The Development of Modus Ponens in Antiquity:[1]
From Aristotle to the 2nd Century AD

SUSANNE BOBZIEN

ABSTRACT

'Aristotelian logic', as it was taught from late antiquity until the 20th century, commonly included a short presentation of the argument forms modus (ponendo) ponens, modus (tollendo) tollens, modus ponendo tollens, and modus tollendo ponens. In late antiquity, arguments of these forms were generally classified as 'hypothetical syllogisms'. However, Aristotle did not discuss such arguments, nor did he call any arguments 'hypothetical syllogisms'. The Stoic indemonstrables resemble the modus ponens/tollens arguments. But the Stoics never called them 'hypothetical syllogisms'; nor did they describe them as ponendo ponens, etc. The tradition of the four argument forms and the classification of the arguments as hypothetical syllogisms hence need some explaining. In this paper, I offer some explanations by tracing the development of certain elements of Aristotle's logic via the early Peripatetics to the logic of later antiquity. I consider the questions: How did the four argument forms arise? Why were there four of them? Why were arguments of these forms called 'hypothetical syllogisms'? On what grounds were they considered valid? I argue that such arguments were neither part of Aristotle's dialectic, nor simply the result of an adoption of elements of Stoic logic, but the outcome of a long, gradual development that begins with Aristotle's logic as preserved in his Topics and Prior Analytics; and that, as a result, we have a Peripatetic logic of hypothetical inferences which is a far cry both from Stoic logic and from classical propositional logic, but which sports a number of interesting characteristics, some of which bear a cunning resemblance to some 20th century theories.

'Aristotelian logic', as it was taught from late antiquity onwards and as it can be found in logic compendia up until the 20th century,[2] commonly

included a short presentation of the argument forms *modus (ponendo) ponens,
modus (tollendo) tollens, modus ponendo tollens,* and *modus tollendo ponens*:
(For brevity, I shall call arguments of these four types '*modus ponens* type
arguments'.)

Diagram 1

(1)	If A, B But A Therefore B	***modus ponendo ponens***
(2)	If A, B But not B Therefore not A	***modus tollendo tollens***
(3)	Either A or B* But A Therefore not B	***modus ponendo tollens***[3]
(4)	Either A or B* But not A Therefore B	***modus tollendo ponens***
		*exclusive and exhaustive 'or'

In late antiquity, arguments of the *ponens* and *tollens* forms were gener-
ally classified as 'hypothetical syllogisms' (συλλογισμοὶ ὑποθετικοί, *syl-
logismi hypothetici*), and they were regarded as valid by virtue of these
forms. However, as far as we know, Aristotle did not discuss such argu-
ments, nor did he call any arguments 'hypothetical syllogisms'.[4] The
Stoics, on the other hand, distinguished certain kinds of arguments that
closely resemble *modus ponens* type argument. But they, too, did not call
these arguments 'hypothetical syllogisms';[5] nor did they describe them as

---

[3] There are modifications of the two disjunctive forms in later ancient texts, with
which however I am not concerned in this paper. The modifications cover disjunctions
that are not exclusive and exhaustive and disjunctions with more than two disjuncts.
I consider the later development of these types of arguments in my 'Hypothetical syl-
logistic in Galen: propositional logic off the rails?' (forthcoming).

[4] The common translation of συλλογισμὸς ἐξ ὑποθέσεως by 'hypothetical syllo-
gism' (e.g. A.J. Jenkinson's translation of the *Prior Analytics*, P. Slomkowski, *Aris-
totle's Topics*, Leiden 1997) is unfortunate in this respect. This will become clear in
the following.

[5] A fact that is frequently disregarded – although the ancients were aware of it (see
[Amm.] *An.Pr.* 68.4 and 14). For Stoic hypothetical *arguments* see my 'The Stoics on
hypotheses and hypothetical arguments', *Phronesis* 43, 1997, 299-312.

ponendo ponens (i.e. as arguments that posit something by positing some-thing), etc. The tradition of the four argument forms and the classification of the corresponding arguments as hypothetical syllogisms hence need some explaining.

In this paper, I offer some explanations by tracing the development of certain elements of Aristotle's logic from Aristotle, via the early Peripatetics, to the logic of later antiquity. I consider the questions: How exactly did the four argument forms arise? Why were there four of them? Why were arguments of these forms called 'hypothetical syllogisms'? On what grounds were they considered valid? I argue that such arguments were neither already part of Aristotle's dialectic,[6] nor simply the result of an adoption of elements of Stoic logic, but that they are the outcome of a gradual development that begins with Aristotle's logic as preserved in his *Topics* and *Prior Analytics*; and that, as a result, we have a Peripatetic logic of hypothetical inferences which – despite appearances – is a far cry both from Stoic logic and from classical propositional logic, but which sports a number of interesting characteristics, some of which bear a cun-ning resemblance to some 20th century theories, and which may well have something to offer to the historian of logic, ancient and modern.

1. *Late Peripatetic and Platonist sources: hypothetical syllogisms*

Let me start with the end-point of the development I am going to recon-struct. From the second century AD onwards, we encounter a terminologi-cal distinction between categorical syllogisms and hypothetical syllogisms.[7] Categorical syllogisms are those that consist – entirely – of categorical propositions whereas hypothetical syllogisms are those that contain at least one hypothetical proposition.[8] Hypothetical syllogisms that contain both categorical and hypothetical propositions are in one tradition called 'mixed hypothetical', or simply 'mixed', syllogisms;[9] They are contrasted with those hypothetical syllogisms which consist solely of hypothetical propositions.[10]

---

[6] As is suggested by Slomkowski 1997.

[7] E.g. Galen *Inst.log.* 8.1-4 (Kalbfleisch), Alc. *Didasc.* 158-9 (Whittaker), Philop. *An.Pr.* 243.11-246.14, [Amm.] *An.Pr.* 68.23-41, Scholium in Amm. *An.Pr.* IX 22–XI 36, Boethius *Hyp.syll.* 1.2.3 (Obertello).

[8] E.g. Boeth. *Hyp.syll.* 1.2.3.

[9] E.g. Alc. *Didasc.* 158.23-7, [Amm.] *An.Pr.* 67.24-30. (Keynes 1924 still calls them 'mixed hypothetical syllogisms'.)

[10] These are called 'wholly hypothetical syllogisms' (οἱ δι' ὅλου (or δι' ὅλων) ὑποθετικοὶ συλλογισμοί, e.g. Alex. *An.Pr.* 326.20; 330.28-30; Philop. *An.Pr.* 243.11-

Importantly, in our sources the distinction between categorical and hypothetical syllogisms is generally presented as derivative of a distinction between categorical and hypothetical *propositions*. Categorical propositions state that something is the case (or holds or belongs) or that something is not the case (or does not hold or not belong); examples are 'some stones are not white', 'Socrates is walking'. Their name is presumably based on the fact that in them something is predicated (κατηγορεῖσθαι) of something.[11] Hypothetical propositions encompass at least conditional propositions (such as 'if it is day, it is light') and various types of disjunctive propositions (e.g. 'either it is day or it is night'). Hypothetical propositions were thought to have two or more categorical propositions as their components (cf. Boeth. *Hyp.syll.* 1.3.5 206.44-5; Amm. *Int.* 3.31-4.2). They are used to make assertions (Galen *Inst.log.* 3.1) and have truth-values (Alex. *An.Pr.* 11.19-20). But one should not jump to the conclusion that they correspond to the complex propositions of Stoic logic or of modern propositional logic, and *a fortiori* one should not assume that their truth-value is a function of the truth-values of their component propositions.[12]

Paradigm cases of categorical syllogisms (συλλογισμοὶ κατηγορικοί) are those in the forms of *Barbara, Celarent*, etc. For instance, an argument in *modus Barbara* is:

> Every α is β.
> Every β is γ.
> Therefore every α is γ.

α, β, and γ are here terms, and such syllogisms are term-logical arguments. The paradigm cases of mixed hypothetical syllogisms are arguments of the form *ponendo ponens*, for instance

> If *p*, then *q*.
> But *p*.
> Therefore *q*.

---

36). In late antiquity their paradigm case is of the kind 'If *p, q*; if *q, r*; therefore, if *p, r*'. For a discussion of these syllogisms see J. Barnes, 'Terms and sentences: Theophrastus on hypothetical syllogisms', *Proceedings of the British Academy* 69, 1983, 279-326 and my 'Wholly hypothetical syllogisms', *Phronesis* 45, 2000, 87-137. Texts which do not mention wholly hypothetical syllogisms usually refer to the mixed ones simply as hypothetical syllogisms.

[11] See Galen *Inst.log.* 2.1-2, Amm. *Int.* 3.9-11, Boeth. *Int.* 186 (Meiser). Cf. Aristotle *An.Pr.* e.g. 25b38-9 '*A* is predicated of all *C*' (τὸ Α κατὰ παντὸς τοῦ Γ κατηγορεῖσθαι). Boethius thus rightly translates κατηγορικός by 'predicative', *Hyp.syll.* 1.1.4.

[12] Thus affirmation and negation are treated as on a par, and the negative particles

p and *q* are here propositions, and such arguments belong to propositional logic.[13] Stoic syllogistic is based on arguments that look just like these.[14] But at least from the second century AD onwards such arguments are also generally accepted as part of Peripatetic and Platonist logic.[15]

For the following, two features of this later ancient syllogistic in particular should be kept in mind: First, that in these sources of later antiquity the term 'hypothetical' in the expression 'hypothetical proposition' denotes a property of the proposition which it has in virtue of its form and independently of its use or function in the argumentational context. (The same holds of the term 'categorical' in the expression 'categorical proposition'.) Second, that the naming of certain syllogisms as 'categorical' or 'hypothetical syllogisms' was thought to be derivative of the fact that they contain categorical or hypothetical propositions.

This later ancient classification of categorical and hypothetical syllogisms may appear straightforward and well thought out. However there are a couple of things that are odd with it.

- Just as to us today the word 'hypothetical' suggests an element of conditionality or of supposition, so did the word ὑποθετικός to the speakers of Greek in antiquity. So why were propositions of the kind 'either *p* or *q*' called 'hypothetical', and – by derivation – why were syllogisms e.g. of the form *tollendo ponens* called 'hypothetical syllogisms'?[16]
- If one thinks that the characteristic of 'mixed' arguments is given by the premiss that is a complex proposition, e.g. 'if *p*, *q*', or 'either *p* or

---

are not connectives; the use of specific connectives for specific types of hypothetical propositions is not necessary; term logic creeps in at unexpected places; and simple conjunctions are not classified as hypothetical propositions.

[13] I use the expression 'propositional logic' in the very loose sense of a logic that is concerned with the relations between whole propositions. I also do not consider here the question whether the name 'sentence logic' would be more apposite, but use the term 'proposition' simply to refer to the primary truth-bearers of the theories.

[14] The forms *ponendo ponens, tollendo tollens, tollendo ponens,* and *ponendo tollens* resemble the forms of the first, second, fourth and fifth of the Stoic indemonstrables, for which see Sext.Emp. *PH* II.157-9, *M* VIII.224-6, Diog.Laert. VII.80-1. For the function of the indemonstrables in Stoic syllogistic see my, 'Stoic Syllogistic' *OSAP* 14, 1996, 133-92, in particular section 1.

[15] There were other types of syllogisms, such as Theophrastus' prosleptic syllogisms (Galen *Inst.log.* 19.1-5, cf. Alex. *An.Pr.* 378.12-379.11) and Galen's relational syllogisms (Galen *Inst.log.* ch. 16), which I here disregard.

[16] Modern propositional logic tends to reserve the name 'hypothetical syllogism' for certain arguments with an 'if-then' functor (or with '→') as main connector in their complex premiss(es).

q', (as the Stoics did), why should they be called *ponendo ponens* (argu-
ments that posit by positing), *ponendo tollens* (arguments that reject by
positing), etc., rather than conditional argument, disjunctive argument,
etc.? Moreover, do not for instance the arguments 'if not *p*, *q*; not *p*;
therefore *q*', 'not both *p* and *q*; but *p*; therefore not *q*'[17] and '*p* and not
q; therefore not *q*' also reject by positing?

These peculiarities, and some related ones, can be sorted out, once one
realizes that the hypothetical syllogisms are in fact not of Stoic prove-
nance but have their origins in Aristotle's logic; and to Aristotle I turn
next.

2. *Aristotle: syllogisms from a hypothesis*

Aristotle classifies and systematizes valid term-logical arguments in Book
I of his *Prior Analytics*. But he does not call them 'categorical syllo-
gisms'.[18] He calls them 'syllogisms'; or 'probative syllogisms' (δεικτικοὶ
συλλογισμοί).[19] We find nothing about 'mixed' arguments in Aristotle's
extant writings. Aristotle has no term for hypothetical propositions. He
does not use the expression 'hypothetical' (ὑποθετικός) at all. He neither
contrasts hypothetical propositions with categorical propositions, nor
hypothetical syllogisms with categorical syllogisms. He does not reflect
on the formal validity of arguments of type *modus ponens*.

However, both in the *Topics* and in the *Prior Analytics* Aristotle men-
tions and talks briefly about arguments which he calls 'syllogisms from a
hypothesis' (συλλογισμοὶ ἐξ ὑποθέσεως).[20] The repeated reference to such

---

[17] The first two cases are in fact sometimes identified with *modus ponendo tollens*,
see my 'Hypothetical syllogistic in Galen: propositional logic off the rails?' (forth-
coming).

[18] Aristotle uses κατηγορικός as synonymous with καταφατικός, in order to
denote – presumably simple – *affirmative* propositions. (So also noted by Alexander,
An.Pr. 256.12-14.) Similarly early Stoic logic (Diog.Laert. VII.70). The change of
meaning of κατηγορικὴ πρότασις to 'simple proposition' generally (as opposed to
affirmative propositions only) is later. Our earliest evidence is Galen *Inst.log* 2.1-2.
The terminological distinction between categorical and hypothetical propositions can
only have been introduced after this change of meaning.

[19] Arist. *An.Pr.* 40b27, cf. 40b24-5, 29a31-2. Alexander identifies them with the *cat-
egorical* syllogisms (*An.Pr.* 262.32, 263.15) and states that Aristotle calls them syllo-
gisms in the unqualified and proper sense (*An.Pr.* 386.13-14 συλλογισμοὺς δὲ ἁπλῶς
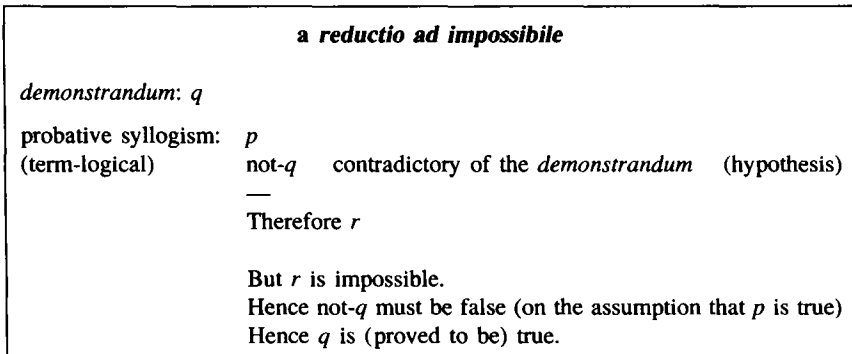καὶ κυρίως λέγει τοὺς κατηγορικούς).

[20] *An.Pr.* 40b23-6, 41a38, 45b15-20, 50a16-b4; *Top.* 108b7-19, 119b35-120a5.

arguments without further explanation allows us to assume that at Aristotle's time, members of the Peripatos were generally familiar with such arguments. In the *Prior Analytics* Aristotle twice contrasts his probative syllogisms with the syllogisms from a hypothesis (*An.Pr.* 40b23-5, 45b13-16).

Among the syllogisms from a hypothesis, Aristotle marks out 'the syllogisms that lead to the impossible' (οἱ εἰς τὸ ἀδύνατον ἄγοντες συλλογισμοί),[21] or *reductiones ad impossibile*, as a special case. These are distinguished from 'the *other* syllogisms from a hypothesis' (οἱ ἄλλοι συλλογισμοί οἱ ἐξ ὑποθέσεως, *An.Pr.* 45b16).[22] Of these latter arguments there were again several types, which Aristotle thought of as in need of classification,[23] although he seems never to have got round to classifying them.

Here is a brief description of the reductions to the impossible, and the hypotheses that occur in them.[24] A reduction to the impossible proves a proposition, say *q*, by showing that the assumption of its contradictory leads to an impossibility. An example is

Diagram 2

a *reductio ad impossibile*		
demonstrandum: *q*		
probative syllogism:	*p*	
(term-logical)	not-*q* contradictory of the *demonstrandum* (hypothesis)	
	⎯⎯	
	Therefore *r*	
	But *r* is impossible.	
	Hence not-*q* must be false (on the assumption that *p* is true)	
	Hence *q* is (proved to be) true.	

---

[21] *An.Pr.* 45a23-4 cf. *An.Pr.* 61a18ff; 62b29ff.

[22] 'Necessarily, every proof and every syllogism proves either that something holds or that it does not hold, and this either universally or particularly, and furthermore either probatively or from a hypothesis. The reduction to the impossible is one kind of <syllogism> from a hypothesis.'.

[23] *An.Pr.* 50a39-b2. Cf. *An.Pr.* 45b15-20: 'In the other syllogisms from a hypothesis (such as those in accordance with a substitution or a quality) the inquiry will be concerned not with the terms of the original *demonstrandum*, but with those which have been taken instead; still, the method of the investigation will be the same. We must examine the syllogisms from a hypothesis and distinguish in how many ways <they are possible>.'.

[24] For more thorough discussions of Aristotle's reductions to the impossible see

It is a moot point what the hypothesis is because of which these arguments count as syllogisms from a hypothesis. However, there can be no doubt that Aristotle called the contradictory of the *demonstrandum* a hypothesis (so e.g. eleven times in *An.Pr.* 63a9-63b11). Plainly, this type of hypothesis in the reductions is a proposition. It has a truth-value, namely falsehood. Usually, it would be a simple or 'categorical' proposition. For instance 'animal holds of every stone' could be such a hypothesis. It is called a 'hypothesis' because of its *function as a supposition in a particular piece of discourse*. It is not asserted, i.e. posited as an assertion, but it is *supposed*, i.e. posited and agreed upon for the purpose of argument only. Thus the hypothesis is not called a 'hypothesis' in virtue of its linguistic or its logical form – as were the hypothetical propositions in later antiquity (above, Section 1). This use of 'hypothesis' for suppositions is not uncommon in antiquity.[25]

In the reductions to the impossible, the conclusion is not proved through a syllogism, or syllogized, but is concluded from a hypothesis (or concluded or proved through the impossible, *An.Pr.* 50a29-32, 61a34-5, 62b38-40). It is concluded from a hypothesis because without hypothesizing the contradictory of the *demonstrandum*, one would never get to the (impossible) conclusion, hence never to the falsehood of the contradictory of the *demonstrandum*, hence never to the *demonstrandum*. Thus the *demonstrandum* is concluded from a hypothesis, viz. by means of an act of supposing.

The arguments that are of special interest for my purpose are Aristotle's '*other* syllogisms from a hypothesis'. The main passages in which Aristotle talks about these come from the *Prior Analytics* and the *Topics*. In the context of the possibility of reducing arguments to the three figures, Aristotle writes:

> Furthermore, we must not attempt to reduce the syllogisms from a hypothesis; for it is not possible to reduce them from the assumptions made. For they[26] have not been proved by a syllogism, but have all been agreed upon by compact: for

---

G. Striker, 'Aristoteles über Syllogismen "aufgrund einer Hypothese"', *Hermes* 107, 1979, 33-50; J. Lear, *Aristotle and Logical Theory*, Cambridge 1980, ch. 3.

[25] Thus, it seems to be this idea of hypothesis qua supposition (of simple or non-simple propositions) which featured in Chrysippus' hypothetical arguments (ὑποθετικοὶ λόγοι) – with the difference that Chrysippus made the context-dependent (suppositional) function of the proposition into a non-relational property, thereby creating an additional kind of complete sayables (λεκτὰ αὐτοτελῆ). Cf. my 'The Stoics on hypotheses and hypothetical arguments' (n. 5 above).

[26] 'They' refers to the syllogisms from an hypothesis. However, here, as elsewhere,

example, if someone should hypothesize that, if there is not one single faculty of contraries, then there is not one single science <of those contraries> either, and should then argue that not every faculty is a faculty of contraries (e.g. <the faculty> of what is healthy and of what is ill <is not>; for then the same thing will at the same time be healthy and ill). He has then shown by proof that not in all cases of contraries is there a single faculty, but he has not proved that there is not a single science of contraries <in those cases>, although it is necessary to accept <this>; however not from a syllogism, but from a hypothesis. This latter <statement>[27] cannot be reduced, but that there is not in all cases of contraries a single faculty can. For this one was surely/perhaps a syllogism,[28] but the other one was a hypothesis. (*An.Pr.* 50a16-28)

And after dealing with reductions to the impossible in a similar way, Aristotle adds:

. . . in the former <type of argument> an advance agreement must have been made, if one is to accept that <the conclusion>, e.g. an agreement that if it has been proved that there is a single faculty of opposites, then one will accept that there is a single science of opposites. (*An.Pr.* 50a33-5)

And in book 1 of the *Topics*:

<The examination of likeness is useful> for syllogisms from a hypothesis, since it is an accepted view that among similars, however things are with one, so they are with the rest. Hence if regarding any of them we are well-equipped to argue about them, we will make an advance agreement that however things are in these cases, so they are with the point at issue; and when we have proved the former, we will also have proved from a hypothesis the point at issue. For we have hypothesized that however things are in these cases, so they are with the point at issue, and have then provided the proof. (*Top.* 108b12-19)

And in book 3:

Further from a hypothesis: you should claim that if it holds of one or does not hold of one, then it does so in the same way to all; for instance, if the soul of human beings is immortal, so are the other souls, and if it is not, nor are the others. If then it is posited that it holds of something, it should be proved that it does not hold of something; for it will follow because of the hypothesis that it holds of nothing. And if it is posited that it does not hold of something, then it should be proved that it does hold of something; for in this way it will follow that it holds of all. (*Top.* 119b35-120a2)

Aristotle uses 'syllogism' in order to denote the conclusion of a syllogistic argument, i.e. the conclusion *qua* being the result of a deduction or valid inference.

[27] Reading τοῦτο rather than τοῦτον. If one reads τοῦτον, this would have to refer to a syllogism from an hypothesis, again, intended as the conclusion of such an argument (see previous note).

[28] It is unclear exactly what probative syllogism Aristotle has in mind.

Following these passages, syllogisms from a hypothesis can be roughly described as follows (cf. Diagrams 3 and 4):

Diagram 3

---

one of the 'other syllogisms from a hypothesis'

q:	the original *demonstrandum*	(τὸ ἐξ ἀρχῆς)
p:	that which is assumed instead	(τὸ μεταλαμβανόμενον)
if p, q:	the hypothesis	(ὑπόθεσις)

the probative syllogism
(part of the syllogism
  from a hypothesis.)

  r

  s

  —   When p is proved, q must be accepted   ⎧ preliminary
               (since if p, q)   ⎨ agreement,

  p ⟶ Now p is proved.   ⎪ based on the
                                     ⎩ hypothesis

               Hence q must be accepted

Diagram 4

---

another of the 'other syllogisms from a hypothesis'

γ holds of β:	the original *demonstrandum*	(τὸ ἐξ ἀρχῆς)
γ holds of α:	that which is assumed instead	(τὸ μεταλαμβανόμενον)
Whatever holds of α, holds of β:	the hypothesis	(ὑπόθεσις)

the probative syllogism
(part of the syllogism
from a hypothesis)

  r

  s When 'γ holds of α' is proved, 'γ holds   ⎧ preliminary
  —————   of β' must be accepted (since whatever   ⎨ agreement,
               holds of α, holds of β)   ⎪ based on the
  γ holds of α ⟶ Now 'γ holds of α' is proved.   ⎩ hypothesis
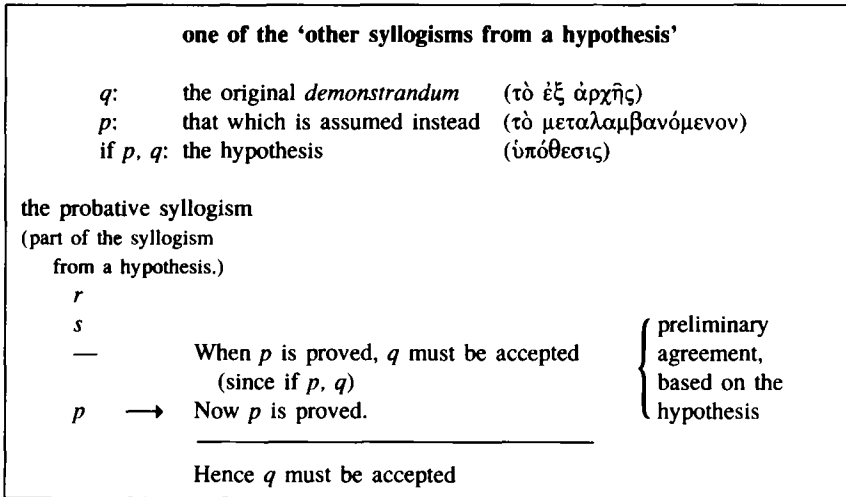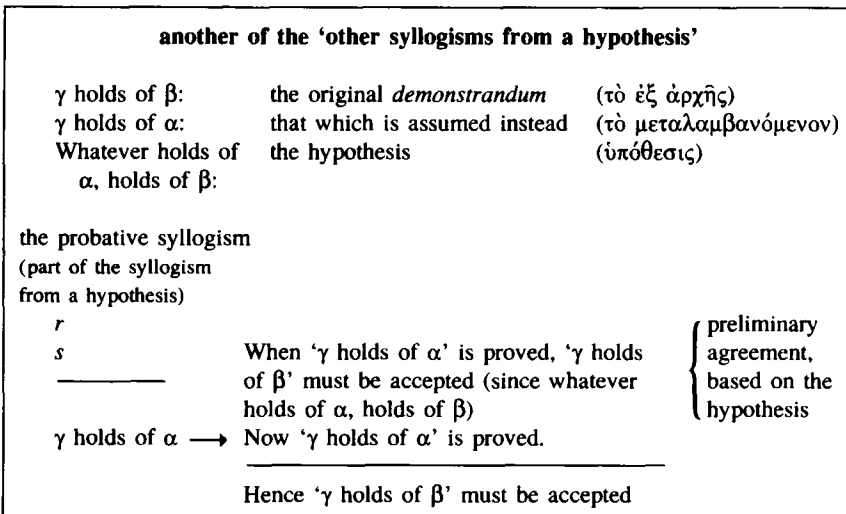
               Hence 'γ holds of β' must be accepted

- One starts with a proposition that is to be proved, say q. q is such that it cannot be proved directly by a term-logical syllogism.

• One then makes an advance agreement (προδιομολογεῖσθαι, *An.Pr.* 50a36; *Top.* 108b15, cf. 110a.37-110b4, διὰ συνθήκης ὡμολογημένοι, *An.Pr.* 50a18-19) that if something else, say *p*, has been proved (or is accepted), *q* needs to be accepted too.[29] I am unsure about the precise ontological status of this agreement. It could be something along the lines of a *contract* that is 'cashed in', as it were, once *p* is proved. It would then have a pragmatic dimension to it. In any case, the agreement seems to differ from a conditional statement or sentence in that it has no truth-value.[30]

• This agreement contains, consists of, or is based on, the hypothesis,[31] which in Aristotle's examples seems to have no uniform logical structure. Thus we find:

> If *a* does not hold of all *b*, then *a* does not hold of all *c* (*An.Pr.* 50a19-20).
> If *a* holds of some *b*, then *a* holds of all *b* (*Top.* 120a4-5)
> However things are in the case of *a*, so they are in the case of *b* (*Top.* 108b13-14; 18-19).

The first two examples are of the conditional form 'if *p*, *q*', the last is not. All three can be understood as saying something about the relation between terms, things, or properties. (In fact, all Aristotle's syllogisms from a hypothesis prove-from-a-hypothesis that something holds or does not hold of something (*An.Pr.* 40b23-5); that is, they are conceived of in terms of term-logical propositions.) The only general requirements seem to be (i) that the hypotheses must be such that they can be

---

[29] *An.Pr.* 50a33-35, *Top.* 120a4-5; cf. the parallel formulations in *Top.* 111b17-23 and 112a24-31, both quoted below. (We find strikingly similar procedures and formulations in texts of modern logic; cf. e.g. Tarski 'On the concept of logical consequence' in his *Logic, Semantics, Metamathematics*, Oxford 1969, at p. 411.)

[30] Or, if it were conceived of as having a truth-value, it would be irrelevant to the question of whether *q* has to be accepted-by-the-partners-of-the-agreement. For when *p* has been proved, *q* has to be accepted-by-the-partners-of-the-agreement (i.e. the agreement that if p is proved, q must be accepted) independently of the truth-value of the conditional 'if *p* is proved, *q* (must be accepted)'. (Cf. F.P. Ramsey's 'habit' in *The Foundations of Mathematics*, 1965, 237-55 and G. Ryle's 'inference-ticket' in *The Concept of Mind*, 1950, p. 121, for similar sounding ideas.)

[31] At *An.Pr.* 41a37-41, Aristotle takes the agreement (ὁμολογία) as a *kind* of hypothesis, whereas elsewhere, the hypothesis is distinguished from the agreement or contract (συνθήκη). This difficulty could be dissolved by assuming that Aristotle distinguished the act of making the agreement or of hypothesizing from the agreement or hypothesis as that which has agreed upon or hypothesized, i.e. as the product of the act, but did not use a consistent terminology.

understood to back up a statement of the kind 'if p is proved, q must be accepted', and (ii) that it should be plausible (πιθανός, *Top.* 110a37). The latter point is self-evident, since otherwise one will hardly get one's interlocutor to agree to it. The reason why these hypotheses are called hypotheses is, again, their *function in the logical discourse*: they are hypothesized or supposed, i.e. assumed and agreed upon by the interlocutors (without further justification or proof), for the purpose of establishing (or refuting) something. Unlike the hypothetical propositions of later antiquity, they are not called hypotheses because they are a type of proposition of a particular form, say of conditional form.

- The newly introduced proposition (p) is called 'that which is taken instead' (τὸ μεταλαμβανόμενον, *An.Pr.* 41a39, 45b18). For it is a substitute that is 'taken instead of' the original *demonstrandum* (q). Instead of the latter, it is *it* which is going to be proved syllogistically (*An.Pr.* 45b15-19; 41a38-40).
- The substituted proposition (p) is then proved by a term-logical syllogism, say 'r; s; therefore p'.
- But then, according to the advance agreement, and based on the hypothesis, q, too, has been proved – although only from a hypothesis.

Thus an Aristotelian syllogism from a hypothesis is a *combination* of (i) a term-logical syllogism and (ii) the agreement which – somehow – is based on a hypothesis, relying on which the partners of the agreement get from the conclusion of the term-logical syllogism to the original *demonstrandum*. Note that the 'form' of the *meta-linguistic* agreement and its 'cashing in' *taken together* display a certain similarity to *modus ponendo ponens*. This is important for what follows.

It is instructive to ask why these Aristotelian arguments were called 'syllogisms from a hypothesis'. Aristotle does not tell us, so we have to infer the answer from the passages in which he mentions or discusses them. These passages suggest that syllogisms from a hypothesis were called 'syllogisms' because they include a term-logical, or 'probative', syllogism, as Aristotle calls them in this context:

- In *An.Pr.* (41a37-41b1) we read '. . . For in each <syllogism from a hypothesis> *the* syllogism is directed towards that which is taken instead, but the original *demonstrandum* is inferred by means of an agreement or some other hypothesis.' The probative syllogism contained in the syllogism from a hypothesis is here referred to as '*the* syllogism': the definite article can only mean that there is exactly one syllogism (proper) involved in a syllogism from a hypothesis, and that this (the probative syllogism) is it.

• In *An.Pr.* 50a17-19 (quoted above) we read that the conclusions of the syllogisms from a hypothesis 'have *not* been proved by a syllogism, *but* have all been agreed upon by compact.' Thus neither the meta-linguistic agreement nor the hypothesis count as a premiss of a syllogism.

Syllogisms from a hypothesis are, hence, not syllogisms proper, i.e. unqualified and in the strict sense (ἁπλῶς, κυρίως);[32] they are syllogisms only in a restrictive, qualified sense.[33] In a proper syllogism, the *demonstrandum* is established – directly – *from* a syllogism (ἐκ συλλογισμοῦ).[34] A syllogism from a hypothesis is called '*from* a hypothesis', since in it the original *demonstrandum* is not established – directly – from a syllogism, but *from a hypothesis* (ἐξ ὑποθέσεως, *An.Pr.* 50a25-6).[35] (In Diagram 3 above, *p* has been inferred from a syllogism, and *q* can then be inferred-from-a-hypothesis.) This hypothesis is not part of any syllogism proper. It is part of an agreement, and only the partners of the agreement are bound to accept the *demonstrandum*. We cannot say that the *demonstrandum*, say *q*, has been syllogized (or deduced, συλλογίζεσθαι) full-stop. Even when *p* (that which is taken instead) has been proved, we can at most say that *q* has been proved *on the agreement* that if *p* is proved, *q* is proved. The partners of the agreement are bound to accept *q* not because of the truth of the hypothesis, but because they entered into the agreement.[36]

---

[32] A syllogism from a hypothesis thus understood would presumably not qualify as a syllogism according to Aristotle's definition of the syllogism (*An.Pr.* 24b18-20, *Top.* 100a25-7), because its conclusion does not follow from necessity. The clause 'it is necessary to accept <the *demonstrandum*>' in *An.Pr.* 50a24-5 means 'necessary relative to having agreed to the hypothesis'.

[33] So also Alexander at *An.Pr.* 42.27-31: '<Aristotle> discusses only categorical syllogisms, because he thinks that only these are syllogisms in the strict sense . . . For he thinks that no syllogism from a hypothesis syllogizes the point at issue.'

[34] Since Aristotle uses συλλογισμός not only to refer to the entire compound of premisses and conclusion, but also to the *result* of the syllogizing, i.e. to the conclusion only (see also above n. 26), we may assume that the conclusion of a syllogizing-from-a-hypothesis (a syllogism from a hypothesis) is syllogized-from-a-hypothesis (a syllogism from a hypothesis).

[35] The name 'syllogism' is borrowed from the probative syllogism it includes, and can be borne only with the qualifying phrase 'from a hypothesis'; in other words, the phrase 'from a hypothesis' functions as a restrictive qualifier.

[36] Can *q* be 'detached' from the hypothesis? The dialectical game can only be won if *q* is accepted 'on its own', as it were. And the agreement is, I take it, an agreement to that effect. Once *p* is proved, *q* must (and hence can) be accepted on its own. Within the game of dialectic, at its end, *q* is free-standing. This does not, however, change the fact that only those who entered in the agreement are bound to accept *q*, and in that sense, *q* is and remains dependent on the agreement.

A syllogism from a hypothesis is thus an argument in which the *demonstrandum* is not directly the conclusion of a probative syllogism, and is hence not (properly) deduced (συλλογίζεσθαι), but is inferred indirectly via an agreed hypothesis, which connects the *demonstrandum* with another proposition which in turn is (properly) deduced. Thus Aristotle's other syllogisms from a hypothesis – whatever else they may be – are certainly not arguments that are valid because of their logical form; nor were they regarded as such.

3. *Aristotle: two passages from the* Topics

Of equal importance for the development of *modus ponens* type arguments are a couple of passages from Aristotle's *Topics* in which he presents some *specific* topoi.[37] Here is the first passage:

> One must examine, regarding the point at issue, [i] what is such that if it is, the point at issue is, or [ii] what is by necessity, if the point at issue is: [i] if one wants to establish something, one must examine what there is such that if *it* is, the point at issue will be (for when the former has been proved to hold, the point at issue will also have been proved to hold); [ii] if, on the other hand, one wants to refute something, one must examine what it is that is if the point at issue is (for when we prove that what follows from the point at issue is not, we will have destroyed the point at issue.) (*Top.* II.4 111b17-23)

In this passage, a relation of consequence is used in two ways: first, a relation of consequence between something *a* and the point at issue *b* is used to establish *b* via *a*; second, a relation of consequence between the point at issue *b* and some other thing *a* is used to refute *b* (or to establish not-*b*) via not-*a*. It is unclear here, whether the point at issue (*a*) and the other thing (*b*) are two terms/predicates, or whether they are whole sentences/states of affairs.[38] I believe it more likely that Aristotle had terms in mind. In that case, 'if *a* is, *b* is' is short for something like 'if *a* holds

---

[37] The importance of these passages has been recently correctly emphasized by Slomkowski 1997. I disagree, though, with his main thesis that Aristotle himself discussed hypothetical syllogisms of type *modus ponens*.

[38] Aristotle uses the relation of consequence (ἀκολούθησις, ἀκολουθεῖν, τὸ ἀκόλουθον) both between terms and between whole sentences; the term 'thing at issue' (προκείμενον) is mostly used for terms, but possibly sometimes (e.g. at *Top.* 120a10) for whole sentences or states of affairs.

(i.e. of a certain thing), *b* holds (i.e. of that thing)'. The second passage follows shortly after the first:

> In the case of things of which hold one and only one of two <predicates>, as for instance of a human being holds either illness or health, if we are well-equipped to argue about the one, that it holds or does not hold, we will also be well-equipped with regard to the remaining one; this converts with regard to both; for [iii] when we have proved that the one <predicate> holds, we will also have proved that the other does not hold; and [iv] when we prove that the one does not hold, we will have proved that the remaining one holds. (*Top*. II.6 112a24-31)

In this passage, a relation of intermediate-less contrariety between two predicate terms, *b*, *c*, with respect to some subject term, *a*, is used in two ways: first, to establish that *b* holds (of *a*) via the fact that *c* does not; second, to establish that b does not hold (of *a*) via the fact that c does. (It is then obvious that the first passage can be understood as presenting some kind of forerunners of the *modi ponendo ponens* and *tollendo tollens*, the second as presenting some kind of forerunners of the *modi tollendo ponens* and *ponendo tollens*.[39])

In Aristotle's extant writings the *topoi* of these two passages are not given a special status, but are just some among many. Moreover, Aristotle nowhere directly draws the connection between these *topoi* and his arguments from a hypothesis. It is however possible to present arguments that exemplify these *topoi* in the form of syllogisms from a hypothesis, and not unlikely that this had been done in the Peripatos at Aristotle's time. In any case, I will do it now.

First, in place of the meta-logical hypothetical agreement of the type 'If that which is taken instead is proved, the *demonstrandum* will be accepted' the two *Topics* passages provide the four variant types of such agreements which one obtains when (in addition to the establishings) one also considers the refutations (or the negations)[40] of the *demonstrandum* and of

---

[39] Alexander's commentary on these passages from Aristotle's *Topics* testifies that these Aristotelian *topoi* were later interpreted as being the ancestor arguments of our quartet of *modus ponens* type arguments. Thus, Alexander explicitly draws the connection to the first two Stoic indemonstrables with *topoi* of the first passage, and the last two Stoic indemonstrables with the *topos* of the second passage (Alex. *Top*. 165.5-166.13, 174.5-175.26).

[40] We can assume that Aristotle considered a refutation of *p* as (materially) equivalent to a proof of not-*p*. This however does not imply that he thought they were the same thing.

that which is taken instead as candidates for such agreements. Here are the four possibilities:[41]

(1')	If it is proved that *a* holds (of something), then it is proved that *b* holds (of that thing).
(2')	If is proved that *a* does not hold (of something), then it is proved that *b* does not hold (of it).
(3')	If it is proved that *a* holds (of something), then it is proved that *b* does not hold (of it).
(4')	If it is proved that *a* does not hold (of something), then it is proved that *b* holds (of it).

Note that in all four cases Aristotle has no qualms about saying that at the end the demonstrandum has been *proved*. (He does not say 'proved' in the case of his surviving examples for 'other' syllogisms from an hypothesis. This may have given the present arguments a superior status to those 'from an agreement' quoted above, or in any event may have contributed to their metamorphosing into hypothetical syllogisms at a later time.)

By analogy with the syllogisms from a hypothesis as described in Aristotle's *Prior Analytics* and *Topics*, the cases (1') to (4') can then be presented as four kinds of arguments in the following way:[42]

Diagram 5

(1''')		
A	It is to be proved that *b* holds (of *c*).	(*demonstrandum*)
B	If *a* holds (of something), *b* holds (of it).	(hypothesis)
C	If it is proved that *a* holds (of *c*), it is proved that *b* holds (of *c*).	(agreement)
D	It is proved that *a* holds (of *c*). (by a probative syllogism that is *part* of the syllogism from hypothesis)	
E	It is proved that *b* holds (of *c*).	(by hypothesis)

---

[41] Or alternatively as:
   (1") If it is established that *a* holds, then it is established that *b* holds.
   (2") If is refuted that *a* holds, then it is refuted that *b* holds.
   (3") If it is established that *a* holds, then it is refuted that *b* holds.
   (4") If it is refuted that *a* holds, then it is established that *b* holds.

[42] The formulations I suggest are meant to give no more than the general idea how the above listed Aristotelian *topoi* could have provided four kinds of syllogism from a hypothesis.

(2''')
   A   It is to be proved that b does not hold (of c).   (*demonstrandum*)
   B   If b holds (of something), a holds (of it).   (hypothesis)
   C   If it is proved that a doesn't hold (of c), it is
       proved that b doesn't hold (of c).   (agreement)
   D   It is proved that a does not hold (of c).   (by a probative syllogism)
   E   It is proved that b does not hold (of c).   (by hypothesis)
(3''')
   A   It is to be proved that b holds (of c).   (*demonstrandum*)
   B   Either a or b holds (of something).   (hypothesis)
   C   If it is proved that a does not hold (of c), it is
       proved that b holds (of c).   (agreement)
   D   It is proved that a does not hold (of c).   (by a probative syllogism)
   E   It is proved that b holds (of c).   (by hypothesis)
(4''')
   A   It is to be proved that b holds (of c).   (*demonstrandum*)
   B   Either a or b holds (of something).   (hypothesis)
   C   If it is proved that a holds (of c), it is proved
       that b does not hold (of c).   (agreement)
   D   It is proved that a holds (of c).   (by a probative syllogism)
   E   It is proved that b does not hold (of c).   (by hypothesis)

Note that in all four cases, *modus ponendo ponens* is used as a kind of
meta-level[43] inference rule to get from the agreement to its 'cashing in'
(lines C, D, E).[44] At the same time, the four arguments show some remote
resemblance to the syllogisms of the four types of *modus ponens* argu-
ments (lines B, D, E). In particular, in case (1''') *modus ponendo ponens*
comes in twice, as it were. However, from a logical perspective there are
a number of important differences between the *modus ponens* type argu-
ments and these four kinds of syllogisms from a hypothesis. If one pos-
tulates a development from (1''') – (4''') to the proper *modus ponens* type
arguments (1) – (4) (Diagram 1 above), one must assume that the fol-
lowing changes happened in the course of this development:

- The elements that are logically relevant for the inference have to
  become whole propositions rather than terms: in particular, the B-lines

---

   [43] I say 'meta-level', since the inference rule warrants the transition from sentences
about a's holding (or not holding) and *about b*'s holding (or not holding) to a sen-
tence *about a*'s holding (or not holding).
   [44] In fact, it seems as if this was the basic inference rule of all Aristotelian syllo-
gisms from a hypothesis. Cf. also Diagram 4 above.

would have to change from 'if something is *a*, it is *b*' ('being *a*, something is *b*') and 'either *a* or *b* holds of something' ('one and only one of *a*, *b*, holds of something') to 'if *p*, then *q*' and 'either *p* or *q*'.

- The *modus ponens* type arguments contain only (modified versions of) the components B, D, E of the syllogisms from a hypothesis; i.e. the stating of the *demonstrandum* (A) is absent, and so is the agreement C.
- D and E are no longer formulated in meta-language: e.g. we have simply '*p*', instead of '*p* is proved'.
- B and D become *premisses* of the argument, E becomes a *conclusion* (and it is no longer presented as the *demonstrandum* with which the argument begins).
- D is no longer understood as being dependent on a probative syllogism '*r, s*, therefore *p*/not-*p*', and *a fortiori* no probative syllogism is *part* of *modus ponens* type arguments.
- The *modus ponens* type arguments are recognized as *valid because of their (modus ponens type) form*.

I now show by means of various passages from Peripatetic and Platonist sources that there is evidence that some such development took in fact place.

As to its temporal location, we can safely assume that, in the first century BC, at the time of the Peripatetics Boethus of Sidon and Aristo of Alexandria this development was by and large completed. For both philosophers wrote on syllogistic;[45] and both must have been familiar with Stoic propositional logic, since it was the prevalent system of logic of the time; moreover, we know from Galen (*Inst.log.* 7.2) that Boethus considered the Stoic indemonstrables both as indemonstrable and as 'first' syllogisms. Thus such arguments must have been integrated in the logic of at least some Peripatetics in the first century BC.

The only Peripatetic philosophers before these two from whom we have some fragments of logic are Theophrastus and Eudemus, both pupils of Aristotle and both earlier than the chief Stoic logician Chrysippus.[46] It

---

[45] Cf. Apul. *Int.* 213.5-10 (Moreschini) for Aristo, Galen *Inst.log.* 7.2 for Boethus.

[46] The early Peripatetic Phaenias of Eresus wrote a *De interpretatione* and *Analytics*, and Strato of Lampsacus composed books on *topoi* (Philop. *Cat.* 7.20, Diog.Laert. V.59-60). No texts are extant, but the titles support the assumption that Aristotle's followers discussed those of his works in which he dealt with syllogisms from a hypothesis.

appears that both of them maintained a position on the kinds of arguments we are interested in that is intermediate between Aristotle and later Peripatetics such as Boethus. Whether these intermediate positions were in fact developed by Theophrastus and Eudemus, or rather by some other early Peripatetic, or perhaps even by Aristotle himself, late in life, is in the end not very important.[47] What matters is that through them we can be witness to the Peripatetic development from the Aristotelian 'syllogisms from a hypothesis' to fully fledged *modus ponens* type arguments.

4. *Eudemus: connecting the passages from* Prior Analytics *and* Topics

There is a passage that suggests that Eudemus considered the syllogisms from a hypothesis in connection with Aristotle's *Topics*, and moreover, that in that context, the above-discussed Aristotelian *topoi* gained special importance. Thus, according to Boethius, Eudemus made the following distinctions of hypotheses:

> Eudemus holds that the hypothesis from which the hypothetical syllogisms obtain their name is said in two ways: for either (a) through a hypothesis of things consistent in themselves something which can in no way happen is accepted in such a way that the argument leads toward the end (destruction?) of the thing, or (b) the consequence which is posited in the hypothesis is revealed (established?, indicated?) by virtue of a connection or by virtue of a division.[48] (Boeth. *Hyp.syll.* 1.2.5 Obertello)

This sentence, although far from clear, is informative in several respects. It suggests that Eudemus distinguished two *meanings* of the word 'hypothesis': one (a) that is related to the hypotheses used in Aristotelian syllogisms to the impossible, and another (b) that seems to be related to the hypotheses used in – all or some of – Aristotle's 'other' syllogisms from a hypothesis. There is no evidence that Aristotle made such a distinction in meaning of the word 'hypothesis'. On the contrary, his use of the phrase

---

[47] Cf. J. Barnes, 'Theophrastus and Hypothetical Syllogistic', in J. Wiesner (ed.), *Aristoteles: Werk und Wirkung* I (Berlin 1985), 559-62 for the – partly contradictory – sources that name Theophrastus, Eudemus, and the Peripatetics in general in the context of syllogisms from a hypothesis.

[48] Hypothesis namque unde hypothetici syllogismi accepere vocabulum duobus (ut Eudemo placet) dicitur modis. aut enim (a) tale adquiescitur aliquid per quamdam inter se consentientium conditionem, quod fieri nullo modo possit, ut ad suum terminum ratio perducatur; . . . aut (b) *in conditione posita consequentia vi coniunctionis vel disiunctionis ostenditur.*

'other syllogisms from a hypothesis' for those syllogisms from a hypothesis which are not syllogisms to the impossible suggests that he regarded both types of arguments as being syllogisms from a hypothesis in the same sense. Eudemus, by contrast, appears to have assumed that the hypotheses in these two types of arguments were hypotheses of different types. The difference seems to consist in what is posited in (or with) each of them: in (a) what is posited is in itself consistent but cannot happen; in (b) what is posited is some sort of consequence.[49] This could simply be the result of Eudemus describing from a different angle, as it were, what Aristotle says in the *Prior Analytics* about syllogisms from a hypothesis.

However, Aristotle's *Prior Analytics* do not provide a model for Eudemus' binary account of the second kind of hypothesis in terms of connection and division. Nothing in the *Prior Analytics* suggests that Aristotle had exactly two types of 'other syllogisms from a hypothesis' in mind. Eudemus' account of the second meaning of hypothesis

> the consequence which is posited in the hypothesis is revealed (established, indicated) (i) by virtue of a connection or (ii) by virtue of a division[50]

can, however, be made sense of in terms of the above Diagram 5, in which I combined the two passages from Aristotle's *Topics* with his account of the 'other syllogisms from a hypothesis'. Thus the consequence (*consequentia*)[51] in the hypothesis could correspond to the element of the *modus ponens* meta-level inference rule (i.e., to the C-lines in the diagram), which allows one to get from 'that which is proved instead' (the D-lines) to the original demonstrandum (the E-lines); and the connection and division could be those expressed in the hypotheses (in the B-lines): connection in (1''') and (2'''), division in (3''') and (4'''). Thus with Eudemus, it seems, two types of hypothesis became predominant in the 'other syllogisms from a hypothesis', and thus two types of 'other syllogisms from a hypothesis':

---

[49] Boethius elaborates on this difference by stating that the first kind of hypotheses, i.e. (a), are simple propositions, whereas the second kind of hypothesis, i.e. (b), are not simple propositions (*Hyp.syll.* 1.2.7).

[50] *coniunctio* and *disiunctio* could be translations of συνέχεια and διαίρεσις. Cf. Philoponus (*An.Pr.* 245.6, cf. 10), who calls the disjunctive premiss of a hypothetical syllogism a ὑπόθεσις κατὰ διαίρεσιν.

[51] Perhaps translating ἀκολούθησις or ἀκολουθία. In the *Hyp.syll.*, Boethius generally does not use the term *consequentia* for conclusions of arguments or for consequents in conditionals, so I assume he does not do so here either.

- those in which the consequence posited in the hypothesis is revealed (established, indicated) by virtue (by the force) of a connection; and
- those in which the consequence posited in the hypothesis is revealed (established, indicated) by virtue (by the force) of a division.

I suggest that the first are those anticipated in Aristotle's *Topics* in the first passage quoted above (*Top*. II.4 111b17-23), and that the second are those anticipated in Aristotle's *Topics* in the second passage quoted above (*Top*. II.6 112a24-31).

In terms of Diagram 5 above, we can express this by saying that the relations between *a* and *b* that underlie the four types of agreements (and make it possible that when the statement in the antecedent is proved, so is the statement in the consequent), were considered as a relation of 'connection' between things in cases (1''') and (2'''), and as a relation of separation or division of things in cases (3''') and (4'''); and that the relation of connection was eventually (perhaps with Eudemus, perhaps only later) expressed in conditional statements, that of separation in some kind of disjunctive statements, in the following way:

Diagram 6

(1'''')	If *a* holds (of some thing), *b* holds (of it).	connection between *a*, *b*
(2'''')	If *b* holds (of some thing), *a* holds (of it).	connection between *a*, *b*
(3'''')	Either *a* holds (of some thing) or *b* holds (of it).	division between *a*, *b*
(4'''')	Either *a* holds (of some thing) or *b* holds (of it).	division between *a*, *b*

What brought about this focus on just those two kinds of hypothesis, connecting and dividing, and led to the neglect of the others Aristotle mentions? All I can offer is conjecture. The examples for 'other syllogisms from a hypothesis' which Aristotle gives in *An.Pr*. 50a16-28 and *Top*. 108b12-19, as well as some other examples in the *Topics*, require a preliminary agreement of the interlocutors. As a result, the original demonstrandum cannot be 'detached' from that agreement,[52] and, although this may be no problem within a game of dialectic, as soon as logic is used to establish or to prove something, this relativization to an agreement may be regarded as insufficient. At some point it may have become clear to the early Peripatetics that in the case of the above kinds of hypothesis (as in the case of the syllogisms to the impossible) no prior agreement is

---

[52] See above n. 36.

required, because nobody could possibly object to the mode of argumentation employed. And they might have found confirmation of this thought in Aristotle's use of the phrases 'we have proved', 'it has been proved' in the relevant passages of the *Topics*.

From Eudemus' distinction between two types of hypothesis that posit a consequence to *modus ponens* type arguments proper, there is a further development in two steps:

- First, and still with the early Peripatetics, the two types of hypothesis in syllogisms from a hypothesis are taken to be two types of *hypothetical premisses* (ὑποθετικὴ πρότασις κατὰ συνέχειαν/κατὰ διαίρεσιν) of such arguments.
- In a second step, with later Peripatetics and with some Platonists, and perhaps under the influence of Stoic logic, from these two types of hypothetical premisses were derived two types of *hypothetical propositions* (ὑποθετικὴ πρότασις κατὰ συνέχειαν/κατὰ διαίρεσιν).

This further development was facilitated, and perhaps blurred, by the fact that in Peripatetic logic the word πρότασις was used both for 'premiss' and for 'proposition'. I now take the two steps of the development in order.

4. *From hypothesis to hypothetical premiss; detachment of the categorical syllogism*

A passage in Alexander's commentary on the *Prior Analytics* (Alex. *An.Pr.* 262-5) allows us a glimpse of the early Peripatetic understanding of the syllogisms from a hypothesis which Theophrastus and Eudemus and their contemporaries discussed. In this passage Alexander comments on Aristotle's mentioning of arguments from a hypothesis in *An.Pr.* I 23 (41a40f).[53] The crucial sentence is said to present the view of the ancients

---

[53] This Alexander passage (as others, e.g. Alex. *An.Pr.* 389-90) includes elements of Aristotle's *Prior Analytics*, of early Peripatetic theory (Theophrastus, Eudemus, etc.), of Stoic propositional logic, and of later Peripatetic logic, and it is quite hard to separate out what comes whence. Alexander explains the early Peripatetic terms by contemporary Stoic and Peripatetic terminology, and also uses this later terminology to paraphrase and explain Aristotle's text. I suggest that – roughly – those terms that are not known from Aristotle and which are glossed by later terms are early Peripatetic. (Alexander writes always already from the perspective of someone with a concept of *modus ponens* type arguments, and his interpretation of Aristotle is thus tainted.)

(ἀρχαῖοι), which is Alexander's usual way of referring to the earlier Peripatetics. The only philosopher mentioned in the context of the passage is Theophrastus, and he is therefore a likely source of the early Peripatetic elements in it. After being told – anachronistically – that the arguments which the Stoics regard as syllogisms (i.e. the 'mixed' arguments) are a subclass of Aristotle's 'other syllogisms from a hypothesis' (*An.Pr.* 262.28-9),[54] we learn that the ancients called these same arguments 'mixed from a hypothetical πρότασις and a probative πρότασις' (μικτοὺς ἐξ ὑποθ-ετικῆς προτάσεως καὶ δεικτικῆς,[55] *An.Pr.* 262.31-2).

If one picks out the non-Stoic bits from the Alexander passage (*An.Pr.* 262-5), including the conditional and disjunctive examples provided, the following picture emerges of the syllogisms from a hypothesis of these Peripatetics. If one takes Alexander's examples as early Peripatetic, the syllogisms appear to have had the following form or structure:[56]

If *Fa, Ga*	hypothetical *protasis* (ὑποθετικὴ πρότασις)	Either *Fa* or *Ga*
But *Fa*	probative *protasis* (δεικτικὴ πρότασις)	But not *Fa*
Therefore *Ga*	the original *demonstrandum* (τὸ ἐξ ἀρχῆς)	Therefore *Ga*

But since Alexander's examples may originate from a later time, alternatively the following could have been their assumed logical structure:[57]

For all *x*: if *Gx, Fx*	hypothetical *protasis*	For all *x*: either *Gx* or *Fx*
But *Ga*	probative *protasis*	But not *Ga*
Therefore *Fa*	the original *demonstrandum*	Therefore *Fa*

---

[54] Οἱ νεώτεροι here must stand for the Stoics or more generally for those who adopted Stoic logic.

[55] The expression δεικτικῆς is glossed, by Alexander or his immediate source, by τοῦτ' ἔστι κατηγορικῆς. This indicates that 'categorical' is the common term at the time of Alexander for what he thinks is expressed (by Theophrastus or some other earlier Peripatetic) by probative (δεικτικῆς). (Cf. n. 19 above.)

[56] As long as there is still a requirement that the second premiss is proved by a cat-egorical syllogism, at least it, and possibly also one of the components of the hypoth-esis, will be of term-logical form.

[57] Cf. my 'Wholly Hypothetical Syllogisms', *Phronesis* 45, 87-137, sections 2 and 4, for some evidence that Theophrastus considered hypothetical propositions of the form 'If anything is *G*, it is *F*'.

The first premiss is called hypothetical (*An.Pr.* 262.21-2); i.e. here we find the expression 'hypothetical' in an early Peripatetic context. It is also still referred to as 'hypothesis' (*An.Pr.* 263.6); the hypothesis is here definitely not formulated in meta-language. The second premiss is called 'probative' and is occasionally still referred to as τὸ μεταλαμβανόμενον (*An.Pr.* 264.18, cf. 21).[58]

The overall linguistic form of Alexander's first illustrative argument resembles *modus ponendo ponens*;[59] and that of the second resembles *tollendo ponens*.[60] But this does not entail that the early Peripatetics understood these arguments in this way; i.e. that they understood them as *formally* valid because of *those* forms. Rather, when one looks at the nomenclature used and examines why it was chosen, it becomes clear that there is still a close affinity to Aristotle's 'arguments from a hypothesis', and that the agreement with the *modus ponens* type arguments is only superficial:

- The *second premiss* is called a 'probative' πρότασις, because it is the conclusion of a 'probative' (δεικτικός) syllogism, i.e. of a categorical syllogism, as is clear from *An.Pr.* 263.4-6; 7-11; 14-17.[61] That is, the name of the probative premiss is not grounded in the fact that its *form* is of a certain kind, e.g. that it is 'simple', or that it has any other non-relational property.[62] It is based on its *function* in the argumentational context, namely its being the conclusion of a probative syllogism. Hence πρότασις in the expression 'probative πρότασις' should be translated by 'premiss' rather than by 'proposition' here: it is *that premiss* of the

---

[58] Alexander's explanation at *An.Pr.* 263.26-36 why the early Peripatetics called this premiss τὸ μεταλαμβανόμενον is likely to be a later attempt of making sense of that use of the term – a phenomenon not uncommon in the Aristotle commentators.

[59] Alex. *An.Pr.* 262.33-5 ... εἰ ἐπιστήμη ἐστιν ἡ ἀρετή, διδακτόν ἐστιν ἀρετή ... ἐπιστήμη ἡ ἀρετή ἐστιν ... ὅτι καὶ διδακτή.

[60] Alex. *An.Pr.* 264.10-12: ἤτοι σῶμά ἐστιν ἡ ψυχὴ ἢ ἀσώματος· ἀλλὰ μὴν οὐ σῶμα· ἀσώματος ἄρα.

[61] In this context, Theophrastus is mentioned as saying that the second premiss is 'that which is in doubt' (ἀμφιδοξούμενον, *An.Pr.* 263.13-14). Hence we can infer that the requirement of a proof of the second premiss is not just Aristotle's view, but also that of some early Peripatetics.

[62] Alexander here glosses 'probative' by 'categorical', and later (e.g. in Galen and Alcinous, see below) 'categorical' in the expression 'categorical πρότασις' denotes the fact that the proposition indicates a simple state of affairs; see below.

syllogism from a hypothesis which has been proved (or is to be proved, or is provable) by a probative syllogism. (Outside the context of the syllogism from a hypothesis, or some other argument, the expression 'probative πρότασις' has no application.)

- The *first premiss* is called a 'hypothetical' πρότασις, because it is or contains the hypothesis.[63] Either way, the hypothetical πρότασις is not called 'hypothetical' because it is a certain kind of complex proposition as in later Peripatetic logic, i.e. not because of its logical *form* (or any other non-relational property). Rather its name is based on its *function* (as a supposition) in the argumentational context – even if it can fulfil this function in this context only because it has a particular form. Hence, again, the expression πρότασις in 'hypothetical πρότασις' should be rendered as 'premiss' rather than as 'proposition'. For outside the syllogism from a hypothesis, the expression 'hypothetical πρότασις' would seem to have no application.

- The conclusion is never called a 'conclusion' (συμπέρασμα, ἐπιφορά), but is throughout referred to as τὸ ἐξ ἀρχῆς. This, too, suggests that the syllogism from a hypothesis is not seen as an independent argument of *modus ponens* type – although Alexander's use of τὸ ἐξ ἀρχῆς may be motivated by Aristotle's text, rather than based on Peripatetic sources.

- The whole argument is called 'hypothetical' or 'from a hypothesis'[64] because it has a hypothetical *premiss*, i.e. because one of its premisses is (or expresses) a certain kind of hypothesis; it is not called 'hypothetical' because one of its premisses is a hypothetical *proposition*, i.e. a proposition of a certain form.

- Finally, why did the early Peripatetics call these arguments 'syllogisms'? They called them 'syllogisms' because they prove something that is not known without a deduction. And the reason why they *prove*

---

[63] If the hypothetical πρότασις is not identical with the hypothesis, it is the premiss in which the hypothesis is expressed. Alex. *An.Pr.* 263.5-6, 32-33, 35-6, 264.26 and 386.27-9 are the relevant passages.

[64] They are called 'hypothetical syllogisms' e.g. in Alex. *An.Pr.* 263.12; 15-16; but since much of the passage is later (see n. 53 above) we cannot infer that the early Peripatetics called these syllogisms 'hypothetical'. (Alex. *An.Pr.* 264.32-265.5 has ἐξ ὑποθέσεως συλλογισμός) If they did call them thus, certainly not in the sense in which they are later defined in Alcinous, Boethius, etc. (see Section 1), since that presupposes the terminological distinction between categorical and hypothetical *propositions*, for which we have no evidence before the second century AD.

something is that their conclusion is – indirectly – established by a pro-
bative syllogism, namely via the second premiss.[65] Hence the name 'syl-
logism' is based not on the *form* of the argument (e.g. *ponendo ponens*),
as one might have expected, but *on its relation to a 'proper', probative
syllogism*. As Alexander reports, if the second premiss were not proved
in that way, the argument would no longer be a syllogism (*An.Pr.* 263.7-
14) – although no doubt it would still have the same form.[66]

Taken together, these points show that these early Peripatetic syllogisms
are very close to Aristotle's 'other syllogisms from a hypothesis', both in
terminology and in the way they are conceived as arguments. Still, there
are also elements that we do not find in Aristotle, and Theophrastus and
his fellow Peripatetics seem to have made a couple of steps forward in
the development of *modus ponens* type arguments.

• Thus, importantly, the syllogism from a hypothesis is no longer viewed
  as a complex of two partial arguments, one formally valid and probative,
  one non-formal. Rather, now it is only one argument, which, however,
  must stand in a certain relation to a probative syllogism.[67] Diagram 6
  illustrates this:

Diagram 6

an early Peripatetic syllogism from a hypothesis			
(probative) syllogism (συλλογισμὸς (δεικτικός))		syllogism from a hypothesis (συλλογισμὸς ἐξ ὑποθέσεως)	
r			
s	If *Ga, Fa*	hypothetical premiss	(ὑποθετικὴ πρότασις)
Therefore *Ga* ⟶	But *Ga*	probative premiss	(δεικτικὴ πρότασις)
	Therefore *Fa*	the original demonstrandum	(τὸ ἐξ ἀρχῆς)

This separation is documented for Theophrastus. For example, we are
told that he considered alternative ways of backing up the second
premisses in such arguments.[68] Thus with the early Peripatetics, the

---

[65] δειχθέντος δὲ τούτου διὰ συλλογισμοῦ, Alex. *An.Pr.* 263.4-5; cf. 7-11; 14-17.
[66] This point is likely to be early Peripatetic, since Theophrastus is mentioned in
An.Pr. 263.13-14.
[67] Alex. *An.Pr.* 262.36-263.6; 264.12-14.
[68] Alex. *AnPr.* 388.17-20; Simpl. *Cael.* 552.31-553.4.

perspective on syllogisms from a hypothesis has shifted, in that one question they ask now is: 'here is a certain kind of argument – in which ways could its premises be justified?'[69]

- The structure of these arguments has also become clearer. For certain of their parts are expressly identified as premises; as premises of different types, viz. hypothetical and probative ones. The difference denoted by their names is that of their different *function* in the inference. There is no comparable difference in function in the premises of term-logical syllogisms.
- The preliminary 'agreement' (and thus the pragmatic element) of Aristotle's syllogisms from a hypothesis has disappeared completely. We do still find the corresponding meta-linguistic deliberation (C in Diagram 5) in the passage, but it is not considered as part of the argument (Alex. *AnPr.* 262.34-5).
- Finally, since Eudemus distinguished two types of *hypothesis*, those by virtue of a connection and those by virtue of a division (Section 4), we would expect Alexander's early Peripatetics to have distinguished two corresponding types of *hypothetical premisses*. And indeed, Galen reports that the "old philosophers" (his way of referring to the early Peripatetics) distinguished between hypothetical premisses in accordance with a connection (ὑποθετικαὶ προτάσεις κατὰ συνέχειαν) and hypothetical premisses in accordance with a division (ὑποθετικαὶ προτάσεις κατὰ διαίρεσιν).[70]

Still, all these changes do not alter the fact that – as far as our sources go – these early Peripatetics did not yet regard their syllogisms from a hypothesis as valid because they displayed forms similar to those of *modus ponens* type arguments; their name 'syllogism' is still based on their

---

[69] The connection with a probative syllogism is recorded in the Latin rhetorical tradition (where we have evidence that the question of whether the probative syllogism is part of the hypothetical syllogism was discussed among some Peripatetics): cf. Boethius *Hyp.syll.* 2.1-6, Cicero *Inv.* 1.57-77, Martianus Capella *De Nuptiis*, 132.29-133.9 (Eyssenhardt); also Quintilian 5.14.5.

[70] Gal. *Inst.log.* 3.4 (cf. 3.1 and 3.3) and 14.2. In the passage quoted, *he* takes πρότασις to mean 'proposition', but this does not entail that the early Peripatetics he quotes did so too. In effect, in the *Institutio Logica*, Galen himself seems to oscillate between taking πρότασις as 'premiss' and as 'proposition'. Cf. on these points my 'Pre-Stoic hypothetical syllogistic in Galen's Institutio Logica', V. Nutton (ed.), Proceedings of the symposium 'Galen beyond Kühn', *BICS* supplement, forthcoming (2002).

dependency on a probative syllogism. Only when the syllogisms from a hypothesis are severed completely from these other arguments, and recognized as valid because of their own form, can they be raised into the status of true syllogismhood. *When* exactly this final separation took place, whether before or after the Stoic logician Chrysippus systematized syllogisms of the 'mixed' kind, I do not know. But, as I said above, presumably by the end of the first century BC it has taken place.[71] In any event, we find traces of the further development in some later sources, even if we cannot date it precisely or attribute it to particular Peripatetic philosophers.

5. *From hypothetical premiss to hypothetical proposition*: modus ponens *appears*

Many post-Stoic Peripatetic and Platonist sources for hypothetical syllogistic are clearly influenced by the Stoic theory of indemonstrable syllogisms. The main indications of this are (i) that they present five (or seven) types of hypothetical syllogisms, including as number three one with the linguistic form of a negated conjuction as first premiss; (ii) that the descriptions of the types closely resemble the Stoic descriptions of their indemonstrables, and sometimes (iii) that multi-disjunct disjunctions are considered. However, there are some later Peripatetic and Platonist texts and passages which seem curiously untouched by Stoic syllogistic, and which can be best made sense of if understood as further developments of the Aristotelian and early Peripatetic theories of the 'other' syllogisms from a hypothesis. These texts characteristically distinguish four hypothetical syllogisms only, have no negated conjunction, describe the syllogisms not in the Stoic way, provide for only two disjuncts, and sometimes contrast this theory with the Stoic one. In this section I concentrate on these texts. They include passages from Alexander's *Topics* commentary, Galen's *Introduction to Logic*, Alcinous' *Didaskalikos*, Ammonius' *De Interpretatione* commentary, Simplicius' *Encheiridion* commentary, Boethius' *On Hypothetical Syllogisms*, and from an anonymous scholium to Aristotle's *Organon* (ed. & transl. Bobzien, *Mnemosyne* 2002).

About the hypothetical προτάσεις (or *propositiones*) we find the following bits of information in these passages:

---

[71] This is also suggested by the fact that Cicero in *Inv.* 1.57-77 presents 'mixed' syllogisms in Peripatetic guise.

Another kind of proposition is that in which we make the assertion not about the being the case (or holding) of things,[72] but about what is when something <else> is, or what is when something <else> is not. Such propositions shall be called 'hypothetical'. Of these those which state that when something else[73] is, necessarily such and such a thing is, shall be called 'by virtue of a connection', and those which state either that when something is not, something is, or when something is, something is not, shall be called 'dividing'.[74] (Gal. *Inst.log.* 3.1)

Hypothetical <is a proposition> which, making some hypothesis, states that something is when something else is, as when we say 'if it is day, it is light'.[75] (Boeth. *Hyp.syll.* 1.1.5 p. 206.42-44)

Every hypothetical proposition is either in accordance with a connection . . . or in accordance with a division: both types are obtained from simple propositions.[76] (Boeth. *Hyp.syll.* 1.3.4)

Hypothetical propositions are those which indicate either a consequence or a conflict.[77] (Alc. *Didasc.* 158.16-17)

In hypothetical propositions truth and falsity depend not on something's being said of something but rather on consequence or conflict.[78] (Alex. *An.Pr.* 11.19-20)

<A hypothetical assertoric sentence> states either that when something is the case, something is the case or that when something is the case, something is not the case, and for this reason it is called 'hypotethical'.[79] (Amm. *Int.* 74.4-5, cf. 3.11-12)

---

[72] Categorical propositions state or indicate that something does or does not hold, cf. Galen *Inst.log.* 3.1 and 2.1-2; Boeth. *Hyp.syll.* 1.1.5.

[73] Thus expressions like 'if *p*, *p*' and 'either *p* or *p*' may be not well-formed according to this account.

[74] Γένος ἄλλο προτάσεώς ἐστιν ἐν αἷς τὴν ἀπόφα[v]σιν οὐ περὶ τῆς ὑπάρξεως ποιούμεθα τῶν πραγμάτων, ἀλλὰ περὶ τοῦ τίνος ὄντος τί ἐστι καὶ τίνος οὐκ ὄντος τί ἐστιν· ὑποθετικαὶ δὲ ὀνομαζέσθωσαν αἱ τοιαῦται προτάσεις, αἱ μὲν, ὅταν τινὸς ἑτέρου ὄντος ἐξ ἀνάγκης εἶναι λέγωσι τόδε τι, κατὰ συνέχειαν, αἱ δὲ, ὅταν ἤτοι μὴ ὄντος εἶναι ἤ [μὴ] ὄντος μὴ εἶναι, διαιρετικαί (*Inst.log.* 3.1). προτάσεως ms, Barnes et al., προτάσεων Mynas, Kalbfleisch; del. μὴ Prantl.

[75] hypothetica est quae cum quadam conditione denuntiat esse aliquid si fuerit aliud, veluti cum ita dicimus: 'si dies est, lux est'.

[76] Omnis igitur hypothetica propositio vel per conexionem fit . . . vel per disiunctionem: uterque enim modus ex simplicibus propositionibus comparatur.

I take Boethius' *propositio hypothetica per conexionem* and *hypothetica propositio per disiunctionem* to be translations of the Greek κατὰ συνέχειαν ὑποθετικὴ πρότασις and κατὰ διαίρεσιν ὑποθετικὴ πρότασις.

[77] Τῶν δὲ προτάσεων αἱ μὲν κατηγορικαί εἰσιν, αἱ δὲ ὑποθετικαί . . . ὑποθετικαὶ δέ εἰσιν αἱ ἀκολουθίαν δηλοῦσαι ἤ μάχην. (Alc. *Didasc.* 158.14-17; cf. Alex. *Top.* 174.5-6, *An.Pr.* 11.19-21, 264.33, Galen *Inst.log.* 4.4, 14.7-11).

[78] ἡ γὰρ ὑποθετικὴ <πρότασις> οὐκ ἐν τῷ τι κατά τινος λέγεσθαι ἀλλ' ἐν ἀκολουθίᾳ ἤ μάχῃ τὸ ἀληθὲς ἤ τὸ ψεῦδος ἔχει.

[79] λέγων ἤ τίνος ὄντος τί ἐστιν ἤ τίνος ὄντος τί οὐκ ἔστι, καὶ διὰ τοῦτο λεγόμενος ὑποθετικός.

Hypothetical assertions originate from predicatives: they indicate either the con-
sequence or the disagreement (διάστασιν) of one predicative proposition with
another.[80] (Amm. *Int.* 3.31-4.1, cf. 73.30-33)

There can be no doubt that in these passages the hypothetical προτάσεις
are understood as propositions in their own right, and not as premisses,
and certainly not as something like a contract. They are presented outside
the context of syllogisms and typically contrasted with categorical προτά-
σεις. They are said to consist of, contain, or come to be from categorical
προτάσεις. We make assertions with them (Galen *Inst.log.* 3.1). They are
taken to have truth-values.[81] They are taken to be used in both 'mixed'
and 'wholly' hypothetical syllogisms, including in the conclusions of the
latter (e.g. Boeth. *Hyp.Syll.* 2.9.1-3.6.4 and Alc. *Didasc.* 159.7-24). Thus
the step (anticipated in Section 4) from a hypothetical πρότασις as a *pre-
miss* that has a particular function in the argument, to the hypothetical
πρότασις as a *proposition* that is of a particular kind, quite independently
of whether or where it occurs in an argument, has taken place – with no
noticeable change in terminology.

Moreover, all these texts distinguish exactly two types of hypothetical
propositions. The first type, whose standard linguistic form is the condi-
tional, is called 'by virtue of a connection' (κατὰ συνέχειαν) and said to
indicate a consequence (ἀκολουθία). The second, whose standard linguis-
tic form is the disjunction, is called 'hypothetical proposition in accor-
dance with a division' (διαιρετικαί; *Inst.log.* 3.4, 14.2: κατὰ διαίρεσιν),
and said to indicate a conflict (μάχη).[82] The pair of expressions 'conse-
quence' and 'conflict' may have been used as replacement for the pair of
expressions 'connection' and 'division'[83] in order to denote the two basic
relations that hold between the things (πράγματα) that are stated or indi-
cated by simple propositions.

---

[80] τὰς ὑποθετικὰς ἀποφάνσεις ἐκ τῶν κατηγορικῶν τὴν γένεσιν ἔχειν, ἄλλης γὰρ
πρὸς ἄλλην κατηγορικῆς προτάσεως ἢ ἀκολουθίαν ἢ διάστασιν σημαίνουσι.

[81] Presumably, a conditional proposition is true when the consequence it indicates
holds, and a disjunctive proposition is true when the conflict it indicates holds.

[82] The pair of logical expressions 'consequent and conflict' may have been adopted
from Stoic logic, and appropriated by Peripatetic and Platonist logicians. The use is
non-Stoic in two respects: (i) ἀκολουθία and μάχη are two basic ontological relations,
not reducible to each other, whereas at least Chrysippus and his followers reduced
ἀκολουθία to μάχη; (ii) the relation is not (as in Stoic logic) between propositions or
truth-bearers, but between πράγματα.

[83] See Alex. *Top.* 165.5-166.13, 174.5-175.26, where both are used in parallel.

Finally, hypothetical propositions are not defined syntactically by means of their linguistic form or their connectives (as the Stoics defined them), nor by means of their truth-conditions, but in terms of what sort of thing they state or what sort of thing we assert with them: they state that something is when something else is, or that something is when something else is not, or that something is not when something else is. That is, they all state something of the kind that . . . when –––; or, in other words, they all contain a conditional element, and *this* is seen as the reason why they are (or why they are called) hypothetical (Galen *Inst.log.* 3.1, Amm. *Int.* 74.4-5).

Thus, we are now in a position to answer our first question from Section 1, i.e. why both the conditional and the disjunctive propositions were classified as hypothetical. The reason is simply that with both types we assert something of the kind that . . . when –––, and thus that both types contain a conditional element.

Historically, I believe, the origin of these later characterizations of the hypothetical propositions lie in the two passages from Aristotle's *Topics* (see Section 3 above). For instance, in the case of the disjunction, the description as 'indicating a division' – ultimately – goes back to the phrase 'things of which holds one and only one of two', together with the disjunctive description 'disease or health' (Arist. *Top.* 112a24-5). On the other hand, the characterization of the hypothetical element picks up 'when we have proved that the one <predicate> holds, we will also have proved that the other does not hold; and when we prove that the one does not hold, we will have proved that the remaining one holds' (Arist. *Top.* 112a27-30).[84] In terms of Diagram 5 above, we can say that the later Peripatetic hypothetical propositions seem to encompass modifications of both the 'hypotheses' in the B-lines (e.g. 'either *Fa* or *Ga*') and the 'agreements' in the C-lines (e.g. 'when *Fa* is proved, not *Ga* is proved').

Now to the hypothetical syllogisms constructed with these hypothetical propositions as first premises. In the sources of the late second and early third century AD, the quartet of *modus ponens* type arguments appears, with the following descriptions:[85]

---

[84] Note that the three cases which Galen mentions at *Inst.log.* 3.1 are exactly the three from the two passages from Aristotle's *Topics* quoted above.

[85] Cf. Alex. *Top.* 165.5-166-13, 174.5-175.26, where all four are used, and Alc. *Didasc.* 159.24-9, where the first two are given, apparently followed by a lacuna.

(1*)   hypothetical syllogism, establishing from a   (συλλογισμὸς ὑποθετικὸς ἐξ
       consequence   ἀκολουθίας κατασκευαστικός)
(2*)   hypothetical syllogism, refuting from a   (συλλογισμὸς ὑποθετικὸς ἐξ
       consequence   ἀκολουθίας ἀνασκευαστικός)
(3*)   hypothetical syllogism, establishing from a   (συλλογισμὸς ὑποθετικὸς ἐκ
       conflict   μάχης κατασκευαστικός)
(4*)   hypothetical syllogism, refuting from a   (συλλογισμὸς ὑποθετικὸς ἐκ
       conflict   μάχης ἀνασκευαστικός)

These four descriptions are important indicators of the ancestry of the syl-
logisms. They are Peripatetic through and through, although not *early*
Peripatetic, since we have ἀκολουθία and μάχη rather than συνέχεια and
διαίρεσις. The use of the terms κατασκευαστικός and ἀνασκευαστικός
places the syllogisms in the context of Aristotle's *Topics* (as do explicitly
the passages from Alexander's *Topics* commentary). Finally, the combi-
nation of the pair ἀκολουθία/μάχη with the pair κατασκευαστικός/ἀνασ-
κευαστικός provides for exactly the four *modus ponens* type hypothetical
syllogisms.[86]

Our second question can now also be answered: the four *modus ponens*
type hypothetical syllogisms are so-called because they contain at least
one hypothetical proposition (see above, Section 1); and those basic hypo-
thetical syllogisms of *modus ponens* type are exactly the four one obtains
from the combinations of κατασκευαστικός and ἀνασκευαστικός with
ἀκολουθία and μάχη. They are those four types which exemplify *modus
ponendo ponens* as meta-logical inference rule, if one makes allowances
for the possible permutations of affirmations and negations. (Or, in the
context of dialectical argumentation, they exemplify the four possibili-
ties of establishing or refuting a problem or a thesis from a hypothetical
proposition.)

How was the validity of these four types of syllogisms justified? In
Alcinous, we obtain as example for a hypothetical syllogism that estab-
lishes from a consequence:

> If the One is a whole and has a limit, then it has a beginning, a middle, and an
> end, and partakes in shape. But the antecedent. Therefore the consequent.[87] (Alc.
> *Didasc.* 159.25-8)

---

[86] These would correspond to those four of which Galen, at *Inst.log.* 14.2, implies
that the ancients, i.e. early Peripatetics, had them.

[87] Εἰ τὸ ἓν ὅλον ἐστὶ καὶ [τὸ] πεπερασμένον, τοῦτο ἀρχὴν καὶ μέσα καὶ τελευτὴν
ἔχον καὶ σχήματος μετέχει· τὸ δὲ ἡγούμενον· τὸ ἄρα λῆγον.

The abbreviations of the second premiss and the conclusion to 'the antecedent' and 'the consequent' show that what is considered as connected by virtue of a consequence are now the things indicated by the 'component *propositions*' of the hypothetical proposition. The abbreviations also indicate that the argument was recognized as having the logical form *ponendo ponens*; and we can hence assume that the argument was considered as valid because of this form. Thus, the hypothetical syllogisms are now, it seems, regarded as valid because of their *modus ponens* type forms (even though the examples still generally have the same subject term in the two component propositions[88]).

Diagram 7

a later Peripatetic hypothetical syllogism (which establishes from a consequence) συλλογισμὸς ὑποθετικὸς (ἐξ ἀκολουθίας κατασκευαστικός)		
1st premiss (λῆμμα)[89]	If *Fa, Ga*	hypothetical proposition (ὑποθετικὴ πρότασις)
2nd premiss (πρόσληψις)	*Fa*	categorical proposition (κατηγορικὴ πρότασις)
conclusion (συμπέρασμα)	Therefore, *Ga*	categorical proposition (κατηγορικὴ πρότασις)

But why were these forms considered valid? Galen remarks that

> The syllogisms that come about from hypothetical propositions are completed by transition from one thing to another by means of consequence or conflict.[90] (*Inst.log.* 14.10)

This suggests that the hypothetical propositions – which indicate either a consequence or conflict, see above – are understood as licensing the

---

[88] I.e. they have the form 'if *Ga, Fa*', 'either *Ga* or *Fa*', or in Alcinous' example 'if *Ga*, then *Fa* and *Ha* and *Ia*', with a conjunction as consequent.

[89] For the use of λῆμμα and πρόσληψις for the premisses (instead of πρότασις) see e.g. Alc. *Didasc.* 158.24-7, Gal. *Inst.log.* chs 4, 5, 14.

[90] οἱ γὰρ ἐξ ὑποθετικῶν προτάσεων γιγνόμενοι οσυλλογισμοὶ κατὰ μετάβασιν ἀφ' ἑτέρου πράγματος ἐφ ἕτερον ἐπιτελοῦν<ται> δι' ἀκολουθίας ἢ μάχης . . . . Galen adds his distinction of complete and incomplete consequence and conflict; for this see my 'Hypothetical syllogistic in Galen: propositional logic off the rails?' (forthcoming). However, unlike his own theory, the quoted sentence allows only for two disjuncts, and has Peripatetic terminology, and he hence presumably took it from a Peripatetic source.

transition from one thing to another. 'Either p or q' licenses the transition from 'p' to 'not q' because it states that when 'p' then 'not q', etc.[91] Thus the understanding of the validity of the forms of the four types of hypothetical syllogisms is not:

if we have 'if p, q' and 'p', then we can infer 'q';

rather it is:

'if p, q' licenses us to infer 'q' from 'p' (when we have 'p').

The hypothetical propositions have taken on – or perhaps rather kept – the function Aristotle's agreement *cum* hypothesis had in his syllogisms from a hypothesis. (This particular understanding of the hypothetical propositions or complex propositions has a modern counterpart in Gilbert Ryle's inference ticket view.[92])

And although in the historical period under consideration the *modus ponens* type arguments were not yet called '*modus (ponendo) ponens*', etc., we can now also attempt an answer to our question, why they eventually obtained these names, rather than being called something like 'disjunctive syllogisms' and 'conditional syllogisms'. For – in line with the above understanding of their form – they can be (and have been) described as follows:

the first posits by positing from a consequence
the second removes by removing from a consequence
the third posits by removing from a conflict
the fourth removes by positing from a conflict[93]

Given that the first parts of these descriptions (posits by positing, etc.) suffice to determine unambiguously which of the four argument types is at issue, it is likely that what happened is that at some point these parts

---

[91] What about *modus tollens*, though? I believe that the transition from 'not q' to 'not-p' was thought to be licensed by 'if p, q' *indirectly*, through proof by antilogism. (For two-premiss arguments a version of antilogism is: 'if an argument which shares a premiss is valid, then an argument with the contradiction of the non-shared premisses as conclusion, and the contradiction of the conclusion as premiss is also valid', cf. e.g. Galen *Inst.log.* 6.6, Alex. *An.Pr.* 29.7-13). This is suggested e.g. by Galen, *Inst.log.* 8.2, where the second hypothetical syllogism is said to be in need of proof.

[92] Cf. G. Ryle, The Concept of Mind, 1950, p. 121.

[93] Cf. the Greek formulations e.g. in [Amm.] *An.Pr.* 68.23-41 ὁ ἐξ ἀκολουθίας τῇ θέσει τοῦ ἡγουμένου τὸ ἑπόμενον τιθείς, etc., a passage where however a Peripatetic or Platonist modification of the Stoic third type of indemonstrables has been added.

alone were taken to denote the argument forms, and that finally they became their actual names.[94]

We thus have evidence from the late 2nd century AD onwards of the group of four *modus ponens* types of arguments, considered as valid in virtue of their (*modus ponens* type) forms. At the same time, both in the names of these arguments, and in the understanding of their component propositions and of their validity, the characteristics of Aristotle's theory of syllogisms from a hypothesis have been preserved.

6. *Appendix: Comparison with modern logic: a* very *rudimentary system of natural deduction*

If one wants to make comparisons with modern logic, what suggests itself is a *very* rudimentary system of natural deduction, interpreted in Gentzen/Prawitz fashion.[95] (However, this system could not be supplemented to become a system of minimal, intuitionist, or classical logic. Rather – like all systems of ancient logic – it would presumably resemble relevance logic.) Here is a brief sketch of such a 'system':

In our *object language*, as it were, we have:

sentence letters:	*p, q, r* . . .
two connectives:	'if . . ., –––', 'either . . ., or –––'

Unusually, perhaps, the sentence letters cover equally affirmations and negations, as well as universal, particular, and singular statements; i.e. the 'system' works with quantified negative and affirmative statements as its 'atomic' elements. Furthermore we need a sign for *rejection*, let us say '*A', for 'A is rejected'.

There are then four *introduction rules* in the system, two for 'if' and two for 'or' (assumptions in square brackets):

---

[94] I do not know when exactly '*modus (ponendo) ponens*', etc., became the names of the four argument forms. (Kant AA IX p. 129 "es wird hier entweder nach dem modo ponente oder dem modo tollente gefolgert"; G. Fr. Meier: *Auszug aus der Vernunftlehre*. in: Kant AA XVI, *Handschriftlicher Nachlass* Vol. 3, p. 745 §393 "modus ratiociniorum hypotheticorum ponens" und "modus ratiociniorum hypotheticorum tollens").

[95] See e.g. D. Prawitz, 'Ideas and Results in Proof Theory', in J.E. Fenstad (ed.), *Proceedings of the Second Scandinavian Logic Symposium* (1971), sections II.1 and 2.

if-I   [A]   [*B]

        | |

        B   *A

      —————   —————

      if A, B   if A, B

or-I   [A]   [*A]   [A]   [*A]

       | | | |

      *B   B   *B   B

     ———————   ———————

     either A or B   either B or A

These introductions can be understood as providing the 'meaning' of the logical constants: 'If A, B' means 'there is a deduction of B from A or there is a deduction of *A from *B'. 'Either A or B' means 'there is a deduction of B from *A, and there is a deduction of *B from A'.[96] Propositions of the forms 'If A, B' and 'Either A or B' are *hypothetical* propositions, since they imply (or assert) deducibility *from an assumption*. Their function thus still corresponds to that of the hypotheses in Aristotle's syllogisms from a hypothesis.

The argument forms of the *modus ponens* type arguments would then be understood as the *elimination rules* of the system as follows:

if-E   if A, B   A   if B, A   *A

    ————————————   ————————————

          B   *B

or-E   either A or B   *A   either A or B   A

    ————————————   ————————————

          B   *B

The eliminations are the syllogisms of the system. They get their *justification* from the 'meaning' of the hypothetical propositions.[97] That is, the validity of the *modus ponens* type arguments is based on the 'meaning' of their hypothetical propositions, which in turn was 'acquired' through their introduction, which in each case is based on the deducibility of an assertion or rejection from an assertion or rejection.

Evidently, this system is not of much use. However, (i), it has to be conceived of as a *supplement* to Aristotelian term-logic, and (ii), the point of interest is not its use, but the particular way in which the logical constants are understood, and how this relates to the justification of the validity of the *modus ponens* type arguments.

Queen's College, Oxford

———————

[96] Cf. Galen *Inst.log.* 3.1, quoted in Section 5 above.
[97] Cf. Galen *Inst.log.* 14.10, quoted in Section 5 above.