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THE DIVIDEND-PRICE RATIO AND EXPECTATIONS OF FUTURE DIVIDENDS AND DISCOUNT FACTORS

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ABSTRACT

A linearization of a rational expectations present value model for corporate stock prices produces a simple relation between the log dividend-price ratio and mathematical expectations of future log real dividend changes and future real discount rates. This relation can be tested using vector autoregressive methods. Three versions of the linearized model, differing in the measure of discount rates, are tested for U. S. time series 1871-1986: versions using real interest rate data, aggregate real consumption data, and return variance data. The results yield a metric to judge the relative importance of real dividend growth, measured real discount rates and unexplained factors in determining the dividend-price ratio.

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What accounts for the variation through time in the dividend-price ratio on corporate stocks? In the context of rational expectations models, the ratio is often interpreted as reflecting changes in the outlook for dividends: when dividends can be forecasted to decrease, the dividend-price ratio should be high. Alternatively, also in the context of a rational expectations model, the ratio is interpreted as reflecting the rate at which future dividends are discounted to today's price: when discount rates are high, the dividend-price ratio is high. In principle, the dividend-price ratio ought to have both of these interpretations at once. Yet their relative importance has never been established, and it is not clear whether the two interpretations together can account for time variation in the dividend-price ratio. We attempt to answer this here using U. S. time series data 1871 to 1986. (For data sources, see the Appendix.)

A simple present value model for stock prices has the following form:

$$P_t = E_t \sum_{k=0}^{\infty} \exp(-\sum_{j=0}^{k} r_{t+j}) D_{t+k} \tag{1}$$

where P_t is the real price per share, D_t is the real dividend per share, and E_t denotes expectation conditional on information publicly available at time t. The discount rate r_t satisfies $\exp(r_t) = E_t[(P_{t+1}+D_t)/P_t]$ and may have several interpretations. In what we will call version I of the model, $\exp(r_t)$ is the ex-ante gross real interest rate on one-period debt, times a constant reflecting a risk premium. In version II of the model, $\exp(-r_t) = kE_t\beta(C_t/C_{t+1})^{\alpha}$ where C_t is per capita real consumption at time t, α is the coefficient of relative risk aversion and k is a constant risk premium. In version III of the model, $\exp(r_t) = \beta\exp(\mu+\alpha V_t)$, where μ is a constant riskless real interest rate, α is the coefficient of relative risk aversion and V_t is the variance, conditional on information available in year t, of the monthly stock returns in that year.

All three versions of the model can be derived, at least approximately, from equilibrium foundations. (See the Appendix for details). In versions I and II, the discount rate for corporate stocks moves through time because the riskless real interest rate changes while the risk premium on stocks is constant; in version III, by contrast, the riskless real rate is constant and the risk premium is time-varying.

The different versions of the model have been studied before, but the present study adds some new perspectives. With regard to version I, Shiller [1981], Mankiw, Romer and Shapiro [1985] and West [1986a], [1986b] have asked whether the volatility of short-term interest rates might help explain the volatility of stock market prices. Version II of the model has been analyzed extensively, following the original theoretical work of Lucas [1978] and Breeden [1979], by Grossman and Shiller [1981], Grossman, Melino and Shiller [1985], Hansen and Singleton [1983], Hall [1985], Mankiw, Rotemberg and Summers [1985] and Mehra and Prescott [1985], among others. The goal of much of this research has been to estimate the coefficient of relative risk aversion α, either from the cross-sectional relation between means and covariances with consumption of different asset returns, or from the time-series relation between forecastable returns and forecastable consumption growth, or from both of these simultaneously. Estimated risk aversion is often implausibly large, especially when cross-sectional information is used. Version III of the model has been proposed, following an exploratory analysis by Merton [1980], by Pindyck [1984], [1986] who argues that much of the variability in stock prices can be explained by the variability of V_t. Against this, Poterba and Summers [1985] have argued that V_t is not persistent enough to account for much variation in stock prices. French, Schwert and Stambaugh [1986] and Campbell [1987] examine the relation between V_t and expected stock returns, but do not develop implications for the dividend-price ratio.

The emphasis of this paper differs from that of much previous work in the area. We are less interested in testing the model and estimating the coefficient of risk aversion, and more interested in accounting for time variation in the dividend price ratio. Our econometric methods reflect this emphasis.

Linearization of the Model

Equation (1) involves an expectation of a complicated nonlinear relation among P_t, D_{t+j} $j=0,1,\ldots$ and r_{t+j} $j=0,1,\ldots$. Some form of linearization will be necessary to pursue the implications of the model that we wish to study. The linearization will introduce an approximation error that could lead to a rejection of the model (1) even if it is true. However, our purpose here is not merely to test the model (1) but to characterize in broad terms how it succeeds and fails; for such a purpose the linearization is useful.

One may divide both sides of (1) by D_{t-1} which is in the information set at time t, and hence can be passed through the expectations operator. Taking logs of both sides of the equation, and using lower case letters to denote natural logs of the corresponding upper case variables, we have:

$$p_t - d_{t-1} = \log E_t S_t^* \tag{2}$$

$$S_t^* = \exp(\Delta d_t - r_t) + \exp(\Delta d_t + \Delta d_{t+1} - r_t - r_{t+1}) + \exp(\Delta d_t + \Delta d_{t+1} + \Delta d_{t+2} - r_t - r_{t+1} - r_{t+2}) + \ldots \tag{3}$$

Since we will linearize the expression, we can pass the log function inside the expectations operator, and defining the log of the dividend price ratio $\delta_t = d_{t-1} - p_t$ we can write:

$$\delta_t = E_t \delta_t^* \tag{4}$$

where $\delta_t^* = -\log S_t^*$. Taking a Taylor expansion of δ_t^* around $\Delta d_{t+j} = g$ $j=0,1,\ldots,$ and $r_{t+j} = r$, $j=0,1,\ldots,$ we find:

$$\delta_t^* \approx \sum_{j=0}^{\infty} \rho^j \, (r_{t+j} - \Delta d_{t+j}) + h \tag{5}$$

where $\rho = \exp(-(r-g))$ and $h = \log(\exp(r-g)-1) - (r-g)/(1-\exp(g-r))$.[1] Thus, the log dividend-price ratio is approximately equal to a constant plus the "present value" of expected current and future values of the one-period discount rate minus the one-period growth in dividends. Note that Δd_{t+j} and r_{t+j} enter symmetrically in (5); all that matters for the dividend price ratio is their difference. Equation (5) represents the combined effect on the dividend-price ratio of expectations both of changes in future dividends and of future interest rates that was noted in the opening paragraph of this paper.

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[1]For finite price p_t r must be greater than g and ρ less than one.

The final step in deriving the linearized model is to pass the log function through the expectations operator in the definition of r_t. For example, in version II of the model $r_t = -k-\log E_t\beta(C_t/C_{t+1})^{\alpha}$; we use the approximation $r_t = -k+\alpha E_t\Delta c_{t+1}$. Since the model is now linear, and since the rational expectation of future ex-ante discount rates equals the rational expectation of future ex-post rates, we can use ex-post measures of real rates in our tests. In what follows, r_t will refer to the ex-post real rate.

It's instructive to note that the linearized model (5) can also be derived in a different way. Calling R_t the return to holding stock for one period, $R_t = (P_{t+1} - P_t + D_t)/P_t$, then $\log(1+R_t) = \log(\exp(\Delta d_t+\delta_t-\delta_{t+1}) + \exp(\delta_t+\Delta d_t))$. Linearizing this expression around $\Delta d_t = g$ and $\delta_t = \delta_{t+1} = \log(\exp(r-g)-1)$, we get an approximation ξ_t to $\log(1+R_t)$, $\xi_t = \delta_t - \rho\delta_{t+1} + \Delta d_t - (1-\rho)h$. If we set $E_t\xi_t = E_t r_t$ we get a rational expectations model which, if solved forward, yields (5).

In Tables 1a and 1b we display several measures of the approximation error for the linearized model (5). In carrying out the approximation, the point of linearization was taken to be the log of one plus the sample average real return on stocks, less the sample average change in log dividends. Table 1a compares ξ_t with R_t and $\log(1+R_t)$. Table 1b compares an approximate "perfect foresight log dividend-price ratio" δ_t^*, constructed using equation (5) and a terminal condition $\delta_T^* = \delta_T$, with an exact δ_t^*, constructed using equation (1) and the same terminal condition[2]. The approximation error is quite small in both tables.

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[2] This terminal condition is used only for evaluating the approximation in equation (5), and not in the empirical work reported below.

Time Series Representations

We will study two representations of each version of the model. In representation (a) real dividends must be differenced to induce stationarity and in representation (b) the dividends are assumed to be stationary around a trend. The reason for including two representations is that evidence is mixed as regards which simple model of the processes is most appropriate for our analysis. A Phillips-Perron [1986] test that the d_t process has a unit root rejects at the 5% level with our full sample period in favor of an alternative that it is stationary around a trend (Table 2). This would suggest that representation (b) is appropriate. However, since we strongly reject the unit root hypothesis for the log dividend-price ratio, this stationarity implies stationarity around a trend for real price as well; yet we do not reject the unit root assumption for price (Table 2).

These internally inconsistent test results are hard to interpret. It's possible that the test lacks power to reject the unit root hypothesis for stock prices because of their smoothness; it's also possible that the test falsely rejects that hypothesis for the dividend series. The Phillips-Perron significance levels are asymptotically correct, but in any finite sample one can add sufficient noise to a process with a unit root to obtain a false rejection.

Time series models involving deterministic trends are currently in disfavor; many people seem to think that they are inherently implausible. But of course in using a deterministic trend in a model we are not asserting that such a trend really will be followed forever. Parsimony dictates that with modest data sets we keep our models simple. An AR(p) model with a deterministic trend might be regarded as an approximation to an ARIMA(p,1,1) model with a moving average component whose root is close to the unit circle, a model in which distant past values are useful in forecasting the distant future.

There is in fact a very concrete reason to consider representation (b) or something like it as well as (a). Unless we incorporate very long lags in the autoregressive representation for the time series, a univariate model that represents dividends in integrated form does not allow forecasted dividends to tend to revert back to their long-run historical values. A short univariate autoregressive forecasting equation in the first-difference of dividends makes forecasts of dividends in the distant future necessarily a function of only the most recent changes of dividends. In fact, distant dividends appear historically to be forecastable fairly well in terms of a long average of past dividends (Shiller [1984]). Of course, we estimate multivariate models but we want to specify these in such a way that an adequate univariate model of dividends is contained as a special case[3].

Note that in representation (a) if Δd_t and r_t are jointly stationary stochastic processes, it follows from the linearized model (4) and (5) that $d_t - p_t$ is stationary, or equivalently, d_t and p_t are cointegrated processes. Econometric techniques have been developed for them by Phillips and Durlauf [1985], Granger and Engle [1986] and Stock [1984]. Our model is

[3] When we tried adding long lags to representation (a) of the models, we got results somewhat resembling those for representation (b). This tends to confirm our view that representation (b) may be a parsimonious way to model "long memory" in the dividend process.

particularly straightforward to deal with since the cointegrating vector is specified in the model and does not require estimation. Ordinary theory of estimation of stationary vector autoregressions is applicable here.

VAR Test Methodology

For each version and representation of the model we define a vector x_t' in such a way that all its elements are known to the public at time t. For version I representation (a) we define $x_t' = [d_{t-1} - p_t, \Delta d_{t-1}, r_{t-1}]$, where all variables are demeaned, and in representation (b) the vector $x_t' = [d_{t-1} - p_t, d_{t-1}, r_{t-1}]$, where all variables are demeaned and d_{t-1} has been detrended. We write $C(L)x_t = u_t$ where the elements of C(L) are p'th-order polynomials in the lag operator L, i.e., we assume that a p'th order vector autoregressive representation for x_t exists. We rewrite the vector autoregressive representation in companion form $z_t = Az_{t-1} + v_t$, so that $E_t z_{t+k} = A^k z_t$. We define the vector e1 such that $e1'z_t = d_{t-1} - p_t$ (demeaned), the vector e2 such that $e2'z_t = \Delta d_{t-1}$ (demeaned) in representation (a) and d_{t-1} (detrended) in representation (b), and the vector e3 such that $e3'z_t = r_{t-1}$.

To state the restrictions of the model in terms of the vector autoregression, we substitute (5) into (4) (disregarding the constant h) and then replace r_{t+j} with $e3'A^{j+1}z_t$. Moreover we replace Δd_{t+j} with $e2'A^{j+1}z_t$ for representation (a) and with $e2'(A^{j+1}-A^j)z_t$ for representation (b). It follows, evaluating the infinite series, that δ_t should equal δ_t' given by:

$$\delta_t' = ((e3' - e2')A+e2'B)(I-\rho A)^{-1}z_t \qquad (6)$$

where B=0 in representation (a) and B = I in representation (b). Since the actual dividend-price ratio is in the information set on which we are conditioning, δ'_t should equal δ_t exactly, except for sampling error. That is, we do not have the usual difficulties in rational expectations models caused by the fact that market participants may have more information than econometricians. We can compare the history of δ'_t and δ_t as a way of evaluating the "fit" of the model. Equivalently, we can compare the elements of the matrix $((e3'-e2')A+e2'B)(I-\rho A)^{-1}$ on the right hand side of (6) with e1'. Both comparisons are made in the tables below.

To write the restriction $\delta_t = \delta'_t$ in terms of model parameters, we can replace δ'_t in (6) with $e1'z_t$, cancel z_t from both sides of the equation, and postmultiply by $(I-\rho A)$ to obtain:

$$e1'(I-\rho A) = (e3' - e2')A + e2'B \qquad (7)$$

Tests of these restrictions on the autoregressive coefficient matrix A using a Wald procedure are reported in the tables below. The restrictions (7) can be interpreted as asserting that a regression of the approximate excess return $\xi_t - r_t$ on information z_t gives z_t a zero coefficient, and the Wald test in fact corresponds in the sample to a standard regression F test of the restrictions.

It is also possible to decompose the behavior of δ'_t into two components: a component δ'_{dt} due to forecasts of change in dividends and a component δ'_{rt} due to forecasts of real interest rates:

$$\delta'_{dt} = (-e2'A+e2'B)(I-\rho A)^{-1}z_t \qquad (8)$$

$$\delta'_{rt} = e3'A(I-\rho A)^{-1}z_t \qquad (9)$$

To study version II of the model, we define in representation (a) the vector $x_t' = [d_{t-1} - p_t, \Delta d_{t-1}, \Delta c_{t-1}]$ where all variables are demeaned and in representation (b) the vector $x_t' = [d_{t-1} - p_t, d_{t-1}, \Delta c_{t-1}]$ where d_{t-1} is also detrended[4]. We assume that there is a vector autoregressive representation for x_t and as above write $C(L)x_t = u_t$, where the elements of C(L) are p'th order polynomials in the lag operator L. Rewriting the vector autoregressive representation in companion form $z_t = Az_{t-1} + v_t$, then $E_t z_{t+k} = A^k z_t$, as above. Then the model (4) and (5) implies:

$$\delta_t' = ((\alpha e3' - e2')A + e2'B)(I - \rho A)^{-1} z_t \qquad (10)$$

$$e1'(I - \rho A) = (\alpha e3' - e2')A + e2'B \qquad (11)$$

Here δ_t', as before, is the theoretical log dividend price ratio, the optimal forecast of the present value of future dividend changes and discount factors. As with version I, we can decompose δ_t' into a component δ_{dt}' due to rational expectations of future dividend changes and a component δ_{rt}' due to rational expectations of future discount factors. δ_{dt}' is defined as in equation (8), and δ_{rt}' is defined as α times the right hand side of equation (9).

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[4] Δc is lagged in the same way as Δd or d. The implicit assumption here is that the consumption data for each year represent consumption on December 31 of the year. Thus, in January of each year (the month in which our price data are drawn) Δc_{t-1} is known but Δc_t is not. There is no fully satisfactory way to handle the unit-averaged consumption data in the context of a theoretical model involving point-of-time consumption data, without going to the continuous time econometrics format, as in Grossman, Melino and Shiller [1985]. We did experiment with including current rather than lagged Δc in the vector, and did not find qualitatively different results.

We can estimate α, the coefficient of relative risk aversion, using the restriction (11). One might at first think that a unique value for α could be found by post-multiplying (11) by $A^{-1}e3$ and solving the resulting expression for α in terms of estimated coefficients. However, the restrictions (11) imply that A is singular. Defining e4 as the vector which is zero except for the second element, which is one, then $(\rho e1' + \alpha e3' - e2' - e4')A = 0$. Our approach was instead to use a method-of-moments estimator for α. Defining λ as the vector of deviations from the restriction (11), $\lambda = e1 - (\rho A)'e1 - A'(\alpha e3 - e2)$, a two-step procedure was used. Defining the vector of parameters of the model besides α as γ, we minimize $\lambda(\alpha,\gamma)'\Omega\lambda(\alpha,\gamma)$. The matrix Ω is taken as $(\partial\lambda/\partial\gamma'\Theta\partial\lambda/\partial\gamma)^{-1}$ where Θ is the variance matrix of the parameter vector γ. In step one, Ω was evaluated at $\alpha = 1$. In step 2, Ω was evaluated at the first round estimate of α.

The resulting estimate of α has the following interpretation. Equation (11) asserts that the prediction at time t of the linearized return $\dot{\xi}_t$ equals (a constant plus) α times the predicted change in log consumption. Our estimate of α is thus analogous to other estimates in the literature that rely on making forecasted returns correspond to forecasted changes in consumption. In Grossman and Shiller [1981] estimation of α along these lines was suggested (in the context of a plot of stock prices and their ex-post rational counterpart) but the discussion was couched in levels; the simple method used here of dealing with nonstationarity (dividing by lagged dividend) was not used and formal estimation in such terms was not attempted.

To study version III of the model, we define the vector $x_t' = [d_{t-1} - p_t,$

$\Delta d_{t-1}, V_{t-1}]$ in representation (a) and $x'_t = [d_{t-1} - p_t, d_{t-1}, V_{t-1}]$ in representation (b) and proceed as with version II.

Results

For all versions of the model, the point of linearization r-g was formed by taking the log of one plus the sample average real return on stocks and then subtracting the sample average change in log dividends. The point of linearization differs slightly across the versions only because the data and/or sample period differed slightly.

Estimates of version I, representation (a) (using first-differenced dividends) appear in Table 3a. The model is rejected at the 6.9% level. Despite this evidence against the model, δ_t does Granger-cause future dividend changes and there is substantial correlation of 0.773 between the theoretical log dividend price ratio δ'_t and the actual dividend price ratio δ_t. Most of this correlation comes from δ'_{dt}, and not δ'_{rt}. While the correlation is substantial, the standard deviation of δ'_t is only 0.417 times that of δ_t. This suggests that there is an element of truth to the model, but that the actual dividend price ratio "overreacts" to the news about future dividends.

For comparison with results in our earlier paper (Campbell and Shiller [1986]) which assumed constant discount factors and studied levels rather than logs of variables, we present also a Wald test of the model in which the time-varying discount factors are suppressed. Now the Wald test rejects at the 0.1% level. Thus it would appear that incorporating time-varying discount rates helps to "save" the efficient markets model. But, apparently it does so largely by bringing in a noisy extraneous variable to destroy the power of the test. The evidence that δ_t Granger causes future real interest rates is very weak (significant at only the 15.6% level). Allowing for time-varying interest rates greatly increases the standard error on the correlation of δ'_t with δ_t, but actually reduces the point estimate. By itself, δ'_{rt} is <u>negatively</u> correlated with δ_t at -0.139.

We now turn to representation (b) of version I, which uses detrended dividends (Table 3b). Some of the results are quite different. In this representation, the optimal forecast of the present value of future dividends discounted at a constant rate is close to a simple trend, reflecting the apparent sharp trend-reverting pattern of real dividends. This means that δ'_t is highly correlated with the detrended dividend; it is somewhat more variable than in representation (a) (its standard deviation is 0.634 times that of δ_t), and has a much lower correlation with δ_t (0.063). This is the kind of "excess volatility" discussed in an earlier paper by one of the authors (Shiller [1981])[5].

Despite these differences, a number of results are common to representations (a) and (b). Although the point estimates of summary statistics look very bad in representation (b), the model is still rejected only at the 6.0% level with the Wald test. It seems still to be the case that the inclusion of ex-post real discount rates helps the model by adding noise to the system; the weak relationship between the dividend-price ratio and future real interest rates is not affected by the specification of the dividend process.

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[5] That paper argued that the stock price displays excess volatility; this does not necessarily imply excess volatility of the dividend-price ratio, but it does imply a low correlation between the theoretical and actual dividend-price ratios.

Plots following the tables show the actual dividend-price ratio and a band from 2 standard errors below the theoretical dividend-price ratio to 2 standard errors above. The theoretical bands differ across representations (a) and (b) as described above, but both representations identify the same historical periods as unexplained by the model: the actual dividend-price ratio was too low in the first decade of the century, too high at the end of the First World War and at most of its subsequent peaks, and too low in the 1960's.

The general character of the results is fairly robust to the point of linearization. In representation (a) when the linearization parameter ρ = 0.900, $\sigma(\delta'_t)/\sigma(\delta_t)$ equals 0.410 and the correlation coefficient $cor(\delta'_t,\delta_t)$ equals 0.761, while when ρ = 0.975, $\sigma(\delta'_t)/\sigma(\delta_t)$ = 0.426 and $cor(\delta'_t,\delta_t)$ = 0.791. In representation (b) when ρ = 0.900, $\sigma(\delta'_t)/\sigma(\delta_t)$ = 0.601 and $cor(\delta'_t,\delta_t)$ = 0.156, while when ρ = 0.975, $\sigma(\delta'_t)/\sigma(\delta_t)$ = 0.688 and $cor(\delta'_t,\delta_t)$ = -0.062.

Because the period around the 1960's look particularly bad for the model, we estimated the model using data 1871-1950 only. The general character of the results is not very different from those reported in Table 3. In representation (a) $\sigma(\delta'_t)/\sigma(\delta_t)$ = 0.556 and $cor(\delta'_t,\delta_t)$ = 0.830, while in representation (b), $\sigma(\delta'_t)/\sigma(\delta_t)$ = 0.777 and $cor(\delta'_t,\delta_t)$ = 0.192.

We also made one more assessment of the approximation error of ξ as a measure of $\log(1 + R_t)$, this time in the context of our vector autoregressive model. It was noted above that the Wald test of the restrictions (7) is nothing more than a standard F test in a regresssion of the approximate excess return $\xi_t - r_t$ on the right hand side variables in the model. In that regression, the R^2 equals 0.077, and the standard deviation of the fitted value equals 0.0502. If instead the true excess return $\log(1+R_t) - \log(1+r_t)$ is regressed on the same variables the R^2 is 0.079 and the standard deviation of the fitted value is 0.0503. There is virtually no difference between the results, and the correlation coefficient between the fitted values is 0.9991.

Results for version II of the model, reported in Tables 4a and 4b, are similar in many ways to those for version I. There is perhaps a little encouragement for the model in that the dividend-price ratio does Granger cause consumption growth in both representations. However the forecastability of consumption growth does not seem to help explain the behavior of the dividend-price ratio. The estimated coefficient of relative risk aversion has the wrong sign in representation (a), and has the right sign but is insignificantly different from zero in representation (b). The model is rejected at about the 2% level in representation (a), and the 0.2% level in representation (b), whether or not we allow time variation in discount rates. The estimated δ'_{rt} has a very low standard deviation, relative to the standard deviation of δ_t, and its correlation with δ_t is imprecisely estimated.

The volatility of returns variable V_t, in version III of the model, does not contribute much toward interpreting actual dividend price ratios. The actual dividend-price ratio δ_t does not Granger-cause V_t. While the risk parameter α has the right sign, it is insignificantly different from zero. The correlation of δ'_{rt} with δ_t is 0.114 in representation (a) (Table 5a) and -0.175 in representation (b) (Table 5b); in both representations the standard deviation of δ'_{rt} is negligible compared to that of δ_t.[6]

Finally, plots for versions II and III of the model identify the same problematic historical periods as did the plots for version I.

<u>Conclusion</u>

In this paper we have tried to explain time variation in corporate stock prices relative to dividends. Our main result is a negative one: there is very little evidence that the dividend-price ratio is driven by rational expectations of observed ex-post one-period discount rates.

The negative conclusion holds whether we measure discount rates from real returns on short debt, aggregate consumption growth, or the volatility of stock returns themselves. Even the weakest implication of a time-varying discount rate model, that the dividend-price ratio should Granger cause future ex-post discount rates, is strongly confirmed only for consumption growth. And however we measure discount rates, the present value of rationally expected future rates moves far too little to explain much variation in the dividend-price ratio.

The standard deviation of the δ'_{rt}, the component of the log dividend-price ratio δ_t attributable to discount rate movements, never exceeds about 17% of the standard deviation of δ_t itself and is never very correlated with it. That δ'_{rt} shows so little variability is not surprising since, as

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[6]Pindyck's analysis [1986] is in some ways similar to ours, and relies on a linearization of the present value relation. His analysis differs from ours in that: a. the first difference of log price rather than the log dividend-price ratio is explained, b. pretax profits (and assumptions about taxes and payout ratios) are employed where we used dividends, c. univariate AR-1 representations for discount factors and pretax earnings are assumed to hold with no superior information, and d. postwar data are employed. He provides estimates of α ranging from 3 to 5, roughly consistent with our estimates for Version III. He does not provide measures of the importance of V_t analogous to ours, except to say that V_t explained about 1/3 of the market decline in 1974.

the tables show, the one-period discount factor measures themselves are always much less variable than the log dividend price ratio, and always show little persistence through time.

There is more support for the view that the dividend-price ratio reflects rational expectations of future dividend growth. The results from representation (a), in which the log dividend process has a unit root, are particularly favorable to this view. In representation (a) we found substantial correlation between the log dividend-price ratio and the appropriate optimal forecast of future dividends. This result is similar to one reported in our earlier paper (Campbell and Shiller [1986]): there we found that the spread between the long-term interest rate and the short-term interest rate tends to be high when short rates can be forecast to increase.

A cynical view of both these results is that they reveal nothing more than that long rates are smoother than short rates, and stock prices are smoother than dividends. Given this smoothness, it's not surprising that actual spreads or ratios correlate somewhat with optimal spreads or ratios. In the present example, the actual dividend price ratio shares the same numerator with the theoretical dividend price ratio, and if the numerator shows some short-run noise not in the denominators, there will be a correlation between actual and theoretical.

However the cynical view cannot account for the finding that the dividend-price ratio strongly Granger causes future dividends. This finding is extremely robust to changes in lag length or time series representation for dividends; in all models we estimated, we found Granger causality at better than the 0.1% level. As one would expect from this result, in all representations the optimal forecast of future dividend growth, δ'_{dt}, places statistically significant weight on the actual dividend-price ratio δ_t.

Even if we reject the cynical view of our results, it is clear that there is considerable variation in the dividend-price ratio which cannot be accounted for by rational expectations of future dividend growth. In both representations (a) and (b) a constant discount rate model is quite strongly rejected, and summary statistics suggest that the actual dividend-price ratio "moves too much". The ratio of the standard deviation of the optimal forecast of dividend growth, to the standard deviation of the actual dividend-price ratio, is significantly less than one at the 5% level in all the models we estimate.

Appendix: Sources of Data and Equilibrium Foundations of the Model

For estimation of version I of the model, the real stock price and real dividend series are the same as in Campbell and Shiller [1986]. P_t is the January Standard and Poor Composite Stock Price Index divided by the Producer Price Index π_t (also for January starting in 1900, annual average before that). D_t is the total dividends per share accruing to index for the calendar year, divided by the annual average producer price index. The real interest rate r_t is $\ln(\pi_t(\pi_{t+1}(1-r_{1t}/200)(1-r_{2t}/200))^{-1})$ where r_{1t} is the January value and r_{2t} is the July value of the prime 4-6-month prime commercial paper rate (6-month starting in 1979) in annual percent. Interest rate data starting in 1938 are from the Board of Governors of the Federal Reserve System, before 1938 from Macaulay [1938], Table 10, pp. A142-60. The sample period for version I is 1871-1986.

For estimation of version II of the model, P_t and D_t are the same as in version I except that they are divided by the (annual average) consumption deflator for nondurables and services rather than by the producer price index. C_t is real per capita consumption of nondurables and services. The consumption deflator and C_t are defined as described in Grossman and Shiller [1981]. The sample period is 1889-1986.

For estimation of version III of the model, P_t and D_t are the same as in version I. We thank James Poterba and Lawrence Summers for providing us with the same V_t series that they used in their paper [1985]; it is an annual series with each observation consisting of the average of squared monthly log gross returns on the Standard and Poor Composite Index for the

12 months of the year. The sample period is 1871-1985.

The model may be derived in general equilibrium as follows. Version II holds in a representative agent economy when the representative individual maximizes $U = E_t \Sigma \beta^j C_{t+j}^{(1-\alpha)}/(1-\alpha)$, and when the conditional covariance between real stock returns and the marginal utility ratio, $Cov_t((P_{t+1}+D_t)/P_t, \beta(C_t/C_{t+1})^{\alpha})$, is constant. The other versions of the model can be obtained by adding assumptions to those of version II, although they may also hold more generally. Version I holds if, in addition to the assumptions above, the conditional covariance between the real return on short-term debt and the marginal utility ratio is constant. Version III is suggested by a constant relative risk aversion framework if the expected marginal utility ratio is constant through time, generating a constant riskless real rate μ satisfying $\exp(-\mu) = E\beta(C_t/C_{t+1})^{\alpha})$, and if we have $1 - Cov_t((P_{t+1}+D_t)/P_t, \beta(C_t/C_{t+1})^{\alpha}) = \exp(\alpha V_t)$. Following Merton [1973], [1980], one can show that this covariance restriction holds up to a linear approximation (it holds exactly in continuous time) if the conditional covariance and variance are constant through time and the stock market return is equal to the return on total invested wealth. Of course, Version III has time-varying covariances and variances, but is suggested by the Merton framework if changes are sufficiently slow.

TABLE 1a

EVALUATION OF THE LOG-LINEAR APPROXIMATION TO STOCK RETURNS

Data Set	Mean Error		Correlation		Variance Ratio	
	(i)	(ii)	(i)	(ii)	(i)	(ii)
1871-1986, PPI deflated	0.0222	0.0051	0.9921	0.9996	0.0166	0.0008
1889-1985, CPI deflated	0.0204	0.0037	0.9926	0.9998	0.0158	0.0005

Notes: The 1871-1986 PPI deflated data are used in versions I and III of the model. The 1889-1985 CPI deflated data are used in version II of the model. (i) compares the approximate return ξ_t to the net simple return R_t. (ii) compares the approximate return ξ_t to the log gross (continuously compounded) return $\log(1+R_t)$. "Correlation" is the correlation of the exact and approximate return. "Variance ratio" is the ratio of the error variance to the variance of the exact return. The statistics shown here are those used in Campbell [1986] to evaluate a linear approximation to bond returns.

TABLE 1b

EVALUATION OF THE LOG-LINEAR APPROXIMATION TO δ_t^*

Data Set, Model Version	Mean Error	Correlation	Variance Ratio
1871-1986, PPI deflated Version I	0.0646	0.9915	0.0175
1889-1985, CPI deflated Version II $\alpha = 1$	0.0158	0.9992	0.0018

Notes: The approximate "perfect foresight log dividend-price ratio" δ_t^* is constructed using equation (5) and a terminal condition $\delta_T^* = \delta_T$. The exact δ_t^* is constructed using equation (1) and the same terminal condition. The ex-post discount rate is adjusted for a constant premium by adding the difference between its mean and the log of the mean gross return on stocks. "Correlation" is the correlation of the exact and approximate δ_t^*. "Variance ratio" is the ratio of the error variance to the variance of the exact δ_t^*. The statistics shown here are those used in Campbell [1986] to evaluate a linear approximation to bond returns.

TABLE 2

UNIVARIATE TESTS FOR UNIT ROOTS

Variable	1871-1986, PPI deflated	1889-1986, CPI deflated
Log Real Dividend	-3.57 (5%)	-3.15 (10%)
Log Real Price	-2.73	-2.51
Log Dividend-Price Ratio	-4.36 (1%)	-3.90 (2.5%)
Ex Post Real Commercial Paper Rate	-7.47 (1%)	----
Real Consumption Growth	----	-11.01 (1%)
Volatility	-5.50 (1%)	----

<u>Notes</u>: Test statistic is $Zt\tilde{\alpha}$ from Phillips and Perron [1986] and as used in Perron [1986]. The statistic is formed from the t statistic on α in the regression $\Delta y_t = \mu + \beta t + \alpha y_{t-1}$, corrected for serial correlation in the equation error using a 4th-order Newey-West [1985] correction. The critical values for the statistic are as reported in Fuller [1976]: 1% -3.96, 2.5% -3.66, 5% -3.41, 10% -3.12.

TABLE 3a

MODEL VERSION I: REPRESENTATION (a), 2 LAGS

Data set 1871-1986, VAR Sample period 1875-1986
Mean return 0.079, mean dividend growth rate 0.013, $\rho = 0.936$

$\sigma(\delta) = 0.275$, $\sigma(\Delta d) = 0.126$, $\sigma(r) = 0.095$
δ_t equation $R^2 = 0.555$, Δd_{t-1} $R^2 = 0.514$, r_{t-1} $R^2 = 0.167$

δ Granger causes Δd at 0.000 level, r at 0.156 level
Δd Granger causes δ at 0.015 level, r at 0.958 level
r Granger causes δ at 0.249 level, Δd at 0.002 level

Linear Wald test of present value model: $\chi_6^2 = 11.689$, P-Value = 0.069
Linear Wald test of present value model imposing constant discount rates: $\chi_6^2 = 22.157$, P-Value = 0.001

Coefficients on	Estimates of δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.585	0.561	0.024
	(0.225)	(0.132)	(0.218)
δ_{t-1}	-0.376	-0.312	-0.064
	(0.165)	(0.155)	(0.122)
Δd_{t-1}	-0.191	-0.274	0.083
	(0.282)	(0.227)	(0.218)
Δd_{t-2}	0.101	0.070	0.031
	(0.161)	(0.086)	(0.151)
r_{t-1}	0.310	-0.177	0.487
	(0.340)	(0.221)	(0.316)
r_{t-2}	0.253	0.391	-0.138
	(0.197)	(0.152)	(0.165)
Joint significance of coefficients	0.000	0.000	0.291

Summary Statistics for Estimates

	δ'_t	δ'_{dt}	δ'_{rt}
σ as ratio to $\sigma(\delta_t)$	0.417	0.405	0.173
	(0.166)	(0.129)	(0.146)
Correlations with δ_t	0.773	0.855	-0.139
	(0.368)	(0.187)	(1.406)
Correlations with δ'_t	-----	0.912	0.274
		(0.171)	(0.820)
Correlations with δ'_{dt}	-----	-----	-0.145
			(1.120)

TABLE 3b

MODEL VERSION I: REPRESENTATION (b), 2 LAGS

Data set 1871-1986, VAR sample period 1875-1986
Mean return 0.079, mean dividend growth rate 0.013, $\rho = 0.936$

$\sigma(\delta) = 0.275$, $\sigma(d) = 0.208$, $\sigma(r) = 0.095$
δ_t equation $R^2 = 0.554$, d_{t-1} $R^2 = 0.845$, r_{t-1} $R^2 = 0.166$

δ Granger causes d at 0.000 level, r at 0.128 level
d Granger causes δ at 0.030 level, r at 0.973 level
r Granger causes δ at 0.165 level, d at 0.000 level

Linear Wald test of present value model: $\chi^2_6 = 12.101$, P-Value = 0.060
Linear Wald test of present value model imposing constant discount rates: $\chi^2_6 = 33.592$, P-Value = 0.000

Estimates of

Coefficients on	δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.235	0.151	0.084
	(0.248)	(0.034)	(0.265)
δ_{t-1}	-0.116	-0.008	-0.109
	(0.102)	(0.033)	(0.110)
d_{t-1}	0.768	0.835	-0.068
	(0.320)	(0.066)	(0.340)
d_{t-2}	0.010	0.025	-0.016
	(0.154)	(0.047)	(0.159)
r_{t-1}	0.381	-0.100	0.482
	(0.301)	(0.056)	(0.322)
r_{t-2}	-0.057	0.034	-0.091
	(0.176)	(0.034)	(0.192)
Joint significance of coefficients	0.000	0.000	0.239

Summary Statistics for Estimates

σ as ratio to $\sigma(\delta_t)$	0.634	0.635	0.164
	(0.202)	(0.035)	(0.107)
Correlations with δ_t	0.063	0.036	0.103
	(0.479)	(0.079)	(1.901)
Correlations with δ'_t	-----	0.967	0.122
		(0.046)	(1.247)
Correlations with δ'_{dt}	-----	-----	-0.137
			(1.317)

TABLE 4a
MODEL VERSION II: REPRESENTATION (a), 2 LAGS

Data set 1889-1985, VAR sample period 1893-1985
Mean return 0.071, mean dividend growth rate 0.011, ρ = 0.941

$\sigma(\delta)$ = 0.280, $\sigma(\Delta d)$ = 0.119, $\sigma(\Delta c)$ = 0.034
δ_t equation R^2 = 0.572, Δd_{t-1} R^2 = 0.456, Δc_{t-1} R^2 = 0.378

δ Granger causes Δd at 0.000 level, Δc at 0.000 level
Δd Granger causes δ at 0.011 level, Δc at 0.000 level
Δc Granger causes δ at 0.961 level, Δd at 0.398 level

Estimate of α = -1.019 (0.793)

Hansen χ^2 test of present value model: χ^2_5 = 13.230, P-Value = 0.021
Linear Wald test of present value model imposing constant discount rates:
χ^2_6 = 14.822, P-Value = 0.022

Coefficients on	Estimates of δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.620	0.531	0.089
	(0.166)	(0.147)	(0.094)
δ_{t-1}	-0.479	-0.382	-0.097
	(0.196)	(0.180)	(0.082)
Δd_{t-1}	-0.282	-0.250	-0.032
	(0.281)	(0.255)	(0.059)
Δd_{t-2}	0.043	0.006	0.037
	(0.132)	(0.122)	(0.042)
Δc_{t-1}	-0.786	-0.664	-0.122
	(0.933)	(0.812)	(0.249)
Δc_{t-2}	-0.622	-0.322	-0.300
	(0.664)	(0.606)	(0.252)
Joint significance of coefficients	0.000	0.000	0.939

Summary Statistics for Estimates

	δ'_t	δ'_{dt}	δ'_{rt}
σ as ratio to $\sigma(\delta_t)$	0.418	0.360	0.069
	(0.132)	(0.125)	(0.060)
Correlations with δ_t	0.758	0.801	0.418
	(0.323)	(0.299)	(0.767)
Correlations with δ'_t	-----	0.995	0.865
		(0.010)	(0.298)
Correlations with δ'_{dt}	-----	-----	0.813
			(0.381)

TABLE 4b

MODEL VERSION II: REPRESENTATION (b), 2 LAGS

Data set 1889-1985, VAR sample period 1893-1985
Mean return 0.071, mean dividend growth rate 0.011, ρ = 0.941

$\sigma(\delta)$ = 0.280, $\sigma(d)$ = 0.199, $\sigma(\Delta c)$ = 0.034
δ_t equation R^2 = 0.560, d_{t-1} R^2 = 0.822, Δc_{t-1} R^2 = 0.402

δ Granger causes d at 0.000 level, Δc at 0.000 level
d Granger causes δ at 0.059 level, Δc at 0.000 level
Δc Granger causes δ at 0.833 level, d at 0.722 level

Estimate of α = 0.320 (0.766)

Hansen χ^2 test of present value model: χ^2_5 = 20.719, P-Value = 0.001
Linear Wald test of present value model imposing constant discount rates: χ^2_6 = 21.111, P-Value = 0.002

Coefficients on	Estimates of δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.140	0.142	-0.002
	(0.042)	(0.036)	(0.015)
δ_{t-1}	-0.033	-0.039	0.006
	(0.041)	(0.034)	(0.024)
d_{t-1}	0.731	0.793	-0.062
	(0.217)	(0.073)	(0.227)
d_{t-2}	0.030	0.033	-0.004
	(0.058)	(0.051)	(0.022)
Δc_{t-1}	-0.047	-0.024	-0.023
	(0.189)	(0.147)	(0.092)
Δc_{t-2}	0.036	-0.021	0.057
	(0.238)	(0.120)	(0.214)
Joint significance of coefficients	0.000	0.000	0.999

Summary Statistics for Estimates

	δ'_t	δ'_{dt}	δ'_{rt}
σ as ratio to $\sigma(\delta_t)$	0.548	0.594	0.048
	(0.156)	(0.041)	(0.178)
Correlations with δ_t	0.122	0.102	0.136
	(0.137)	(0.081)	(0.361)
Correlations with δ'_t	-----	0.999	-0.951
		(0.003)	(0.119)
Correlations with δ'_{dt}	-----	-----	-0.958
			(0.086)

TABLE 5a

MODEL VERSION III: REPRESENTATION (a), 2 LAGS

Data set 1871-1985, VAR sample period 1875-1985
Mean return 0.077, mean dividend growth rate 0.013, $\rho = 0.937$

$\sigma(\delta) = 0.276$, $\sigma(\Delta d) = 0.126$, $\sigma(V) = 0.003$
δ_t equation $R^2 = 0.555$, Δd_{t-1} $R^2 = 0.430$, V_{t-1} $R^2 = 0.393$

δ Granger causes Δd at 0.000 level, V at 0.438 level
Δd Granger causes δ at 0.089 level, V at 0.638 level
V Granger causes δ at 0.575 level, Δd at 0.336 level

Estimate of $\alpha = 5.818$ (5.447)

Hansen χ^2 test of present value model: $\chi^2_5 = 12.697$, P-Value = 0.026
Linear Wald test of present value model imposing constant discount rates:
$\chi^2_6 = 16.222$, P-Value = 0.013

Coefficients on	Estimates of δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.504	0.496	0.009
	(0.168)	(0.148)	(0.039)
δ_{t-1}	-0.377	-0.357	-0.021
	(0.163)	(0.147)	(0.033)
Δd_{t-1}	-0.087	-0.134	0.047
	(0.213)	(0.193)	(0.044)
Δd_{t-2}	0.140	0.130	0.011
	(0.080)	(0.074)	(0.018)
V_{t-1}	21.584	14.095	7.489
	(10.256)	(8.798)	(4.848)
V_{t-2}	11.434	11.194	0.241
	(6.534)	(6.047)	(1.993)
Joint significance of coefficients	0.000	0.000	0.826

Summary Statistics for Estimates

	δ'_t	δ'_{dt}	δ'_{rt}
σ as ratio to $\sigma(\delta_t)$	0.477	0.421	0.079
	(0.146)	(0.125)	(0.056)
Correlations with δ_t	0.625	0.687	0.114
	(0.313)	(0.313)	(0.358)
Correlations with δ'_t	-----	0.992	0.751
		(0.011)	(0.264)
Correlations with δ'_{dt}	-----	-----	0.663
			(0.326)

TABLE 5b
MODEL VERSION III: REPRESENTATION (b), 2 LAGS

Data set 1871-1985, VAR sample period 1875-1985
Mean return 0.077, mean dividend growth rate 0.013, ρ = 0.937

$\sigma(\delta)$ = 0.276, $\sigma(d)$ = 0.208, $\sigma(V)$ = 0.003
δ_t equation R^2 = 0.555, d_{t-1} R^2 = 0.818, V_{t-1} R^2 = 0.418

δ Granger causes d at 0.000 level, V at 0.459 level
d Granger causes δ at 0.099 level, V at 0.192 level
V Granger causes δ at 0.461 level, d at 0.561 level

Estimate of α = 5.083 (4.602)

Hansen χ^2 test of present value model: χ^2_5 = 25.831, P-Value = 0.000
Linear Wald test of present value model imposing constant discount rates: χ^2_6 = 29.410, P-Value = 0.000

Coefficients on	Estimates of δ'_t	δ'_{dt}	δ'_{rt}
δ_t	0.130	0.151	-0.020
	(0.038)	(0.031)	(0.027)
δ_{t-1}	-0.024	-0.032	0.008
	(0.034)	(0.032)	(0.021)
d_{t-1}	0.948	0.830	0.118
	(0.105)	(0.058)	(0.097)
d_{t-2}	-0.011	0.038	-0.049
	(0.057)	(0.043)	(0.042)
V_{t-1}	6.295	0.801	5.494
	(4.060)	(1.388)	(4.473)
V_{t-2}	1.942	2.418	-0.476
	(1.846)	(1.266)	(1.719)
Joint significance of coefficients	0.000	0.000	0.932

Summary Statistics for Estimates

σ as ratio to $\sigma(\delta_t)$	0.715	0.649	0.083
	(0.061)	(0.031)	(0.066)
Correlations with δ_t	-0.008	0.013	-0.175
	(0.065)	(0.066)	(0.287)
Correlations with δ'_t	-----	0.997	0.823
		(0.004)	(0.147)
Correlations with δ'_{dt}	-----	-----	0.780
			(0.172)

REFERENCES

Breeden, Douglas T., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics 7:265-96, 1979.

Campbell, John Y., "A Defense of Traditional Hypotheses about the Term Structure of Interest Rates," Journal of Finance 41:183-193, March 1986.

Campbell, John Y., "Stock Returns and the Term Structure", forthcoming Journal of Financial Economics, 1987.

Campbell, John Y., and Robert J. Shiller, "Cointegration and Tests of Present Value Models," NBER Working Paper No. 1885, April 1986.

French, Kenneth R., G. William Schwert and Robert F. Stambaugh, "Expected Stock Returns and Volatility", unpublished paper, University of Chicago, revised March 1986.

Granger, Clive W. J., and Robert F. Engle, "Cointegration and Error-Correction: Representation, Estimation and Testing", forthcoming Econometrica, 1986.

Grossman, Sanford J., Angelo Melino and Robert J. Shiller, "Estimating the Continuous Time Consumption Based Asset Pricing Model," NBER Working Paper No. 1643, 1985.

Grossman, Sanford J., and Robert J. Shiller, "The Determinants of the Variability of Stock Market Prices," American Economic Review, 71:222-7, May 1981.

Hall, Robert E., "Real Interest Rates and Consumption," NBER working paper No. 1694, August 1985.

Hansen, Lars P., and Kenneth J. Singleton, "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," Journal of Political Economy, 91:249-65, 1983.

Robert E. Lucas, "Asset Prices in an Exchange Economy," Econometrica 46:1429-1445, 1978.

Macaulay, Frederic, Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856, NBER, New York, 1938.

Mankiw, N. Gregory, Julio Rotemberg, and Lawrence Summers, "Intertemporal Substitution in Macroeconomics," Quarterly Journal of Economics, 1985.

Mankiw, N. Gregory, David Romer, and Matthew D. Shapiro, "An Unbiassed Reexamination of Stock Market Volatility," Journal of Finance, 40:677-87, May 1985.

Mehra, Raj and Edward Prescott, "The Equity Premium: A Puzzle," Journal of Monetary Economics 15:145-162, March 1985.

Merton, Robert C., "On Estimating the Expected Return on the Market", Journal of Financial Economics 8:323-361, 1980.

Newey, Whitney K. and Kenneth D. West, "A Simple, Positive Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," NBER Technical Working Paper No. 55, 1986.

Perron, Pierre, "Trends and Random Walks in Macroeconomic Time Series: Further Evidence from a New Approach," Departement de sciences economiques et C.R.D.E., Universite de Montreal, 1986.

Phillips, Peter C.B. and Stephen N. Durlauf, "Multiple Time Series Regression with Integrated Processes", Cowles Foundation Discussion Paper No. 768, November 1985.

Phillips, Peter C.B. and Pierre Perron, "Testing for Unit Roots in Time Series Regression," Cowles Foundation Discussion Paper, June 1986.

Pindyck, Robert S., "Risk, Inflation, and the Stock Market," American Economic Review, 74:335-51, June 1984.

Pindyck, Robert S., "Risk Aversion and the Determinants of Stock Market Behavior," NBER Working Paper No. 1921, May 1986.

Poterba, James M., and Lawrence H. Summers, "The Persistence of Volatility and Stock Market Fluctuations," NBER Working Paper No. 1462, April 1985.

Shiller, Robert J., "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", American Economic Review 71:421-436, June 1981.

Shiller, Robert J., "Stock Prices and Social Dynamics", Brookings Papers on Economic Activity 2:457-498, 1984.

Stock, James H., "Asymptotic Properties of Least Squares Estimates of Cointegrating Vectors," unpublished paper, Harvard University, November 1984.

West, Kenneth D. [1986a], "Dividend Innovations and Stock Price Volatility", NBER Working Paper No. 1833, February 1986.

West, Kenneth D. [1986b], "A Specification Test for Speculative Bubbles", unpublished paper, Princeton University, revised June 1986.

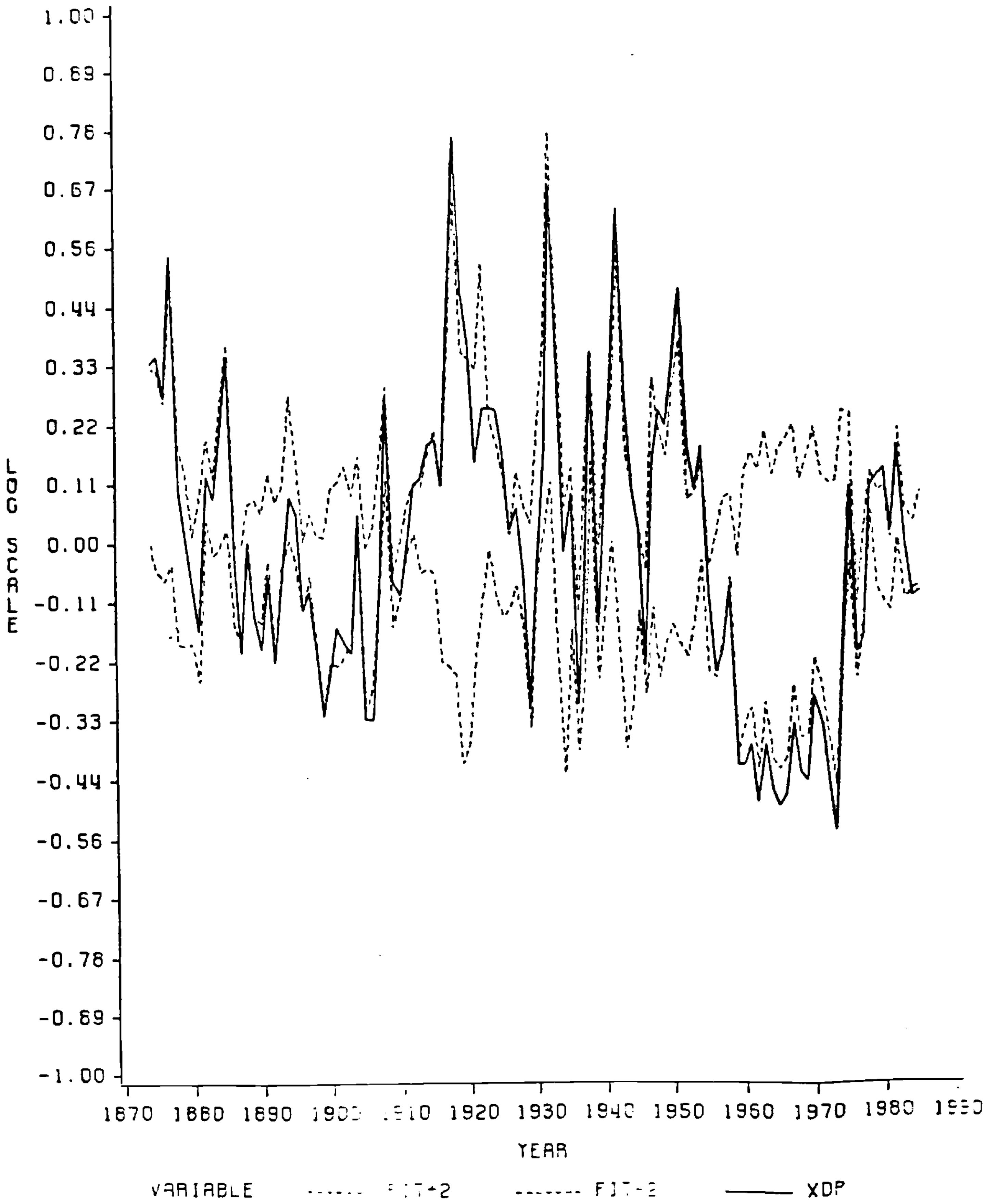
Notes for Figures.

Each figure shows the demeaned log dividend-price ratio δ_t (solid line, labelled XDP below the figures) and a band around the theoretical demeaned log dividend-price ratio δ'_t (dashed lines, labelled FIT+2 and FIT-2 in the figures). δ'_t is the optimal linear forecast with information available at time t of δ^*_t, the weighted average of future dividend changes and future real interest rates. The upper edge of the plotted band is 2 standard errors above δ'_t, and the lower edge is 2 standard errors below.

There are six figures, one for each version and time series representation of the model. Representation (a) is labelled "cointegrated" in the figures, and representation (b) is "trend-stationary".

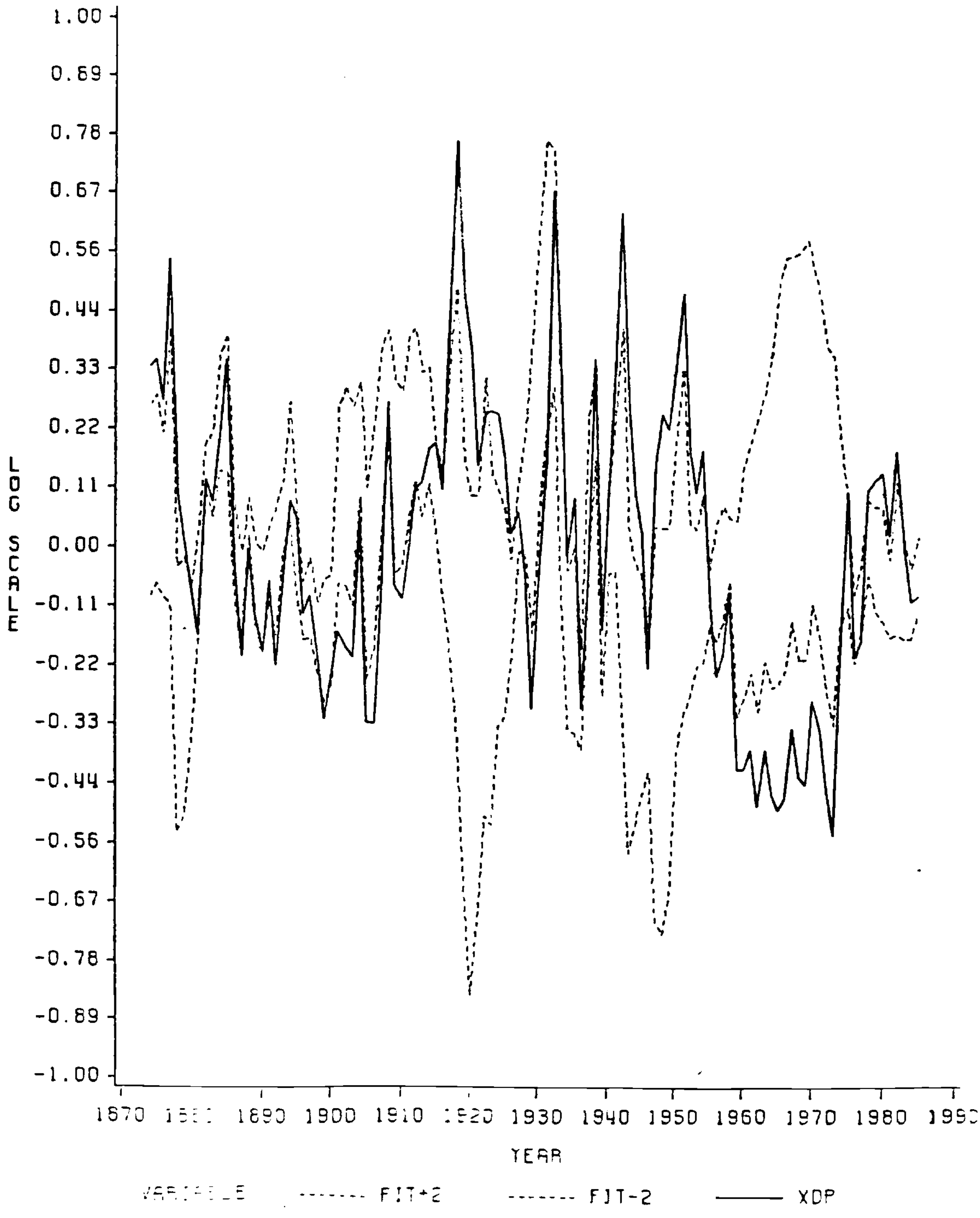
LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 1. COINTEGRATED



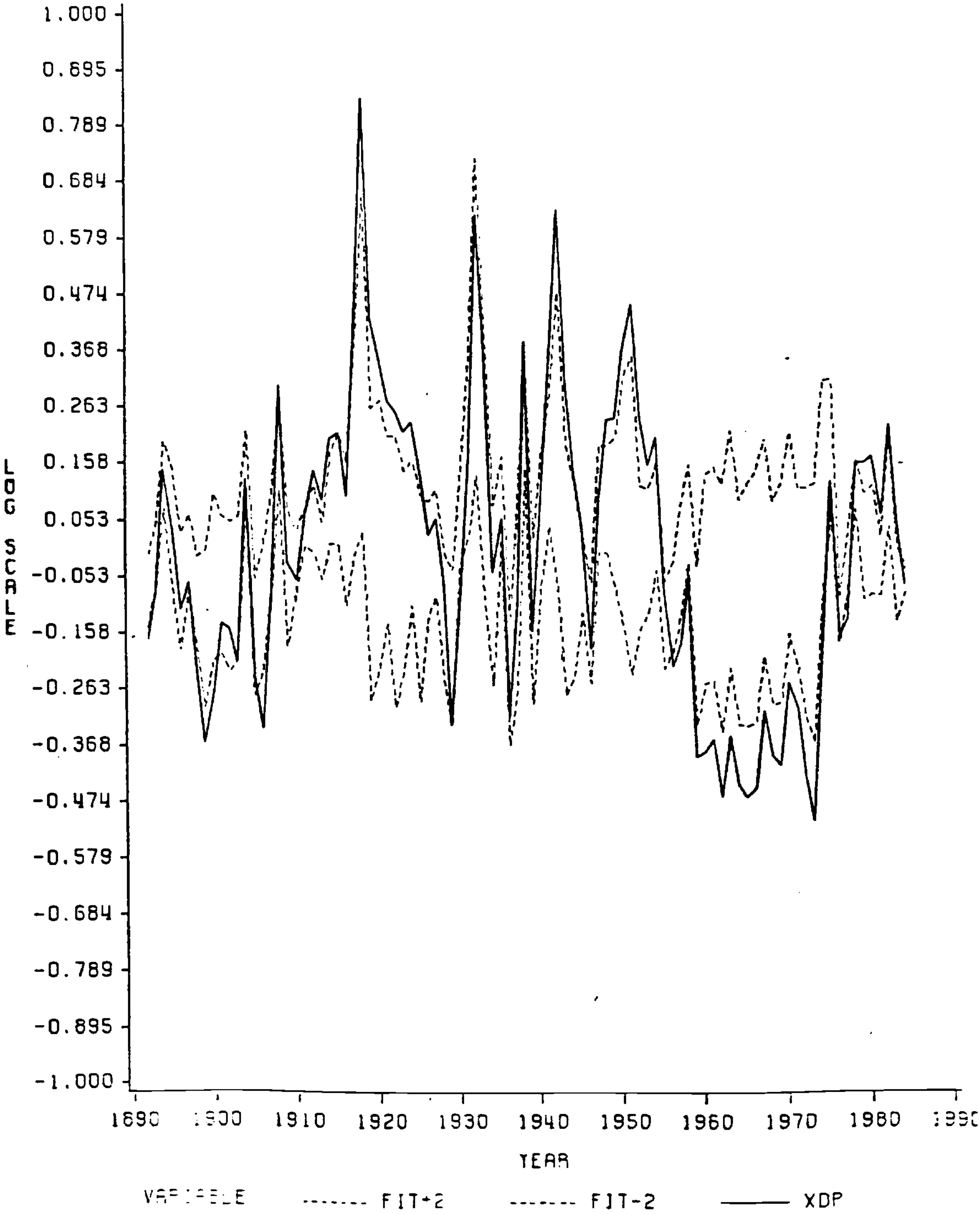
LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 1. TREND-STATIONARY



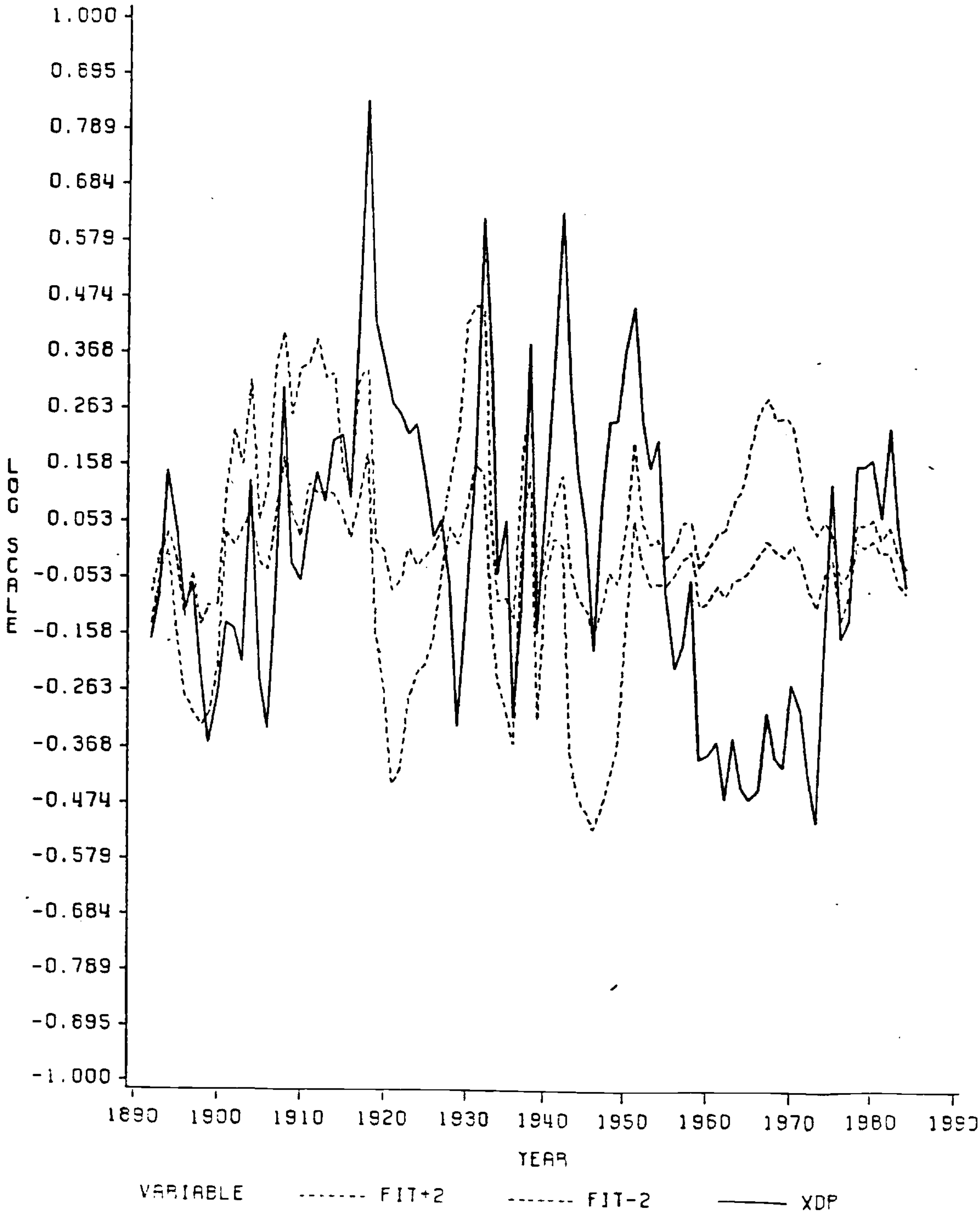
LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 2, COINTEGRATED



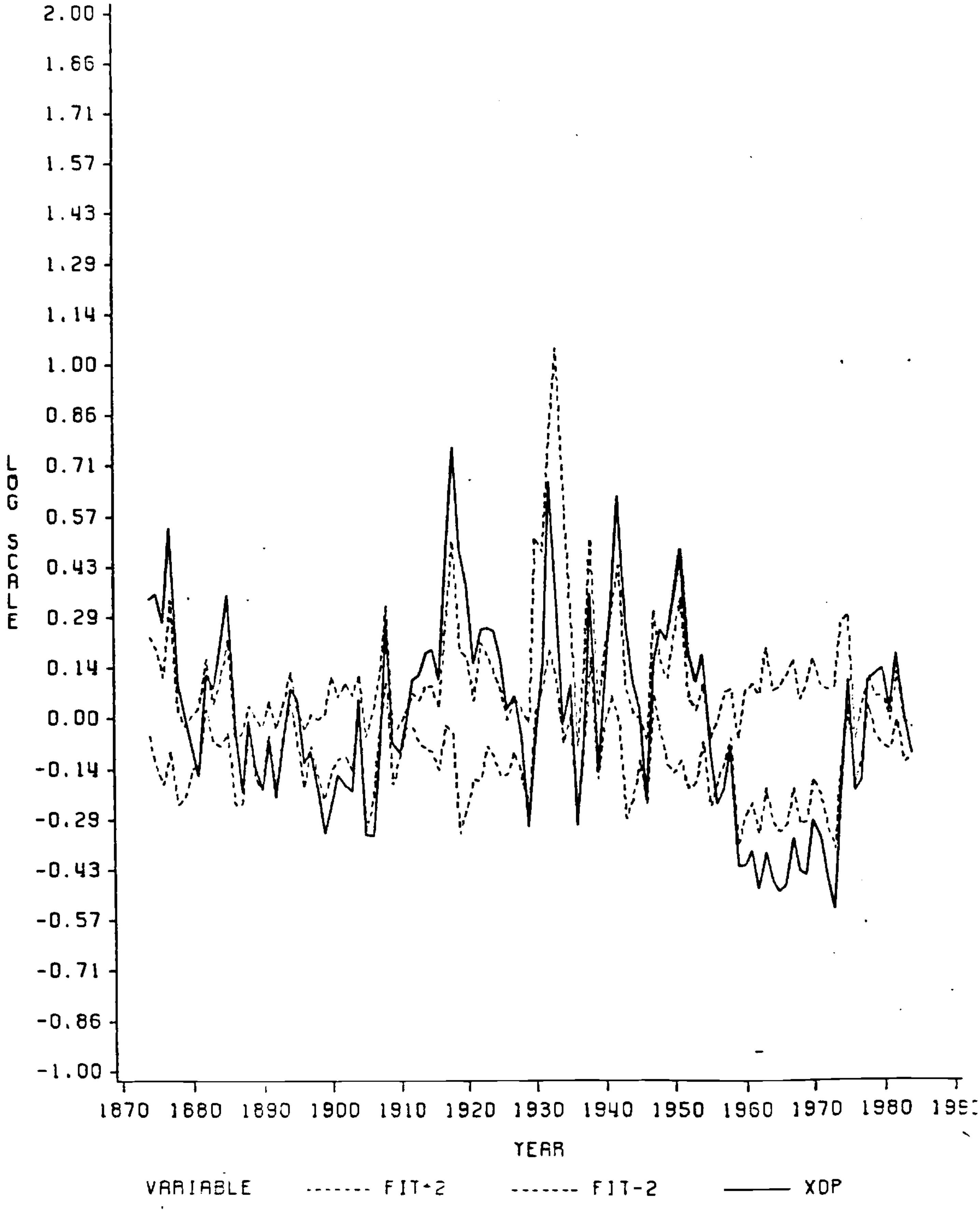
LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 2. TREND-STATIONARY



LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 3, COINTEGRATED



LOG D/P RATIO AND VAR FORECAST BAND

MODEL VERSION 3. TREND-STATIONARY