The difference between "significant" and "not significant" is not itself statistically significant*

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Abstract

A common error in statistical analyses is to summarize comparisons by declarations of statistical significance or non-significance. There are a number of difficulties with this approach. First is the oft-cited dictum that statistical significance is not the same as practical significance. Another difficulty is that this dichotomization into significant and non-significant results encourages the dismissal of observed differences in favor of the usually less interesting null hypothesis of no difference.

Here, we focus on a less commonly noted problem, namely that changes in statistical significance are not themselves significant. By this, we are not merely making the commonplace observation that any particular threshold is arbitrary—for example, only a small change is required to move an estimate from a 5.1% significance level to 4.9%, thus moving it into statistical significance. Rather, we are pointing out that even large changes in significance levels can correspond to small, non-significant changes in the underlying variables. We illustrate with a theoretical and an applied example.

Keywords: multilevel modeling, multiple comparisons, replication, statistical significance

1 Introduction

A common statistical error is to summarize comparisons by statistical significance and to draw a sharp distinction between significant and non-significant results. The approach of summarizing by statistical significance has a number of pitfalls, most of which are covered in standard statistics courses but one that we believe is less well known.

Among the well known pitfalls are the oft-cited point that statistical significance does not equal practical significance. For example, if the estimated effect of a drug is to decrease blood pressure by 0.10 with a standard error of 0.03, this would be statistically significant

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but probably not practically significant (or so we suppose, given our general knowledge that blood pressure values are typically around 100). Conversely, an estimated effect of 10 with a standard error of 10 would not be statistically significant, but it has the possibility of being practically significant.

A second problem with the automatic use of a binary significant/non-significant decision rule is that it encourages practitioners to ignore potentially important observed differences in favor of the usually less interesting null hypothesis.

Related to this last point is the lesser-known problem, which is the topic of this article, that changes in statistical significance are not themselves significant. By this, we are not merely making the commonplace observation that any particular threshold is arbitrary—for example, only a small change is required to move an estimate from a 5.1% significance level to 4.9%, thus moving it into statistical significance. Rather, we are pointing out that even large changes in significance levels can correspond to small, non-significant changes in the underlying variables. We illustrate with two examples.

2 Theoretical example: comparing the results of two experiments

Consider two independent studies with effect estimates and standard errors of 25 ± 10 and 10 ± 10. The first study is statistically significant at the 1% level, and the second is not at all statisticall significant, being only one standard error away from 0. Thus it would be tempting to conclude that there is a large difference between the two studies. In fact, however, the difference is not even close to being statistically significant: the estimated difference is 15, with a standard error of $\sqrt{10^2 + 10^2} = 14$.

Assessing the statistical significance of the differences between study results is not merely an academic curiosity. Consider a third independent study with much larger sample size that yields an effect estimate of 2.5 with standard error of 1.0. This third study attains the same significance level as the first study, yet the difference between the two is itself also significant! Both find a difference but the magnitude of that difference is much different. Does the third study replicate the first study? If we restrict attention only to judgments of significance we might say yes, but if we think about the effect being estimated we would say no, as noted by Utts (1991). In fact, the third study finds an effect size much closer to that of the second study but now because of the sample size it attains significance.

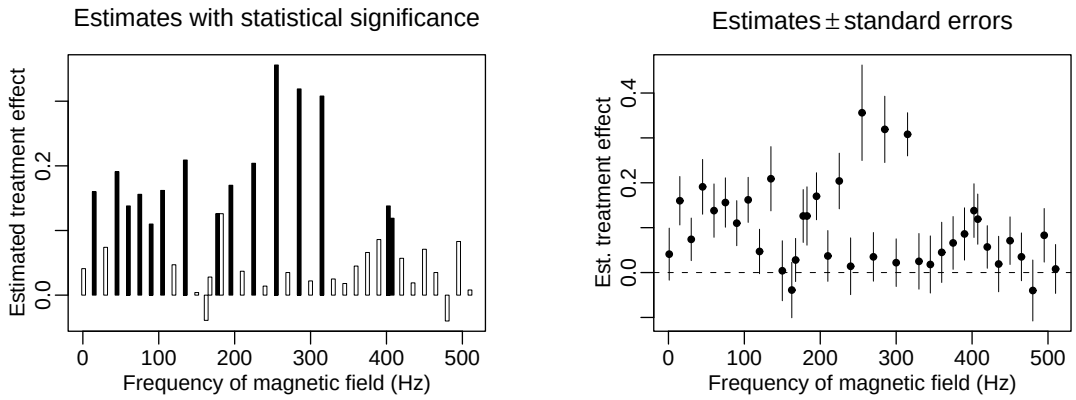

Figure 1: (a) Estimated effects of electromagnetic fields on calcium efflux from chick brains, shaded to indicate different levels of statistical significance, adapted from Blackman et al. (1988). A separate experiment was performed at each frequency. (b) Same results presented as estimates ± standard errors. As discussed in the text, the first plot, with its emphasis on statistical significance, is misleading.

3 Applied example: comparison of several related experiments

The issue of comparisons between significance and non-significance is of even more concern in the increasingly-common setting where there are a large number of comparisons. We illustrate with an example of a laboratory study with public health applications.

In the wake of concerns about the health effects of low-frequency electric and magnetic fields, Blackman et al. (1988) performed a series of experiments to measure the effect of electromagnetic fields at various frequencies on the functioning of chick brains. At each of several frequencies of electromagnetic fields (1 Hz, 15 Hz, 30 Hz, ..., 510 Hz), a randomized experiment was performed to estimate the effect of exposure, compared to a control condition of no electromagnetic field. The estimated treatment effect (the average difference between treatment and control measurements) and the standard error at each frequency were reported.

Blackman et al. (1988) summarized the estimates at the different frequencies by their statistical significance, using a graph similar to Figure 1a with different shading indicating results that are more than 2.3 standard errors from zero (that is, statistically significant at the 99% level), between 2.0 and 2.3 standard errors from zero (statistically significant at the 95% level), and so forth. The researchers used this sort of display to hypothesize that one process was occurring at 255, 285, and 315 Hz (where effects were highly significant), another

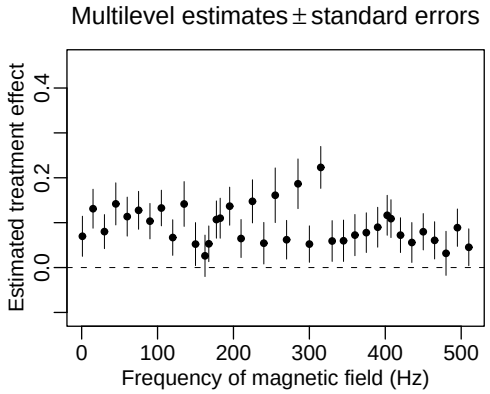

Figure 2: Multilevel estimates and standard errors for the effects of magnetic fields, partially pooled from the separate estimates displayed in Figure 1. The standard errors of the original estimates were large, and so the multilevel estimates are pooled strongly toward the common average which is near of 0.1.

at 135 and 225 Hz (where effects were only moderately significant), and so forth. The estimates are all of relative calcium efflux, so that an effect of 0.1, for example, corresponds to a 10% increase compared to the control condition.

The researchers in the chick-brain experiment made the common mistake of using statistical significance as a criterion for separating the estimates of different effects, an approach does not make sense. At the very least, it is more informative to show the estimated treatment effect and standard error at each frequency, as in Figure 1b. This display makes the key features of the data clear. Though the size of the effect varies it is just about always positive and typically not far from 0.1.

One way to handle the multiple-comparisons aspect of this problem is to fit a multilevel model of the sort used in meta-analysis. If at each frequency j, we label the estimated effect and standard error as y_j and σ_j, then the simplest multilevel model is $y_j \sim \mathrm{N}(\theta_j, \sigma_j^2)$, $\theta_j \sim \mathrm{N}(\mu, \tau^2)$, and the resulting Bayesian estimates for the effects θ_j are partially pooled toward the average of all the data (see, for example, Gelman et al., 2003, chapter 5). The posterior estimates and standard errors are shown in Figure 2.

The multilevel model can be seen as a way to estimate the effects at each frequency j, without setting "non-significant" results to zero. Some of the apparently dramatic features of the original data as plotted in Figure 1a—for example, the negative estimate at 480 Hz and the pair of statistically-significant estimates at 405 Hz—do not stand out so much in the multilevel estimates, indicating that these features could be easily explained by sampling

variability and do not necessarily represent real features of the underlying parameters.

4 Discussion

It is standard in applied statistics to evaluate inferences based on their statistical significance at the 5% level. There has been a move in recent years toward reporting confidence intervals rather than p-values, and the centrality of hypothesis testing has been challenged (see Krantz, 1999, for a review of these issues), but even when using confidence intervals it is natural to check whether they include zero. Statistical significance, in some form, is a way for us to assess the reliability of statistical findings. However, as we have seen, comparisons of the sort, "X is statistically significant but Y is not" can be misleading.

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