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The distances of cataclysmic variables

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Summary. A method is described for determining the distance of a cataclysmic variable by making use of the $K(2.2\,\mu m)$ magnitude of the secondary star. This is achieved by calibrating surface brightness in this band as a function of effective temperature. The method is relatively insensitive to the temperature and evolutionary state of the secondary. We apply the method to determine the distances of a number of dwarf novae and nova-like variables as well as the recurrent nova T CrB and the AM Her type binary VV Pup.

1 Introduction

Up to now there has been no general method for determining the distances and hence luminosities of cataclysmic variables. Distances of classical novae can be obtained from studies during outburst but these methods are clearly not applicable to dwarf novae or nova-like variables. Trigonometric parallaxes have been measured for a few dwarf novae but are inadequate at present, to place significant constraints on the distance. Kraft & Luyten (1965) obtained a mean absolute magnitude of +7.5 for dwarf novae at minimum from a study of proper motions and radial velocities. However, it seems clear that there is considerable dispersion about this mean value, so we cannot adopt this value for any individual dwarf nova.

The most promising method of determining the distances of cataclysmic variables is to make use of the properties of the secondary stars. Until recently such an approach was limited by the fact that the secondaries were only detectable in long period systems ($P > 6$ hr). However, this limit applies only to observations in the blue region of the spectrum which was used for early spectroscopic observations. When observations are extended to longer wavelengths the secondaries can be detected in shorter period systems. Wade (1979) showed that the secondary of U Gem ($P = 4.25$ hr) began to dominate the light of the system at wavelengths longer than $0.7\,\mu m$. Recent observations of Z Cha ($P = 107$ min) show that the secondary is visible in this system in the $J(1.25\,\mu m)$ and $K(2.2\,\mu m)$ bands (Bailey *et al.* 1981). This is the first detection of the secondary in one of the ultra-short period group of dwarf novae, and means that the method described here can be applied to at least some cataclysmic variables over the whole period range observed.

One method of obtaining the distance would be to estimate the mass, spectral type or temperature of the secondary and to use the known properties of the main sequence to give the corresponding absolute magnitude and hence the distance. This method has two main disadvantages.

(i) The main sequence relations may not be valid for the secondaries of cataclysmic variables.

(ii) The absolute magnitude is such a steep function for lower main sequence stars that this method is very sensitive to small errors in the estimate of mass, spectral type or temperature.

The method to be described here avoids both these difficulties. The approach we adopt makes use of the surface brightness of the star which we calibrate as a function of effective temperature. (In practice we use the $V-K$ colour as a convenient effective temperature parameter.) Combined with a determination of the radius of the star (which is constrained by the fact that it must fill its Roche lobe) and the observed apparent magnitude of the secondary, the distance can be determined. We might reasonably expect a relation between surface brightness and effective temperature to be much more widely applicable than relations involving radius or luminosity separately, as this relation is determined by local properties of the stellar atmosphere rather than by the internal structure of the star. Although the method could, in principle, be applied at any wavelength there are a number of advantages in using a long wavelength band. (In practice the $K(2.2\,\mu m)$ band is the longest wavelength at which good quality data on these systems can be obtained.)

(1) There is a much greater likelihood of detecting the secondary at a long wavelength.

(2) For cool stars the surface brightness in the K band is a very much weaker function of effective temperature than that in the V band, so the temperature need not be determined so accurately.

(3) Interstellar extinction corrections are negligible in the K band for the stars considered here.

2 Surface brightness calibration

We make use of the K surface brightness parameter (S_K) which we define by

$$S_K = K + 5 - 5\log d + 5\log(R/R_\odot), \qquad (1)$$

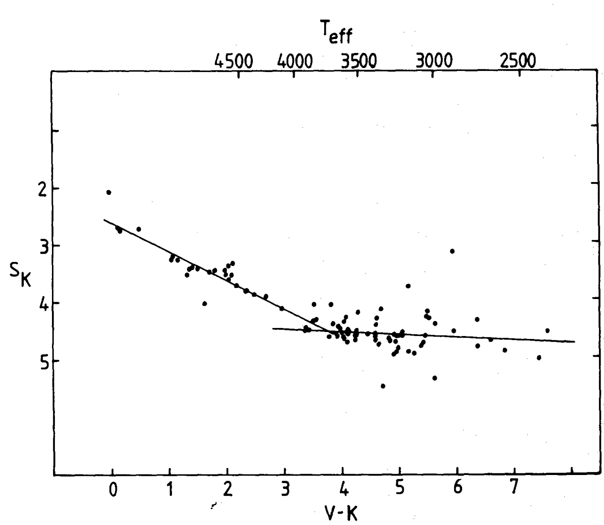

Figure 1. K surface brightness parameter plotted against $V-K$ colour for nearby stars. The lines represent the fit to the data given by equation (2). The effective temperature is given according to the calibration by Veeder (1974).

where K is the observed K magnitude, d is the distance in parsecs and R is the radius of the star. For a star of one solar radius S_K is equivalent to the absolute K magnitude.

Lacy (1977) has estimated radii for a large number of nearby stars and S_K has been determined for all stars in Lacy's list for which infrared photometry is available. For this purpose photometry has been taken from Glass (1974), Johnson (1965), Johnson *et al.* (1966), Mould & Hyland (1976) and Veeder (1974). The results are shown in Fig. 1 where S_K is plotted against the V–K colour index which we use as an effective temperature parameter. The effective temperature according to Veeder's (1974) calibration is also shown.

The plotted data appear to show a distinct change in slope at around V–K = 3.5 and can be closely represented by two straight line segments as follows

$$S_K = 2.56 + 0.508\,(V\text{–}K) \qquad \text{for } (V\text{–}K) < 3.5,$$
$$S_K = 4.26 + 0.058\,(V\text{–}K) \qquad \text{for } (V\text{–}K) > 3.5. \qquad (2)$$

Most of the points lie within ~0.3 mag of these lines. A few points show much larger discrepancies. In view of the large sample of stars used it is not unreasonable that errors in the photometry could be responsible for the discrepant points.

For $(V\text{–}K) > 3.5$ the slope of the line is very small. This somewhat surprising result suggests that we can reasonably make the approximation $S_K = 4.55$ for all M-type stars. However, the physical basis for this is not clear. In Fig. 2 the fitted lines are compared with the results of assuming the star radiates as a blackbody at the T_{eff} given by Veeder's calibration, and with the model atmospheres for M dwarfs calculated by Mould (1976). These results do not show the change of slope observed in our empirical fit. Despite this the difference between the empirical and theoretical values of S_K does not exceed 0.3 mag except for $(V\text{–}K) > 6$ which represents extreme M dwarfs ($T_{\text{eff}} < 2900$ K). A possible reason for the discrepancy is that the Barnes–Evans relation used by Lacy (1977) to obtain the radii breaks down for such cool stars. However, the blackbody calculations may also be unreliable in this region as there are strong H_2O bands in the near infrared spectrum, and no model atmospheres have been calculated cooler than 3000 K. For the present purposes we will continue to use the empirical calibration given by equation (2) noting that it may be somewhat less reliable for $(V\text{–}K) > 6$.

The secondaries of cataclysmic variables differ from single stars of the same mass and radius in their surface gravity, which is variable over the surface. They may also be signifi-

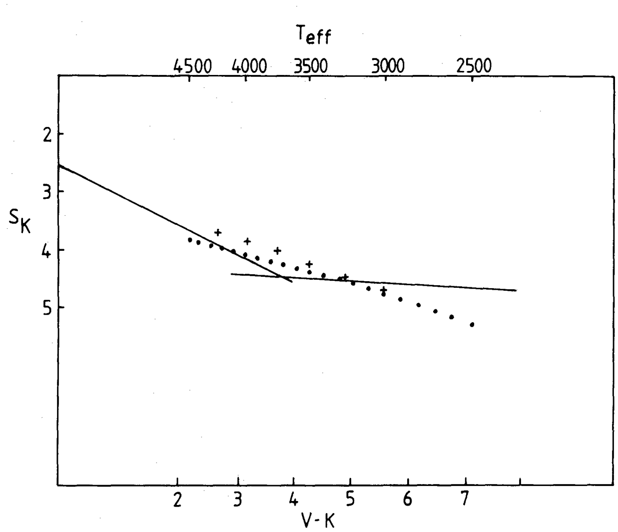

Figure 2. Comparison of the empirical fit to the data of Fig. 1, with the prediction of blackbody models (dots) and M dwarf model atmospheres from Mould (1976) (crosses).

cantly evolved, which would lead to a different surface gravity from that of a main sequence star of the same temperature. Thus it is important to investigate how S_K varies with surface gravity. We can do this by looking at data on stars of different luminosity classes. Almost all late type stars with well determined parallaxes are dwarfs. However, S_K can also be determined for any star with measured angular diameter since equation (1) can be rewritten

$$S_K = K + 0.155 + 5 \log \phi',$$

where ϕ' is the angular diameter in milliarcsec.

In this way S_K has been determined for 10 late type giants and supergiants from the compilation of Barnes & Evans (1976). All the stars have $6.5 > (V-K) > 3.5$ and give a mean S_K of 4.31 only marginally different from the value for the nearby dwarfs.

The fact that the surface brightness at a given $V-K$ is only marginally different for supergiants and dwarfs whose surface gravities differ by a factor of 1000 or more, indicates that no problems should result from the departure of surface gravity from main sequence values in the secondaries of cataclysmic variables.

3 Radii

The fractional radius R_2/a of a Roche lobe filling secondary in a cataclysmic binary can be represented by (Paczynski 1971)

$$R_2/a = 0.462\ (1+q)^{-1/3} f(q),$$

where $q = M_1/M_2$ (M_1 is the mass of the white dwarf, M_2 that of the secondary) and $f(q)$ is a function (tabulated in Table 1) which is within a few per cent of 1 except for very small values of q. Combined with Keppler's law for the binary system this gives the relation

$$(R_2/R_\odot)^3 = 7.34\ (M_2/M_\odot)\, P^2 f(q)^3, \tag{3}$$

where P is the period in days.

Equations (1), (2) and (3) then form the basis of the method for determining distances. To apply the method to any system we need to make use of four parameters.

(1) The period of the binary. This is accurately known for most well studied cataclysmic binaries.

(2) The K magnitude of the secondary. For long period systems the infrared colours usually indicate that the secondary contributes most of the radiation at K. For shorter period systems other parts of the system such as the accretion disc and bright spot may also contribute significantly to the K flux, and the various components must be separated (e.g. by an analysis of the eclipse light curves).

Table 1. The function $f(q)$.

q	$f(q)$
0.4	1.115
0.6	1.074
0.8	1.049
1.0	1.032
2.5	0.990
5.0	0.981
10.0	0.983
20.0	0.990

(3) The secondary mass, and

(4) The $V-K$ colour of the secondary. These two parameters present more problems as the V magnitude of the secondary is not easily measurable for short period systems, and masses are not generally well determined. Fortunately our procedure turns out to be surprisingly tolerant of quite large errors in these parameters. Even a factor of 2 error in the mass, or a 1 magnitude error in $V-K$ (on the steep part of the calibration) lead to only 25 per cent errors in the distance.

Although, in principle, the mass ratio q is a fifth parameter, the assumption $f(q) = 1$ will not usually result in errors of more than a few per cent. In the following analyses we have corrected for $f(q)$ only for the systems where q is well determined.

4 Application to individual systems

We will now apply the method to a number of cataclysmic variables. The results are summarized in Table 2.

U Gem

Wade (1979) and Stauffer, Spinrad & Thorstensen (1979) have shown that an M type star begins to dominate the spectrum redward of 0.7 μm, while the light curves at J and K (Frank *et al.* 1981) show the 'ellipsoidal variable' type curve expected for a lobe filling secondary. Hence we may assume the observed mean K magnitude of 10.85 is that of the secondary. We assume $V-K = 5$ for the secondary corresponding to a spectral type of M5 as found by Wade (1979). Adopting a mass of $0.35 M_\odot$ (Smak 1976) gives $d = 78$ pc. Wade (1979) using observations in a different spectral region and adopting the same value of M_2 has obtained $d = 76$ pc. The excellent agreement between the two methods provides further support for the accuracy of our S_K calibration.

SS Cyg

Szkody (1977) has measured $K = 9.37$ for the system at quiescence and we will assume that this applies to the secondary. Stover (1979) finds that the secondary contributes 40 per cent of the flux in a spectral region corresponding to the Johnson B band. Since $B \sim 12.3$ at quiescence this implies $B-K = 3.9$ for the secondary indicating a K5 star with $V-K = 2.7$. Stover (1979) has given improved radial velocity curves for both components of SS Cyg. These results in limits on the secondary mass of $0.25–0.86 M_\odot$ based on the absence of eclipses and the requirement that the white dwarf mass should not exceed the Chandrasekhar limit. The preferred value of $0.77 M_\odot$ is that at which the secondary fits Lacy's (1977) mass–radius relation for the main sequence. The resulting distance of 95 pc is close to Kiplinger's (1979) lower limit of 111 pc. The higher value of 143 pc given by Kiplinger results from assuming a temperature of 5000 K for the secondary which is substantially higher than that indicated by the $V-K$ colours.

EM Cyg

Infrared light curves of this sytem are given by Jameson, King & Sherrington (1980). They find that the secondary dominates the system in the infrared and has $K = 11.8$ and $V-K = 2.15$. EM Cyg is both an eclipsing binary and double lined spectroscopic binary, so its mass is well determined as $0.90 M_\odot$ (Robinson 1974) with limits of $0.73 M_\odot$ and $1.07 M_\odot$

Table 2. Adopted parameters and derived distances.

Star	Type*	P(day)	K	$V-K$	$M_2/M_\odot$	q	S_K	Distance	Lower limit	Upper limit
Z Cha	DN	0.074499	13.8	5.0	0.17		4.55	134 pc		
SS Cyg	DN	0.2749	9.37	2.7	0.77 (0.25–0.86)	1.6	3.93	95 pc	56 pc	113 pc
EM Cyg	DN	0.290909	11.7	2.15	0.90 (0.73–1.07)	0.78	3.65	352 pc	285 pc	429 pc
U Gem	DN	0.176906	10.85	5.0	0.35		4.55	78 pc		
AE Aqr	NL	0.411655	8.73	3.02	0.74 (0.55–1.15)	1.26	4.09	84 pc	66 pc	112 pc
RW Tri	NL	0.231883	12.4	3.25	0.47		4.21	247 pc		
UX UMa	NL	0.196671	12.8	4.9	0.40		4.54	216 pc		
T CrB	RN	227.6	4.82	5.4	2.6 (2.2–3.0)	0.71	4.57	1180 pc	970 pc	1430 pc
VV Pup	AM	0.069747	14.0	5.0	0.18		4.55	144 pc		

*DN = Dwarf nova; NL = nova-like variable; RN = recurrent nova; AM = AM Herculis type.

(Jameson *et al.* argue for a value near the lower limit). The distance of 352 pc derived here agrees well with that obtained by Jameson *et al.* (1980).

Z Cha

Preliminary analysis of the infrared light curves of this eclipsing dwarf nova shows that the secondary contributes most of the radiation at K. The value of $M_2 = 0.17 M_\odot$ is close to values found by Ritter (1980), Vogt (1979) and Bailey (1978). The infrared colours of the secondary are consistent with those of an M star and we will adopt $V-K = 5$ (corresponding to an M5 star). The resulting distance is 134 pc.

AE Aqr

The observations of this long period system indicate that the secondary dominates the light except in the blue. Patterson (1979) gives the V magnitude of the secondary as 11.75, while Szkody (1977) has measured $K = 8.73$. Patterson (1979) gives the mass as $0.74 M_\odot$, with extreme limits of 0.55–$1.15 M_\odot$ being set by the absence of eclipses and the requirement that the mass of the white dwarf should not exceed the Chandrasekhar limit. The resulting distance of 84 pc is substantially less than the 150 pc obtained by Patterson (1979). The discrepancy can be traced to Patterson's adoption of a K2 spectral type for the secondary, while the $V-K$ colour indicates a cooler star (K6–K7).

RW Tri

Infrared light curves of this nova-like variable have been obtained by Longmore *et al.* (1981). The data used for this system has been taken from the analysis of these light curves by Frank & King (1981). The resulting distance of 247 pc is rather higher than Frank & King's estimate but within their quoted limits.

UX UMa

The data for this star are based on the observations and analyses of the system reported by Frank *et al.* (1981). The resulting distance of 216 pc is in excellent agreement with that obtained by Frank *et al.*

T CrB

This recurrent nova differs from the other stars discussed here in that the secondary is a lobe filling giant. Nevertheless, as S_K shows little dependence on luminosity class the method is equally applicable to such systems. The K magnitude is given as 4.82 by Feast & Glass (1974). With $V = 10.2$ and $M_2 = 2.6 M_\odot$ (Paczynski 1965) we obtain $d = 1180$ pc.

VV Pup

When this AM Herculis binary is in its faint state ($V \sim 18$) a late M star dominates the spectrum in the red (Greenstein and Oke 1978; Liebert *et al.* 1978). We assume the K magnitude of 14 in the faint state (Szkody & Capps 1980) applies to the secondary and adopting $V-K = 5$ and $M_2 = 0.18 M_\odot$ (which assumes the secondary obeys a main sequence mass–radius relation) we find $d = 144$ pc. This is just consistent with the upper limit of 150 pc according to Liebert *et al.* (1978).

5 Accuracy of the distance determinations

From the discussion in Section 3 we can estimate that the uncertainty in the surface brightness calibration amounts to ±30 per cent and hence introduces a ±15 per cent uncertainty into the distance. In each individual case errors in the parameters used add to this uncertainty. Normally the most uncertain parameter is the secondary mass M_2. For systems which are double lined spectroscopic binaries well defined observational limits can be placed on the secondary mass. In these cases we have estimated upper and lower limits on the distance by calculating distances corresponding to the extreme values of the mass, and then increasing the upper limit by 15 per cent and decreasing the lower limit by 15 per cent to allow for the uncertainty in the calibration.

For the other systems it is not possible to obtain well defind limits on the secondary mass at present, so it would be misleading to attempt to quote upper and lower limits on the distance in these cases. We note that masses substantially different from the values we quote would indicate substantial deviations from the main sequence. Also the distance can be easily recalculated for any other values of M_2 using the fact that d scales as $M_2^{1/3}$.

6 Conclusions

We have described a method for determining the distances of cataclysmic variables using the K magnitude of the secondary star. This is achieved by making an empirical calibration of K surface brightness against $V-K$ colour using data on nearby late type stars. The empirical calibration is shown to be in good agreement with blackbody calculations and M dwarf model atmospheres.

The method is applied to a number of cataclysmic variables. For systems which have previous distance estimates the agreement is good in most cases. Where significant discrepancies arise these can be attributed to different adopted temperatures for the secondary. Previous distance methods based on the optical brightness of the secondary are very sensitive to the temperature of the star whereas the method described here is relatively insensitive.

This approach makes it possible to compare the luminosities of different types of cataclysmic variables and thus obtain information on accretion rates for example. We defer such an analysis until further observations make it possible to discuss a larger sample of objects.

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