The Determination of Poisson's Ratio and of the Absolute Stress-variation of Refractive Index

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XXII. *The Determination of Poisson's Ratio and of the Absolute Stress-variation of Refractive Index.* *By* F. TWYMAN, *F.Inst.P., and* J. W. PERRY.

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ABSTRACT.

The Paper describes a ready method of determining the stress-optical coefficients, the Hilger interferometer being employed. Young's modulus of elasticity and Poisson's ratio are determinable simultaneously.

AFTER the discovery by Brewster * of the phenomenon of double refraction in strained media, the first to make quantitative determinations of the effect was Neumann,† who also developed a theory of the phenomenon. Further experiments were carried out by Kerr, ‡ establishing the laws of the propagation of light in strained media. Absolute measurements were made later by Pockels,§ whose investigations resulted in the production of a glass which, when subjected to any strain within the elastic limits, remained optically almost isotropic. Filon|| later investigated by various methods the birefringence and its dispersion in a bent beam, and advantageously proposed the use of two constants in specifying the stress-optical effect in such a beam when bent, called by him the stress optical coefficients, and defined as

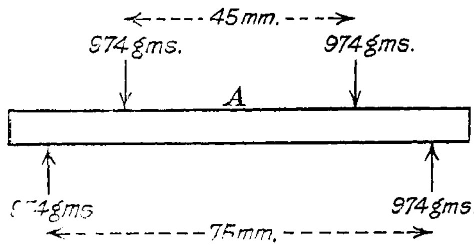

FIG. 1.

the " additional retardation per unit thickness per unit tension " for ordinary and extraordinary rays respectively. Filon specifies these constants in terms of " brewsters," a unit proposed ¶ also by him (1 brewster $=1\cdot10^{-10}$ cm.^{2}/gm.).

In the course of an investigation concerning the optical properties of a certain melting of glass, it became necessary to determine approximately the stress-optical coefficients. It is the purpose of the following to indicate the method employed, which differed from any known to have been published, and is applicable in connection with determinations of other small refractive index variations. ** No great accuracy is claimed for the measurements here recorded, but the method is susceptible of refinement.

A strip of the glass was prepared, of dimensions $80\times15\cdot1\times2\cdot48$ mm., of which the two $80\times2\cdot48$ mm. faces were accurately plane parallel, and one $80\times15\cdot1$ mm.

* Phil. Trans. (1816).

† Abh. d. Berliner Akad. (1841). II.

‡ Phil. Mag., 5, XXVI. (1888).

§ Ann. d. Phys., 4, 7, 745.

|| Phil. Trans., A. CCVII. (1907) ; *see also* Proc. Camb. Phil. Soc., XI., 478, XII., 55, 314.

¶ Proc. Roy. Soc., A., LXXXIII., 572.

** As, for instance, the variations of thickness and of mean refractive index from point to point of nearly plane parallel glass plates.

face polished accurately plane. It was supported on two pieces of soft wire (electric fuse wire) 75 mm. apart, so that a load (of 1948 gms.) could be applied by resting a weight symmetrically on two similar pieces of wire 45 mm. apart, as is shown in Fig. 1.

Compressive stress at A : 94·66 kg./cm.^{2}.

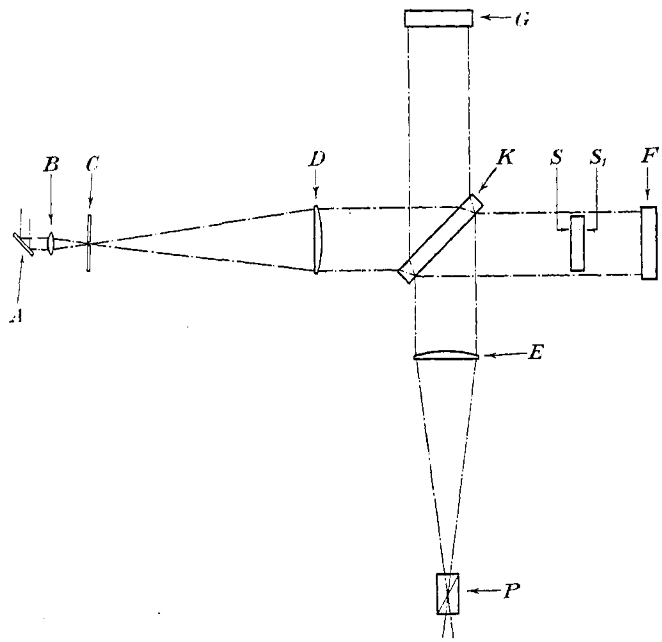

FIG. 2.

ARRANGEMENT (*a*).

The whole was placed in a Hilger prism interferometer* (Fig. 2), in the beam usually occupied by the prism, and the interferometer illuminated by the green Hg radiation $\lambda 5461 \mathring{A}.U.$ The interferometer was so adjusted that uniform coloration was produced, with the strip unloaded, over the whole area presented by the strip.

On loading the strip interference bands as shown in Fig. 3 were produced. These are due to the composite effect of alteration in the length of glass path occasioned by the stress and variation of refractive index also occasioned by the stress. Since, moreover, the stress in general occasions double refraction, the appearance is a complex one, but it is readily simplified by admitting to the observer's eye light polarised either in a horizontal or in a vertical plane.

ARRANGEMENT (*b*).

Interference bands can also be obtained by covering the interferometer mirrors F and G and observing by means of the light reflected from the surfaces s and s' of the strip.

* Phil. Mag., VI., 35, 1.

If now n be the normal refractive index of the glass, $2t$ the length of glass path in the direction traversed by the light, Δn and Δt the variations in n and t due to loading the strip in the manner indicated, and m the number of complete bands observed from A to B (Fig. 3), to the approximation aimed at in the present determination we can write for the case of arrangement (a) :

$$\tfrac{1}{2}m_a\lambda=\Delta(n-1)t=(n-1)\Delta t+t\Delta n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (a)$$

while for the case of arrangement (b) :

$$\tfrac{1}{2}m_b\lambda=\Delta nt=n\Delta t+t\Delta n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (b)$$

from which two equations Δn and Δt are easily determinable.

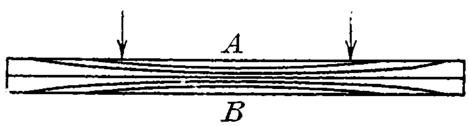

FIG. 3.

Particulars of the glass : Melt 683.* Derby Crown Glass Co.

$n_D=1{\cdot}5759 \qquad \nu=41{\cdot}3$

The following results were obtained :—

Arrangement.	Ordinary ray.			Extraordinary ray.		
	m_0	Δn_0	Δt	m	Δn_e	Δt
(a)	8	0·000110	1·65λ	4·3	0·000044	1·60λ
(b)	11·3			7·5		
Compressive stress at A						94·66 kg./cm.².

Thus, for any known load, Δn for both ordinary and extraordinary rays is determinable immediately from the observations. It will also be noticed that Δt is twice determined. The two values found should, of course, be identical; that they are not so is due to the fact that m was measured by counting the bands simply as seen by the unaided eye at P, the accuracy so obtained sufficing for the immediate purpose of the experiment. More accurate means of counting the bands will readily suggest themselves to one desiring to adapt the method to more precise determinations.

It would, of course, be necessary then to correct for several sources of error, which in the more refined application of the method would no longer be negligible, arising from the prismatic form of the strip when bent. It may be shown that to

* We have to thank the courtesy of the Derby Crown Glass Co., Ltd., for letting us have, and permitting us to publish, the theoretical composition of their melt 683 as determined from the batch used. No analysis of the actual glass was made.

Theoretical Formula :

$100\,SiO_2.\ \ 17\,PbO.\ \ 11\,Na_2O.\ \ 5\,K_2O.$

Theoretical Percentage Composition :

SiO_2	54·83
PbO	34·64
Na_2O	6·23
K_2O	4·30

the order of approximation aimed at in the present measurement such sources of error may be ignored (the greatest correction—*i.e.*, that to the value of $m_{ob}\lambda$, necessitated is approximately $0{\cdot}08\lambda$, and is thus smaller than the probable observational error).

The stress-optical coefficients as thus determined for the glass as above specified are as follows :—

$$C_0 = -5{\cdot}81 \qquad\qquad C_e = -2{\cdot}32$$

The above measurements enable us, since Δt is determined, to calculate Poisson's ratio if Young's modulus E is known for the glass. Failing a more precise knowledge of E it is possible to determine it on the same apparatus by applying the method originated by Cornu,* elaborated by Straubel,† and quite recently employed by Jessop ‡ in the determination of Poisson's ratio for specimens of plate glass. The procedure is as follows :—

A right-angled prism is so mounted under the loaded strip that light proceeding from the half-silvered mirror K (Fig. 2) is reflected normally from the under-surface of the strip, and passes back to the mirror K to be reflected to the observer's eye at P. The beam which is reflected from the curved under-surface of the strip interferes with the beam reflected from the upper surface of the prism ; the curvature of the strip can, therefore, be determined by counting the interference fringes so formed and observed by the eye at P. Because of the large number of fringes with the original load, the total load was reduced to 1·006 kg., thereby rendering possible a more accurate counting. The accuracy in E thus secured is estimated at about ± 3 per cent.

The following observations were made, y being measured from the centre of the strip along its major axis :—

y	7·5	10·0 mm.
$m\lambda$	6·75	11·75 λ

The radius of elastic curvature R is then given by

$$R = \frac{y_2{}^2 - y_1{}^2}{(m_2 - m_1)\lambda}$$

and Young's modulus E is calculated from

$$E = \frac{12\ RM_2}{td^3}$$

where M is the bending moment, the suffix being to distinguish this from the earlier value. E was thus found to be 6,320 kg./mm.².

Returning now to the observations firstly made, Poisson's ratio σ is to be derived

from $$\sigma = \frac{E}{12} \cdot \frac{d^2}{M_1}\ \Delta t\ ;$$

thus is obtained the value $\sigma = 0{\cdot}196.$

DISCUSSION.

Mr. J. Guild inquired what fraction of a band could be counted in the determination of E.

The Author replied that the bands were counted to the nearest one-tenth.

* C.R., LXIX., 333 (1869).
† Wied. Ann., LXVIII., 369 (1899).
‡ Phil. Mag., VI., 42 (1921).