



The direct adaptive trajectory control of robot manipulators via fuzzy logic algorithm

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ABSTRACT

The paper is addressed to study the trajectory control of robotic mechanisms, particularly, to the transient performance of path tracking control systems. The coefficients and terms of the robot dynamic equations may be partly unknown and some dynamic effects may be unmodeled. One general scheme, which is a combination of the generalization of the computed torque method and the velocity gradient technique, provides a simple way to obtain some known and unknown parameter adaptive control laws, in particular, the exponential path tracking adaptive control algorithms with the important properties of robustness to bounded disturbances and unmodeled dynamics. The algorithms require the on-line measurements of the tracking error and its first derivative. The persistent excitation assumptions for desired paths are not required. The fuzzy logic control strategy is applied to obtain implementable algorithms on the base of the developed adaptive laws. The approach includes the attractive features of both strategies: robustness from direct adaptive control, and simple implementability from fuzzy logic control.

INTRODUCTION

The highly nonlinear and coupled nature of robot dynamics causes necessity of development of effective controllers, which can cope with nonlinear properties and the effects of centrifugal, Coriolis and gravity forces. The control problems become complicated in the face of uncertainty on the parameters describing the grasped



payload, such as moments of inertia or exact position of the center of mass in the end-effector, or even its mass in the case of the set of payloads. The development of high-speed and high-precision industrial robots are encouraged to design adaptive controllers for path tracking control with ability to provide high-quality transient performance, e.g. Ortega and Spong [1]. Recently, Song, Middleton and Anderson [2] suggested to provide exponential stability to the path tracking control systems. The exponential stable systems possess the properties of exponential convergence and robustness to bounded disturbances and unmodeled dynamics. The authors of paper [2] offered several interesting schemes designed on the basis of direct adaptive control strategy without the requirement of persistent excitation. Their adaptive control laws do not require the measurement of acceleration. The control depends only on the tracking error in the opposite to the composite adaptive control scheme by Slotine and Lee [3] which requires on-line calculations of prediction error by time consuming technique. Furthermore, the composite adaptive controller ensures exponential path tracking under the condition of persistent excitation as the most of existent adaptive control schemes.

Adaptive control strategy can be regarded as strict mathematical theory. For instance, direct adaptive control is based on the theory of ordinary differential equations, functional analysis and Lyapunov function method. The stability and the robustness of system performance must be proved mathematically. Furthermore, there are several common schemes for adaptive algorithm design reviewed by Bortsov, Polyakhov and Putov [4]. One of most effective scheme is the velocity gradient method [5],[6] pioneered by Fradkov. But the implementation issue of developed algorithms is still their weak point.

On the other hand, fuzzy control algorithms are nonmathematical ones, but it has been proved that these algorithms can be simply implemented whenever the dynamic properties of controlled system are not well defined or modelled, e.g. Sugeno [7], Tong [8], Hwang and Lin [9].

The first purpose of this paper is the study on the design of the exponential path tracking adaptive control algorithms via velocity gradient method. We design attractive algorithms offered in [2] and some new control laws.

The second purpose of our paper is an attempt to combine the direct adaptive and fuzzy control principles to obtain simple and well implementable control algorithm. We develop our controller on the base of very attractive bang-bang algorithm, in which the switches depend on the tracking error e and its first derivative \dot{e} .

The paper consists of five sections. The design of the parameter adaptive controllers for robotic applications are discussed in the introduction. The second section includes the description of basic dynamic models and properties which are important for the adaptive control design. The parameter adaptive control design is considered in the third section. The fuzzy adaptive algorithm is the subject of the fourth section. The results are analyzed and needed investigations are outlined in the conclusion.

THE PROPERTIES OF RIGID ROBOT DYNAMICS

The nonlinear dynamics of rigid robot is presented by well known Lagrange equations:

$$H(q; \hat{p})\ddot{q} + C(q, \dot{q}; \hat{p})\dot{q} + G(q; \hat{q}) + f_d = \tau, \quad (1)$$

where $q \in R^n$ is the vector of generalized coordinates, $\hat{p} \in R^m$ is the vector of equivalent system parameters, $H(q; \hat{p}) \in R^{n \times n}$ is the inertia matrix, $C(q, \dot{q}; \hat{p}) \in R^{n \times n}$ is the matrix, which reflects Coriolis and centrifugal forces, $G(q; \hat{q} \in R^n$ is the gravity force vector, f_d is the vector which represents unmodelled dynamic effects.

Assume that the bounds of q, \dot{q}, \hat{p} and f_d are given as follows:

$$q_i^- \leq q_i \leq q_i^+, (-\infty < q_i^- \leq q_i^+ < \infty), i = 1, \dots, n, \quad (2)$$

$$\dot{q}_i^- \leq \dot{q}_i \leq \dot{q}_i^+, (-\infty < \dot{q}_i^- \leq \dot{q}_i^+ < \infty), i = 1, \dots, n, \quad (3)$$

$$\hat{p}_j^- \leq \hat{p}_j \leq \hat{p}_j^+, (-\infty < \hat{p}_j^- \leq \hat{p}_j^+ < \infty), j = 1, \dots, m, \quad (4)$$

$$f_{di}^- \leq f_{di} \leq f_{di}^+, (-\infty < f_{di}^- \leq f_{di}^+ < \infty), i = 1, \dots, n, \quad (5)$$

Direct adaptive control design is based on the fact that the members of the equation (1) are not independent, but connected by some important relations (properties) [7].

Property 1.

For all q and \hat{p} , the matrix $H(q; \hat{p})$ is always symmetric positive defined (PSD).

Property 2.

For all q, \dot{q}, \hat{p} , for $\forall z \in R^n$ and for proper definition of matrix $C(q, \dot{q}, \hat{p})$ we have the following relation:

$$z^T[\dot{H}(q, \hat{p}) - 2C(q, \dot{q}; \hat{p})]z = 0 \quad (6)$$

Property 3.

For $\forall r \in R^n$ we have the following linear parametrization relation:

$$H(q; \hat{p})\dot{r} + C(q, \dot{q}; \hat{p})r + G(q; \hat{p}) = \Phi(q, \dot{q}, r, \dot{r})\hat{p} \quad (7)$$

where $\Phi(q, \dot{q}, r, \dot{r}) \in R^{n \times m}$ is a known matrix independent of equivalent parameters of the system.

Stronger relation than (7) was exploited in [2]:

$$H(q; \hat{p})x + C(q, \dot{q}; \hat{p})y + G(q; \hat{p}) = Y(q, \dot{q}, x, y)\hat{p}, \quad (8)$$

where $\dot{y} \neq x (x \in R^n, y \in R^n)$, $Y(q, \dot{q}, y, x)$ is a known matrix independent of the system parameters and, generally, not equal to matrix $\Phi(\cdot)$ from (7).

ADAPTIVE EXPONENTIAL PATH TRACKING CONTROL DESIGN

Our objective is to design a control torque such that actual trajectory of system, defined by q and \dot{q} , converges exponentially to the desired trajectory, defined by continuous and bounded functions q^* and \dot{q}^* , i.e. for tracking error $e(t) = q(t) - q^*(t)$ and its first derivative $\dot{e}(t) = \dot{q}(t) - \dot{q}^*(t)$ we demand

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (9)$$

$$\lim_{t \rightarrow \infty} \dot{e}(t) = 0 \quad (10)$$

with the convergence rate is at least $e^{-\lambda t} (\lambda > 0)$, where λ can be arbitrarily specified.

The velocity gradient method [5, 6] is our tool for control design. It is briefly described here.

Let us have an equation for generalized plant:

$$\dot{x} = F(x, \theta, t), t \geq 0 \quad (11)$$

where $x \in R^n$ is a state vector, $F(\cdot)$ is a continuous and continuously differentiable by x and θ vector-function.

We will consider design of control algorithms in the following form:

$$\theta = \theta(x_0^t, \theta_0^t, t), \quad (12)$$

which is providing the minimization of a performance criterion:

$$Q = Q(x_0^t, \theta_0^t, t). \quad (13)$$

It is possible to use two general forms of the performance criterion.

Case 1: a local form

$$Q = Q(x(t), \theta(t)), \quad (14)$$

where $Q(x, t) \geq 0$ is a scalar function.

Case 2: an integral form

$$Q = \int_0^t q(x(s), \theta(s), s) ds. \quad (15)$$

For both cases function $\omega(x, \theta, t)$ is the velocity of function Q change on the trajectories of the system (11), i.e.

$$\omega(x, \theta, t) = \frac{d}{dt} Q(x, t). \quad (16)$$

The velocity gradient method (VGM) provides the movement along the gradient of the function $\omega(x, \theta, t)$ by θ :

$$\frac{d}{dt} [\theta(t) + \psi(t)] = -\Gamma \nabla_{\theta} \omega(x, \theta, t), \quad (17)$$

where $\psi(\cdot) : \psi^T \cdot \nabla_{\theta} \omega \geq 0$, Γ is SPD.

It is possible to write the equation (17) in the integral form:

$$\theta = -\psi(x, \theta, t) - \int_0^t \Gamma \nabla_{\theta} \omega(x(s), \theta(s), s) ds. \quad (18)$$

The conditions of stability of the algorithms (17) and (18) have been given in [6].

The design procedure can be described by the following five steps.

1. To give the control goal and the performance criterion.
2. To choose the update parameter vector.
3. To form the velocity gradient vector.
4. To choose one of the possible forms (17) or (18) for VGM.
5. To check the validity of stability conditions.

To apply this method for design of exponential stable control algorithms we can exploit the following fact: if the system related with (11) by the expression:

$$\dot{\psi} = e^{\lambda t} x, \lambda > 0, \quad (19)$$

is stable (bounded or asymptotically stable), the system (11) is exponentially stable [2].

In the control law design, we start with a modification of computed torque method:

$$\tau = H(q; p)x + C(q, \dot{q}; p)y + G(q; p) - K_D w, \quad (20)$$

$$w = \dot{e} + D e, D = \text{diag}(d_1, \dots, d_n), \quad (21)$$

$$x = \ddot{q}^* - \lambda w - D \dot{e}, \quad (22)$$

$$y = \dot{q}^* - D e, \quad (23)$$

where D is SPD, $\lambda > 0$ is a scalar.

Using the property 3 (see (8) with (20)-(23) we obtain

$$\tau = \sum_{i=1}^m Y_i(q, \dot{q}, x, y) p_i - K_D w, \quad (24)$$

where $Y_i(\cdot) \in R^n, i = 1, \dots, m$ are known vectors.

The combination of (1) and (24) yields the equation for closed-loop system:

$$H(q; \hat{p})(\dot{w} + \lambda w) + C(q, \dot{q}; \hat{p})w + f_d = \sum_{i=1}^m Y_i(q, \dot{q}, x, y)(p_i - \hat{p}_i) - K_D w, \quad (25)$$

Using an auxiliary function:

$$s = e^{\lambda t} w$$

the equation (25) can be rewritten as

$$H(q; \hat{p}) \dot{s} = -C(q, \dot{q}; \hat{p}) s - e^{\lambda t} f_d + \sum_{i=1}^m e^{\lambda t} Y_i(q, \dot{q}, x, y)(p_i - \hat{p}_i) - K_D s \quad (26)$$

A performance criterion is chosen as

$$Q = \frac{1}{2} [s^T H s]. \quad (27)$$

Thus

$$\omega(q, \dot{q}, p) = s^T \left(-e^{\lambda t} f_d + \sum_{i=1}^m e^{\lambda t} Y_i(q, \dot{q}, x, y)(p_i - \hat{p}_i) - K_D s \right), \quad (28)$$

and

$$\nabla_p \omega = \sum_{i=1}^m e^{2\lambda t} w^T Y_i(q, \dot{q}, x, y). \quad (29)$$

We can define a member of the sum in the right part of (29) as

$$P_i(w, \lambda) = e^{2\lambda t} w^T Y_i(q, \dot{q}, x, y). \quad (30)$$

On the base of VGM in the form (18) and the condition

$$\psi^T \nabla_p \omega \geq 0 \quad (31)$$

we can obtain several adaptive algorithms:

Algorithm 1: $\psi \equiv 0$,

$$p_i = -\gamma_i \int_0^t P_i(w, \lambda) d\sigma, \quad (32)$$

Algorithm 2: $\psi = \nabla_p \omega$,

$$p_i = -\gamma_{2i} P_i(w, \lambda) - \gamma_{1i} \int_0^t P_i(w, \lambda) d\sigma, \quad (33)$$

Algorithm 3: $\psi = \text{sign} \nabla_p \omega$

$$p_i = -\gamma_i \text{sign}(P_i) \quad (34)$$

where $i = 1, \dots, m$.

Using $\gamma_{1i} \equiv 0$ in (33) leads to the proportional adaptive law :

$$p_i = -P_i(w, Y_i, \lambda), \quad (35)$$



Taking into account the bounds of \hat{p}_i given by (2.4) and demanding symmetry of adaptive law, we obtain the bang-bang proportional algorithm from (35) as

$$p_i = \begin{cases} \hat{p}_i^+ & \text{if } P_i(w, Y_i, \lambda) < -\theta \\ \hat{p}_i^+ - \frac{\hat{p}_i^+ - \hat{p}_i^-}{2\theta} (P_i(w, Y_i, \lambda) + \theta) & \text{if } -\theta < P_i(w, Y_i, \lambda) < \theta \\ \hat{p}_i^- & \text{if } P_i(w, Y_i, \lambda) > \theta \end{cases} \quad (36)$$

where $\theta > 0$ is a small constant, which can be arbitrary specified.

The algorithms (32), (33) and (36) are somewhat similar to that of Song, Middilton and Anderson, but they have been obtained on the base of VGM common scheme. The algorithms (34), (35) are new ones. The algorithms (32), (33) have the integral(I) and proportional+integral(PI) forms respectively. Using VGM we can also obtain PID adaptive law [11].

From (34) we obtain the bang-bang algorithm as

$$p_i = \begin{cases} \hat{p}_i^+ & \text{if } P_i(w, Y_i, \lambda) < 0 \\ \hat{p}_i^- & \text{if } P_i(w, Y_i, \lambda) > 0 \end{cases} \quad (37)$$

where $i = 1, \dots, m$.

FUZZY ADAPTIVE ALGORITHM

The basis for fuzzy adaptive algorithm is (37). We assume that $\text{sign}Y_i > 0$. The inputs of proposed fuzzy controller are E and \dot{E} which are fuzzified variables of the path tracking error e and path velocity error \dot{e} respectively. We can measure e and \dot{e} and do not have to use approximation algorithms to obtain \dot{E} . The output of our controller δU_i is the fuzzified change of $\delta p_i = p_i(t) - p_i(t - 1)$. All the universes of discourse for $E, \dot{E}, \delta U_i$ are arranged from -1 to 1, and scaling factors K_1, K_2, K_3 for $E, \dot{E}, \delta U_i$ must be chosen to fit the universe of discourse for each variable.

Each fuzzy variable is quantized into seven qualitative fuzzy variables: Positive Big (PB), Positive Medium (PM), Positive Small (PS), Zero (ZE), Negative Small (NS), Negative Medium (NM), Negative Big (NB). The triangular membership function for each fuzzy variable is chosen. We are able to use the center of gravity method [7] to form a look-up table for the algorithm (37).

The two degrees of freedom, planar robot arm with two revolute joints is chosen as a plant to examine the validity of control strategy. The set of test tasks includes: the pick up of the bounded but unknown payload, the trajectory control with bounded values of mass parameters of the robot links with or without external disturbances.

The software developed includes the simulation programs, and the rule and fuzzy set editors. It is able to create a codification file, which is downloaded via the serial port or programmed in an EPROM memory of an 8096 microcontroller. It executes the following scan cycle:

- 1) To assign membership values
- 2) To evaluate the rules to determine the membership values of the consequence sets
- 3) To calculate the value of the outputs.



The algorithm does not impose constraints on the number of inputs, outputs, sets or rules, but those are imposed by the hardware because of a bounded memory. Anyway the available memory is suitable for most control applications. It is important to remark that the rules must have the following syntax:

IF ... (AND ... (AND ... (...))) THEN ... (AND ... (...))

which is sufficient for most applications.

CONCLUSION

We outlined the process of the fuzzy adaptive control design for robot path tracking applications. Our objective was to obtain implementable control algorithm with low cost of computations and proper performance characteristics. The software for the research and implementation of suggested algorithm was developed. The results of simulation and the experiments with microprocessor controller, which are in progress, are able to show us the real advantages and needed modifications of our control strategy. We are also examining the possibility to use a fuzzy process model like an observer in adaptive control.

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