# The Determination of the Quark-Gluon Mixed Condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ from Lattice QCD

Takumi Doi,[1,*)] Noriyoshi Ishii,[2] Makoto Oka[1] and Hideo Suganuma[1]

[1]*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*
[2]*Radiation Laboratory, The Institute of Physical and Chemical Research (RIKEN), Wako 351-0198, Japan*



(Received March 14, 2003)

We study the quark-gluon mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$, using the $SU(3)_c$ lattice QCD with the Kogut-Susskind fermion at the quenched level. We generate 100 gauge configurations on the $16^4$ lattice with $\beta = 6.0$, and perform the measurement of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ at 16 points in each gauge configuration for each current quark mass of $m_q = 21, 36, 52$ MeV. Using the 1600 data for each $m_q$, we find $m_0^2 \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle \simeq 2.5$ GeV$^2$ at the lattice scale of $a^{-1} \simeq 2$ GeV in the chiral limit. The large value of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ suggests its importance in the operator product expansion in QCD. We study also chiral restoration at finite temperature in terms of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$, which is another chiral order parameter. We present the lattice QCD results of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ at finite temperature.

## §1. The importance of the quark-gluon mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$

The non-perturbative nature of quantum chromodynamics (QCD) is characterized by its nontrivial vacuum structure such as various condensates. Among various condensates, we emphasize here the importance of the quark-gluon mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle \equiv g\langle\bar{q}\sigma_{\mu\nu}G^A_{\mu\nu}\frac{1}{2}\lambda^A q\rangle$. First, the mixed condensate represents a direct correlation between quarks and gluons in the QCD vacuum. In this point, the mixed condensate differs from $\langle\bar{q}q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ even at the qualitative level. Second, this mixed condensate is another chiral order parameter of the second lowest dimension and it flips the chirality of the quark as

$$g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle = g\langle\bar{q}_R\ (\sigma_{\mu\nu}G_{\mu\nu})\ q_L\rangle + g\langle\bar{q}_L\ (\sigma_{\mu\nu}G_{\mu\nu})\ q_R\rangle. \qquad (1\cdot1)$$

Third, the mixed condensate plays an important role in various QCD sum rules, especially in the baryons,[1),2)] the light-heavy mesons[3)] and the exotic mesons.[4)] In the QCD sum rules, the value $m_0^2 \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle \simeq 0.8 \pm 0.2$ GeV$^2$ has been proposed as a result of the phenomenological analyses.[5)–8)] However, in spite of the importance of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$, there was only one preliminary lattice QCD study,[9)] which was performed with insufficient statistics (only 5 data) using a small ($8^4$) and coarse lattice ($\beta = 5.7$).

Therefore, we present the calculation of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ in lattice QCD with a larger ($16^4$) and finer ($\beta = 6.0$) lattice and with high statistics (1600 data). We perform the measurement of $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ as well as $\langle\bar{q}q\rangle$ in the $SU(3)_c$ lattice at the quenched level, using the Kogut-Susskind (KS) fermion to respect chiral symmetry. We generate 100 gauge configurations and pick up 16 space-time points for each

*) E-mail: doi@th.phys.titech.ac.jp

configuration to calculate the condensates. With this high statistics of 1600 data for each quark mass, we perform a reliable estimate for the ratio $m_0^2 \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle$ at the lattice scale of $a^{-1} \simeq 2$ GeV in the chiral limit.[10)]

## §2. Lattice formalism

Since both of the condensates $\langle\bar{q}q\rangle$ and $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ are chiral order parameters, they are sensitive to explicit chiral-symmetry breaking. Therefore, we adopt the KS fermion, which preserves the explicit chiral symmetry in massless quark limit, $m = 0$. On the other hand, the Wilson and the clover fermions would not appropriate for our study, because these fermions explicitly break chiral symmetry even for $m = 0$.

The action for the KS fermion is described by spinless Grassmann fields $\bar{\chi}, \chi$ and the gauge link-variable $U_\mu \equiv \exp[-iagA_\mu]$. When the gauge field is set to be zero, the $\mathrm{SU}(4)_f$ quark-spinor field $q$ with spinor $i$ and flavor $f$ is expressed by $\chi$ as

$$q_i^f(x) = \frac{1}{8}\sum_\rho (\Gamma_\rho)_{if}\ \chi(x+\rho), \quad \Gamma_\rho \equiv \gamma_1^{\rho_1}\gamma_2^{\rho_2}\gamma_3^{\rho_3}\gamma_4^{\rho_4}, \quad \rho \equiv (\rho_1,\rho_2,\rho_3,\rho_4), \tag{2·1}$$

where $\rho$ with $\rho_\mu \in \{0,1\}$ runs over the 16 sites in the $2^4$ hypercube. When the gluon field is turned on, we insert additional link-variables in Eq. (2·1) in order to respect the gauge covariance. Hence, the flavor-averaged condensates are expressed as

$$a^3\langle\bar{q}q\rangle = -\frac{1}{4}\sum_f \mathrm{Tr}\left[\langle q^f(x)\bar{q}^f(x)\rangle\right] = -\frac{1}{2^8}\sum_\rho \mathrm{Tr}\left[\Gamma_\rho\Gamma_\rho^\dagger\ \langle\chi(x+\rho)\bar{\chi}(x+\rho)\rangle\right], \tag{2·2}$$

$$\begin{aligned} a^5 g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle &= -\frac{1}{4}\sum_f\sum_{\mu,\nu} \mathrm{Tr}\left[\langle q^f(x)\bar{q}^f(x)\rangle\sigma_{\mu\nu}G_{\mu\nu}\right] \\ &= -\frac{1}{2^8}\sum_{\mu,\nu}\sum_\rho \mathrm{Tr}\Big[\ \mathcal{U}_{\pm\mu,\pm\nu}(x+\rho)\ \Gamma_{\rho'}\Gamma_\rho^\dagger\ \langle\chi(x+\rho')\bar{\chi}(x+\rho)\rangle\ \sigma_{\mu\nu}\ G_{\mu\nu}^{\mathrm{lat}}(x+\rho)\ \Big], \\ & \rho' \equiv \rho \pm \mu \pm \nu, \end{aligned} \tag{2·3}$$

where the sign $\pm$ is taken such that the sink point $(x+\rho') = (x+\rho\pm\mu\pm\nu)$ belongs to the same hypercube of the source point $(x+\rho)$. Here, in order to respect the gauge covariance, we have used, in Eq. (2·3),

$$\mathcal{U}_{\pm\mu,\pm\nu}(x) \equiv \frac{1}{2}[\ U_{\pm\mu}(x)U_{\pm\nu}(x\pm\mu) + U_{\pm\nu}(x)U_{\pm\mu}(x\pm\nu)\ ], \tag{2·4}$$

where the definition of $U_{-\mu}(x) \equiv U_\mu^\dagger(x-\mu)$ is used.

On the gluon field strength $G_{\mu\nu}$, we adopt the clover-type definition on the lattice,

$$G_{\mu\nu}^{\mathrm{lat}}(s) = \frac{i}{16}\sum_A \lambda^A\ \mathrm{Tr}\left[\lambda^A\{U_{\mu\nu}(s) + U_{\nu\,-\mu}(s) + U_{-\mu\,-\nu}(s) + U_{-\nu\,\mu}(s)\} - \lambda^A\{\mu\leftrightarrow\nu\}\right], \tag{2·5}$$

which contains no $\mathcal{O}(a)$ discretization error. This is also an advanced point which is absent in Ref. 9).

## §3. The lattice QCD results

We calculate the condensates $\langle\bar{q}q\rangle$ and $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ using the SU(3)$_c$ lattice QCD at the quenched level. We perform the Monte Carlo simulation with the standard Wilson action at $\beta = 6.0$ on the $16^4$ lattice. The lattice unit $a \simeq 0.10$ fm is obtained so as to reproduce the string tension $\sigma = 0.89$ GeV/fm.[11] We use the quark mass $m = 21, 36, 52$ MeV, i.e., $ma = 0.0105,\ 0.0184,\ 0.0263$. For the fields $\chi$, $\bar{\chi}$, the anti-periodic condition is imposed. We measure the condensates on 16 different physical space-time points $x$ in each configuration as $x = (x_1, x_2, x_3, x_4)$ with $x_\mu \in \{0, 8\}$ in the lattice unit. For each $m$, we calculate the flavor-averaged condensates, and average them over the 16 space-time points and 100 gauge configurations.

Figure 1 shows the values of the bare condensates $a^3\langle\bar{q}q\rangle$ and $a^5 g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ against the quark mass $ma$. We emphasize that the jackknife errors are almost negligible, due to the high statistics of 1600 data for each quark mass. From Fig. 1, both $\langle\bar{q}q\rangle$ and $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ show a clear linear behavior against the quark mass $m$. Therefore, we fit the data with a linear function and determine the condensates in the chiral limit. The results are summarized in Table I.

We check the reliability of our lattice QCD results by considering the finite volume artifact. In order to estimate this artifact, we carry out the same calculation

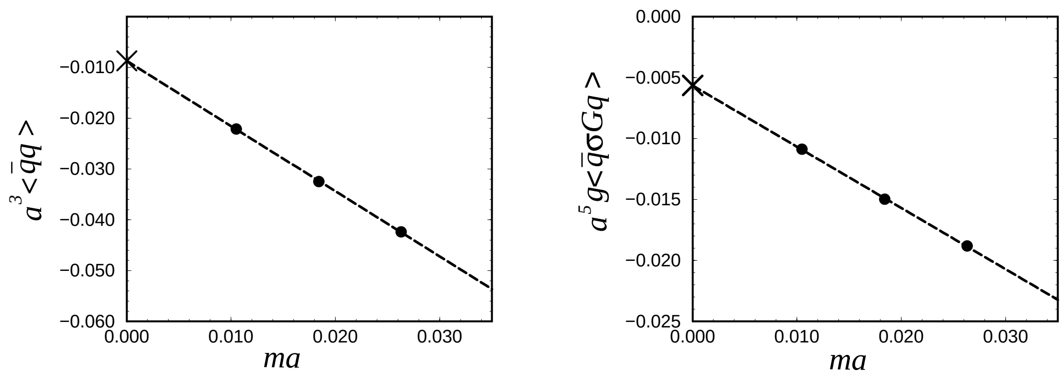


Fig. 1. The bare condensates $a^3\langle\bar{q}q\rangle$ and $a^5 g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ plotted against the quark mass $ma$. The dashed lines denote the best linear extrapolations, and the cross symbols correspond to the values in the chiral limit. The jackknife errors are hidden in the dots.

Table I. The numerical results of $a^3\langle\bar{q}q\rangle$ and $a^5 g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ for various $ma$. The last column denotes their values in the chiral limit obtained by the linear chiral extrapolation.

| | $ma = 0.0263$ | $ma = 0.0184$ | $ma = 0.0105$ | chiral limit |
|---|---|---|---|---|
| $a^3\langle\bar{q}q\rangle$ | −0.04240(16) | −0.03247(15) | −0.02212(16) | −0.00872(17) |
| $a^5 g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ | −0.01882(15) | −0.01498(14) | −0.01088(14) | −0.00565(14) |

imposing the periodic boundary condition on the Grassmann fields $\chi$ and $\bar{\chi}$, instead of the anti-periodic boundary condition, keeping the other parameters same. We obtain that the results with different boundary conditions almost coincide within about 1% difference, and thus we conclude that the physical volume $V \sim (1.6 \text{ fm})^4$ in this simulation is large enough to avoid the finite volume artifact.[10)]

The values of the condensates in the continuum limit are to be obtained after the renormalization, which, however, suffers from the uncertainty of the non-perturbative effect. As a more reliable quantity, we provide the ratio $m_0^2 \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle$, which is free from the uncertainty from the wave function renormalization of the quark.

Now, we present the estimate of $m_0^2$ using the bare results in SU(3)$_c$ lattice QCD as

$$m_0^2 \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle \simeq 2.5 \text{ GeV}^2. \qquad (\beta = 6.0 \text{ or } a^{-1} \simeq 2 \text{ GeV}) \qquad (3\cdot1)$$

The large value of $m_0^2$ suggests the importance of the mixed condensate in OPE. Although we do not include renormalization effect, this result itself is determined very precisely.[10)]

## §4. Discussion and outlook

For comparison with the standard value in the QCD sum rule, we change the renormalization point from $\mu \simeq \pi/a$ to $\mu \simeq 1$ GeV corresponding to the QCD sum rule. Following Ref. 9), we first take the lattice results of the condensates as the starting point of the flow, then rescale the condensates perturbatively. We adopt the anomalous dimensions at the one-loop level,[12)] and choose the parameters $\Lambda_{\rm QCD} = 200 - 300$ MeV and $N_f = 0$ corresponding to quenched lattice QCD. We obtain $m_0^2|_{\mu=1\rm GeV} \equiv g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle|_{\mu=1\rm GeV} \sim 3.5 - 3.7 \text{ GeV}^2$. Comparing with the standard value of $m_0^2 = 0.8 \pm 0.2 \text{ GeV}^2$ in the QCD sum rule, our calculation results in a rather large value. (Note that the instanton model has made a slightly larger estimate as $m_0^2 \simeq 1.4 \text{ GeV}^2$ at $\mu \simeq 0.6$ GeV.[13)]) For a more definite determination of $m_0^2$, the non-perturbative renormalization scheme may be desired.

We again emphasize that the mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ plays the important roles in various contexts in quark hadron physics. Hence, it is preferable to perform further studies. In particular, the thermal effects are interesting in relation to chiral restoration, because the mixed condensate is another chiral order parameter. Considering also the on-going experiments for the finite-temperature QCD in the RHIC project, we investigate the thermal effects on the mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$. We perform the calculation using the $16^3 \times N_t$ lattices with $N_t = 16, 12, 10, 8, 6, 4$ at $\beta = 6.0$. Figure 2 shows the lattice QCD results for the mixed condensate at finite temperature. We find a drastic change of the mixed condensate around the critical temperature $T_c$, which reflects chiral-symmetry restoration.[14)]

In summary, we have studied the quark-gluon mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ using SU(3)$_c$ lattice QCD with the KS fermion at the quenched level. For each quark mass of $m_q = 21, 36, 52$ MeV, we have generated 100 gauge configurations on the $16^4$ lattice with $\beta = 6.0$. Using the 1600 data for each $m_q$, we have found $m_0^2 \equiv$

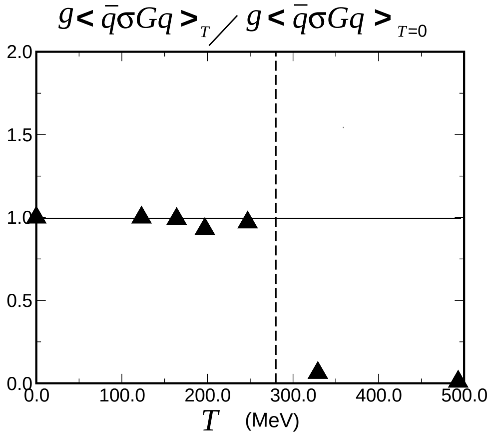


Fig. 2. The quark-gluon mixed condensate $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle_T/g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle_{T=0}$ plotted against the temperature $T$. The jackknife errors are hidden in the triangles. The vertical dashed line denotes the critical temperature $T_c \simeq 280$ MeV at the quenched level.

$g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle \simeq 2.5$ GeV$^2$ in the chiral limit at the lattice scale corresponding to $\beta = 6.0$ or $a^{-1} \simeq 2$ GeV. We have also investigated $g\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ at finite temperature in lattice QCD.

## Acknowledgements

We would like to thank Dr. H. Matsufuru for his useful comments on the programming technique. This work is supported in part by the Grants for Scientific Research (Nos. 11640261, 12640274 and 13011533) from the Ministry of Education, Culture, Sports, Science and Technology, Japan. T. D. acknowledges the support of the JSPS Research Fellowships for Young Scientists. The Monte Carlo simulations have been performed on the NEC SX-5 supercomputer at Osaka University.

### References

1) B. L. Ioffe, Nucl. Phys. B **188** (1981), 317 [Errata; **191** (1981), 591].
2) H. G. Dosch, M. Jamin and S. Narison, Phys. Lett. B **220** (1989), 251.
3) H. G. Dosch and S. Narison, Phys. Lett. B **417** (1998), 173 and references therein.
4) J. I. Latorre, P. Pascual and S. Narison, Z. Phys. C **34** (1987), 347.
5) V. M. Belyaev and B. L. Ioffe, Sov. Phys. -JETP **56** (1982), 493.
6) L. J. Reinders, H. R. Rubinstein and S. Yazaki, Phys. Lett. B **120** (1983), 209.
7) A. A. Ovchinnikov et al., Sov. J. Nucl. Phys. **48** (1988), 721 [Yadern. Fiz. **48** (1988), 1135].
8) S. Narison, Phys. Lett. B **210** (1988), 238.
9) M. Kremer and G. Schierholz, Phys. Lett. B **194** (1987), 283.
10) T. Doi, N. Ishii, M. Oka and H. Suganuma, Phys. Rev. **D** (2003), in press, hep-lat/0211039.
11) T. T. Takahashi, H. Suganuma, Y. Nemoto and H. Matsufuru, Phys. Rev. D **65** (2002), 114509.
12) S. Narison and R. Tarrach, Phys. Lett. B **125** (1983), 217.
13) M. V. Polyakov and C. Weiss, Phys. Lett. B **387** (1996), 841.
14) T. Doi, N. Ishii, M. Oka and H. Suganuma, Nucl. Phys. A **721** (2003), 934c, hep-lat/0212006; *Proc. Int. Conf. on "Quark Confinement and the Hadron Spectrum V"*, Italy, 2002, hep-lat/0212025.