wayleaves is not so much their cost as the trouble and delay in obtaining them, and that they can seldom be bought outright. The anomalous and conservative provisions of English law in this matter have been recently discussed both by Mr. Welbourn in his paper on high-tension overhead lines $\dagger$ and by Mr. C.

* C. J. Beaver. "Cables." fournal I.E.E., vol. 53, p. 57, 1915 . B. Welbourn. "British Practice in the Construction of Hightension Overhead Transmission Lines." Ibid., vol. 52, p. 177, 1914 . C. Vernier. "The Laying and Maintenance of Transmission Cables." Ibid., vol. 47, p. 3 13, 1911 .
$\dagger$ Ante.

Vernier in his Address to the Newcastle Local Section,*: and the matter had also been previously raised by Mr . W. B. Woodhouse as a result of his experience with the Yorkshire Electric Power Company. $\dagger$ When a suitable time arrives the need for the legal changes which they advocate should not be lost sight of.
(b) The inductance of a 3-phase circuit, other factors remaining constant, varies as the logarithm of the dis. tance between the conductors; the higher inductance of overhead lines is therefore inherent in their construction, since this distance must be many times the corresponding distance in a cable, owing partly to the lower insulating value of air as compared with impregnated paper, but more particularly to the necessity for preventing the wires being short-circuited by birds, or by unequal swinging due to wind, or by unequal sagging due to snow. This higher inductance does not affect the voltage-drop when the power factor of the system is not a lagging one, but it is unusual for the amount of synchronous plant on the system to be sufficient to ensure this. The question of voltage-drop is dealt with more fully in a later section, but by reference to Fig. io the increase in voltage-drop at 50 cycles with various lagging power factors is clearly shown. ${ }_{\ddagger}$ It will be noted that it is quite serious, particularly in the case of heavy-section lines, and consequently the use of overhead mains means either a reduced radius of transmission for a given voltage, or extra capital cost in providing additional copper. To some extent this disadvantage is minimized by the fact that for economy overhead mains should be run at a lower current density than cables (see section on the economical section of mains and Fig. 6), and also by the fact, noted previously, that it is often possible to carry overhead lines over a shorter route than the equivalent cable.
In certain special cases the inductance of overhead mains may be of definite benefit. This is, for example, the case in continuous-current traction systems supplied by rotary converters which can be arranged to draw a leading current from the line at heavy loads. It will be seen , by again referring to Fig. io that with quite a moderate leading power factor the total voltage-drop in an overhead line feeding a rotary-converter sub-station may be zero.
(c) If a cable and an overhead line of equal section are connected in parallel the higher inductance of the overhead line causes the total current to divide unequally between the two circuits. This not only prevents the two circuits being run at their most economical current density, but also, owing to the resistance losses being dependent on the square of the current, they are greater than if the current were divided equally. Further, owing to the difference in the inductance of the parallel circuits, there is a phase difference between the currents in the two branches. This results in the arithmetical sum of the currents in the branches being greater than the total current, thus causing additional resistance losses and a reduction in the carrying capacity of the circuits. To take a concrete example, assume that 300 amperes at 6,000 volts and 50 cycles is to be transmitted through a 0.15 sq . in.

[^0]cable and a 0.15 sq. in. overhead line in parallel. The actual current in the cable will be 210 amperes, and in the overhead line 105 amperes, while the resistance losses are increased by 24 per cent as compared with the losses if both circuits were either cable or overhead line.
(d) It is at once evident that, so far as the power station and the mains back to the power station are concerned, the wattless current produced by the extra inductance of an overhead line will be as deleterious as if it were produced by consumers' apparatus.

## I(b). Switchgear.

The following remarks are directed to the more important or novel features which should influence the choice of the most suitable apparatus for sub-station use. Similar principles apply, but with greater force, to powerstation switchgear.

Switchgear fulfils several functions, but the primary one is to isolate faulty apparatus, and consequently to interrupt or prevent heavy short-circuit currents. This is the determining factor in its design. The problem is therefore one of considerable difficulty, since it is largely beyond mathematical calculation and the chief data to work upon are those obtained from actual experience. With many problems great assistance can be obtained from experiments, but this is difficult with switch design, as the conditions cannot be adequately reproduced on a small scale and it is not usually commercially practicable to risk experiments on an actual system.

As the tendency is for distribution systems to supply a denser load it follows that the maximum short-circuit current is continually increasing ; for not only is there a larger kilowatt capacity of running plant on the system but also a greater number of feeds into a fault. Moreover, the conditions are further aggravated by higher distribution voltages, without any corresponding reduction in the resistance of the faults. It therefore follows that switch design is a gradual process of feeling one's way ; methods which have given good service on one size of system being tried on a larger one and modifications introduced if the more onerous conditions show up weaknesses.

The breaking of a heary short-circuit can be best likened to the detonation of an explosive, and if the switch is badly designed the tank will be blown off and the whole switch wrecked. This was at one time thought to be due to the large quantities of white-hot oil vapour generated by the arc rising to the surface, and spontaneously igniting on meeting the air. It has, however, been proved by experience that a switch designed to withstand the maximum explosive pressure of the gases may be unable to withstand a short-circuit with unlimited power behind it, and it now seems probable that the chief explosive effect is to be found in the rapid generation of gas beneath the surface of the oil. Switches for heavy duty are usually designed with air cushions and vent pipes open to the atmosphere in order to remove the gases, but these measures do not materially relieve the pressure in the tank, as apparently the inertia of the liquid causes an exceedingly high local pressure which is transmitted hydraulically to all parts before the oil has an opportunity to move at its free surface. It is therefore necessary either to reduce the intensity of the explosion or to
build the switch strong enough to withstand it. Since it becomes an expensive matter to build a switch to withstand a pressure of many hundreds of pounds per square inch the former alternative has received much attention. If the arc can be drawn out more quickly it follows that, other things being equal, extra resistance is introduced and the circuit is more quickly broken, thus cutting down the rate of heat generation and limiting the period during which it is being generated. Recent practice consequently tends towards quickening the break by decreasing the inertia of the moving parts and by accelerating their movement by means of powerful springs. Attempts have also been made to use the magnetic forces of the current for this purpose, since such an arrangement has the advantage that the more severe the short-circuit the greater is the speed of break. One of the most interesting suggestions in this connection is that of drawing out two arcs electrically in series, but arranged parallel close to each other in opposite directions, so that there is a repulsive effect between them proportional to the product of the currents in the two arcs, that is to say proportional to the square of the current. As the mass of the vapour carrying the current is so small the repulsive force will produce very rapid outward movement of the arcs in opposite directions, thus causing them to lengthen, which is in addition to and superimposed upon the lengthening due to the separation of the contacts.*

Perhaps the most obvious way of reducing the violence of the explosion is to cut down the short-circuit current by reactance coils. Except in connection with generators the use of these coils is attended by many disadvantages, as pointed out in the discussions on the recent papers read before the Institution on this subject. $\dagger$ In any case it does not seem the correct procedure to use measures of a palliative nature at the present stage of switch design, since if there is one statement which can be made with more certainty than another it is that successful design has not reached its ultimate development. If used at all reactance coils should only be looked upon as an extra precaution kept in reserve to deal with unforeseen difficulties.

Switchgear is the most vital part of the system, because a fault on any part from the busbars to the far side of the switches is a busbar fault which will not only shut down the sub-station concerned but will also seriously derange the whole system, since even if it were possible automatically to isolate the particular sub-station all the "through" connections at the sub-station would be interrupted. In practice the only means of circumscribing the effects of a busbar fault are to divide the system into a large number of isolated networks, thereby involving uneconomical working, or to divide the system into sections by means of graded overload gear, which latter can seldom be relied upon to discriminate properly. It is accordingly imperative to make the switchgear a sound job, and any saving in capital expenditure is dearly bought if it in any way increases the risk of breakdown, particularly as the cost of switchgear is relatively small compared with that of the mains. One very common error is to save money

[^1] Newcastle Local Section." Ibid., vol. 53, p. 102, 1915.
by proportioning switchgear according to the capacity of the apparatus which it controls ; this entirely overlooks the primary duty of switchgear, namely, that of dealing with short-circuits, the severity of which may be as great on small apparatus as on large.

Faults on switchgear fall into three categories :-
(1) Faults under short-circuit conditions.
(2) Failure of apparatus under normal conditions.
(3) Faults due to mistakes in operation.
(1) The means for preventing the failure of switches under short-circuit have already been considered, but if for any reason the switch is unable to operate properly an explosion may occur when breaking a short-circuit, and it is therefore desirable to limit the ensuing damage as far as possible by preventing the explosion affecting other panels, and by discharging any gases which may be produced in such a way that they do not cause short-circuits either on the damaged panel or on others. Even though a switch should explode it will often still interrupt the circuit, and it should therefore be isolated from other apparatus on the panel and all the leads to it should be insulated in order to prevent them being short-circuited by the products of the explosion. Other effects of a short-circuit, even on switchgear other than that immediately involved, are the large mechanical forces* and the fusing of connections by the heavy currents. The latter effect is more particularly liable to occur in current-transformer primary windings, which should, therefore, always be of heavy section.
(2) Failure of apparatus under normal conditions may occur owing to such causes as plaster falling on busbars, and short-circuits due to mice, rats, etc., but such dangers can be readily guarded against once they are realized. Another fruitful cause of trouble is the potential transformer ; these should not only be most carefully constructed but their number should be strictly limited, and under no circumstances should they be connected to the busbars without the intervention of an oil switch.
(3) Mistakes in operation probably account for more switchgear faults than any other cause and they are liable to be made even by the most careful operator, so that so-called "foolproof" arrangements should not be considered as disparaging to the operating staff and undoubtedly justify their slight extra expense, quite apart from the increased safety for operators and other persons who may be working on the switchgear.
The first and most obvious precaution is an interlock between each oil switch and its corresponding isolating switches, in order to prevent the isolating switches being either opened or closed unless the oil switch is open, thus preventing any possibility of making or breaking circuit on an isolating switch.
The second precaution is an arrangement to facilitate the routine earthing of feeders, which is responsible for numerous mistakes owing partly to the fact that the person who is carrying out the earthing cannot be at both ends of the feeder at once and has therefore to rely on a second party to see that the feeder is dead from the far end. By using permanent earthing switches it is a simple matter to interlock at the earthing end so as to ensure that the

[^2] castle Local Section, Yournal I.E.E, vol. 53, p, 102, 1915.
earthing switches cannot be closed until the oil switch is opened, and vice versa; it is, however, difficult to protect against earthing a feeder which is alive from the far end, and all that can be done is to operate the earthing switches from a safe distance and to make the earth as quickly and definitely as possible.
The third and equally important precaution is to guard all live apparatus either by screens or by making the switchgear ironclad. Although it may sound difficult it is really a simple matter so to interlock the guarding arrangements with the switchgear mechanism that it is quite impossible to obtain access to live conductors or to make any such conductors alive while access can be obtained to them. Briefly, the guarding arrangements should fulfil the following conditions:-
(a) That access cannot be obtained to the feeder ends unless they are earthed.
(b) That access cannot be obtained to the oil switch unless it is isolated or earthed from both sides.
(c) That access cannot be obtained to the busbars or gear in connection with them unless all the circuits connected to the particular section of busbars are isolated.

## II(a). The Economical Section of Mains.

Although it is generally known that the economical cross-section of a main is that at which the sum of the annual charges and the value of the energy lost is a minimum, it is seldom that any practical use is made of the formula. This is partly owing to the difficulty of calculating the losses and of placing the correct value upon them when calculated, and also partly because there is an impression that the resulting cross-section would work out to a figure inconsistent with that required by considerations of carrying-capacity and voltage-drop. In the following investigation an attempt is made to apply the formula to the special case of high-pressure distribution systems, and this seems to show that for this case at any rate the results are commercially useful.
The annual charges which have been taken for underground and overhead mains at various voltages are shown in Fig. I. These are based on average commercial prices, allowing interest at the rate of 5 per cent per annum,* and depreciation at 2 per cent for underground mains and 3 per cent for overhead mains, suitable allowance being made for trench work in the former case and for wayleave charges in the latter. With compound interest at 5 per cent the rates allowed for depreciation are sufficient to enable the underground mains to be replaced after $22 \frac{1}{2}$ years and the overhead mains after $17 \frac{1}{2}$ years, allowing a scrap value of 20 per cent in each case. It is probable that these figures considerably underestimate the life of mains, but conservative values have been taken so that there may be no question of the importance of capital charges being minimized $\dagger$. It will be noticed that in the case of overhead mains only one curve is given, as up to 20,000 volts the

* This is assumed to be a reasonable figure for a sound industrial concern operating on a fairly large scale; municipalities and similar bodies would usually be able to obtain their capital at a somewhat less rate of interest.
$\dagger$ No allowance is made for repairs, as these consist almost entirely of jointing and labour, which are independent of the cross-section of the main.
only difference is in the insulators, and this is so small that it is usually worth while to make the line suitable for 20,000 volts, although it may only be intended to operate it at a lower voltage. Higher voltages than 20,000 will not be considered, as the design and manufacture of apparatus for such higher voltages is not sufficiently standardized to enable definite annual charges to be assumed.
The only losses which are important enough to be taken into account in this investigation are the resistance losses in the copper conductors; and in estimating their value it is necessary, as in all estimates of the cost of producing electricity, to take account of the effect of the addition of the load considered to the normal load of the system. It is not the load factor of the particular type of load which matters, but its effect on the load factor of the system ; in fact a load with quite a poor load factor may be distributed over the day in such a manner that it actually improves


Fig. I.-Annual Cost per Mile of Interest and Depreciation for Various Types of 3-phase Mains.
the load factor of the system considered as a whole. Unfortunately, resistance losses constitute a load which is far from beneficial ; not only is there no diversity between the curve of losses and the main load curve, but the peaks of the former are accentuated owing to the fact that resistance losses vary as the square of the load. Of course the losses in different parts of the system will have different load factors, depending on the load factors of the currents in the particular parts, but considering them as a whole it is safe to assume that on the average the curve of total losses is proportional to the square of the main load curve.

In order to obtain definite figures as to the value of the energy lost, certain fundamental data must be assumed, and accordingly a system has been taken having a maximum load of not less than 50,000 kilowatts and an average load factor of 50 per cent ; figures which fairly closely correspond to what may be expected if the system deals with the general demand over a reasonably extended area.

In Fig. 2 the full-line curve shows the assumed load curve of the typical system with a load factor of 50 per cent, and the dotted curve shows to an enlarged scale the shape of the corresponding loss curve, which has a load factor of only 35 per cent. From an analysis of the fixed and running charges the cost per unit of generating these


Fig. 2.-Load Curve of Typical System with 50 per cent Load Factor and Corresponding Curve of Losses (to Enlarged Scale) with a Load Factor of 35 per cent.
losses at the power stations can be calculated, and to this must be added the fixed charges on the increased capacity of the distribution system necessary to transmit these losses. They do not directly increase the carrying capacity of the distribution system since they are manifested as


Fig. 3.- Extreme Forms of Load Curve for a given Number of Units at 50 per cent Load Factor.
voltage-drop, but this causes the load to be supplied at a lower voltage and thus incidentally increases the current that the distribution system has to carry. Since the power stations can seldom be located at the exact electrical centre of gravity of the load, the losses have to be transmitted, on the average, over say two-thirds of the distribution system and should therefore bear two-thirds of
the fixed charges of the latter. There are of course further losses in transmitting the primary losses, but these are quantities of the second order and may be neglected. Taking the cost of the distribution system at two-thirds that of the power stations, the value of resistance losses on the typical system considered is about ${ }^{\circ} 25 \mathrm{~d}$. per unit.
Since the resistance losses over a given time are proportional to the mean square of the current, it is very necessary in calculating them to take account of the shape of the current load-curve. This will be readily seen by considering the two extreme forms of a 50 per cent load-curve shown in Fig. 3. In the first case the R.M.S. current is equal to the average current, while in the second case it is $\sqrt{ } 2$ times it for the particular load factor of 50 per cent ; or more generally for any load factor it is $\sqrt{ }(100 \div$ load factor $)$ times the average current. The load curves which are met with in practice lie between these extremes, and for the typical 50 per cent load curve given in Fig. 2 the R.M.S. current is $I \cdot 18$ times the average current. The full lines


Fig. +.-Effect of Shape of Load Curve on R.M.S. Current and Load Factor of Losses.
in Fig. + show the extreme values of this multiplier for different load factors, and the dotted line the approximate values for the usual type of load curve; corresponding extreme and usual values for the load factor of the losses are also indicated.

On the system which we are considering, the load factor of the mains nearest to the power stations will not be much less than the system load-factor, viz. 50 per cent, but for mains on the outskirts it will be much less, probably as low as 30 per cent. Over the whole network the average would probably be about 40 per cent, and it will be seen from Fig. 4 that the corresponding R.M.S. current would be $r 25$ times the average current. Calculating the losses on this basis and taking their value as 0.25 d . per unit, the annual value of the losses per mile of 3 -phase main is fo. $000296 \mathrm{I}^{2} / \mathrm{A}$, where I is the maximum current in amperes and $A$ is the cross-sectional area of the conductor of each phase in square inches. The value is expressed for convenience in terms of the maximum current, since mains are usually laid to suit a given maximum current.

By adding the curves of annual charges and value of losses for mains of various sizes for any particular maximum current, a curve is obtained of the type shown in Fig. 5, which refers to a 6,000 -volt underground cable required to deal with a maximum current of 100 amperes. The most interesting feature of this curve is that the most economical section is not very definite; it is actually 0.09 square inch, but for $2 \frac{1}{2}$ per cent increase in the total annual costs the section can be increased $44^{\circ} 5$ per cent, or decreased 27.2 per cent, the corresponding figures for 5 per cent increase in annual costs being $65^{\circ} 5$ per cent and $36 \cdot 1$ per cent respectively. It follows that it is not sound practice to cut the section of mains too fine, more especially since it is a most expensive matter after a main is once laid to increase its carrying capacity if this should prove too small.

In order to see to what extent the results are dependent on the particular system load-factor which has been selected, a corresponding curve has been calculated in exactly the same way but assuming a system load factor of only 40 per cent. This is shown dotted in Fig. 5 and


Fig. 5.-Combined Annual Cost of Interest, Depreciation, and Energy Losses per mile of 6,000-volt Underground Main carrying a Maximum Load of 100 Amperes.
gives a reduction of only 2 per cent in the most economical section and 2 per cent in the minimum total annual cost per mile, compared with the original curve. The difference between the two curves is so small because the reduction in the number of units lost at the lower load factor is nearly balanced by the extra value of the losses per unit due to the lower load factor of the losses. A further curve has also been added-shown chain dotted-to show the extent to which the results are affected by the price of copper and lead. The original curve corresponds to basis prices of $£ 60$ and $£ 15$ per ton respectively, while the chain-dotted curve corresponds to basis prices of $£ 80$ and $£_{20}$ per ton. This shows that the influence of ordinary variations in metal prices is negligible.
By plotting a series of curves similar to the full-line curve in Fig. 5, the curves shown in Fig. 6 have been obtained. The full lines in Fig. 6 give, for underground and overhead mains at various voltages, the most economical cross-section for any given maximum current, while the dotted lines show the increased cross-section corresponding to 5 per cent extra annual cost. It will be


Fig. 6.-Most Economical Cross-sections of 3-phase Mains for Various Maximum Currents.


Fig. 7. - Minimum Total Annual Cost per Mile of 3-phase Mains for Various Maximum Currents.
noticed that up to 11,000 volts in the case of underground mains, and up to 20,000 volts in the case of overhead mains, the voltage makes no difference to the economical cross-section ; and from Fig. I it will be seen that this is because between those limits the annual charges happen to differ by approximately constant quantities which are independent of the cross-section.
To show how the economical cross-sections compare with sections settled by considerations of the safe carry-ing-capacity, a further curve has been plotted showing the carrying-capacity of armoured 3 -core 3,000 -volt cable on the basis of the Rules of the Verband Deutscher Elektrotechniker.* As the Rules refer to continuous loads, and the maximum load is usually not maintained for a sufficient time to heat up the cable fully, the curve has been plotted on the assumption that the equivalent heating current is the mean between the R.M.S. current and the maximum current, i.e. 0775 of the maximum current in the present instance with a 40 per cent feeder load-factor. The reduction for voltages up to 11,000 is not more than io per cent, but unfortunately no figures are available at present for 20,000 -volt cables. It will be seen that except for $20,000-$ volt cables the economical section gives an overload margin of as much as 100 per cent in nearly all cases, and even for 20,000 -volt cables it is evident that a considerable margin exists, so that it is quite permissible to settle the cross-section of mains on the basis of maximum economy. That it does not pay normally to operate cables at the maximum current density allowed by heating-limits is of interest, since it means reduced voltage-drop per mile and consequently an increased radius of distribution for a given voltage.

From the series of curves used in determining the economical sections it is also readily possible to find the minimum annual costs of interest charges, depreciation, and losses per mile of main for given maximum currents, on the assumption that the economical sections which correspond to those currents are used. If an allowance for repairs is added, the resulting figures will give the total minimum annual cost per mile of main, thus providing a definite basis on which the relative economy of alternative arrangements of mains can be determined. The curves in Fig. 7 have been obtained in this manner, the allowance for repairs being taken as a certain percentage of the cost of a 0.15 sq . in. main, since repairs consist almost entirely of jointing and labour and their cost is therefore independent of the cross-section. The percentages which have been taken are i per cent for underground mains and 2 per cent for overhead mains, but these must only be looked upon as approximations, since the cost of repairs depends on so many conditions.

## II(b). The Lay-out of the Distribution System.

Except in special circumstances it is usually essential for each sub-station to have at least two separate sources of supply, and, if the supply to the sub-station is not to be interrupted by a failure of one source, some form of discriminating protective device must be installed on each feeder in order to isolate it automatically in the event of its breakdown. It is, however, not so generally recognized that it is of equal importance that a fault on one feeder must not interfere with the supply through the sound

[^3]feeders however severe the fault may be. The only forms of protection in commercial use which meet these conditions under all circumstances are the balanced-current protective system with pilot wires and the split-conductor protective system. Both have the further advantage that the isolation of the faulty feeder is practically instantaneous and can be effected with quite a low value of the fault current so that the disturbance to the general system is a minimum. Full particulars of these protective systems and of their advantages are given in Mr. Wedmore's recent paper,* the split-conductor system being finally recommended as the more suitable for feeder protection.

It is probable that one or other of them would be universally used if it were not that there is often an impression that they involve extra capital cost which is not justified by the extra security they give. Undoubtedly they increase the cost per mile of a main of given section, but this is counterbalanced by the saving effected by the possibility of using an interconnected system which allows of :-
(a) A reduction in the cost of mains due to the saving in spare feeders.
(b) A reduction in the cost of mains due to the possibility of replacing a number of small feeders by a few large ones which are cheaper per ampere of carrying-capacity.
(c) A reduction in the amount of switchgear required.
(d) A reduction in the total annual cost of mains, owing to it being possible to take advantage of the diversity between the demands of different sub-stations.

The extent to which a system may be safely interconnected by the use of these devices is shown by Fig. 8, which illustrates diagrammatically the high-pressure distribution system on the North-East Coast. No less than 350 sub-stations are connected to this system, and it is fed by 15 power stations, many of which utilize waste energy in the form of exhaust steam and coke-oven gas. The whole of the feeders shown are normally in commission and interconnected, the older ones being equipped with balanced-current protection and the more recent ones with split-conductor protection. As showing the reliability of both these forms of protection the operating records of this system show that over a period of time, selected quite at random, faults occurred on 23 feeders equipped with automatic protection, and that in 22 cases the faulty feeder was instantaneously isolated without causing an interruption of supply to a single sub-station, except in one instance where the sub-station in question was given a non-duplicate supply through the faulty feeder. In the remaining case although the protective gear operated satisfactorily one of the feeder switches failed to open due to a mechanical fault ; this was equivalent to a busbar fault and brought out the overload gear at two sectioning points, thus limiting the trouble to this section of the system.

In the following investigation an attempt is made to give definite figures for the saving effected by an interconnected system. These figures show that the saving is not only sufficient to balance the cost of the special protective devices, but that a system so equipped is actually cheaper than systems protected by less efficient methods which do not give the same freedom from interruption of supply.

* E. B. Wedmore. "Automatic Protective Switchgear for Alternating.current Systems." Fournal I.E.E., vol. 53, p. 157, 1915.


Fig. 8.-North-East Coast High-pressure Distribution System.
Note-Generating stations shown cross-hatched.

In order that the results may be on the conservative side the extra cost of the special protective devices has been taken at to per cent on the switchgear, including building accommodation, and at two shillings per yard on the mains, which is sufficient to cover the cost of balancedcurrent protection and decidedly more than sufficient to cover the cost of split-conductor protection. In the first place a comparison will be made between the three types of system that are available in the case of the supply to a number of sub-stations the supply to which must be reasonably free from interruption. These are :-
(1) An interconnected system equipped with balancedcurrent or split-conductor protection, which ensures complete continuity of supply to all sub-stations in the event of a fault on a feeder.
(2) A simple radial system with duplicate feeders direct from the power station to each sub-station, equipped with time-limit overloads at the power-station end and reversepower relays at the sub-station end. This limits the risk of interruption to the sub-station fed by the faulty feeder and has been adopted for important supplies such as that to the London Underground Railways. Owing to the defects of the reverse-power relay it cannot ensure complete continuity of supply to the sub-station affected
(3) A series radial system with duplicate feeders to each sub-station protected as in (2), but with direct feeders from the power station to only half the sub-stations, each of these in turn feeding one of the remaining sub-stations. This involves graded overload gear, and a faulty feeder may shut down both the sub-stations in connection with it and will probably shut down one of them ; it is therefore only permissible where continuity of supply is not of such vital importance.
For the purpose of the comparison a typical case is assumed of an area of 25 square miles with the power station at the centre and a sub-station with a maximum load of 500 kilowatts at 0.8 power factor and a load factor of 30 per cent situated at the centre of each square mile, the distribution being effected by underground mains at 6,000 volts. It is assumed that owing to the diversity between the various sub-station loads the power-station load-factor will be 50 per cent, and the average feeder load-factor on an interconnected system 40 per cent. The average maximum feeder currents are therefore deduced by assuming the sub-station maximum demand to be reduced in the ratio of the sub-station load-factor to the average feeder load-factor. This only holds for the interconnected system, but in the first instance the same reduced sub-station demands will be taken for other types of system, i.e. the advantage which an interconnected system gains from its utilization of diversity is neglected.

The annual cost per switch panel is taken from Fig. 12, and the annual cost of the mains from Fig. 7 , the proper deductions being made in the case of types (2) and (3) for the omission of the special protective devices. Table $I$ gives the comparison, while the diagrammatic lay-outs of the three types of system are shown in Fig. 9 (a), (b), and (c).
In certain cases where momentary interruption of supply is not of great importance a "tee" system has been used, arranged as shown in Fig. 9 (d). Each sub-station is normally given a non-duplicate supply from one of the feeders, and arrangements are made for the sub-station to be changed over to the other feeder in emergency, thus
involving a complete temporary interruption in the supply to all the sub-stations fed by the faulty feeder. In a modification of this system both tees into the sub-station are normally closed through switches equipped with timelimit gear, but the sub-station busbars are sectioned by a special switch. The sectioning switch may be either left open or it may be closed and equipped with instantaneous overload gear so that it will immediately trip in case of a feeder fault. In either case supply is automatically maintained to half the sub-station busbars, and by opening the faulty feeder switch and closing the sectioning switch complete supply to the sub-station can be resumed.

Table 2 gives a comparison of these systems with the interconnected system, and it is interesting to note that the latter is still cheaper in spite of the great sacrifices in security entailed by the "tee" systems.

The foregoing comparison proves that the interconnected system is the most economical for the particular case which has been selected as typical ; but in order to make the investigation complete it is necessary to show that this superiority still holds under other conditions, and accordingly the interconnected system will be compared with the cheapest system giving reasonable security of supply-the series radial system-under various modified conditions which may obtain in practice. These are :-
(I) The same number of sub-stations distributed one mile apart in a ring round the power station, or in line one mile apart with the power station at the centre, as shown in Fig. $9(e),(f)$, $(g)$, and ( $h$ ).
(2) The area of supply extended or decreased with the same number of sub-stations and the same total load on the system, i.e. the sub-stations situated at the increased and decreased spacings of one per 4 square miles and four per square mile respectively, instead of one per square mile, the load per sub-station remaining the same.
(3) The area of supply extended to, say, 49 square miles with the same density of load and a correspondingly increased number of sub-stations supplied from the central power station, as şhown in Fig. 9 (i) and ( $j$ ). A further extension of the area of supply to 8 r square miles has also been worked out, but it has not been considered necessary to complicate Fig. 9 with details of this, as the lay-outs are similar to those for the 49 square miles. There is no need to consider a reduction in the area of supply as this is already quite small.
(4) The original distribution of sub-stations but with increased or reduced loads per sub-station, the alternative loads considered being $250,1,000$, and 2,000 kilowatts. The arrangement of feeders remains the same for the series radial system, as a cable capable of dealing with the load of two $2,000 \cdot k i l o w a t t$ sub-stations in emergency is not too large to be handled. For the interconnected system it is necessary to run more feeders from the power station to deal with the heavier loads, the feeder arrangements which would be adopted for the 1,000 - and $2,000-$ kilowatt sub-stations being shown in Fig. 9 (k) and ( $l$ ) respectively.

The details of the comparison are given in Table 3.
The outstanding features of this comparison are :-
(a) The economy of the interconnected system is generally maintained.
(b) The saving effected by an interconnected system rapidly increases as the area of supply is enlarged. This


Fig. 9.-Diagrammatic Lay-outs of Distribution Systems.
is important as only relatively restricted areas have been considered.
(c) The saving is greatest for lightly loaded sub-stations and decreases as the sub-station loading increases. In the
to diversity has been ignored, and secondly it will be found on investigation that a point has been reached at which distribution at so low a voltage as 6,000 is uneconomical. The question of the most economical distribu-

Table i.

| Type of System |  |  | Reference to Flgs. | No. of Switches | Mileage of Mains | $\underset{6}{\text { Annual }} \text { Costs. }$ | Percentage Increased Cost over Interconnected System |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Interconnected | -•• | - | 9 (a) | 64 | $32^{\circ} \mathrm{O}$ | 4,752 | - |
| Simple radial ... | ... | - | 9 (b) | 96 | $93^{\circ} 7$ | 7,664 | 6r 3 |
| Series radial ... | ... | - | 9 (c) | 96 | $6 \mathrm{I} \cdot 2$ | 6,075 | $27 \cdot 8$ |

Table 2.

| Type of System |  | Reference to Figs. | No. of Switches | Mileage of Mains | Annual Costs. | Percentage Increased Cost over Interconnected System |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Interconnected .. | . | 9 (a) | 64 | $3^{\circ} \mathrm{O}$ | 4,752 | - |
| Semi-duplicate "tee" | $\cdots$ | 9 (d) | 80 | $48 \cdot 0$ | 5,478 | 153 |
| Change-over "tee"... | ... | 9 (d) | 56 | 48.0 | 5,216 | $9 \cdot 8$ |

Table 3.

| Nature of Modification to the Typical System | Type of System | $\underset{\text { Fig. } 9}{\text { Reference to }}$ | $\underset{S w i t c h e s}{\text { No. of }}$ | Mileage of Mains | $\underset{6}{\text { Annual }}$ Cost. | Percentage Increased Cost over Corresponding Inter- connected connected System |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sub-stations distributed in ring | Interconnected Series radial | $\stackrel{(e)}{f})$ | $\begin{aligned} & 64 \\ & 96 \end{aligned}$ | $\begin{array}{r} 54.6 \\ 115.9 \end{array}$ | $\begin{array}{r} 8,179 \\ \mathbf{1 0 , 9 7 7} \end{array}$ | $3 \overline{2}$ |
| Sub-stations distributed in line | Interconnected Series radial | $\begin{aligned} & (g) \\ & (h) \end{aligned}$ | $\begin{aligned} & 66 \\ & 96 \end{aligned}$ | 84 168 | $\begin{aligned} & 11,508 \\ & 15,644 \end{aligned}$ | $35 \cdot 9$ |
| Sub-stations spaced one per $\{$ 4 square miles | Interconnected Series radial | $\begin{aligned} & (a) \\ & (c) \end{aligned}$ | 64 96 | $\begin{array}{r} 64.0 \\ 122.4 \end{array}$ | $\begin{array}{r} 8,736 \\ 11,103 \end{array}$ | 27.1 |
| Sub-stations spaced four per $\{$ square mile | Interconnected Series radial | $\begin{aligned} & (a) \\ & (c) \end{aligned}$ | 64 96 | $\begin{aligned} & 16.0 \\ & 30.6 \end{aligned}$ | $\begin{aligned} & 2,760 \\ & 3,562 \end{aligned}$ | 29.0 |
| Area of supply increased to $\{$ 49 square miles | Interconnected Series radial | $\begin{aligned} & (i) \\ & (j) \end{aligned}$ | $\begin{aligned} & 128 \\ & 192 \end{aligned}$ | $\begin{array}{r} 69.0 \\ 1669 \end{array}$ | $\begin{aligned} & 11,108 \\ & 16,135 \end{aligned}$ | $45^{-2}$ |
| Area of supply increased to $\{$ 8I square miles | Interconnected Series radial | 二 | $\begin{aligned} & 192 \\ & 320 \end{aligned}$ | $\begin{aligned} & \mathbf{1 2 0} \cdot 8 \\ & 347 \end{aligned}$ | $\begin{aligned} & 20,521 \\ & 33,124 \end{aligned}$ | $\overline{61 \cdot 4}$ |
| Sub-station load reduced to 250 kilowatts | Interconnected Series radial | $\begin{aligned} & (a) \\ & (c) \end{aligned}$ | $\begin{aligned} & 64 \\ & 96 \end{aligned}$ | 32 62 | $\begin{aligned} & 3,960 \\ & 5,295 \end{aligned}$ | 337 |
| $\underset{\text { I,000 kilowatts }}{\text { Sub-station load to }}\{$ | Interconnected Series radial | $\binom{(k)}{c}$ | $\begin{aligned} & 64 \\ & 96 \end{aligned}$ | 353 $6 r^{2} 2$ | $\begin{aligned} & 6,533 \\ & 7,721 \end{aligned}$ | 18.2 |
| $\underset{\substack{\text { Sub-station load increased to } \\ 2,000 \\ \text { kilowatts }}}{ }$ | Interconnected Series radial | $\binom{()}{c}$ | $\begin{aligned} & 80 \\ & 96 \end{aligned}$ | 53.2 $6 \mathrm{r} \cdot 2$ | $\begin{aligned} & \mathbf{r o , 4 6 9} \\ & 10,572 \end{aligned}$ | $\overline{\text { ro }}$ |

case of the most heavily loaded sub-station it would appear at a first glance that while the interconnected system may have other advantages its direct economy is very small. This is only apparent, for in the first case the saving due
tion woltage will be considered further at a later stage, but it may be noted now that with a sub-station loading as high as 2,000 kilowatts the distribution system costs 37.1 per cent more at 6,000 volts than it would at 11,000 volts,
while if a pressure of 11,000 volts were adopted the interconnected system would show a saving of 21.8 per cent compared with a series radial system at the same voltage.

The possible further savings which may be effected if diversity is taken into account have been considered, and without going into details of the calculation it may be taken as adding something of the order of $4 \frac{1}{2}$ per cent to the cost of a simple radial system, and rather less to the cost of a series radial system.

When comparing an interconnected system with other types, two further points must also be borne in mind.
(I) It is a very difficult matter exactly to forecast sub-station maximum loads or, as in the case of railways, definitely to fix the allocation between the various substations. If the sub-stations are fed independently it is obvious that in proportioning the feeders to them allowance must be made for the assumed maximum loads being exceeded or varied, while if the sub-stations are intercon-
(a) The higher the voltage the less is the proportionate cost of the protective devices, which is practically independent of the voltage.
(b) The higher the voltage the smaller is the section of the mains for given loads, and consequently the greater the advantage offered by interconnection in reducing the total length of mains and increasing their average capacity.

This is shown very clearly by Table 4 , which gives the comparison at various distribution voltages between the annual costs of interconnected and series radial systems for the typical distribution of sub-stations and a load of 2,000 kilowatts per sub-station.

## II (c). The Most Suitable Distribution Voltage.

In settling the distribution voltage the primary consideration is that it shall be sufficiently high to ensure that the voltage variation at the boundaries of the supply area can be kept within a reasonable amount without

Table 4.

| Distribution Voltage | Interconnected System |  | Series-radial System |  | Percentage Increased Cost of Series Radial System over Corresponding Interconnected System |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Reference to Fig. 9 | $\underset{\neq}{\text { Annual }} \text { Cost. }$ | Reference to Fig. 9 | $\underset{f_{6}}{\text { Annual }} \text { Cost. }$ |  |
| 6,000 | (l) | 10,469 | (c) | 10,572 | $1 \times 0$ |
| 11,000 | (k) | 7,634 | (c) | 9,295 | 21.8 |
| 20,000 | (a) | 7,984 | (c) | 10,756 | $34 \cdot 8$ |

Table 5.

| Type of Main | Amperes per Square Inch at Maximum Load |  | Resistance Voltage-drop between Phases per Mile at Maximum Load |  |
| :---: | :---: | :---: | :---: | :---: |
|  | With the Economical Loading | With Decreased Loading Corresponding to 5 per cent Extra Cost | With the Economical Loading | With Decreased Loading Corresponding to 5 per cent Extra Cost |
| Underground mains up to ix,000 volts ... | 910 | 635 | $68 \cdot 1$ | 47.5 |
| 20,000-volt underground mains ... .. | 1,190 | 800 | 88.8 | 59.9 |
| Overhead mains up to 20,000 volts .... | 800 | 540 | $59^{\prime} 9$ | $40 \cdot 4$ |

nected the particular distribution between the sub-stations does not greatly matter so long as the total system load is unaltered.
(2) The greater part of the saving effected by an interconnected system is in the trunk feeders, and in consequence it is more marked the farther the power stations are removed from the centres of load. In the above comparisons the power station has been taken at the electrical centre of gravity of the load and therefore at the least favourable position for the interconnected system.

So far the investigation has been limited to the comparatively low voltage of 6,000 ; but, speaking generally, the higher the system voltage the greater is the economy to be obtained by interconnection. This is obvious if it is remembered that-
putting an excessive amount of copper in the feeders. It is usually desirable that the voltage variation should not exceed 5 per cent either side of the normal, i.e. a total of io per cent, and from this about $2 \frac{1}{2}$ per cent should be deducted for transformer voltage-drop, leaving a permissible maximum drop of $7 \frac{1}{2}$ per cent in the highpressure mains. Fig. 6 shows that under the same conditions the current density in the mains, and therefore the resistance voltage-drop per mile, is approximately independent of their section. The actual figures taken from Fig. 6 are given in Table 5.

As the power factor of most distribution systems is less than unity it is also necessary to take account of the inductive voltage-drop. This is not independent of the section, but given the current density from Table 5 and
the frequency of the system it can be readily calculated from the size and spacing of the conductors. Fig. io shows, for several typical sizes of both underground and overhead mains at various voltages, the relative amount by which the inductive drop increases the total voltage-drop at various power factors. The calculations are based on a frequency of 50 cycles per second and on a constant current density, since the economical value for the latter is independent of the power factor. They also assume the use of ordinary 3 -conductor mains; if arranged for split-conductor protection the inductive drop would be appreciably reduced.
It is evident that the maximum radius of distribution and area of supply at a given voltage will vary according to the power factor of the system and to the various factors which determine the inductive drop, but in order to give


Fig. 10.-Effect of Inductive Drop at Different Power Factors for Various Mains.
an approximate idea of the relative figures for the maximum radius and area at various voltages the curves given in Fig. if have been prepared on the following assumptions :-
(I) Permissible voltage-drop 7.5 per cent.
(2) Average power factor 0.8 .
(3) A network comprising equal lengths of underground and overhead mains.
(4) An average cross-section of main of 0.15 square inch.
(5) A frequency of 50.

It is interesting to note that, although it is more economical to run higher voltage cable at an increased current density, the curves of both area and radius have still a steep upward tendency at 20,000 volts. This would be further accentuated if allowance were made for the fact that the average cross-section of main tends to decrease at higher voltages, with a corresponding decrease
in the value of the inductive drop relative to the resistance drop.
It is, however, not sufficient to settle the distribution voltage on the basis of permissible voltage-drop alone. It


Fig. in.-Radius and Area of Distribution at Various Voltages.
Permissible drop $7 \frac{1}{2}$ per cent.
Frequency $50 \sim$.
Average power factor 0.8 .
Equal lengths of underground and overhead mains.
is also important to choose that voltage which gives the cheapest distribution system, and from this point of view it may often pay to use a voltage much higher than that which is required by the conditions of voltage-drop.


Fig. 12.-Annual Cost of Switchgear per Panel.
Generally speaking, the higher the loads which have to be dealt with the higher is the economical voltage ; but it is also necessary to take into consideration the proportion between the total number of switches required and the total mileage of mains, as the cost of switchgear increases with the voltage. Fig. 12 gives fairly safe
figures for the annual cost per switch at different voltages on the basis of annual charges of 8 per cent on the switchgear and the corresponding building accommodation, and these figures have been used in the following investigation.

Taking a system comprising 24 sub-stations evenly spaced as in Fig. 9 (a), and allowing an average of $2 \frac{1}{2}$ switches per sub-station for controlling the step-down transformers, the total annual cost of switchgear and mains has been calculated for various distribution voltages and various sub-station loadings. In addition the spacing of the sub-stations has also been varied so as to give several comparative proportions between the number of switches and the mileage of mains. From these results the series of curves given in Fig. 13 $a$ have been plotted showing the most economical voltage under the varying conditions.


Fig. I3a.-Economical Distribution Voltage.

It will be seen by reference to Fig. 7 that the annual costs of mains of varying sections lie approximately on straight lines, and it is therefore permissible, by plotting the curves in Fig. i3a to a base of average demands, to make the results applicable to any arrangement of network whatever its type or extent. In order to find the most economical voltage for a given distribution of loads, the procedure would then be to assume a voltage which it is anticipated will be about correct, to lay out the distribution system on this basis, calculate from this the average number of kilovolt-amperes per mile of main and the number of switches per mile of main, and then from these two figures find from the curves in Fig. i3a what the most economical voltage would be. If the original voltage which had been assumed should prove to have been so incorrect that the arrangement of feeders and the number of switches would be appreciably altered by the adoption of the revised voltage, it may be desirable to lay out the system afresh with the revised voltage and afterwards check the results again with the curves in Fig. 13a, the method being thus one of trial and error. As a rule,
however, the first voltage assumed should be sufficiently close to the correct figure to enable the latter to be obtained by the first trial. The economical voltage obtained in this way must of course be always checked to ensure that it also meets the requirements of voltage-drop.
In order to give some idea of the extent to which the economical voltage can be departed from, the curves given in Fig. $1^{3}$ b have also been calculated; these show the upper and lower limits of voltage corresponding to an increase of 5 per cent in the cost of the distribution system. The limits are fairly wide, but, if they are exceeded, the extra cost of the distribution system increases at a cumulatively rapid rate. In practice it is advisable to adopt a voltage in the neighbourhood of the upper limit in order to keep down the voltage-drop as much as possible, and also in order to make allowance


Fig. 13b.-Upper and Lower Limits of Distribution Voltage for 5 per cent increased Cost of Distribution System.
for the increase in the density of load which will usually occur.
As might be expected, Fig. 13 shows that the heavier the loads to be supplied and the more they are concentrated the higher is the most economical voltage. It should be noticed that the curves show no signs that at 20,000 volts the maximum economical voltage has been reached, provided the system loading is heavy enough, and hence for the larger systems of the future we may expect that for economical reasons alone the distribution voltage will be raised above 20,000 volts.

These calculations only refer to underground mains; with overhead mains the economical voltage will be higher, since there is so little difference between their cost at various voltages.

In order to see what are typical figures in actual practice for the number of switches per mile of main and the average number of kilovolt-amperes per mile of main, the following table has been prepared for two typical systems for which the author has had access to the necessary data, and as a matter of interest the corresponding economical
and limiting values for the distribution voltage have been deduced from Fig. 13 .

Table 6.

| System voltage actually adopted | 5.500 | 20,000 |
| :---: | :---: | :---: |
| Number of switches per mile of main ... ... ... ... ... | $2 \cdot 63$ | 0*725 |
| Average maximum kilovolt-amperes per mile of main ... ... ... | 450 | 3,500 |
| Most economical system voltage ... | 5,700 | 16,000 |
| Upper and lower limits for system voltages with 5 per cent extra cost of distribution system | $\begin{aligned} & 9,500 \\ & 3,200 \end{aligned}$ | $\begin{aligned} & \text { 20,000 } \\ & \text { 12,000 } \end{aligned}$ |

## Conclusion.

So many factors are involved that it has only been possible to attempt a general survey of the problem of high-pressure distribution, but it is hoped that this may prove useful as a starting-point in the detailed consideration of any particular case.

Theoretical calculations must, however, be used with caution. The results of the graph and the slide rule should always be considered in conjunction with the results of practical experience and the whole sifted by the exercise of judgment, accuracy in which is the true test of an engineer. The following paragraph in Professor Arthur Schuster's address to the British Association is so pertinent
to the subject that perhaps its quotation may be permitted :-
"Why does a theory ever fail, though it may be sound in reasoning ? It can only do so because every problem involves a much larger number of conditions than those which the investigator can take into account. He therefore rejects those which he believes to be un-essential, and if his judgment is at fault he goes wrong. But the practical man will often fail for the same reason. When not supported by theoretical knowledge he generalizes the result of an observation or experiment, applying it to cases where the result is determined by an altogether different set of conditions. To be infallible the theorist would have to take account of an infinite number of circumstances and his calculations would become unmanageable, while the experimenter would have to perform an infinite number of experiments, and both would only be able to draw correct conclusions after an infinite lapse of time. They have to trust their intuition in selecting what can be omitted with impunity, and if they fail, it is mainly due to the same defect of judgment. And so it is in all professions : failure results from the omission of essential considerations which change the venue of the problem."

In conclusion the author wishes to record his obligations to the Newcastle-upon-Tyne Electric Supply Company, Ltd., and associated power companies on the North-East Coast, and to their consulting engineers, Messrs. Merz and McLellan, for permission to use much of the data on which the conclusions of the paper are based.


[^0]:    * Gournal I.E.E., vol. 52, p. 17, 1914.
    $\dagger$ Ilbid., vol. 44, p. 802, 1910 .
    $\ddagger$ The curves given in Fig. ro are calculated for ordinary 3 -conductor mains. If arranged for split-conductor protection the inductive drop is reduced.

[^1]:    * Hunter and Shand, British Patent No. 11,586 (1912).
    t K. M. Faye.Hansen and J. S. Peck. "Current-limiting Reactances on large Power Systems." fournal I.E.E., vol. 52, p. 511, 1914: E. P. Hollis. "Reactance and Reactance Coils in Power Circuits." Ibid. vol. 52, p. 254, 1914. P. V. HUNTER. "Address as Chairman of the

[^2]:    * These were discussed in Mr. P. V. Hunter's Address to the New-

[^3]:    * Gournal I.E.E., vol. 52, p. 786, 1914.

