The distribution of calibrated likelihood-ratios in speaker recognition

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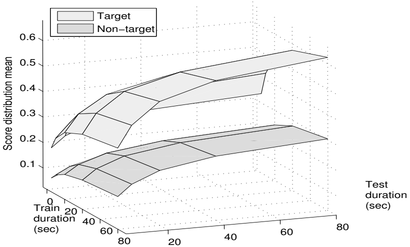
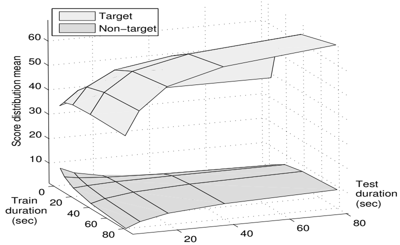
[1] First published at Interspeech 2013

Inspiration for this work

- We had these badly-behaving scores[2] depending on utterance duration
- We tried to design universal calibration transformations
- Question arose: where do calibrated scores hang out?
- What is their distribution?



(a) Cosine Kernel



(b) Normalized Cosine Kernel

[2] Mandasari *et al.*, Interspeech 2011

What is calibration?

Traditionally:

- The capability to set a threshold correctly

Nowadays:

- The ability to give a proper probabilistic statement about identity
  - ...to produce (log) likelihood ratio scores for every comparison
  - ...that lead to optimal Bayes' decisions

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Bayes' decision

Priors + likelihoods → posteriors

Posteriors + costs → expected costs

Minimize expected costs → decision

The forensic motivation of the Likelihood Ratio

Use the log Likelihood Ratio as weight of evidence in court

- Using Bayes's rule, separate contributions
  - Forensic Expert, w.r.t. the material they know about
  - The other evidence / circumstances of the case

  to compute the posterior probability that suspect is the perpetrator

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  to compute the posterior probability that suspect is the perpetrator $H_p = \neg H_d$
- Mathematically,

$$\underbrace{\frac{P(H_p \mid E, I)}{P(H_d \mid E, I)}}_{\text{judge/jury wants to know}} = \underbrace{\frac{P(E \mid H_p, I)}{P(E \mid H_d, I)}}_{\text{given by expert}} \times \underbrace{\frac{P(H_p, I)}{P(H_d, I)}}_{\text{other evidence}}$$

From scores to likelihood ratios

- A likelihood ratio can be treated like a score
  - All analysis tricks work: ROC, DET, EER, decision cost functions…
- But can we transform a score into a LR?
- This is a process known as calibration: giving meaning to probabilistic statements

From scores to likelihood ratios

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problem statement

But what is the definition of *calibrated* scores / LRs?

Definition of *Calibrated Likelihood Ratios*

Our definition[3]

The LR of the LR is the LR

or, for the mathematically inclined

$$\mathrm{LR} = \frac{P(\mathrm{LR} \mid H_p)}{P(\mathrm{LR} \mid H_d)}$$

[3] Proof in paper, short version in Mandasari *et al.*, IEEE-TASLP (2013, accepted)

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which happens to be equivalent to

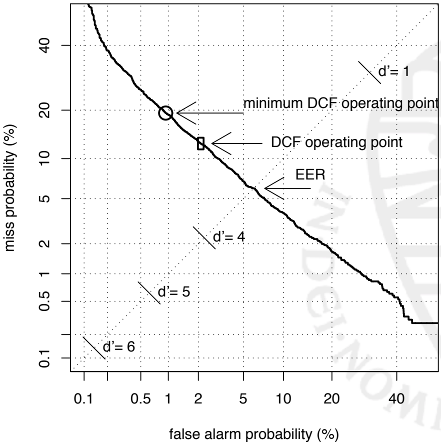
$$\log \mathrm{LR} = \log \frac{P(\log \mathrm{LR} \mid H_p)}{P(\log \mathrm{LR} \mid H_d)}$$

The LLR of the LLR is the LLR

[3] Proof in paper, short version in Mandasari *et al.*, IEEE-TASLP (2013, accepted)

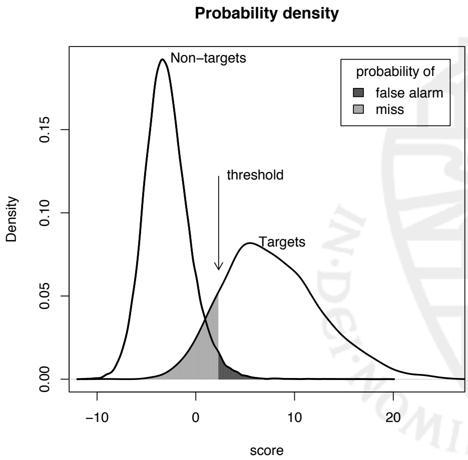
More inspiration: Why are DET curves straight?

- If score distributions are Gaussian, then DET curve is straight
  - Slope is ratio of standard-deviations of the score distributions
- If DET is straight, score distributions are not necessarily Gaussian
  - but can be made Gaussian by warping of score axis



For reference: these are the score distributions

- Clearly not Gaussian
- But *still* leading to a straight DET curve
- non-targets: $d(x)$ (different)
- targets: $e(x)$ (equal)



Can Gaussian Scores be Well Calibrated?

Let's try

- Gaussian non-targets $d(x) = \mathcal{N}(x \mid \mu_d, \sigma_d^2)$
- calibration definition for LLR:

$$x = \log \frac{e(x)}{d(x)}$$

$$\text{targets } e(x) = e^x d(x)$$

Now use the expression for the normal distribution $\mathcal{N}$, and see what the targets $e(x)$ look like

$$e(x) = e^x d(x) = \frac{1}{\sqrt{2\pi}\sigma_d} e^{x-(x-\mu_d)^2/2\sigma_d^2}$$

Math 101

Expanding the exponent for target distribution $e(x)$:

$$
\begin{aligned}
& -\frac{x^2 - 2\mu_d x + \mu_d^2}{2\sigma_d^2} + \frac{2\sigma_d^2 x}{2\sigma_d^2} \\
&= -\frac{x^2 - 2(\mu_d + \sigma_d^2)x + \mu_d^2}{2\sigma_d^2} \\
&= \underbrace{-\frac{\left(x - (\mu_d + \sigma_d^2)\right)^2}{2\sigma_d^2}}_{\text{Gaussian form}} + \underbrace{\frac{2\mu_d\sigma_d^2 + \sigma_d^4}{2\sigma_d^2}}_{\text{Normalisation constant}}
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Gaussian form

- if $\mu_e = \mu_d + \sigma_d^2$
- with $\sigma_e = \sigma_d$
- normalization requires $-2\mu_d = \sigma^2$

Conclusions of this little exercise

- Consider non-target distribution $d(x)$ and target score distribution $e(x)$
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...the calibration definition tells us

- $e(x)$ is normally distributed as well
- Variances are the same for $d(x)$ and $e(x)$
- The means are symmetric around 0,

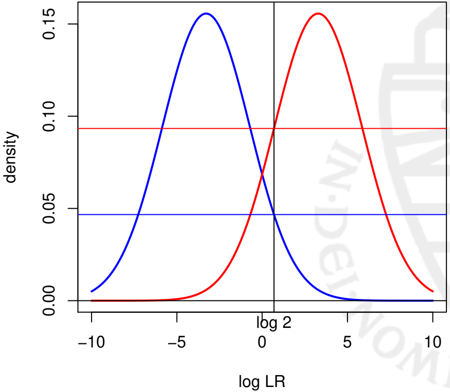
$$\mu_d = -\mu_e$$

- Variance and mean are related

$$\sigma^2 = 2\mu$$

Example of well-calibrated scores

- $\mathrm{LR} = 2$
  density scores around 2 is $2\times$ as high for targets (red) as for the non-targets (blue)



Example of well-calibrated scores

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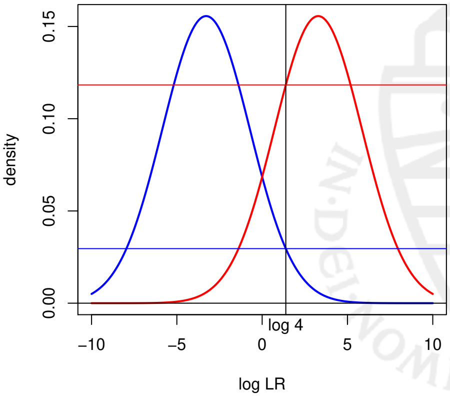
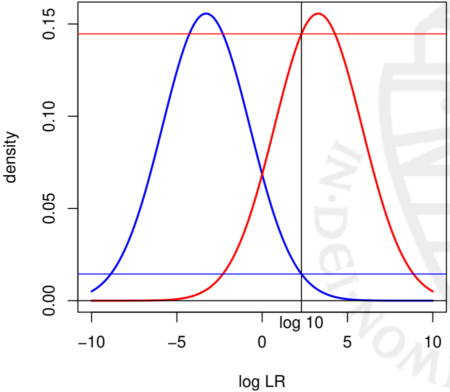


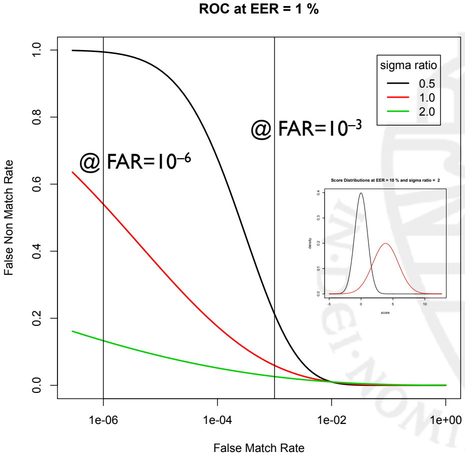
Example of well-calibrated scores

- $\mathrm{LR} = 2$
  density scores around 2 is 2× as high for targets (red) as for the non-targets (blue)
- $\mathrm{LR} = 10$



Some direct consequences

- Well calibrated straight DET curves must be of 45° slope
- Preferred "flat" straight DET curves can't arise from calibrated scores
  - highly-discriminative systems have flat DET curves,
  - fingerprint, iris, . . .



All relations are known, now

From this model of scores all other characteristics follow, e.g.,

- Equal Error Rate $E_=$
  - Threshold at 0
  - Integrate the miss error:

$$E_= = \int_{-\infty}^{0} \mathcal{N}(x \mid \sigma, \mu)\, dx$$
$$= \Phi(-\mu/\sigma) = \Phi(-\sqrt{\mu/2})$$

  - $\Phi(z)$ cumulative normal distribution
- Cost of LLR C_{llr}

$$C_{\text{llr}} = \frac{1}{\log 2} \int_{-\infty}^{\infty} \mathcal{N}(x \mid \mu, \sigma) \log(1 + e^{-x})\, dx$$

  - C_{llr} depends only on $E_=$

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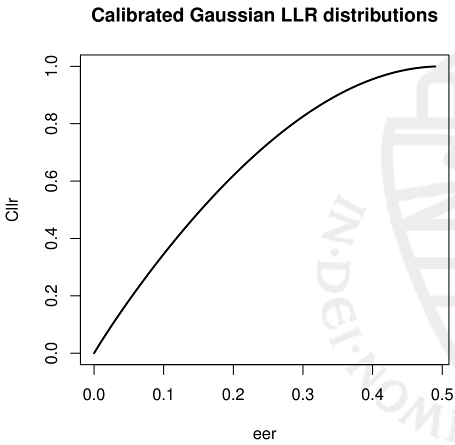
Approximate relation:

$$C_{\mathrm{llr}} \approx 1 - (2E_= - 1)^2$$



Application: a new way of doing calibration

Calibration is the process of fixing scores so that they can be interpreted better as log likelihood ratios

- Traditionally, this is done in speaker recognition by an affine transformation of score s

$$x = as + b$$

- parameters a and b found by logistic regression using a development set of trials

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New calibration method:

Find a and b by constraining the transformed scores to satisfy the Gaussian LLR conditions for μ and σ

Math 101 again

Raw score means and variances $m_{d,e}$, $s^2_{d,e}$.

- Transformed target mean: $am_e + b = \mu$
- Transformed non-target mean $am_d + b = -\mu$
- Weighted variance $v = (1-\alpha)s^2_d + \alpha s^2_e$
- Transformed variance $\sigma^2 = a^2 v = 2\mu$

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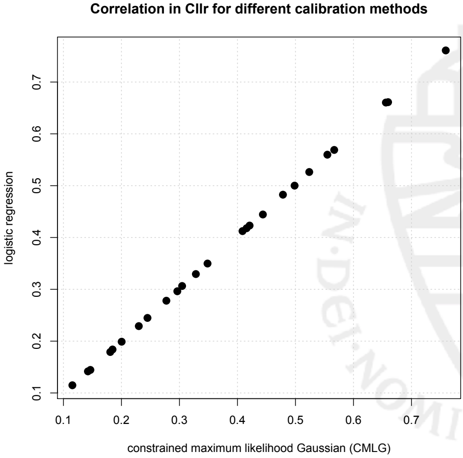
. . . results in solution

- $a = \dfrac{m_e - m_d}{v}$
- $b = -a\dfrac{m_e + m_d}{2}$
- This is a closed-form solution!

Constrained Maximum Likelihood Gaussian: CMLG

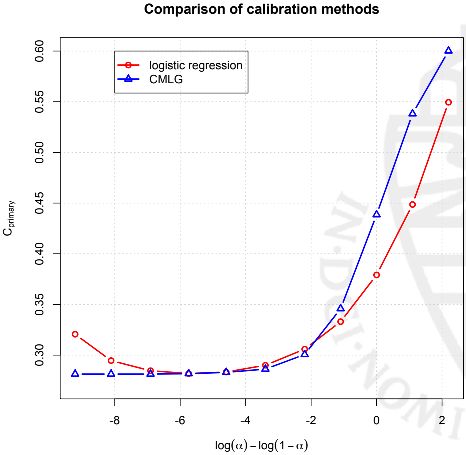
First calibration experiment: Miranti's scores

- RUN i-vector PLDA system
- calibrate on SRE-2008, evaluate using C_{llr} on SRE-2010
- 25 different duration-combinations, to sample range of performances
- Two linear calibration methods
  - y Logistic regression
  - x This method (CMLG)



Second experiment: Niko's scores

- Agnitio Research's SRE-2012 system and scores
- Calibrated using their dev-set
- Evaluated using C_{primary}
  - official SRE-2012 metric
  - sensitive to low-FA range
- Contrasting
  - Niko + GD Interspeech 2013
  - This method CMLG



Conclusions

- We can prove that "the LLR of the LLR is the LLR"
  - ...already in exam questions course Forensic Linguistics...
- Well calibrated Gaussian non-target scores imply
  - Gaussian target scores
  - with same variance
  - and opposite mean
  - and a variance that is equal to the difference in means
- We can use it to find calibration parameters
  - as a closed-form solution
  - that gives same performance as logistic regression, for
    - two different systems
    - two different evaluation data bases
    - two different calibration-sensitive evaluation metrics