The direct application of FFT algorithms on the synthesis of non-uniformly spaced arrays

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Abstract

In previous work, we presented one efficient method applying the FFT (Fast Fourier Transform) algorithms to the computation of non-uniformly spaced antenna array factors, based on the Fourier relation between the array factor and its source distribution. Using the grid in the spatial domain, the element positions of one non-uniformly spaced array are set to another equi-spaced array with the smaller spacing. Then, the conventional IFFT (Inverse FFT) algorithm is used to compute its array factor. According to the reciprocal property of the Fourier transform, the direct Fourier transform can be applied in the synthesis problem of non-uniformly spaced arrays. If the array factor of one non-uniformly spaced array is completely given in the respective region, the correspondent array excitation can be obtained by directly applying the FFT after using the sampling theorem. If it is not, the synthesized array will not be exactly as desired after directly applying the FFT algorithm. In this case, the array element positions are chosen to be those where the array element distribution is concentrated. To achieve the more approximated array factor, we can use some methods to modify the array element currents, for example, the matrix relation between the array factor samples and the array element currents. Thus, in this paper we show the possibility of the FFT algorithm application directly in the synthesis of non-uniformly spaced arrays and demonstrate how to synthesize one non-uniformly spaced array from the samples of array factors under the Fourier relation. Of course, this method can also be used in the synthesis of uniformly spaced arrays, which is viewed as the particular problem. Due to the application of the FFT algorithm, the advantages such as fast computation, easy use and so on are obvious. Furthermore, this method permits the exact array source distribution of a non-uniformly spaced array to be obtained, given the corresponding array factor.

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1 Introduction

In all antenna array applications, the synthesis work to generate specified radiation patterns is a very important task. The Fourier relation between the array factor and its source distribution was established by one of authors and is applied in the analysis and synthesis of antenna arrays. As one powerful computation tool, the FFT algorithms are also used. As it is well known, the general applications of the FFT algorithms are just suited to a set of data with equal distance. In [1], we gave a method to apply the FFT algorithms on the analysis of non-uniformly spaced arrays. In that method, using the general sampling theory, the non-uniformly spaced array is converted to another with equal spacing on a grid in space domain. If there is no spatial position error for the array elements, the array factor, obtained by using IFFT (Inverse FFT) algorithms, will be the desired. According to the properties of Fourier transform [2], if a non-uniformly spaced array factor is given, we can use the FFT to realize the synthesis work.

There are two cases to be considered: in the first case, the array factor corresponds to a finite non-uniformly spaced array and it is given in all interval of the variable; in the second, the source distribution will be an approximation of the desired one. To the first case, it will be presented a procedure, based on FFT that permits to obtain the corresponding source distribution. To the second case, if the array factor is not completely specified, we can apply the FFT algorithm to get the element positions and currents of the non-uniformly spaced array to be synthesized and then, modify the element currents so that the synthesized array factor will be more approximated to the desired one, if this requirement is necessary. In the following, we will give the direct application of FFT algorithms on the mentioned synthesis problems under the Fourier relation and for linear arrays. The synthesis examples will also be given to show the application results.

2 Synthesis procedure

From the Fourier Relation between the array factor and its source distribution, the source distribution is the Fourier transform of array factor unless a constant. For the given array factor $\underline{F}(\beta_x)$ in β_x domain, its source distribution can be obtained by

$$\underline{S}(x) = \frac{1}{2\pi}\mathcal{F}\left[\underline{F}(\beta_x)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\underline{F}(\beta_x)e^{-j\beta_x x}d\beta_x \, , \qquad (1)$$

where $\mathcal{F}$ represents the Fourier transform, θ the angle between the x direction and the point of the far field, $\beta_x=\beta\cos(\theta_x)=\beta\cos(\theta)$ and $\beta=2\pi/\lambda$, λ being the wavelength. The visual window is $-\beta\leq\beta_x\leq\beta$.

Once the relation is the Fourier transform, as one powerful computation tool, the FFT algorithms can be used. For a uniformly spaced array, if the equal spacing is d, for applying the FFT algorithm, the chosen interval in β_x domain must be one with period equal to $2\pi/d$, for example the interval $[-\pi/d \ \pi/d]$ [3]. For a non-uniformly spaced array, after using the method presented in [1], the non-uniformly spaced array is converted to a uniformly spaced array with equal spacing d_q whose array factor is the same as that of the non-uniformly spaced array. For example, for an array factor defined in the interval $[-\pi/d_q \ \pi/d_q]$, in β_x domain, its continuous source distribution can be obtained through the Fourier transform. To use the FFT algorithms, firstly, the array factor has to be periodic. The periodicity of array factor can be realized by the convolution of this array factor with one Dirac impulse train. According to the property of Fourier transform, in space domain, the discrete array distribution will be obtained by the multiplication of continuous source distribution with one Dirac impulse train with equal spacing d_q which is the inverse Fourier transform of the impulse train in β_x domain. The process is shown in Fig. 1, where ⊗ represents the convolution and • the multiplication.

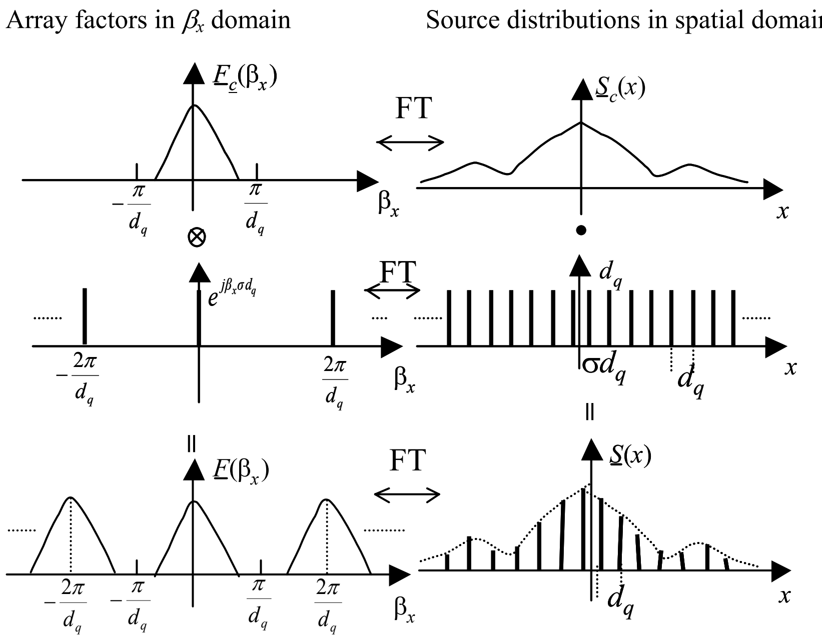


Figure 1: Computation of a discrete array using the Fourier transform.

From the process shown in Fig. 1, for an array factor defined in the interval $[-\pi/d_q \ \pi/d_q]$, its discrete array distribution is

$$\underline{S}(x) = \left[\frac{1}{2\pi}\int_{-\frac{\pi}{d_q}}^{\frac{\pi}{d_q}} \underline{F}_{\underline{c}}(\beta_x) e^{-j\beta_x x} d\beta_x\right] \sum_{m=-\infty}^{\infty} d_q \delta\left[x-(m+\sigma)d_q\right]$$
$$= \frac{d_q}{2\pi}\sum_{m=-\infty}^{\infty}\left[\int_{-\frac{\pi}{d_q}}^{\frac{\pi}{d_q}} \underline{F}_{\underline{c}}(\beta_x) e^{-j\beta_x (m+\sigma) d_q} d\beta_x\right]\delta\left[x-(m+\sigma)d_q\right] \quad (2)$$

In this equation, $\delta(x)$ is the Dirac impulse function and σd_q is the element position shift relative to the origin in x-axis with $0\leq\sigma<1$. The element position shift just has the effect on the phase of array factor, as seen in the figure.

As we know, the FFT algorithms are used to the calculation of discrete values and, generally, the array factor is given as a continuous function. Through the sampling theory, we can sample the array factor $\underline{F}(\beta_x)$. The sampling process can be carried out multiplying the array factor with a Dirac impulse train. In Fig. 2 a)~c), it is shown the discretization of $\underline{F}(\beta_x)$ in the interval $[-\pi/d_q\ \pi/d_q]$ by the use of a Dirac impulse train with equal spacing $2\pi/(Pd_q)$, where P is the total sample number in one period.

On the other side, generally, the discrete array distribution is limited. Assume that it is limited in the interval $[Ap1\ Ap2]$. Thus in order to satisfy the Nyquist theorem, to the equal spacing in the Dirac impulse train, it is needed that $Pd_q\geq Ap2\text{-}Ap1$. The sampled array factor is given by

$$\begin{aligned}\underline{F}_s(\beta_x) &= \underline{F}(\beta_x)\sum_{k=-\infty}^{\infty}\delta\left[\beta_x-\tfrac{2\pi}{Pd_q}(k+\tau)\right]\\ &= \sum_{k=-\infty}^{\infty}\underline{F}\left[\tfrac{2\pi}{Pd_q}(k+\tau)\right]\delta\left[\beta_x-\tfrac{2\pi}{Pd_q}(k+\tau)\right],\end{aligned}\tag{3}$$

where τ is the sample shift fraction relative to the origin in the β_x domain with $0\leq\tau<1$.

Now, the FFT algorithms can be applied to synthesize the discrete array. The discrete values of array factor are those in a chosen period. After the FFT application, the equal spacing is d_q and it is defined in the interval $[Ap1\ Ap2]$. If the array factor $\underline{F}(\beta_x)$ is one of non-uniformly spaced array, in the synthesized array distribution, besides existing non zero values in some element positions, the others will be zero, as shown in Fig. 2 d). After removing the zeros, we obtain the synthesized non-uniformly spaced array.

In the analysis and synthesis of antenna arrays, using the sampling theory, the adapted FFT and IFFT algorithms were developed in [3]. The synthesis FFT algorithm for discrete arrays is given in the following expression

$$\begin{aligned}\underline{S}\left[(m+\sigma)d_q\right] &= \frac{1}{P}e^{-j\tau\frac{2\pi}{P}(m+\sigma)}FFT\left\{\underline{F}\left[\frac{2\pi}{P}(k+\tau)\right]e^{-j\sigma\frac{2\pi}{P}k}\right\}\\ &= \frac{1}{P}e^{-j\tau\frac{2\pi}{P}(m+\sigma)}\sum_{k=<P>}\underline{F}\left[\frac{2\pi}{Pd_q}(k+\tau)\right]e^{-j\frac{2\pi}{P}km}e^{-j\sigma\frac{2\pi}{P}k},\\ &\begin{cases}-\dfrac{P}{2}\leq m\leq\dfrac{P-2}{2} & P\quad even\\ -\dfrac{P-1}{2}\leq m\leq\dfrac{P-1}{2} & P\quad odd\end{cases}\end{aligned}\tag{4}$$

In the next section, this algorithm will be applied to the synthesis examples in two different cases.

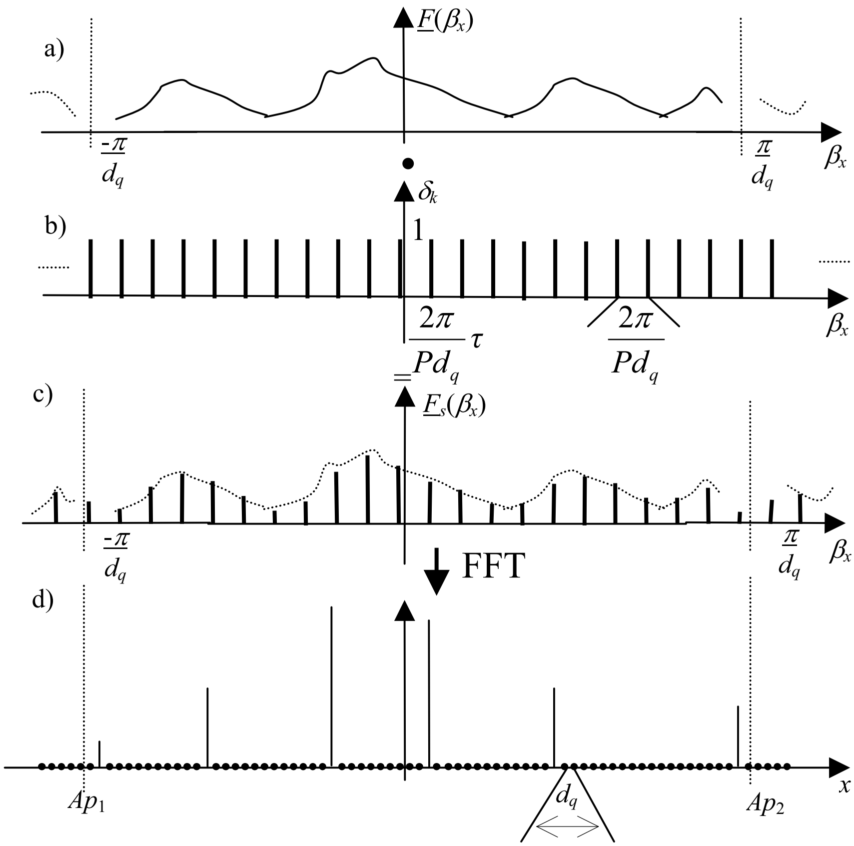
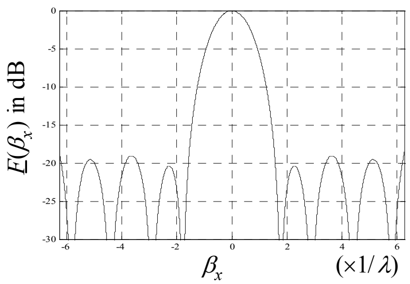


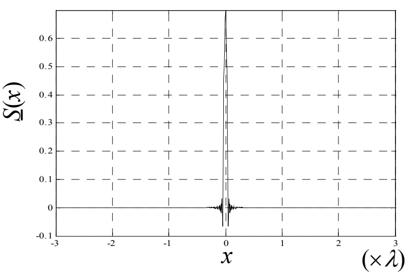
Figure 2: Sampling of array factor to FFT application: a) The continuous array factor; b) The Dirac impulse train; c) The sampled array factor and d) the synthesized array by applying FFT.

3 Synthesis examples

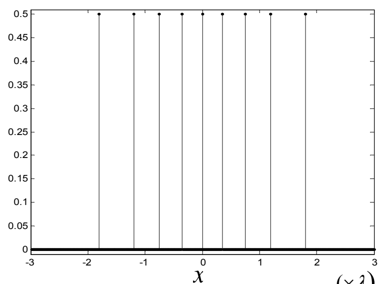
In the direct application of FFT algorithms, the main problem is the determination of the interval of synthesized array factor. As we have described before and in [1], if the equal spacing is d_q, after converting the non-uniformly spaced array into one uniformly spaced in the grid of x-axis, the array factor should be completely given in the interval $[-\pi/d_q\ \pi/d_q]$ in β_x domain. In this case, we can obtain the correspondent non-uniformly spaced array. Thus for a given array factor, first, we have to determine the corresponding period in β_x domain. Let us consider one example.



a) Desired array factor in the visual window



b) Array source distribution



c) Synthesized array

Positions x_n	Currents I_n
0	0.5
$\pm 0.35\lambda$	0.5
$\pm 0.75\lambda$	0.5
$\pm 1.2\lambda$	0.5
$\pm 1.8\lambda$	0.5

Figure 3: The desired array factor and synthesized array.

The array factor is defined by the following expression and is the approximation to the Chebyshev pattern with SLL (Sidelobe Level) equal to 20dB in the visual window, as shown in Fig. 3. a).

$$\underline{F}(\beta_x) = 0.5\frac{\sin(2.1\beta_x)}{\sin(0.3\beta_x)} + 2\cos(0.2\beta_x)\cos(0.55\beta_x) - \cos(0.6\beta_x) \quad (5)$$

If we just consider the array factor in the visual window, the source distribution in the interval [-3λ 3λ] is shown in Fig. 3 b). In this case, the source distribution is almost continuous. However, to non-uniformly spaced arrays, the array factor is defined in all β_x domain from -∞ to +∞. Through expanding the array factor or by analysing the eqn (5), we can determine the array factor period. It is in the interval [-100π/λ 100π/λ]. Correspondingly, the equal space d_q in spatial domain should be 0.01λ. After applying directly the FFT algorithm given in eqn (4), we obtain its corresponding array distribution. In Fig. 3 c), for the good representation, the array distribution in the interval [-3λ 3λ] is given. Outside of this interval, the array values are zeros. After removing the zero values, the synthesized array element values are given in the text box.

We see that the array is of the uniform amplitude but with non-uniform spacing. Its array factor is exactly the same as that given in eqn (5).

In some cases, the period of the given array factor maybe is very large or does not exist. Thus, the array factor is not completely specified. Obviously, the synthesized array will not be the exact non-uniformly spaced array after applying the FFT algorithms. But the array amplitude distribution is concentred in some positions in x-axis. If the more information of array factor is given, these positions will be more approximated to those exact positions. Thus we can choose these positions as the synthesized array element positions. Consequently, through the application of FFT algorithms, we can obtain the element amplitudes and phases, respectively. The synthesized array factor may not be a good approximation. In this case, using the summation expression of the array factor, we can choose some critical samples from the desired array factor and obtain a set of equations. If the array element number is N and N array factor values are chosen, with N known element positions, we can obtain N element currents with amplitudes and phases. The solution can be simply obtained by doing the matrix inverse. For the success to achieve the better synthesis result, the choice of array factor values plays an important roll. In some cases, maybe it is needed to do iteratively. Especially, when the synthesized array elements are complex and not symmetric, the choice way will become more complicated. More discusses can be encountered in [4].

Now lets see one application of the synthesis process described above. The information of the array factor, given in eqn (5), is achieved only in the interval $[-10\pi/\lambda\ 10\pi/\lambda]$. If we just use them to apply the FFT algorithm, in space domain, the array distribution will be limited in one restricted interval. To get the array distribution in one larger interval, we just need to set one larger P used in eqn (4) or to pad zeros in the set of array factor samples, and then apply the FFT algorithm. After using FFT algorithm and zero padding, the obtained array distribution in the interval $[-3\lambda\ 3\lambda]$ is shown in Fig. 4 a). Inside the interval, the array distribution is concentred in some peak values and outside of it, the array distribution tends monotonically to zero. From this distribution, we can take the array element positions and the corresponding currents from the peak values. If not using the zero padding, the array element positions are $x_n=\{0\ \pm0.3\lambda\ \pm0.8\lambda\ \pm1.2\lambda\ \pm1.8\lambda\}$ and the corresponding currents $I_n=\{0.3665\ 0.3934\ 0.3295\ 0.5356\ 0.5124\}$. For this array, the array factor is calculated. The desired array factor and that synthesized in the visual window are shown in Fig. 4 b). The desired array factor is represented with a continuous line and the approximated array factor with a discontinuous line. By comparison, we see that between of them, there is a great difference. As a result, it is necessary to modify the element current values. Because the desired array factor in the visual window is desired, we choose the desired array factor samples in that interval. Due to the array element number, 9 samples with equal distance from the desired array factor are chosen. They are shown in Fig. 4 c) by '*'. After resolving the equations, the new element currents are $I_n'=\{0.0579\ 0.7727\ 0.4692\ 0.4775\ 0.5017\}$. To the array with new element currents and positions given before, its array factor in the visual window is shown in Fig. 4 c) with discontinuous line. For the comparison, in this figure, the desired array factor in visual window is also given by continuous line. We see that they are very approximated. The difference between of them is given in Fig. 4 d). The maxim difference absolute value is 0.0542 and the rms value 0.001.

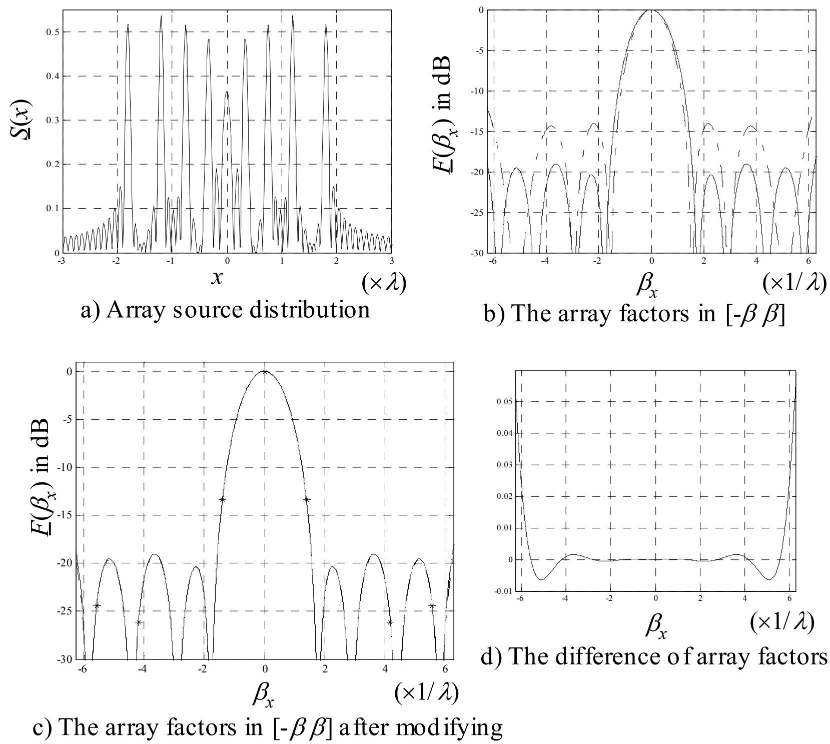


Figure 4: The synthesis when the array factor is not completely given.

Of course, when modifying the element current values, the different array factor samples conduct to different current values. The choice way can be depended on the synthesis requirements. In this example, we see that the difference between the desired and synthesized array factors is very small. Thus we don't renew the element currents any more.

In order to show the application of the presented method, another example is given. In [5], the cosecant pattern is synthesized using general alternative-projection method and some synthesized non-uniformly spaced arrays are given. For one with 20 elements and with nominal spacing λ (wavelength), in [1], the analysis result is given using IFFT algorithms. For this array factor, the array element position values have three decimal digits in the scale of λ. Thus, according to the element position relation between the non-uniform array and its equivalent uniformly spaced array, it is needed that the spacing d_q equals to 0.001λ without spatial position error. Correspondingly, to get the complete array factor in β_x domain, the interval should be $[-\pi/d_q\ \pi/d_q]=[-1000\pi/\lambda\ 1000\pi/\lambda]$. As the array element number is 20 and the nominal spacing is λ, we sample the array factor with 20 samples in every interval whose length is equal to that of the visual window, that is, $2\pi/\lambda$. Like this, 20000 samples of array factor with equal spacing can be obtained. After applying FFT algorithm with P=20000, the element amplitudes of the synthesized array in the interval [-10λ 10λ] are shown in Fig. 5. Outside of this interval, the element amplitudes are zero. Inside the interval, there are 20 non-zero values that should be the synthesized array element amplitudes. After removing the zeros in the obtained array distribution, the amplitudes and phases of synthesized array are given in the table 1. By the comparison with the values given in [5], we find that they are exactly the same.

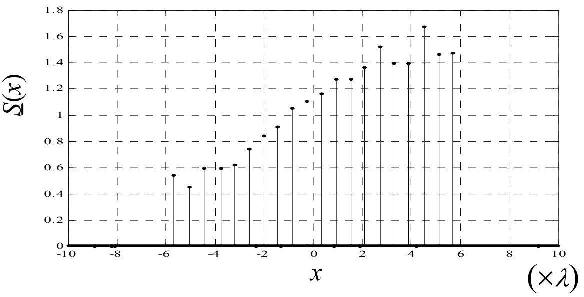


Figure 5: Synthesized array element distribution.

Table 1: The element parameters.

x	-5.707	-5.030	-4.429	-3.751	-3.185	-2.613	-2.004
$\|I\|$	0.54	0.45	0.59	0.59	0.62	0.74	0.84
φ	-35.88	-47.25	-40.30	-42.77	-34.72	-32.94	-29.53
x	-1.441	-0.838	-0.242	0.329	0.947	1.533	2.116
$\|I\|$	0.91	1.05	1.10	1.16	1.27	1.27	1.36
φ	-21.69	-17.56	-9.09	-1.40	4.30	14.70	23.32
x	2.737	3.317	3.899	4.542	5.13	5.705	
$\|I\|$	1.52	1.39	1.39	1.67	1.46	1.47	
φ	35.60	55.11	68.77	86.46	132.97	-159.91	

In this table, the $|I|$ is the element amplitude and φ is the element phase in degree. The element positions are in the scale of the wavelength λ.

In the practice computation, due to numeric precision, there maybe appears a little difference in the numeric values obtained by the computer. The little difference can be ignored and the synthesis results are not affected significantly.

4 Conclusion

In this paper, it was presented the direct application of FFT algorithms to synthesize the non-uniformly spaced antenna arrays, based on the duality property of Fourier transform. Two cases of the synthesis problem were discussed. If the complete array factor is given, we can directly obtain the synthesized array; if not, firstly, we use FFT algorithms to determine the synthesized array element distribution and choose the element positions and then determine the element currents. The advantages of FFT algorithms are well known. Another example of practical interest is: for instant, if there just exists some information of the array factor, such as the graphical representation or a set of samples, using the presented method, we can achieve good synthesis results. The method can also be applied in the synthesis of uniformly spaced arrays. In this case, the array factor is defined in one explicit period.

In the analysis of non-uniformly spaced arrays, as we have described in [1], other non-uniform FFT algorithms can also be applied. But since the inverse process doesn't exist, in the synthesis problems, those methods can't be used in the direct way.

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