concerned with the fact that in order to make use of this replacement we must consider a system in which our electron is one of N indistinguishable electrons.

On the Value of the Cosmical Constant.

(Abstract.)

It is argued that, owing to the curvature of space, a term $i\sqrt{N}/R$ should appear in the wave equation for an electron, N being the number of electrons in the universe and R the radius of the universe in an eigenstate, and having therefore the dimensions calculated for an Einstein world. The units are here supposed to be such that the electrostatic energy of two electrons at a distance r is simultaneously represented by a term i/r. The theoretical term is identified with the term ordinarily attributed to the proper-mass of the electron. From this identification we find the cosmical constant $\lambda = 9{\cdot}79\,.\,10^{-55}$, which gives a speed of recession of the spiral nebulæ 528 km. per sec. per megaparsec. The observed speed according to Hubble is 465 km. per sec. per megaparsec.

The Diffraction of Electrons in Gases.

By F. L. Arnot, Ph.D., Lecturer in Natural Philosophy, The University, St. Andrews.

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Introduction.

The new wave mechanics has been eminently successful in correlating and accounting for the large amount of experimental data on the periodic properties of the atom. The applications of the new theory to aperiodic phenomena, though not nearly so numerous, have been attended with no less success; indeed, it is in these experiments on free electrons, in which diffraction patterns similar to those produced by beams of X-rays and light are obtained, that the wave nature of electrons is so clearly and objectively demonstrated.

Such diffraction effects have been obtained by a number of investigators* by scattering beams of homogeneous electrons in crystals, thin films and by a

* Davisson and Germer, G. P. Thomson, Thomson and Reid, Rupp, Kikuchi, Rose, and others. For literature see "The Wave Mechanics of Free Electrons," by G. P. Thomson.

ruled grating. Similar effects are obtained when electrons are scattered by complex molecules,* owing to the symmetrically situated nuclei in the molecular structure, and when α-particles are scattered by helium† owing to interference between the waves of the scattered incident particles and recoiling helium nuclei.‡

Recently Bullard and Massey§ and the author‖ have described a new class of diffraction phenomena which are produced when slow electrons are scattered by single atoms. In these experiments the diffraction patterns observed are due to interference between the secondary wavelets originating from different portions of a single atom, and so may be regarded as the electrical analogue of the diffraction of light by a random distribution of small particles.

In the experiments of the author mercury vapour was used as the scattering gas, and electrons having energies of from 8 to 800 volts, corresponding to wave-lengths of from 4·0 to 0·4 Å.U. were employed. All the scattering curves showed distinct maxima and minima, maxima of four different orders being obtained. Pearson and Arnquist¶ have subsequently published similar results over a smaller range of velocities.

The present paper deals with an extension of this work to gases. The angular distributions of the elastically scattered electrons in Xe, Kr, Ar, Ne, H_2, N_2, CH_4 and CO have been measured for several different velocities of the primary beam between 30 and 800 volts, corresponding to wave-lengths of from 2·0 to 0·4 Å.U. The lower velocity curves of the rare gases all show distinct diffraction patterns, but at the higher velocities the maxima and minima in Ar and Ne have completely disappeared.

Apparatus.

The apparatus used in this work was the same as that employed in the work on mercury vapour. Full details of the apparatus and an account of the experimental procedure have been given in two previous papers.** It will be sufficient to reproduce here a diagram of the essential parts, fig. 1.

* Mark and Wierl, 'Naturwiss,' vol. 18, p. 205 (1930); R. Wierl, 'Ann. Physik,' vol. 8, p. 521.

† Chadwick, 'Proc. Roy. Soc.,' A, vol. 128, p. 114 (1930); Blackett and Champion, 'Proc. Roy. Soc.,' A, vol. 130, p. 380 (1931).

‡ Mott, 'Proc. Roy. Soc.,' A, vol. 126, p. 259 (1930).

§ Bullard and Massey, 'Proc. Roy. Soc.,' A, vol. 130, p. 579 (1931).

‖ Arnot, 'Proc. Roy. Soc.,' A, vol. 130, p. 655 (1931).

¶ Pearson and Arnquist, 'Phys. Rev.,' vol. 37, p. 970 (1931).

** Arnot, 'Proc. Roy. Soc.,' A, vol. 130, p. 655 (1931); *ibid*, vol. 129, p. 361 (1930).

These are an electron gun consisting of the tungsten filament, F, entirely enclosed within two cylinders containing two holes, S_1 and S_2, each of 1 mm.

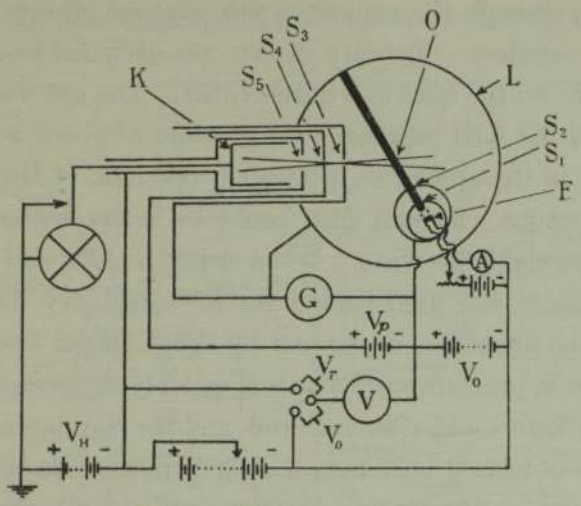

FIG. 1.—Diagram of Apparatus.

in diameter. The gun could be rotated about the axis, O, through nearly 360° by means of a small ground glass joint.

The receiving system consisted of three concentric cylinders containing the slits, S_3, S_4, S_5, and the concentric Faraday cylinder, K, within the inner shielding cylinder. The dimensions of these slits were respectively 8 mm. × 0·2 mm., 5 mm. × 0·2 mm., 2 mm. × 2 mm., and the three slits were equally spaced 5 mm. apart. The distance of S_3 from the scattering point, O, was 12 mm. The inside of the collision chamber was lined with the large nickel cylinder, L.

A small difference of potential, V_p, of 9 volts was applied to the first two slits, S_3 and S_4, to stop positive ions from entering the Faraday cylinder. A potential of $V_r = (V_0 + V_p - 4)$ volts, where V_0 is the velocity of the primary electron beam, was applied between the next two slits, S_4 and S_5, so as to stop all but the elastically scattered electrons from being received. And finally a potential, V_H, of 12 volts was applied between the slit, S_5, and the Faraday cylinder to prevent any secondary emission of electrons from the Faraday cylinder. The glass work was of pyrex and the whole apparatus up to within a few centimetres of the ground glass joints was baked out at a temperature of 500° C., the joints being kept below room temperature by means of water coolers. The apparatus was connected through a liquid air trap to a diffusion pump backed by a Hyvac pump, and a good " sticking vacuum " was obtained on a sensitive McLeod gauge.

The gas was allowed to leak into the apparatus through a fine glass capillary which was internally sealed into the inlet tube. The gas was not circulated, but after passing through the apparatus was pumped off into the atmosphere through another capillary. Mercury vapour was excluded from the apparatus by liquid air traps on the inlet and outlet tubes. The gas was contained in a large bulb behind the inlet capillary at a pressure of about 5 mm. This produced a pressure in the apparatus of about 0·003 mm. of Hg which was read on the McLeod gauge. Several runs could be taken without the pressure showing any appreciable decrease. When using Xe, Kr and Ne, however, a smaller gas container was used behind the inlet capillary, which resulted in the pressure in the apparatus decreasing by about 10 per cent. during a run. Since the decrease in pressure with time was sensibly uniform, the pressure was read immediately before and after each run, and the readings corrected accordingly. Pressures of from 0·0025 mm. to 0·0035 mm. of Hg were used.

The relation between the scattered current and the pressure was measured for several different angles of scattering, and was found to be linear over the above range of pressures. The scattering was therefore due to single collisions. At angles below 30° the linear relationship ceased to hold for pressures above 0·006 mm. in argon, due to the onset of multiple scattering.

Results.

(1) *The Angular Distribution of the Scattered Electrons.*—The results obtained for the monatomic rare gases Xe, Kr, Ar and Ne, are shown in fig. 2. The number at the side of each curve represents the energy in volts of the primary electron beam. I am indebted to Dr. Aston for the loan of the xenon and krypton, both of which gases were at least 99 per cent. pure. The argon and neon were obtained from the British Oxygen Company, the argon being supplied as 99 per cent. pure, and the neon as 98 per cent. pure. The curves for electrons of different velocities in Xe, Kr and Ar have all been displaced in a vertical direction, but the base line for each curve is shown in the figure. In the case of Ne, the curves for the three highest velocities, 205, 412 and 830 volts, are all drawn on the same base line, as are also the curves for the two lowest velocities, 29 and 42 volts.

The results obtained for the diatomic gases H_2, N_2, CO, and for the more complicated gas, CH_4, are shown in fig. 3. The hydrogen, nitrogen, and methane were obtained from commercial cylinders. The methane was subsequently found to contain 15 per cent. of nitrogen, but this should not affect the general shape of the curves. The carbon monoxide was produced by

dropping concentrated sulphuric acid on to sodium formate *in vacuo*, the gas being dried by passing through a trap immersed in liquid air. The curves for N_2 and CO have all been displaced in a vertical direction by an equal amount,

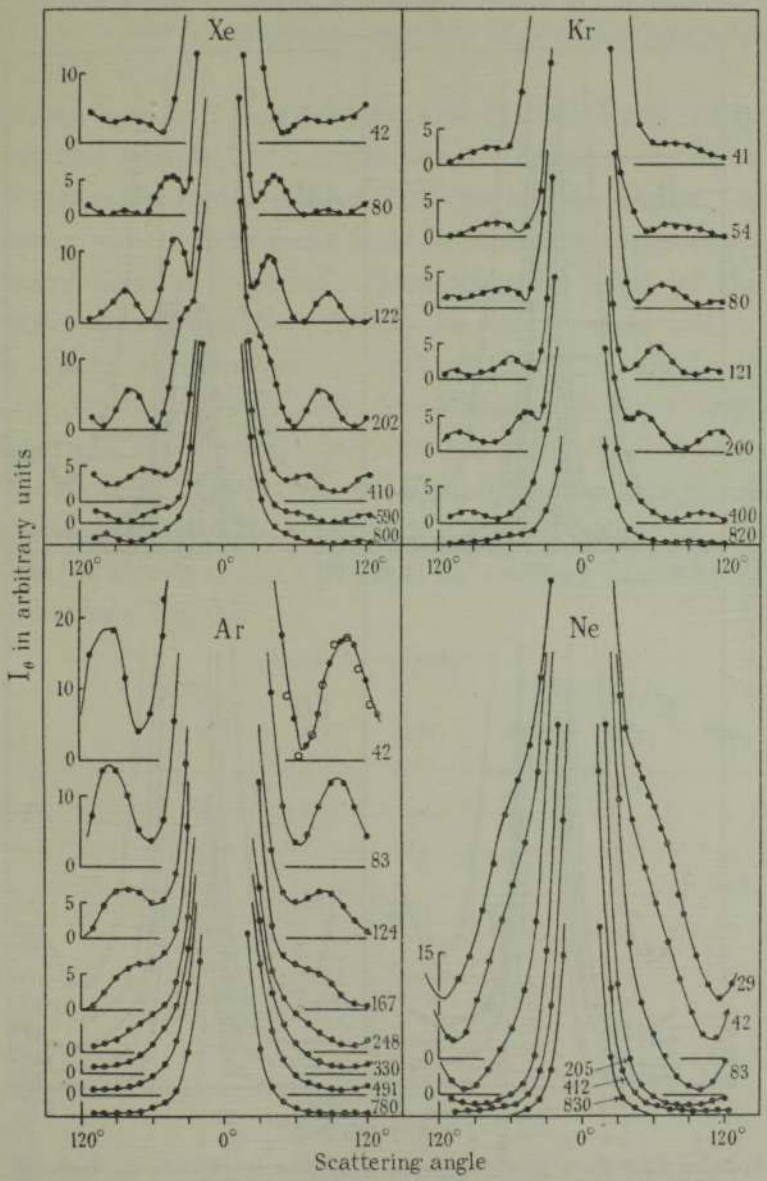

FIG. 2.—Angular Distribution of Scattered Electrons in Xenon, Krypton, Argon and Neon.

the base line of each curve being two units of ordinate above the base line of the curve immediately below it. The curves for H_2 and CH_4 have not been displaced at all.

All the curves in figs. 2 and 3 have been reduced to a standard pressure of

10^{-3} mm. of Hg at 18° C. and a standard electron beam current of 7·4 microamps. The ordinates of both figures represent I_θ in arbitrary units, where I_θ is defined thus. If a beam of electrons passes through a gas containing N atoms per unit volume, then NI_θ is the proportion of the beam elastically scattered

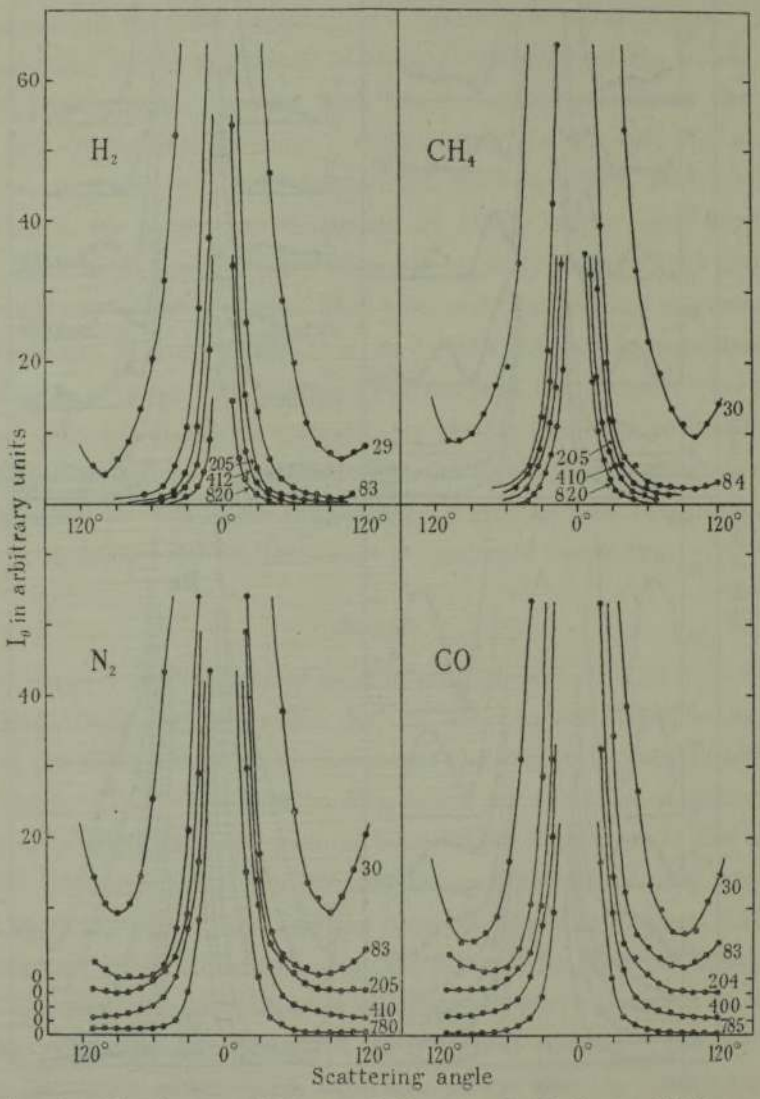

FIG. 3.—Angular Distribution of Scattered Electrons in Hydrogen, Methane, Nitrogen and Carbon Monoxide.

in the direction θ, per unit solid angle and per unit length of the beam. I_θ has the dimensions of an area.

The experimental points from which the curves in figs. 2 and 3 are drawn are given in Tables I to VI. In using these tables the appropriate angular

correction given at the foot of each column must be applied. For example, the values of I_θ given in the third column of Table I for 80-volt electrons in xenon are for the following angles, 13°, 18°, 23°, 28° and so on to 118°, the angular correction for this column being −2°. This method of tabulating the results has been adopted solely to obviate the necessity of providing a separate column of angles for each velocity.

It will be seen from fig. 2 that the rare gases all show clearly marked diffraction maxima and minima, the angular interval between successive maxima becoming larger as the atomic number of the gas become smaller. As was the case in mercury vapour,* the peaks move in to smaller angles as the velocity of the primary beam is increased. Since the wave-length of the incident electrons varies inversely as their velocity we should expect this to occur on any simple theory of diffraction by rigid spheres. The points denoted by circles on the 42-volt curve in argon are Bullard and Massey's points† for 40-volt electrons in argon fitted to my curve at 103°. The agreement s seen to be very satisfactory. It will be seen that the diffraction maxima and minima have completely disappeared in the higher velocity curves in Ne and Ar. This is due to the refractive index of the atom becoming nearly equal to unity for very short wave-lengths.

Table I.—Xenon.

V_0.	42.	80.	122.	202.	410.	590.	800.
Angle.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°							
15	—	70·0	31·4	31·9	—	—	24·5
20	80·0	22·5	13·1	18·6	22·4	15·7	15·0
25	41·6	5·60	4·96	13·0	—	—	—
30	22·2	1·98	5·62	13·1	9·6	5·9	6·2
35	10·9	2·93	7·87	11·6	—	—	—
40	5·20	4·55	9·10	9·60	4·4	2·5	2·63
45	3·63	5·37	8·60	6·23	—	—	—
50	1·64	4·85	5·80	3·08	3·04	1·76	1·74
55	1·75	3·20	—	1·17	—	—	—
60	2·60	1·79	0·80	0·60	3·46	1·39	1·04
70	3·40	0·00	0·00	2·75	3·57	0·86	0·50
80	3·11	0·51	2·64	5·40	2·02	0·32	0·31
90	2·89	0·71	3·96	4·36	1·44	0·13	0·20
100	3·55	0·22	2·17	1·70	1·60	0·56	0·32
110	3·85	0·56	0·0	0·53	3·00	1·08	0·53
120	5·36	1·48	0·0	1·74	3·70	1·12	0·18
Angle correction	0°	−2°	−2°	0°	+2°	+3°	+3°

* Arnot, ' Proc. Roy. Soc.,' A, vol. 130, p. 655 (1931).
† Bullard and Massey, ' Proc. Roy. Soc.,' A, vol. 130, p. 579 (1931).

Table II.—Krypton.

V_0.	41.	54.		80.	121.	200.	400.	820.
Angle.	I_θ.	I_θ. (Expt.)	I_θ. (Theor.)	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°								
10	—	—	—	—	—	—	—	65
15	—	63	—	—	—	—	—	—
20	230	—	32·4	73	—	47·3	24·3	13·8
25	—	24·6	—	—	—	—	—	—
30	81	—	12·5	21·7	10·5	10·2	10·15	5·35
35	—	6·3	—	—	4·1	—	—	—
40	22·8	—	2·04	3·65	1·64	4·84	5·33	2·12
45	—	1·41	—	—	—	4·69	—	—
50	5·73	—	0·255	0·91	2·11	5·21	3·00	1·08
60	3·10	1·53	1·61	2·41	3·96	4·45	1·49	0·83
70	3·00	1·87	2·18	3·22	4·45	2·43	0·74	0·36
80	3·05	1·81	1·55	2·70	2·54	0·70	0·65	0·28
90	2·79	0·98	0·77	1·56	0·97	0·45	1·08	0·53
100	2·10	0·19	0·248	0·55	0·77	1·54	1·36	0·52
110	1·37	0·00	0·044	0·86	1·24	2·61	1·13	0·24
120	1·12	—	—	0·85	1·13	2·66	0·48	0·15
Angle correction	0°	0°	0°	−2°	−4°	−2°	0°	0°

Table III.—Argon.

V_0.	42.	83.	124.	167.	248.	330.	491.	780.
Angle.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°								
15	—	—	—	—	—	—	—	56
20	160	210	154	114	89	65	50	26
30	73	65	38	29	23	19·5	18	9·2
40	34	24·5	12·4	11·7	11·2	10·5	9·0	4·0
50	17·3	8·7	6·5	7·7	7·6	6·9	5·0	2·0
60	5·9	3·6	5·1	6·3	5·4	4·2	2·8	1·1
70	2·1	4·7	5·7	5·8	4·0	2·8	1·8	0·68
80	6·5	8·6	6·7	5·0	2·8	1·8	1·2	0·48
90	13·6	11·9	6·2	3·8	1·9	1·1	0·87	0·45
100	16·6	11·9	4·4	2·0	1·1	1·0	0·75	0·42
110	16·0	8·5	2·5	0·9	1·0	1·0	0·86	0·41
120	11·1	4·4	1·0	0·5	1·7	1·3	1·17	0·41
Angle correction	−1°	0°	−1°	0°	−1°	0°	0°	0°

Table IV.—Neon.

V_θ.	29.	42.	83.	205.	412.	830.
Angle.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°						
10	127	—	—	88	49	26·6
20	81	83	96	55	18·0	8·2
30	46·4	51	41·6	18·6	6·2	2·5
40	41·3	34	21·4	8·1	3·2	1·1
45	39·5	—	—	—	—	—
50	37·0	27	13·0	4·2	1·9	—
55	35·5	—	—	—	—	—
60	32·5	22	8·6	2·8	1·4	—
65	30·2	—	—	—	—	—
70	27·1	16·6	5·3	1·9	0·93	—
75	23·1	—	—	—	—	—
80	20·4	11·2	2·6	1·6	0·87	—
90	14·6	6·7	1·2	1·5	0·71	—
100	10·5	3·5	0·8	1·6	0·73	—
110	8·6	3·0	1·9	2·1	0·80	—
120	10·7	6·4	4·7	2·4	0·74	—
Angle correction	+6°	+2°	+1°	0°	+4°	+4°

Table V.—Hydrogen and Methane.

	Hydrogen.					Methane.				
V_θ.	29.	83.	205.	412.	820.	30.	84.	205.	410.	820.
Angle.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°										
10	—	(99)	53·5	33·6	14·5	(700)	178	(153)	73	35·2
15	—	—	—	—	6·32	—	70	—	32·5	17·2
20	152	25·6	15·2	7·3	3·52	229	40	30·5	18·0	10·2
25	—	—	—	—	—	—	20	—	11·6	6·2
30	85·3	13·0	5·0	3·0	1·30	104	11·8	12·0	7·5	3·7
35	—	—	—	—	—	—	—	—	5·5	2·0
40	46·7	6·2	2·0	1·36	0·50	53	6·8	5·8	3·7	1·3
50	28·8	3·6	1·3	0·72	—	33	5·6	3·0	1·9	0·8
60	19·9	2·2	0·87	—	—	23	3·1	2·2	1·3	—
70	11·3	1·6	0·63	—	—	18·4	2·8	1·7	0·8	—
80	8·3	1·3	0·41	—	—	13·5	2·6	1·4	—	—
90	7·1	1·0	0·38	—	—	11·2	2·3	—	—	—
100	6·4	0·8	—	—	—	9·7	2·1	—	—	—
110	7·2	1·6	—	—	—	11·3	2·8	—	—	—
120	8·1	—	—	—	—	14·2	3·1	—	—	—
Angle correction	+1°	0°	−1°	−1°	−1°	+1°	0°	−2°	−3°	−2°

Table VI.—Nitrogen and Carbon Monoxide.

	Nitrogen.					Carbon monoxide.				
V_0.	30.	83.	205.	410.	780.	30.	83.	204.	400.	785.
Angle.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.	I_θ.
°										
10	660	290	224	137	86	(433)	350	225	—	—
20	243	56	53	35·8	23·1	152	108	57	38·5	24·6
30	118	20	18	16·4	8·2	75	36·2	18·5	15·4	8·1
40	68	8·7	9·0	7·8	2·5	38·6	14·2	9·0	7·3	2·7
50	38	5·4	6·8	3·8	1·5	26·6	8·3	7·0	3·5	1·5
60	23·8	4·0	4·9	2·3	0·9	13·2	6·4	4·7	2·1	0·87
70	13·6	3·6	3·3	1·8	0·5	9·9	5·0	3·3	1·6	0·59
80	11·4	2·7	2·6	1·4	0·4	6·8	3·9	2·2	1·2	0·39
90	9·4	2·9	2·5	1·0	0·5	6·4	3·8	2·3	0·70	0·33
100	11·7	3·5	2·2	0·8	0·5	6·9	4·4	2·02	0·59	0·33
110	15·5	4·6	2·4	0·7	0·5	11·0	5·4	2·05	0·55	0·20
120	20·5	6·2	2·4	0·6	0·5	14·7	7·1	2·14	0·49	0·14
Angle correction	0°	0°	−1°	−1°	−1°	+1°	0°	−1°	−1°	−1°

Referring now to fig. 3, we see that the molecular gases H_2, CH_4, N_2 and CO all show no diffraction maxima. In the lower velocity curves of each of these gases, however, a minimum occurs in the region of 90° which moves in to smaller angles as the velocity of the electrons is increased, at the same time becoming fainter until it finally disappears altogether, the higher velocity curves falling off monotonically like those of the rare gases. After the minimum is passed the curves probably continue to rise steadily as the angle is increased to 180°, for Ramsauer and Kollath* have recently shown that at much lower velocities the backward scattering may exceed the forward scattering.

The similarity of the scattering curves of CH_4 to those of H_2 is very interesting, for Bullard and Massey† find that at velocities below 30 volts the scattering curves of CH_4 resemble those of Ar. It is also well known that the total effective cross section curve for CH_4 resembles those of the rare gases for velocities below 30 volts. The similarity of the cross section curves, and also of the scattering curves, of methane and argon for electrons of velocities below about 30 volts can be explained‡ if we assume that the four electrons of the four

* Ramsauer and Kollath, 'Ann. Physik,' vol. 10, p. 143 (1931).

† Bullard and Massey, 'Proc. Roy. Soc.,' A, vol. 133, p. 637 (1931).

‡ See Karl Darrow, 'Bell. Tel. System Monograph' B, 527, p. 15 (1930).

H-atoms of the CH_4 molecule form with the four outer electrons of the C-atom a closed shell of eight electrons similar to the outer shell of the rare gas atoms. Then, since electrons of less than 30 volts energy will be mainly scattered by this outer shell, we should expect the cross section and angular scattering curves of methane to be similar to those of the rare gases. Electrons of energy greater than 30 volts will, however, penetrate this outer shell without suffering much deflection, and we should, therefore, no longer expect the scattering curves to resemble those of argon, for the inner structure of methane is probably quite different to that of argon. It would be possible then to account for the similarity between the angular scattering curves and also the cross section curves (see p. 628) of methane and hydrogen by assuming that the electrons are scattered in methane mainly by the four hydrogen nuclei of the CH_4 molecule. If this were true we should expect the cross section for CH_4 to be about twice that of H_2 for electrons of greater than 30 volts energy. It will be shown on p. 628 that this is actually the case.

A very interesting idea emerges from the above discussion, namely, that we may be able to examine the structure of complex molecules by analysis of the diffraction patterns exhibited in the angular scattering curves of these molecules in much the same way as the structure of crystals has been determined by analysis of X-ray diffraction patterns. By using electrons of increasing velocity we could probe deeper and deeper into the molecule, examining the structure of each successive shell of electrons either by comparing the diffraction patterns obtained with those obtained from simpler atoms of known structure; or, when the theory of electron scattering has been more fully developed, by direct analysis of the diffraction patterns obtained from the molecule alone.

(2) *The Effective Cross Sections for Elastic Collisions.*—If we multiply the ordinates of figs. 2 and 3 by $2\pi \sin\theta$ we obtain curves which represent the proportion of the primary beam scattered in all azimuths per unit angle, as a function of the angle of scattering. The area under these curves,

$$Q = 2\pi \int_0^{\pi} I_\theta \sin\theta \, . \, d\theta,$$

gives the effective cross section for elastic collisions for electrons of different velocities. The total effective cross section for elastic collisions of all the atoms in a cubic centimetre at 1 mm. pressure and 0° C. is then given by

$$W = \frac{2{\cdot}71 \times 10^{19}}{760} Q \text{ cm.}^2 \text{ per cm.}^3.$$

Whereas on the classical theory the scattering per unit solid angle at 0° is infinite, on the quantum theory it is finite, so that the curves representing the scattering per unit angle should all pass through the origin at 0°. When the measurements are taken in to sufficiently small angles these curves do actually bend over so as to extrapolate through the origin. Fig. 4 shows the scattering

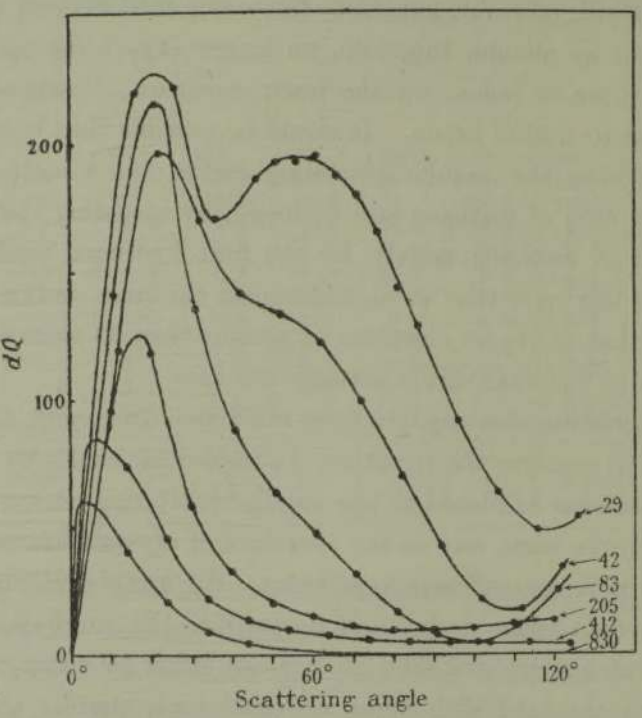

FIG. 4.—Scattering per unit angle in Neon.

per unit angle in Ne obtained by multiplying the ordinates of the neon curves in fig. 1 by $2\pi \sin \theta$. This figure clearly shows how the effective elastic cross section decreases as the velocity of the electrons increases.

In fig. 5 the elastic cross section in arbitrary units is plotted against the velocity of the electrons in $\sqrt{\text{volts}}$. It should be realised that these curves indicate only very roughly the variation of the elastic cross section with velocity for the following reasons :—

(1) The curves representing the scattering per unit angle have only been integrated from 0° to 120° instead of from 0° to 180°, *i.e.*, elastic collisions in which the electrons are scattered through angles greater than 120° are not included.

(2) There is an uncertainty in the extrapolation of some of these curves to 0° owing to readings not having been taken down to sufficiently small angles to determine the point where the curves bend over.

In spite, however, of these defects it appears worth while to consider these cross section curves for elastic collisions in relation to the total effective cross section

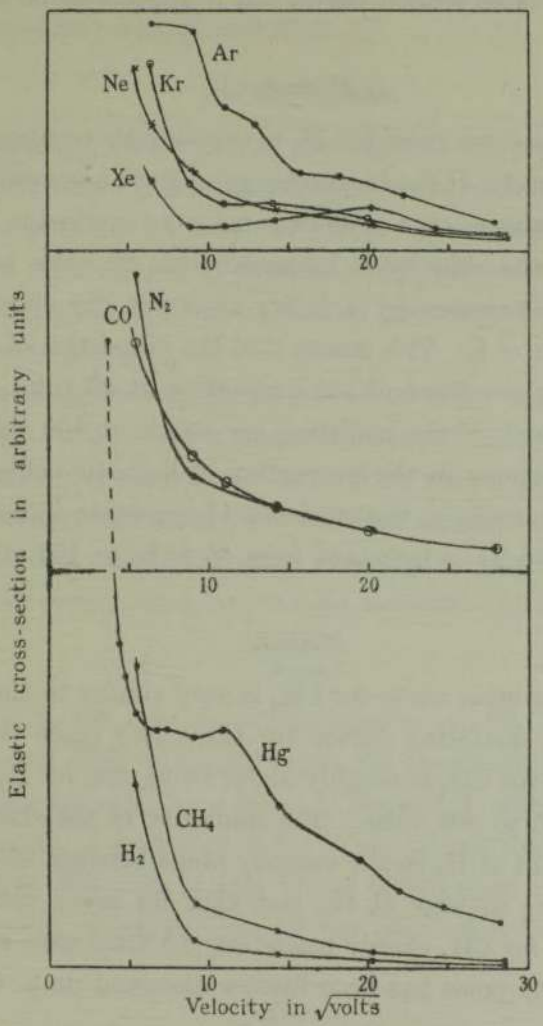

FIG. 5.—The Elastic Cross Section Curves.

curves, or absorption coefficient curves, which include both elastic and inelastic collisions.* The latter curves will be referred to briefly as " the total cross section curves," while the former will be called the " elastic cross section curves."

Mercury Vapour.

Fig. 5 also contains the elastic cross section curve for Hg vapour calculated from the author's results for Hg vapour which have already been published.†

* For literature and summary of results see Kollath, ' Physik. Z.,' vol. 31, p. 985 (1930).
† Arnot, ' Proc. Roy. Soc.,' A, vol. 130, p. 655 (1931).

The elastic cross section curve shows the slight maximum and steep rise found by Brode* and others for the total cross section. The whole curve is very similar to the total cross section curve, but it appears to be shifted to higher velocities.

Hydrogen.

The elastic cross section curve for H_2 is very similar to Normand's curve for the total cross section.† It shows the change in slope observed by Normand at about 81 volts, but this change of slope is far more marked in the elastic cross section curve. As the velocity is increased from 25 volts to 100 volts, the total cross section decreases by a factor of 3 but the elastic cross section decreases by a factor of 9. This means that the proportion of elastic collisions at 100 volts is only one-third of the proportion at 25 volts. Therefore, not more than 33 per cent. of the collisions are elastic at 100 volts. We should expect this large increase in the proportion of inelastic collisions from other considerations; for instance, the efficiency of ionisation increases by a factor of 2·7 when the velocity is increased from 25 volts to 100 volts.‡

Methane.

The elastic cross section curve for CH_4 is very similar to that for H_2, as are also the respective scattering curves for these two gases (see fig. 3). The elastic cross section for CH_4 is roughly about twice that for H_2 throughout the velocity range of 25 to 800 volts. The similarity of the elastic cross section curve for CH_4 to that of H_2 in the velocity range between 25 and 800 volts is especially interesting in view of the fact that for lower velocities the total cross section curve for CH_4 closely resembles the total cross section curves of the rare gases. This point has been further discussed on p. 625.

Nitrogen and Carbon Monoxide.

The elastic cross section curves for N_2 and CO are both almost identical and are of about the same absolute value, as are also the total cross section curves (Normand, *loc. cit.*). There are not sufficient points on the curve to decide whether the small maximum found by Normand in N_2 at 144 volts is due to elastic or inelastic collisions. As the velocity is increased from 36 to 400 volts the total cross section decreases by a factor of about 4 and the elastic cross

* Brode, 'Proc. Roy. Soc.,' A, vol. 125, p. 134 (1929).
† Normand, 'Phys. Rev.,' vol. 35, p. 1217 (1930).
‡ Compton and van Voorhis, 'Phys. Rev.,' vol. 27, p. 729 (1926).

section decreases by a factor of about 5·6. This means that the proportion of elastic collisions at 400 volts is about 71 per cent. of the proportion at 36 volts, *i.e.*, the ratio of elastic to inelastic collisions does not vary very much throughout this range of velocities.

The Rare Gases.

The most interesting feature about the elastic cross section curves for the rare gases is the presence of a small maximum (or change of slope) which appears in all the curves between 100 and 400 volts. The maximum is most clearly defined in Xe, in which gas it appears at about 400 volts. In Kr it appears at about 144 volts, while in Ar and in Ne its presence is indicated merely by a change in slope of the curve.

Another interesting feature of the elastic cross section curves of the rare gases is that at 50 volts Ar has a greater elastic cross section than Kr, and Kr a greater one than Xe, whereas we know from the results of Ramsauer that where the total cross section curves reach their maximum value, in the neighbourhood of 10 volts, the above order of the gases is reversed, Xe having a greater total cross section than Kr, and Kr a greater one than Ar.

Neon.

Dealing now with the rare gases individually, we find that for Ne, as the velocity is increased from 36 to 400 volts, the total cross section as found by Normand (*loc. cit.*) decreases by a factor of about 2·7, whereas the elastic cross section decreases by a factor of 5·3. This means that the proportion of elastic collisions at 400 volts is only about 50 per cent. of the proportion at 36 volts. Therefore not more than 50 per cent. of the collisions are elastic at 400 volts.

Argon.

For Ar we find that, as the velocity is increased from 36 to 400 volts, the total cross section (Normand's curve) decreases by a factor of about 3·5 and the elastic cross section decreases by about the same factor. Thus the ratio of elastic to inelastic collisions does not vary much throughout this range of velocities. Normand's curve also shows the change in slope observed at about 225 volts.

Krypton and Xenon.

I have not been able to find any data on the total cross section for Kr and Xe within the range of 36 to 400 volts, and so no conclusions regarding the proportion of elastic to inelastic collisions can be drawn.

We see from the above that in the two lightest gases used, H_2 and Ne, the proportion of elastic collisions decreases considerably as the velocity is increased, while in the heavier gases N_2, CO and Ar, the ratio of elastic to inelastic collisions does not vary much throughout the entire velocity range of 36 to 400 volts.

The actual figures given will probably need to be revised when the elastic cross sections have been more accurately determined, but the main features outlined above should not be seriously affected.

Before leaving this subject it may be well to clear up a point which has caused some confusion in the past. It has long been known that remarkably consistent measurements of the total effective cross section, or absorption coefficient, have been obtained with widely different types of apparatus. The usual methods involve measuring the decrease in intensity of an electron beam after travelling a fixed distance through the gas. The geometry of the final collecting slits determines the maximum angle through which an electron may be deflected without being measured as absorbed. It would be expected, therefore, that measurements made with apparatus having different sized slits would give widely different results if a large fraction of the total number of scattered electrons were deflected through small angles. Since, however, consistent results are obtained, it must be concluded that only a small fraction of the scattered electrons are deflected through small angles.* This conclusion has appeared to some writers† to be in direct disagreement with the early measurements‡ made on electron scattering. This error has arisen because the results of electron scattering experiments are always plotted so as to show the scattering *per unit solid angle*. When the results are plotted so as to show the scattering between θ and $\theta + d\theta$, or *per unit angle*, we see that the number of electrons scattered through small angles is only a very small fraction of the total number of scattered electrons. This is clearly shown in fig. 4, which gives the scattering *per unit angle*.

Discussion.

Mott§ has shown that the elastic scattering of electrons by atoms can be treated as the diffraction of de Broglie waves by a static field of force, Huygens' principle being employed to determine the resultant amplitude of the scattered

* See also M. C. Green, 'Phys. Rev.,' vol. 36, p. 239 (1930).

† Brode, 'Phys. Rev.,' vol. 35, p. 504 (1930); Gaertner, 'Ann. Physik,' vol. 8, p. 135, (1931).

‡ *e.g.*, Arnot, 'Proc. Roy. Soc.,' A, vol. 125, p. 660 (1929).

§ Mott, 'Proc. Roy. Soc.,' A, vol. 127, p. 658 (1930).

wave. From the amplitude of the scattered wave we can then deduce the number of electrons scattered in any direction. The method employed is to find a solution of the general wave equation of Schrödinger such that the solution represents an incident wave falling on the atom together with a diffracted wave emerging from the atom. This solution is in the form of an integral equation. From this equation it is seen that the scattered wave is formed by the interference of secondary wavelets, each of these secondary wavelets orginating from an element of volume (dx, dy, dz) of the atomic field, with an amplitude at a distance R from the scattering centre of

$$\frac{1}{\mathrm{R}}\frac{2\pi m}{h^2}\,\mathrm{V}\,(xyz)\,\psi\,(xyz)\,dx\,dy\,dz \tag{1}$$

for an incident wave of unit amplitude.

In order to proceed further, Born assumed that the incident wave is not appreciably distorted within the atom. With this assumption (1) reduces to

$$\frac{1}{\mathrm{R}}\frac{2\pi m}{h^2}\,\mathrm{V}\,(xyz)\,dx\,dy\,dz. \tag{2}$$

Changing over to spherical polar co-ordinates we find that the amplitude of the resultant wave at a large distance R from the scattering centre, and in a direction making an angle θ with the direction of the incident wave is

$$\left.\begin{aligned} &\mathrm{R}^{-1}f\,(\theta) \\ \text{where}\quad &f\,(\theta)=\frac{8\pi^2 m}{h^2}\int_0^\infty \frac{\sin\mu r}{\mu r}\,\mathrm{V}\,(r)\,r^2\,dr \\ \text{and}\quad &\mu=4\pi\sin\tfrac{1}{2}\theta/\lambda \end{aligned}\right\}, \tag{3}$$

λ being the de Broglie wave-length of the incident electrons. The scattered intensity I_θ, defined on p. 620 is then given by

$$\mathrm{I}_\theta=[f\,(\theta)]^2. \tag{4}$$

In order to evaluate the integral in (3), we require to know V (r), which is the potential energy of one of the incident electrons at a distance r from the centre of the field of force of the scattering atom. Approximate expressions for V (r) have been given by Thomas,* Fermi,† and a more accurate one by Hartree.‡ Bullard and Massey,§ using the Thomas-Fermi expression for V (r), have

* Thomas, ' Proc. Camb. Phil. Soc.,' vol. 23, p. 542 (1926).

† Fermi, ' Z. Phys.,' vol. 48, p. 73 (1928).

‡ Hartree, ' Proc. Camb. Phil. Soc.,' vol. 24, p. 89 (1928).

§ Bullard and Massey, ' Proc. Camb. Phil. Soc.,' vol. 26, p. 556 (1930).

evaluated (3) and so determined I_θ. They have shown that $I_\theta/Z^{2/3}$ is a function of $v \sin \frac{1}{2}\theta/Z^{1/3}$ only, where v is the velocity of the incident electrons and Z the atomic number of the scattering atoms. Thus the scattering produced by all atoms for all velocities and all angles can be represented on a single curve. A table is given from which this curve can be constructed.

Born's formula (3) gives a scattering curve for all atomic fields which falls off monotonically with increasing angle, and therefore fails to explain the results given in this paper for low velocities, which show maxima and minima. This failure of Born's formula for low velocities is partly due to the assumption, made in deducing it, that the incident wave is not distorted by the field of the atom.

It will be seen, however, from fig. 2 that as the velocity is increased the maxima and minima gradually disappear, and it might therefore be expected that Born's formula might fit the higher velocity curves of fig. 2. In fig. 6

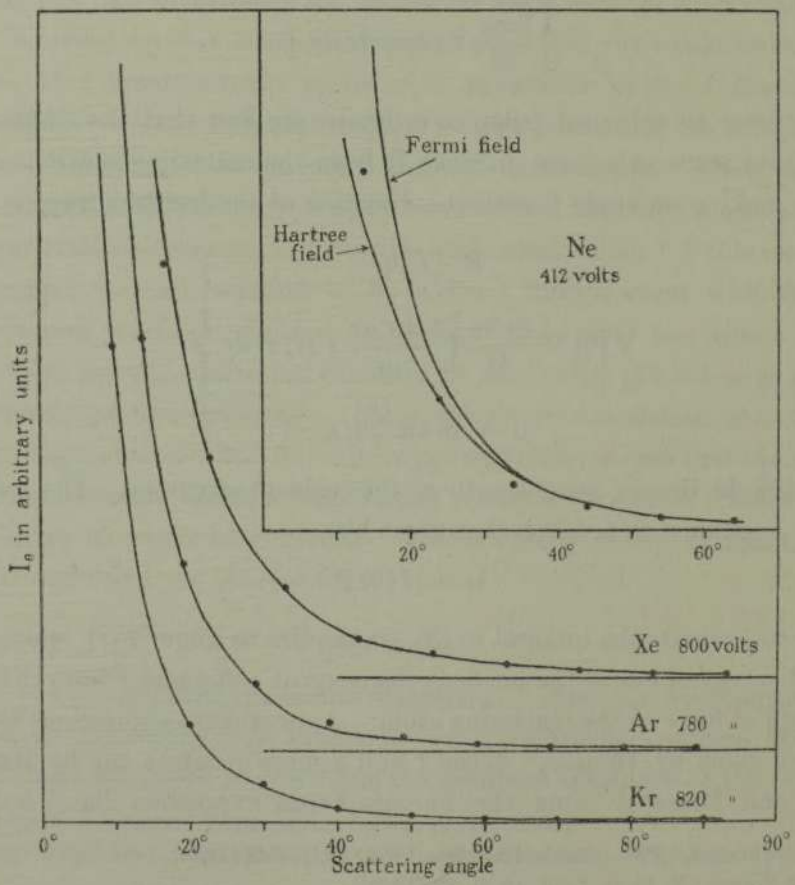

FIG. 6.—Experimental Points fitted to the Born Scattering Curves for Xenon, Krypton Argon and Neon.

the Born scattering curves for 800-volt electrons in Xe, 780-volt electrons in Ar, and 820-volt electrons in Kr, are constructed from the table given in Bullard and Massey's paper. My experimental points have been fitted to each of these curves, and it will be seen that the agreement is remarkably good, especially in view of the fact that in Xe and Kr the maximum beyond 90° has not quite disappeared. It should be mentioned here that the order in which the curves are placed in fig. 6 is not significant, as they are not all drawn to the same scale.

The Born scattering curves, using respectively the Thomas-Fermi and the Hartree fields, have also been drawn in fig. 6 for 412-volt electrons in Ne. It will be seen that the experimental points fit the Hartree field curve more closely than they do the Thomas-Fermi field curve. This is only to be expected, since the Thomas-Fermi field, though differing only very slightly from the Hartree field for heavy atoms like Xe and Kr, is not at all accurate for light atoms.

The conclusion to be drawn from fig. 6 is that the Born approximation is justified for these relatively high velocities, and consequently that the de Broglie waves of electrons of these velocities are not seriously distorted by the atomic field of the atom.

Massey has applied Born's formula to the case of molecular hydrogen* and nitrogen,† and has informed me that satisfactory agreement is obtained with my experimental curves shown in fig. 3. The calculations, however, are not able to account for the minimum at about 90° observed in the lower velocity curves in these gases.

As has been stated above, Born's formula fails at the lower velocities, and is unable to account for the maxima and minima observed in the scattering curves of the rare gases shown in fig. 2. For the explanation of these curves we must seek a more exact solution of the "integral wave equation" representing the incident and scattered waves. Faxen and Holtsmark‡ have given the exact solution of this equation, using, instead of the approximate expression (2), the expression (1) which takes into account the distortion of the incident wave within the atom. By this means Holtsmark§ has successfully accounted for the Ramsauer effect in argon and krypton.

* Massey, 'Proc. Roy. Soc.,' A, vol. 129, p. 616 (1930).

† *Unpublished.*

‡ Faxen and Holtsmark, 'Z. Phys.,' vol. 45, p. 307 (1927).

§ Holtsmark, 'Z. Phys.,' vol. 55, p. 437 (1929); *ibid.*, vol. 66, p. 49 (1930).

The expression obtained for the scattered intensity is

$$I_\theta = \frac{a^2}{p^2} \sum_{l=0}^{\infty} \sum_{l'=0}^{\infty} (2l+1)(2l'+1) \cos(\delta_l - \delta_{l'}) \sin\delta_l \sin\delta_{l'} P_l(\theta) P_{l'}(\theta), \quad (5)$$

where

$$p = \frac{2\pi mva}{h}$$

and a is the " radius " of the hydrogen atom $= 0{\cdot}53 \times 10^{-8}$ cm. Holtsmark,* using the Hartree field with a correction applied for polarisation, has calculated the " phase variations," δ, in Kr for values of l from 0 to 4 for several values of p.

The angular scattering curve shown in fig. 7 has been calculated from equation (5) above for a velocity of 54 volts (corresponding to $p = 2{\cdot}0$) using

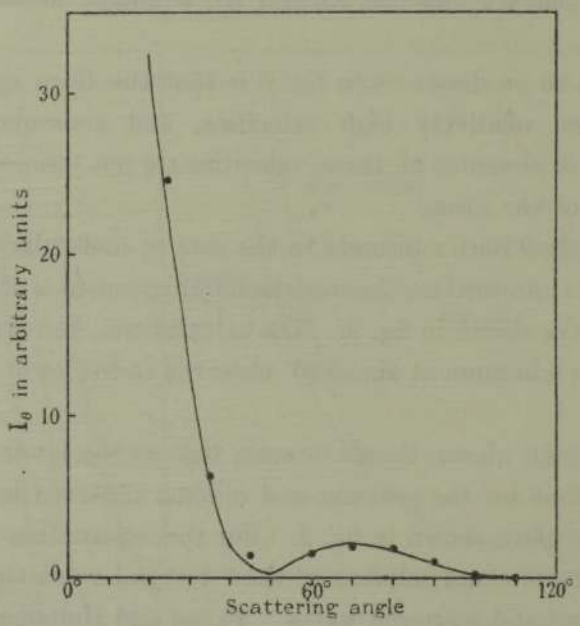

Fig. 7.—Experimental Points fitted to Holtsmark's Theoretical Curve for 54-volt Electrons in Krypton.

the values of δ from $l = 0$ to $l = 4$ given for the atomic field No. 2 in Table III of Holtsmark's paper. The values of I_θ from which this curve is drawn are given in Table II, column 4, and Holtsmark's absolute values for I can be obtained by multiplying these values by $33{\cdot}8 \times 10^{-18}$ cm.^{2} My experimental points for 54-volt electrons are shown fitted to this curve in fig. 7. The good agreement obtained shows that the experimental results containing maxima

* Holtsmark, 'Z. Phys.,' vol. 66, p. 49 (1930).

and minima, which Born's formula is unable to account for, are satisfactorily explained, at any rate in this particular case, by the exact solution of the wave equation.

Allis and Morse,* using the exact solution of the wave equation and a simplified atomic field in place of the Hartree field, have calculated the total effective cross section for elastic collisions for a number of different atoms. The cross sections are obtained from equation (5) by integrating over the whole sphere. Their method, which is much simpler to apply than Holtsmark's owing to the idealised atomic field, gives curves which agree very well with the experimental ones as regards size and form. They have since calculated from equation (5) the angular scattering curves for a number of different voltages in argon. These calculations are not yet published, but they have kindly informed me that satisfactory agreement is obtained with my experimental curves, shown in fig. 2.

It is probable, however, that even the exact solution of the wave equation will fail to account successfully for the experimental results at still lower velocities owing to the electron exchange effect. The possibility of interchange between the atomic and colliding electrons has been considered by Oppenheimer† and by Mott,‡ who have shown that interference effects are to be expected, and recently Massey and Mohr§ have shown that the exchange effect is of fundamental importance in the scattering of electrons of from 4 to 15 volts energy in helium.

When the contributions to the scattering due to the exchange terms have been worked out and included in the expression for the scattering found from the exact solution of the wave equation, and when the Hartree fields have been evaluated for the different atoms, and suitably modified so as to account correctly for the polarisation forces, then we may look with some confidence to a full explanation of the diffraction effects in the monatomic gases described in this paper.

Summary.

The angular distributions of the elastically scattered electrons in Xe, Kr, Ar, Ne, H_2, N_2, CH_4 and CO have been measured for several different velocities of the primary beam between 30 and 800 volts. Diffraction effects are observed

* Allis and Morse, 'Z. Phys.,' vol. 70, p. 567 (1931).

† Oppenheimer, 'Phys. Rev.,' vol. 32, p. 361 (1928).

‡ Mott, 'Proc. Roy. Soc.,' A, vol. 125, p. 222 (1929).

§ Massey and Mohr, 'Proc. Roy. Soc.,' A, vol. 132, p. 605 (1931), and later unpublished work.

in all these gases, the curves of the rare gases showing several maxima and minima, while the lower velocity curves for the molecular gases all have a minimum in the region of 90°.

In the higher velocity curves the diffraction maxima and minima have practically disappeared in the case of Xe and Kr, and have completely disappeared in the case of the other gases, showing that the refractive index of the atom is practically unity for these short wave-lengths.

The high velocity curves of the rare gases all agree well with the Born scattering formula. The 54-volt curve in Kr is found to be in very good agreement with the theoretical scattering curve calculated from Faxen and Holtsmark's scattering formula.

The effective cross section curves for elastic collisions have been determined by graphical integration of the angular distribution curves. The cross section curves give information concerning the variation with velocity of the proportion of elastic to inelastic collisions. The work is being extended to other gases and vapours, and to smaller and larger scattering angles.

This work was carried out during my last year at the Cavendish Laboratory, Cambridge; and it gives me great pleasure to acknowledge here the kindly assistance given me by Professor Lord Rutherford, Mr. P. M. S. Blackett, Dr. J. Chadwick, and other members of the laboratory.