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THE DYNAMIC BEHAVIOR OF THE REAL EXCHANGE RATE IN STICKY PRICE MODELS

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ABSTRACT

Existing empirical evidence suggests that real exchange rates exhibit hump-shaped dynamics. I show that this is a robust fact across nine large, developed economies. This fact can help explain why existing sticky-price business cycle models have been unable to match the persistence of the real exchange rate. The recent literature has focused on models driven by monetary shocks. These models yield monotonic impulse responses for the real exchange rate. It is extremely difficult for models that have this feature to match the empirical persistence of the real exchange rate. I show that in response to a number of different real shocks a two-country sticky-price business cycle model yields hump-shaped dynamics for the real exchange rate. The hump-shaped dynamics generated by the model are a powerful source of endogenous persistence that allows the model to match the long half-life of the real exchange rate.

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1 Introduction

Since the breakdown of the Bretton Woods system of fixed exchange rates, the real exchange rates of the world's largest economies have been highly volatile. Furthermore, swings in these real exchange rates have been highly persistent. A large recent literature has studied whether the volatility and persistence of real exchange rates can be understood in the context of sticky price models with staggered price setting. This literature was pioneered by V.V. Chari, Patrick J. Kehoe and Ellen R. McGrattan (2002). They concluded that such models can explain the volatility of the real exchange rate but that they can not match its persistence. A number of subsequent papers have sought to address this "persistence anomaly" by introducing various forms of strategic complementarity and asymmetry as well as sticky wages and persistent monetary policy (Paul Bergin and Robert C. Feenstra, 2001; Gianluca Benigno, 2004; Jan J.J. Groen and Akito Matsumoto, 2004; Jens Sondergaard, 2004; Hafedh Bouakez, 2005). While these features increase the persistence of the real exchange rate considerably, they are not sufficient to match the half-life of the real exchange rate seen in the data.

Existing empirical evidence suggests that real exchange rates exhibit hump-shaped dynamics (John Huizinga, 1987; Martin S. Eichenbaum and Charles L. Evans, 1995; Yin-Wong Cheung and Kon S. Lai, 2000). I show that this is a robust empirical fact for nine large, developed economies. I estimate an autoregressive model for the real exchange rate of each economy. The estimated short term dynamics cause impulses to be amplified for several quarters before they start dying out. Figure 1 illustrates this by plotting the estimated response of the U.S. real exchange rate to a unit sized impulse. After the impulse, the real exchange rate keeps rising for over a year. It takes the real exchange rate 10 quarters to fall below the initial size of the impulse. After this short term amplification, the real exchange rate mean reverts quite rapidly; falling below 1/2 the size of the impulse in 18 quarters and below 1/4 of the size of impulse in less than 26 quarters.

These hump-shaped dynamics can help explain why existing sticky-price business cycle models have been unable to match the persistence of the real exchange rate. Following Chari, Kehoe and McGrattan (2002), the literature has mostly focused on the response of the real exchange rate to monetary shocks. I present a two country sticky-price model with staggered price setting and show that in response to a monetary shock the model implies an exponentially decaying response for the real exchange rate. Even with very large amounts of strategic complementarity, the rate of decay of the real exchange rate is such that the model is nowhere close to matching the empirical persistence of real exchange rates.

Empirical work on vector autoregression models suggests that only a small fraction of the variability of most macroeconomic aggregates is due to monetary shocks (Lawrence J. Christiano, Eichenbaum and Evans, 1999). I show that in response to several different types of real shocks—productivity shocks, labor supply shocks, government spending shocks, shocks to the world demand for home goods and cost-push shocks—my model implies hump-shaped dynamics for the real exchange rate. These hump-shaped dynamics are a powerful source of endogenous persistence that allow it to easily generate a half-life equal to the estimated half-life of the U.S. real exchange rate. Contrary to conventional wisdom, I show that these real shocks generate slightly more real exchange rate volatility in the model than does the monetary shock. My model is therefore able to match the persistence of the real exchange rate, its humped dynamics as well as the volatility of the HP-filtered real exchange rate relative to HP-filtered output.

The paper proceeds as follows. Section 2 presents the empirical analysis. Section 3 presents the model. Section 4 presents the theoretical results. Section 5 concludes.

2 Empirical Evidence

In this section, I extend the analysis of Cheung and Lai (2000) by studying the dynamics of the trade weighted real exchange rate of nine large, developed economies. I obtain data on these trade weighed real exchange rates from the Bank of International Settlements.[1] I also use data on aggregate consumption for the 9 economies I study. I obtain data on aggregate consumption from the International Financial Statistics database published by the International Monetary Fund. The empirical specification I adopt is an AR(p) model with an intercept but no time trend. This model may be written in augmented Dickey-Fuller regression form as

$$q_t = \mu + \alpha q_{t-1} + \sum_{j=1}^{p} \psi_j \Delta q_{t-j} + \epsilon_t, \tag{1}$$

where q_t is the log of the real exchange rate, μ, α and ψ_j are parameters and ϵ_t is an error term. I calculate median unbiased estimates of μ, α and ψ_j using the grid-bootstrap method described in

[1]These real exchange rates are trade weighted using manufacturing trade for 27 economies. They are published at a monthly frequency. I constructed a quarterly series by using the first month of each quarter. My sample period is 1975:1 to 2006:3. Marc Klau and San Sau Fung (2006) describe how these real exchange rate series are constructed.

Bruce E. Hansen (1999).[2] Point estimates of other statistics—such as the half-life—are calculated from the point estimates for α and ψ_j. I calculate confidence intervals and P-values using a conventional bootstrap.

My primary interest is the extent to which the impulse response of the real exchange rate is hump-shaped. It is useful to define scalar measures of how hump-shaped an impulse response function is. As building blocks toward such measures, I calculate the "up-life", half-life and "quarter-life" of the real exchange rate series I study. I follow the recent empirical literature on the real exchange rate in defining the half-life as the largest time T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$, where $IR(T)$ denotes the impulse response of the real exchange rate at time T.[3] I define the up-life and the quarter-life analogously. The up-life is the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. The quarter-life is the largest time T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Just as the half-life is meant to measure the time it takes for the impulse response to fall below half (the size of the impulse), the up-life is the time it take for the impulse response to fall below one and the quarter-life is the time it take for the impulse response to fall below a quarter.

I consider an impulse response that dies out at a constant exponential rate as the benchmark "no hump" case. Such a process will have an up-life of zero. A non-zero up-life can, therefore, be viewed as evidence that the process has a hump-shaped impulse response. This fact suggests that one sensible measure of the degree of hump in the impulse response is the ratio of the up-life to the half-life (UL/HL). The UL/HL is a measure between 0 and 1. It measures the fraction of time before the impulse response falls below 1 out of the total time before it falls below 1/2.

Another feature of an impulse response that dies out at a constant exponential rate is that it takes the process the same amount of time to fall from 1/2 to 1/4 as it take to fall from 1 to 1/2. In other words, the half-life is equal to the quarter-life minus the half-life (HL = QL - HL). For a process that has a hump-shaped impulse response, the half-life is larger than the quarter-life minus the half-life (HL > QL - HL). Or written slightly differently 2HL - QL > 0. These facts suggest that 2HL - QL, or equivalently the difference between HL and QL - HL, can be viewed as a measure

[2] Hansen (1999) uses the grid-bootstrap method to calculate confidence intervals, i.e., to estimate the 5th and 95th quantile of the distribution of the statistics of interest. I use this same method to estimate the 50th quantile of the statistics I am interested in. These estimates of the 50th quantile are median unbiased point estimates. Hansen's grid-bootstrap method is closely related to the method proposed by Donald W.K. Andrews and Hong-Yuan Chen (1994).

[3] The impulse response is defined as $IR(t) = \partial(E_s q_t - E_{s-1} q_t)/\partial\epsilon_s$, where E_s denotes the expectations operator conditional on information known at time s. It is the moving average representation of the process estimated for the real exchange rate.

of the degree of hump in the impulse response.

The first issue that arises in estimating equation (1) is the choice of lag length. I considered a range of values for p from 1 to 8. For values of p smaller than 4, the shape of the estimated impulse response function is quite sensitive to the chosen lag length. However, for values of p between 4 and 8 the estimated impulse response is virtually identical. From this I conclude that a lag length of at least 4 is needed to flexibly estimate the impulse response. I choose to set $p = 5$.

Table 1 presents results for the U.S. real exchange rate. The half-life estimate I obtain is consistent with the results of Christian J. Murray and David H. Papell and the earlier literature surveyed by Kenneth Rogoff (1996). The point estimate is 4.5 years and therefore within the "consensus range" of 3 to 5 years. Also, consistent with Murray and Papell (2002), the 90% confidence interval for the half-life is very wide. Even 30 years after the breakdown of Bretton Woods, it is not possible to estimate the half-life of the real exchange rate with much precision.

Figure 1 plots the impulse response of the U.S. real exchange rate. It exhibits a pronounced hump. Rather than dying out exponentially, the impulse response rises further—peaking at about 1.2—before it starts dying out. The impulse response doesn't fall below 1 (the size of the impulse) until 10 quarters after the impulse. Table 1 reports that the up-life of the U.S. real exchange rate is 2.4 years, which implies that the UL/HL is 0.53. In other words, 53% of the time that it takes the real exchange rate to fall below 1/2 it is actually above one.

A comparison of the quarter-life and the half-life shows that once the real exchange rate starts reverting towards its mean it does so quite quickly. I estimate the quarter-life of the U.S. real exchange rate to be 6.4 years. This implies that the QL - HL—the time it takes the real exchange rate to fall from 1/2 to 1/4—is only 1.9 years. The literature on the dynamics of the real exchange rate has tended to interpret the half-life as its rate of mean reversion. The results discussed above show that this is misleading. The rate of mean reversion of the real exchange rate is far from being constant. The half-life measures the rate of mean reversion in the short run. It is therefore heavily affected by the short term dynamics of the real exchange rate. The QL - HL, however, measures the rate of mean reversion further out, when the short term dynamics have mostly died out. The results in table 1 show that the rate of mean reversion of the real exchange rate is very slow initially but becomes substantially faster after the short term dynamics die out.

Table 2 reports results for trade weighted real exchange rates of Canada, the Euro Area, France, Germany, Italy, Japan, Switzerland, the U.K. as well as the U.S. For all 9 of these economies, the half-life is larger than QL-HL. The median half-life is 3.7 years while the median QL-HL is 1.9 years. For 8 of these 9 economies, UL/HL is positive. The median UL/HL is 0.44. Table 2 reports P-values for three sets of hypothesis tests. The null hypotheses tested for each economy are: $\alpha = 1$, UL/HL = 0 and HL < QL-HL. The statistical significance of all three hypotheses varies greatly from economy to economy. The median P-value for $\alpha = 1$ is 5%, while the median P-value for UL/HL = 0 and HL < QL-HL are 18% and 8%, respectively.

Earlier evidence of hump-shaped dynamics in the real exchange rate includes Eichenbaum and Evans (1995) and Cheung and Lai (2000). Eichenbaum and Evans (1995) estimate an identified VAR that includes the real exchange rate. They show that the real exchange rate exhibits hump-shaped dynamics in response to their identified monetary policy shocks. They refer to this result as "delayed over-shooting". Jon Faust and John H. Rogers (2003) estimate VARs under a range of alternative identifying assumptions. They argue that the delayed over-shooting result is sensitive to the choice of identifying assumptions. Cheung and Lai (2000) estimate ARMA models for four bilateral U.S. real exchange rates and find evidence hump-shaped dynamics in all four cases. My results differ from Cheung and Lai (2000) in two ways. First, I consider trade weighted real exchange rates for 9 economies. Second, I employ median unbiased estimation methods.

3 The Model

The model I employ to understand the dynamics of the real exchange rate is a two country model in the tradition of Maurice Obstfeld and Rogoff (1995). It incorporates a number of features that have been developed in the subsequent literature such as staggered price setting, local currency pricing, home biased preferences and heterogeneous factor markets. The core of the model consists of five equations. Aggregate consumption in each country evolves according to consumption Euler equations:

$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}), \tag{2}$$

$$c_t^* = E_t c_{t+1}^* - \sigma(i_t^* - E_t \pi_{t+1}^*). \tag{3}$$

The dynamics of inflation in each country are governed by New Keynesian Phillips curves:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\zeta[\phi_H c_t^M + \phi_F c_t^{M*}] + \kappa\gamma_q q_t - \eta_t, \tag{4}$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa\zeta[\phi_F c_t^M + \phi_H c_t^{M*}] - \kappa\gamma_q q_t - \eta_t^*, \tag{5}$$

and international risk-sharing implies that

$$c_t - c_t^* = \sigma q_t. \tag{6}$$

Here c_t denotes home consumption, π_t denotes home CPI inflation, i_t denotes the home short-term nominal interest rate, q_t denotes the real exchange rate and η_t is a composite of five different types of shocks: productivity shocks, labor supply shocks, government spending shocks, shocks to the world demand for home goods and cost-push shocks. All variables denote percentage deviations from a steady state with balanced trade. Foreign variables are denotes with asterisks. Superscript M and $M*$ denote the following weighted averages: $c_t^M = \phi_H c_t + \phi_F c_t^*$ and $c_t^{M*} = \phi_F c_t + \phi_H c_t^*$, where ϕ_H is the steady state fraction of total spending allocated to domestic goods and ϕ_F is the corresponding fraction allocated to imports.

A fully microfounded model that yields these equations up to a log-linear approximation is presented in detail in the Appendix to this paper.[4] This model features a continuum of household types each of which consumes and supplies labor. Each type of household consumes a basket of all goods produced in the world economy but supplies a differentiated labor input. Household preferences are biased in favor of home goods. There is a continuum of monopolistically competitive firms. Each firm demands labor and produces a differentiated good. Goods prices are sticky. The opportunity to revise prices arrives randomly as in Guillermo Calvo (1983). Firms are able to price to market and their prices are sticky in the local currency. Households have access to complete financial markets. The government in each country finances spending though lump-sum taxation of households.

To close the model, one must specify a monetary policy for each country. Recent work has stressed the importance of the systematic component of monetary policy as opposed to monetary shocks in shaping macroeconomic dynamics. I assume that the central bank in each country sets nominal interest rates according to John B. Taylor (1993) type rules:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\psi_c c_t + (1-\rho_i)\psi_\pi \pi_t + \epsilon_t, \tag{7}$$

$$i_t^* = \rho_i i_{t-1}^* + (1-\rho_i)\psi_c c_t^* + (1-\rho_i)\psi_\pi \pi_t^* + \epsilon_t^*, \tag{8}$$

[4]This appendix is available on my website.

where ϵ_t and ϵ_t^* denote home and foreign monetary policy shocks, respectively. In keeping with recent empirical work, I include a lagged interest rate term in the central banks' interest rate rule (Richard H. Clarida, Jordi Galí and Mark Gertler, 1998, 2000).

Finally, I assume that all four exogenous shocks—η_t, η_t^*, ϵ_t and ϵ_t^*—follow AR(1) processes. Given initial conditions, equations (2)-(8) and the processes for the exogenous shocks constitute a fully specified general equilibrium model of the world economy.

4 Theoretical Results

The theoretical question that I address in this section is whether the model described above can replicate the stylized facts about the dynamics of the real exchange rate discussed in section 2. The model consists of a set of linear equations with expectations terms. This type of model may be solved using standard methods based on the work of Olivier J. Blanchard and Charles M. Kahn (1980).[5] To aid comparison with earlier work, I use values for the parameters of the model that correspond closely values used in the recent literature. I list the values of the parameters in table 3.[6]

My main theoretical results are presented in table 4. The first row of this table repeats, for convenience, the key empirical features of real exchange rates established in section 2. The second row reports results for the model presented in section 3 under the assumption that there exists a perfectly frictionless economy-wide labor market in each country and business cycles are due to monetary policy shocks.[7] This "homogeneous labor market" specification of the model is designed to correspond to the benchmark model in Chari et al. (2002). The results in table 4 confirm that it does. The real exchange rate is much less persistent than in the data. This is true whether one measures persistence by the half-life of the impulse response—0.6 years versus 3.7 years in the data—or by the autocorrelation of the series after it as been HP-filtered—0.49 versus 0.78 in the data. As Chari et al. (2002) emphasize, this model can, however, match the volatility of the

[5] I use code described in Christopher A. Sims (2001).

[6] Let me briefly describe the rationale behind a few of the parameter values: I follow Chari et al. (2002) in choosing $\sigma = 1/5$. This value is chosen to roughly match the relative volatility of the real exchange rate and consumption. The value $\omega = 3$ results from assuming a Cobb-Douglas production function with a labor share equal to 2/3, disutility of labor that yields a Frisch elasticity of labor supply equal to 1/5 and a steady state labor supply of 1/4. The value $\phi_H = 0.94$ is chosen to roughly match the fraction of total spending allocated to domestic goods in the U.S.

[7] The structure of the labor market affects the model through the parameter ζ. This is discussed in more detail below and in the Appendix to this paper.

HP-filtered real exchange rate relative to HP-filtered consumption.[8]

A large number of papers have in recent years argued that one reason why simple, largely frictionless models—such as the model used by Chari et al. (2002)—are unable to match the persistence of key business cycle variables is that they seriously underestimate the degree of strategic complementarity in price setting (Taylor, 1999; Bergin and Feenstra, 2000; Michael Woodford, 2003). In the model presented above, the parameter ζ is a measure of the average degree of strategic complementarity of firm pricing decisions. If $\zeta < 1$, the pricing decisions of firms are strategic complements on average. If, however, $\zeta > 1$, firm pricing decisions are strategic substitutes on average. Under the assumption of homogeneous labor markets, $\zeta = \omega + \sigma^{-1} = 8$. This specification of the model therefore implies a substantial degree of strategic substitutability.

Bergin and Feenstra (2001) and Sondergaard (2004), attempt to solve the problem of generating persistence in the real exchange rate by increasing the degree of strategic complementarity in the model. They find that increasing the degree of strategic complementarity increases the persistence of the real exchange rate somewhat. But they are unable to match the persistence seen in the data. The third row of table 4 reports results for my model under the assumption that the labor market in each country is highly segmented. All other assumptions are the same as before. In this "heterogeneous labor market" case, $\zeta = (\omega + \sigma^{-1})/(1 + \omega\theta) = 0.26$, implying a large degree of strategic complementarity. In this respect this specification is meant to resemble the models used in Bergin and Feenstra (2001) and Sondergaard (2004). The results for this model confirm that increasing the degree of strategic complementarity in the model increases the persistence of the real exchange rate. However, the real exchange rate is still substantially less persistent than in the data.

The fourth row in table 4 reports results for a calibration of the model that I have dubbed "extreme". It is extreme in that I have set $\zeta = 0.01$. As the name suggests, this is not meant to be a realistic calibration. Rather, I have included it to illustrate that even given very extreme assumptions about the degree of strategic complementarity the model does not fit the empirical features of the real exchange rate. In this case, the half-life of the real exchange rate is only 1.4 and the autocorrelation of the HP-filtered real exchange rate is only 0.65.

[8] I study the volatility of the real exchange rate relative to consumption because consumption plays a more central role in the model than output. However, in the model, the volatility of consumption and output are very similar. For an extensive discussion of the HP-filter and other filtering methods, see Marianne Baxter and Robert G. King (1999). I use code written by Baxter and King to filter the data.

Another striking shortcoming of the three specifications of the model discussed above is the fact that they totally fail to capture the humped shape of the impulse response the real exchange rate. For all three of these specifications, UL/HL is 0.00 and QL - HL and HL are almost identical. Figure 2 plots the impulse response of the real exchange rate to a home monetary policy shock in the heterogeneous factor markets model. The impulse response dies out exponentially like an AR(1) processes.

Next consider the behavior of the model in response to Phillips curve shocks. In the Appendix to this paper, I show that at least five different types of disturbances appear in the model as shocks to the Phillips curve. These are productivity shocks, labor supply shocks, government spending shocks, shocks to the world demand for home produced goods and cost-push shocks. The fact that all these different disturbances enter the model in the same way—as shocks to the Phillips curve—implies that they all have the same implications regarding the dynamics of consumption, inflation, interest rates and the real exchange rate. For the purpose of analyzing the dynamics of the real exchange rate, I therefore need not make any assumptions about the relative importance of these five types of disturbances.[9]

The fifth and sixth row of tables 4 report results for the model with homogeneous and heterogeneous labor markets, respectively, when business cycles are driven by Phillips curve shocks. The dynamics of the real exchange rate differ in two ways from what they are when business cycles are driven by monetary policy shocks. First, in this case the model is able to match the persistence of the real exchange rate in the data quite well. The half-life of the real exchange rate is between 3.3 and 4.1 years depending on the degree of strategic complementarity, while it is 3.7 years in the data. The autocorrelation of the HP-filtered real exchange rate is between 0.82 and 0.84 compared with 0.78 in the data.

Second, the model also generates a hump-shaped response of the real exchange rate to Phillips curve shocks. The UL/HL is roughly 0.40 in the model, while it is 0.44 in the data, and the difference

[9]It is important to note that, while the five shocks that I lump together as Phillips curve shocks imply the same dynamic behavior for consumption, inflation, the interest rate and the real exchange rate, they don't all imply identical behavior for other variables such as output. For example, a positive productivity shock and a negative government spending shock both imply that inflation will fall and consumption will rise but they have different implications for output. Output will rise in response to a positive productivity shock but fall in response to a negative government spending shocks. By writing the model the way I have, I have been able to solve for the dynamics of the real exchange rate without making any reference to the dynamics of output. The impulse response of the real exchange rate in response to a Phillips curve shock is therefore consistent with a wide range dynamics for output (and other variables) depending on the relative importance of the five shocks that make up the Phillips curves shock in my model.

between QL-HL and HL is between 1.2 and 1.5 years, while it is 1.8 years in the data. Figure 3 plots the response of the real exchange rate to a home Phillips curve shock in the heterogeneous labor markets case. The response of the real exchange rate to a monetary policy shock is plotted as well for comparison. Clearly the qualitative feature of the impulse response are very different and much more in line with the empirical impulse response in figure 1.

Conventional wisdom says that real shocks cannot generate the same level of volatility in the real exchange rate as monetary shocks can. This notion—while intuitively appealing—is not supported by models such as the model I analyze in this paper. In these models, the volatility of the real exchange rate relative to consumption is determined largely by the households' elasticity of intertemporal substitution. The last column in table 4 shows that the real exchange rate is actually slightly more volatile relative to consumption when business cycles are due to real shocks than when they are due to monetary policy shocks.

Chari et al. (2002) emphasize the fact that their model is able to match the volatility of the HP-filtered real exchange rate relative to HP-filtered output if the coefficient of intertemporal substituion is assumed to be 1/5. The last column in table 4 shows that my model also matches this statistic regardless of which shocks drive the business cycle. The fact that this class of models is able to match this particular statistic has been interpreted to mean that they can explain the large volatility of the real exchange rate. This interpretation ignores the fact that the HP-filter assigns the vast majority of the volatility of the real exchange rate to its "trend". Figure 4 plots the U.S. real exchange rate along with its HP-filter "trend". According to the HP-filter, most of the large movements in the U.S. real exchange rate over the last 30 years—such as the large appreciation and subsequent depreciation in the 1980's—have been movements in the "trend".[10]

4.1 Understanding the Humped Dynamics of the Real Exchange Rate

To understand why Phillips curve shocks yield a hump-shaped impulse response for the real exchange rate while monetary policy shocks do not, it is helpful to take a closer look at the structural equations of the model. If the home consumption Euler equation—equation (2)— is "solved for-

[10] Diego Comin and Gertler (2006) find that conventional business cycle filters assign a sizable amount of cyclical variation to the trend when they are applied to macroeconomic quantities such as output and consumption.

ward", it yields

$$c_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - E_{t+j}\pi_{t+1+j}). \tag{9}$$

Risk-sharing implies that $q_t = \sigma^{-1}(c_t - c_t^*)$. Due to the large amount of home-bias that I have assumed (in order to match the empirical ratio of imports to consumption), home shocks have very muted effects on foreign variables and vice versa.[11] This implies that the impulse response of the real exchange rate is close to being a scaled version of the impulse response of home consumption when the impulse in question is a shock to the home country. Shocks that imply hump-shaped impulse responses for consumption will therefore also imply hump-shaped impulse responses for the real exchange rate.[12]

If consumption is to be hump-shaped, the sum on the right hand side of equation (9) must be hump-shaped. Considering for concreteness a shock that raises home consumption, this means that while the sum on the right hand side of equation (9) must become negative on impact the first few elements of the sum must be positive. This pattern implies that the sum will become more negative for a few periods as the positive terms drop out of the sum. In other words, for consumption to be hump-shaped, the impulse response of the real interest rate must be shaped roughly as in figure 5.

The crucial difference between monetary policy shocks and Phillips curve shocks is that monetary policy shocks lead inflation and consumption to move in the same direction on impact while Phillips curve shocks lead these variables to move in opposite directions on impact. This is illustrated in figures 6 and 7. Figure 6 plots the response of home consumption and home inflation to a home monetary policy shock. A positive monetary policy shock increases consumption. The boom in consumption, in turn, causes inflation to rise. As the shock dissipates, consumption and inflation return to their steady state values monotonically.

Figure 7 plots the response of home consumption and home inflation to a home Phillips curve shock. A positive Phillips curve shock, in contrast, increases consumption and decreases inflation on impact. As the shock dissipates, inflation rises above trend due to the boom in consumption.

[11] My results are not very sensitive to the high degree of home-bias I assume. Decreasing the degree of home-bias weakens my results somewhat—i.e., makes the real exchange rate less volatile and less hump-shaped. But even if I calibrate the home-bias to match the import share in consumption for a small country such as Sweden my results don't change significantly.

[12] In a model in which utility is not time separable or not separable between consumption and leisure, the risk-sharing condition would become $q_t = \sigma^{-1}(\lambda_t - \lambda_t^*)$, where $\lambda_t = \partial U/\partial C_t$. This is why adding habit formation to the model does not yield a hump-shaped response of the real exchange rate to monetary shocks. In such a model, the response of consumption to a monetary shock is hump-shaped but the response of marginal utility is not hump-shaped.

Both series then return to steady state. The Phillips curve shock therefore causes a non-monotonic impulse response for inflation which yields a similar non-monotonic impulse response for the real interest rate. It is this non-monotonic impulse response of the real interest rate that causes consumption and the real exchange rate to be hump-shaped.

In my model—as in most other models in the literature—relative consumption and the real exchange rate are highly correlated. In the data, however, these variables are roughly uncorrelated. At present there are no fully satisfactory solutions to this problem in the literature. However, my main results regarding the hump-shaped response of the real exchange rate to Phillips curve shocks carry over to a model with habit formation in which the correlation of relative consumption and the real exchange rate is substantially lower (around 0.45). In the model with habit formation, the real exchange rate is proportional to the ratio of marginal utility in the two countries but marginal utility is no longer proportional to consumption. My results also carry over to a model in which international trade in financial assets is limited to non-contingent one period bonds.

5 Conclusions

I document empirically that the real exchange rates of nine large, developed economies have exhibited hump-shaped dynamics in the post Bretton Woods era. I argue that this fact can help explain why existing sticky-price business cycle models have been unable to match the persistence of the real exchange rate. I present a two country sticky price model with staggered price setting and show that in response to a monetary shock the model implies an exponentially decaying dynamics for the real exchange rate. Even with very large amounts of strategic complementarity, the rate of decay is such that the model is unable to matching the empirical persistence of real exchange rates. I then show that in response to several different types of real shocks the model implies humped dynamics for the real exchange rate. The hump-shaped dynamics generated by the model are a powerful source of endogenous persistence that allow it to easily generate a half-life equal to the estimated half-life of the U.S. real exchange rate.

A Household Behavior and Market Structure

The world consists of two countries. In each country there is a continuum of household types indexed by x. The home country households have indexes on the interval $N_H = [0, 1]$. The foreign country households have indexes on the interval $N_F = (1, 2]$. Home households of type x seek to maximize a discounted sum of utilities represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t(x), \xi_t) \right] \right\}, \tag{10}$$

where β is a discount factor, ξ_t is a country specific vector of shocks to the household's preferences, C_t denotes household consumption of a composite consumption good, $L_t(x)$ denotes the households' supply of differentiated labor input x. The function $u(C_t)$ is increasing and concave while $v(L_t(x), \xi_t)$ is increasing and convex in $L_t(x)$. There are an equal (large) number of households of each type x.

The consumption index in equation (10) is

$$C_t = \left[\phi_{H,t}^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \phi_{F,t}^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{11}$$

where $\eta > 0$ denotes the elasticity of substitution between home and foreign goods and the $\phi_{j,t}$'s are preference parameters that determines households' relative preference for home versus foreign goods. If $\phi_{H,t} > \phi_{F,t}$, households preferences are biased toward home produced goods. It is analytically convenient to normalize $\phi_{H,t} + \phi_{F,t} = 1$. I allow the home bias in preferences to vary exogenously over time and refer to such variation as shocks to the world demand for home goods. I assume for simplicity that households in both countries have the same degree of steady state home bias, i.e., $\phi_H^* = \phi_F$.

The subindices, $C_{j,t}$, are in turn CES indices of the differentiated goods produced in the two countries. These indices are given by

$$C_{H,t} = \left[\int_{N_H} c_t(z)^{\frac{\theta_t-1}{\theta_t}} dz \right]^{\frac{\theta_t}{\theta_t-1}}, \quad \text{and} \quad C_{F,t} = \left[\int_{N_F} c_t(z)^{\frac{\theta_t^*-1}{\theta_t^*}} dz \right]^{\frac{\theta_t^*}{\theta_t^*-1}}. \tag{12}$$

Here the differentiated goods are indexed by z. The consumption by the representative household in the home country of good z in period t is denoted by $c_t(z)$ and $\theta_t > 1$ and $\theta_t^* > 1$ denote the elasticity of substitution at time t between the differentiated goods produced in the home country and foreign country, respectively. I assume that θ_t and θ_t^* vary exogenously. These variations may be interpreted as variation in the monopoly power of firms in the two countries. In the recent literature on monetary policy, these shocks have been referred to as "cost-push" shocks.

All goods produced in the economy are non-durable consumption goods purchased and consumed immediately by households. Investment and capital accumulation play no role in the model. To the extent that capital is used in production, each firm in the economy is endowed with a fixed amount of non-depreciating capital. Labor is immobile and there are a fixed number of firms operating in each country.

Each country has a government. These governments operate fiat currency systems denominated in "home currency" and "foreign currency", respectively. There are independent central banks that conduct monetary policy in each country by controlling the short term nominal interest rate in the domestic currency. The governments finance spending by lump sum taxes.

Households face a decision in each period about how much to consume of each of the differentiated goods produced in the world. The representative household seeks to maximize the value of the composite consumption good, C_t, that it can purchase given its income and given the prices it faces. Prices in the home country are denominated in home currency and are denoted by $p_t(z)$. Prices in the foreign country are denominated in foreign currency and are denoted by $p_t^*(z)$. The demand for home produced good z that results from this optimization by the home and foreign households is

$$c_t(z) = C_{H,t}\left(\frac{p_t(z)}{P_{H,t}}\right)^{-\theta_t} \quad \text{and} \quad c_t^*(z) = C_{H,t}^*\left(\frac{p_t^*(z)}{P_{H,t}^*}\right)^{-\theta_t}, \tag{13}$$

where

$$C_{H,t} = \phi_{H,t}C_t\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \quad \text{and} \quad C_{H,t}^* = \phi_{H,t}^*C_t^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}. \tag{14}$$

Demand for foreign produced goods is given by analogous expressions. In these equations $P_{H,t}$, $P_{H,t}^*$, P_t and P_t^* are price indexes given by

$$P_{H,t} = \left[\int_{N_H} p_t(z)^{1-\theta_t}dz\right]^{\frac{1}{1-\theta_t}} \quad , \quad P_{H,t}^* = \left[\int_{N_H} p_t^*(z)^{1-\theta_t}dz\right]^{\frac{1}{1-\theta_t}}, \tag{15}$$

$$P_t = \left[\phi_{H,t}P_{H,t}^{1-\eta} + \phi_{F,t}P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \text{and} \quad P_t^* = \left[\phi_{H,t}^*P_{H,t}^{*1-\eta} + \phi_{F,t}^*P_{F,t}^{*1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{16}$$

P_t and P_t^* will be referred to as the home and foreign country price levels, respectively. For simplicity, I assume that the demand of the home and foreign governments—denoted by $g_t(z)$, $g_t^*(z)$, $G_{j,t}$, $G_{j,t}^*$, G_t and G_t^*—is given by analogous equations to equations (13) and (14).

Agents in both countries have access to complete financial markets. There are no impediments to international trade in financial securities. Home households of type x face a flow budget constraint given by

$$P_t C_t + E_t[M_{t,t+1} B_{t+1}(x)] \leq B_t(x) + W_t(x) L_t(x) + \int_{N_H} \Phi_t(z) dz - T_t, \tag{17}$$

where $B_{t+1}(x)$ is a random variable that denotes the state contingent payoff of the portfolio of financial securities held by households of type x at the beginning of period $t+1$, $M_{t,t+1}$ is the stochastic discount factor that prices these payoffs in period t, $W_t(x)$ denotes the wage rate received by home households of type x in period t, $\Phi_t(z)$ is the profit of firm z in period t and T_t denotes lump sum taxes.[13]

A necessary condition for equilibrium in this model is that there exist no arbitrage opportunities. It follows from the absence of arbitrage opportunities that all portfolios of financial securities that pay off in period $t+1$ may be priced in period t using a unique stochastic discount factor, $M_{t,t+1}$, as in equation (17). In order to rule out "Ponzi schemes," households' portfolios of financial wealth must always be large enough that future income suffices to avert default.

Home households choose C_t, $L_t(x)$ and $B_t(x)$ in order to maximize expression (10) subject to equation (17). An optimal plan must satisfy

$$u_c(C_t) = P_t \Lambda_t, \tag{18}$$

$$M_{t,T} \Lambda_t = \beta^{T-t} \Lambda_T, \tag{19}$$

$$v_l(L_t(x), \xi_t) = W_t(x) \Lambda_t, \tag{20}$$

where Λ_t denotes the marginal utility of nominal income of households at time t, that is, the Lagrange multiplier of the constrained optimization and subscripts on the functions u and v denote partial derivatives. These three equations should hold for all periods t and all subsequent periods T. The optimal plan must also satisfy a standard trasversality condition.

Foreign households solve an analogous problem. Their optimal plan must satisfy

$$u_c(C_t^*) = P_t^* \Lambda_t^*, \tag{21}$$

[13] In equation (17) financial assets are denominated in the home currency and $M_{t,t+1}$ denotes the home currency nominal stochastic discount factor. It is important to note that the financial assets in equation (17) cannot generally be denominated in "goods". If goods are not freely traded internationally and don't have the same exchange rate adjusted price in the two countries, as will be assumed below, the same good in different countries must be viewed as two different goods. Financial assets can in this case be denominated in "goods for delivery in home country" or "goods for delivery in foreign country" but not "goods".

$$M_{t,T}\frac{\Lambda_t^*}{\mathcal{E}_t} = \beta^{T-t}\frac{\Lambda_T^*}{\mathcal{E}_T}, \tag{22}$$

$$v_l(L_t^*(x), \xi_t^*) = W_t^*(x)\Lambda_t^*, \tag{23}$$

as well as a trasversality condition. Here $\mathcal{E}_t$ denotes the nominal exchange rate, i.e., the home currency price of foreign currency. Notice that the stochastic discount factor in equation (22) is the same stochastic discount factor as in equation (19). This simply reflects the fact that assets are traded on global markets in which all agents face the same prices.

From equation (18)-(19) and (21)-(22) it follows that

$$\frac{u_c(C_T)}{u_c(C_t)} = \frac{M_{t,T}}{\beta^{T-t}}\frac{P_T}{P_t} \quad \text{and} \quad \frac{u_c(C_T^*)}{u_c(C_t^*)} = \frac{M_{t,T}}{\beta^{T-t}}\frac{\mathcal{E}_T P_T^*}{\mathcal{E}_t P_t^*}. \tag{24}$$

Combining these equations yields

$$Q_t = \frac{u_c(C_t^*)}{u_c(C_t)} \tag{25}$$

where $Q_t = \mathcal{E}_t P_t^*/P_t$ is the real exchange rate at time t and for simplicity $Q_0 = 1$.

B Firm Behavior

In each country there is a continuum of firm types indexed by z. The home country firms have indexes on the interval $N_H = [0, 1]$. The foreign country firms have indexes on the interval $N_F = (1, 2]$. Firms of type z specializes in the production of a differentiated good, $y_t(z)$. There are an equal (large) number of firms of each type.

In the following two subsections, I will describe two environments and the resulting firm behavior in each environment. I will refer to these two environments as the heterogeneous factor markets model and the homogeneous factor markets model. In both the heterogeneous factor markets model and the homogeneous factor markets model, I assume that firms are able to price discriminate between consumers in the two countries. In other words, they price-to-market (see, e.g., Krugman, 1987). Furthermore, firms denominate the price of their good in the home and foreign country in the local currency of each country. In other words, they practice local-currency pricing (see, e.g., Devereux, 1997). Prices are sticky in both countries. Price setting is assumed to be synchronized within each firm type but staggered between firm types.[14] In each period firms of type z can change their prices with probability $1 - \alpha$. With probability α they must keep their prices unchanged.

[14] See Woodford (2003, section 3.1.) for an argument for why this assumption is reasonable.

This model of price stickiness was first proposed in Calvo (1983). The fact that a firm's ability to change its prices is independent of the state of the economy makes this model simple and tractable.

B.1 The Heterogeneous Factor Market Model

All inputs to production except labor are fixed for each firm. Firms of type z must hire labor of type $x = z$. Other types of labor are not useful in the production of goods of type z. In other words, the labor market is highly segmented. This may be due to the fact that specific skills are required to produce each type of good. In this case, x denotes the skills each type of household is endowed with or has invested in. The production function of firms of type z is

$$y_t(z) = A_t f(L_t(z)) \tag{26}$$

where A_t denotes an exogenous technology factor and $L_t(z)$ denotes the amount of labor input used by firms of type z in period t. The function f is increasing and concave. It is concave because there are diminishing marginal returns to labor given the fixed amount of other inputs employed at the firm. Firms act to maximize their value in domestic currency.

In order to maximize profits a home country firm of type z that is able to change its prices at time t chooses $p_t(z)$, $p_t^*(z)$ and $L_T(z)$ to maximize

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \Phi_T(z), \tag{27}$$

where

$$\Phi_T(z) = p_t(z)(C_{H,T}+G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + \mathcal{E}_T p_t^*(z)(C^*_{H,T}+G^*_{H,T}) \left(\frac{p_t^*(z)}{P^*_{H,T}}\right)^{-\theta_T} - W_T(z)L_T(z) \tag{28}$$

subject to the constraint that it produces at least as much as it sells,

$$(C_{H,T} + G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + (C^*_{H,T} + G^*_{H,T}) \left(\frac{p_t^*(z)}{P^*_{H,T}}\right)^{-\theta_T} \leq A_T f(L_T(z)). \tag{29}$$

Necessary conditions for an optimal plan are

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}(C_{H,T} + G_{H,T}) P_{H,T}^{\theta_T} (1-\theta_T)[p_t(z) - \frac{\theta_T}{\theta_T - 1} S_T(z)] = 0, \tag{30}$$

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}(C^*_{H,T} + G^*_{H,T}) P_{H,T}^{*\theta_T} (1-\theta_T)[\mathcal{E}_T p_t^*(z) - \frac{\theta_T}{\theta_T - 1} S_T(z)] = 0, \tag{31}$$

for each period t at which firms of type z are able to change their prices,

$$W_t(z) = A_t f_l(L_t(z)) S_t(z) \tag{32}$$

for all t and equation (29) with equality for all t. Here $S_t(z)$ is the marginal cost of production, i.e. the Lagrange multiplier of the firm's constrained optimization problem. Foreign firms solve an analogous optimization problem.

Combining equations (18), (20) and (32) in order to eliminate $\Lambda_t(z)$ and $W_t(z)$ gives

$$\frac{S_t(z)}{P_t} = \frac{v_l(L_t(z), \xi_t)}{A_t f_l(L_t(z)) u_c(C_t)}. \tag{33}$$

Notice that $L_t(z) = f^{-1}(y_t(z)/A_t)$. Using this relation, $S_t(z)/P_t$ can be written without reference to $L_t(z)$ as

$$\frac{S_t(z)}{P_t} = \frac{v_l(f^{-1}(y_t(z)/A_t), \xi_t)}{A_t f_l(f^{-1}(y_t(z)/A_t)) u_c(C_t)}. \tag{34}$$

Here the marginal costs of firms of type z have been written in terms of their level of output and the level of domestic consumption. This is useful since it simplifies the model by eliminating both $W_t(z)$ and $L_t(z)$.

B.2 The Homogeneous Factor Markets Model

There exists a fixed amount of non-depretiating capital in the economy that is owned by the firms. For simplicity, I assume that firms can rent their capital stock to other firms but not sell it. All workers are identical from each firm's perspective. Firms are therefore indifferent regarding which workers they hire and all workers receive the same wage W_t in equilibrium. The production function of firms of type z is

$$y_t(z) = A_t f(L_t(z), K_t(z)) \tag{35}$$

where A_t denotes an exogenous technology factor and $L_t(z)$ denotes the amount of labor input used by firms of type z in period t and $K_t(z)$ denotes the amount of capital used by firms of type z in period t. The function f is increasing in both its arguments and homogeneous of degree one. Firms act to maximize their value in domestic currency.

In order to maximize profits a home country firms of type z that are able to change its prices at time t chooses $p_t(z)$, $p_t^*(z)$, $L_T(z)$ and $K_t(z)$ to maximize (27) where

$$\Phi_T(z) = p_t(z)(C_{H,T} + G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + \mathcal{E}_T p_t^*(z)(C_{H,T}^* + G_{H,T}^*) \left(\frac{p_t^*(z)}{P_{H,T}^*}\right)^{-\theta_T}$$

$$-W_T L_T(z) - R_T(K_T(z) - K(z)) \quad (36)$$

subject to the constraint that it produces at least as much as it sells,

$$(C_{H,T} + G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + (C^*_{H,T} + G^*_{H,T}) \left(\frac{p^*_t(z)}{P^*_{H,T}}\right)^{-\theta_T} \leq A_T f(L_T(z), K_T(z)), \quad (37)$$

where R_T denotes the rental rate on capital in period T and $K(z)$ denotes the capital endowment of firms of type z.

Necessary conditions for an optimal plan are equations (30)-(31) for each period t at which firms of type z are able to change their prices,

$$W_t = A_t f_l(L_t(z), K_t(z)) S_t(z) \quad (38)$$

$$R_t = A_t f_k(L_t(z), K_t(z)) S_t(z) \quad (39)$$

for all t and equation (37) with equality for all t. Notice that equations (38)-(39) imply that

$$\frac{W_t}{R_t} = \frac{f_l(L_t(z), K_t(z))}{f_k(L_t(z), K_t(z))}.$$

Since f is homogeneous of degree one, this implies that all firms choose the same labor-capital ratio in period t even though they produce different amounts. This, in turn, implies that equation (38) can be rewritten as

$$S_t = \frac{W_t}{A_t f_l(h_t, 1)},$$

where h_t denotes the common labor-capital ratio of all firms. Notice that this equation implies that the marginal cost of all firms is equal. I have denoted this common marginal cost as S_t.

Combining this last equations with equations (18), (20) and (38) in order to eliminate $\Lambda_t(z)$ and W_t yields

$$\frac{S_t}{P_t} = \frac{v_l(L_t, \xi_t)}{A_t f_l(h_t, 1) u_c(C_t)}, \quad (40)$$

where L_t is the amount of labor supplied by the representative household. Unlike in the heterogeneous markets case, all households supply the same amount of labor when the labor market is homogeneous.

C Log-Linearization of Heterogeneous Factor Markets Model

In this section, I work out a log-linear approximation of the heterogeneous factor markets model. A log-linear approximation of the homogeneous factor markets model may be derived in an analogous fashion.

First, consider the left equation in (24). The expectation of the $T = t+1$ version of this equation may be written

$$I_t = E_t\left[\frac{1}{\beta}\frac{u_c(C_t)}{u_c(C_{t+1})}\frac{P_{t+1}}{P_t}\right],$$

since the gross short term nominal interest rate is given by $I_t = 1/E_t M_{t,t+1}$. A log-linear approximation of this equations is

$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}), \tag{41}$$

where $\sigma = -u_c/u_{cc}C$, lower case letters denote percentage deviations from steady state of the same upper case letters unless otherwise noted, uppercase letters without a time subscript denote steade state values and $\pi_t = \log(P_t/P_{t-1})$. The foreign consumption Euler equation yields an analogous log-linear approximation.

A log-linear approximation of equation (25) is

$$c_t - c_t^* = \sigma q_t. \tag{42}$$

Log-linear approximations of the equations in (16) are

$$\phi_H p_{H,t} + \phi_F p_{F,t} = 0, \tag{43}$$

$$\phi_F p_{H,t}^* + \phi_H p_{F,t}^* = 0, \tag{44}$$

where $p_{j,t} = \log(P_{j,t}/P_t)$ and I have made use of the fact that the normalization $\phi_{H,t} + \phi_{F,t} = 1$ implies that all relative prices are 1 in steady state. Notice that these last two equations imply that

$$\pi_t = \phi_H \pi_{H,t} + \phi_F \pi_{F,t} \tag{45}$$

$$\pi_t^* = \phi_F \pi_{H,t}^* + \phi_H \pi_{F,t}^* \tag{46}$$

A log-linear approximation of equation (15) is

$$\pi_{H,t} = \frac{1-\alpha}{\alpha}(p_{h,t} - p_{H,t}). \tag{47}$$

$$\pi_{F,t} = \frac{1-\alpha}{\alpha}(p_{f,t} - p_{F,t}). \tag{48}$$

where $\pi_{j,t} = \log(P_{j,t}/P_{j,t-1})$.

Define c_t^M and c_t^{M*} as $c_t^M = \phi_H c_t + \phi_F c_t^*$ and $c_t^{M*} = \phi_F c_t + \phi_H c_t^*$, respectively and M and $M*$ superscripts on other variables denote the analogous weighted averages. Given this notation, a log-linear approximation of (29), (34) and their foreign counterparts are

$$y_{t,T} = c_T^M + g_T^M + (\theta - \eta)p_{H,T}^M - \theta p_{h,t}^M + \theta \sum_{\tau=t+1}^{T} \pi_\tau^M + \phi_{H,T}^M,$$

$$y_{t,T}^* = c_T^{M*} + g_T^{M*} + (\theta - \eta)p_{F,T}^{M*} - \theta p_{f,t}^{M*} + \theta \sum_{\tau=t+1}^{T} \pi_\tau^{M*} + \phi_{F,T}^{M*},$$

$$s_{t,T} = \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A}\right) y_{t,T} - \frac{u_{cc}C}{u_c} c_T + \frac{v_{l\xi}}{v_l}\xi_T - \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A} + 1\right) a_T,$$

$$s_{t,T}^* = \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A}\right) y_{t,T}^* - \frac{u_{cc}C}{u_c} c_T^* + \frac{v_{l\xi}}{v_l}\xi_T^* - \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A} + 1\right) a_T^*.$$

where $s_{t,T}$ denotes the percent deviation from steady state of the real marginal cost in period T of the firms that set their prices in period t, $y_{t,T}$ denotes the percent deviation from steady state in period T of the level of output of firms that set their prices in period t and $\Psi = 1/f_l(f^{-1}(y/A))$. Also, I assume that $C = C^* = Y$.

Combining these last four equations to eliminate $y_{t,T}$ and $y_{t,T}^*$ yields

$$s_{t,T} = \omega(c_T^M + g_T^M) + \omega(\theta - \eta)p_{H,T}^M - \omega\theta p_{h,t}^M + \omega\theta \sum_{\tau=t+1}^{T} \pi_\tau^M + \phi_{H,T}^M + \sigma^{-1} c_T - \tilde{a}_T, \tag{49}$$

$$s_{t,T}^* = \omega(c_T^{M*} + g_T^{M*}) + \omega(\theta - \eta)p_{F,T}^{M*} - \omega\theta p_{f,t}^{M*} + \omega\theta \sum_{\tau=t+1}^{T} \pi_\tau^{M*} - \phi_{H,T}^M + \sigma^{-1} c_T^* - \tilde{a}_T^*, \tag{50}$$

where

$$\omega = \left(\tfrac{v_{ll}Y}{v_l f_l A} + \tfrac{\Psi_y Y}{\Psi A}\right) \quad \text{and} \quad \tilde{a}_t = (\omega + 1)a_t - \tfrac{v_{l\xi}}{v_l}\xi_t$$

and where we use the fact that $\phi_{H,t}^M = -\phi_{F,t}^{M*}$.

Log-linear approximations of equations (30) and (31) and their foreign counterparts are given by

$$p_{ht} = (1 - \alpha\beta) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t(s_{t,t+j} - \hat{\theta}_{t+j}) + \sum_{j=1}^{\infty} (\alpha\beta)^j E_t \pi_{t+j}, \tag{51}$$

$$p_{ht}^* = (1 - \alpha\beta) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t(s_{t,t+j} - q_{t+j} - \hat{\theta}_{t+j}) + \sum_{j=1}^{\infty} (\alpha\beta)^j E_t \pi_{t+j}^*, \tag{52}$$

$$p_{ft}^* = (1 - \alpha\beta) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t(s_{t,t+j}^* - \hat{\theta}_{t+j}^*) + \sum_{j=1}^{\infty} (\alpha\beta)^j E_t \pi_{t+j}^*, \tag{53}$$

$$p_{ft} = (1 - \alpha\beta) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t(s_{t,t+j}^* + q_{t+j} - \hat{\theta}_{t+j}^*) + \sum_{j=1}^{\infty} (\alpha\beta)^j E_t \pi_{t+j}, \tag{54}$$

where $\hat{\theta}_t = (\theta/(\theta-1)^2)\theta_t$.

Combining equations (47), (49) and (51) yields

$$\pi_{H,t} + \frac{1-\alpha}{\alpha}p_{H,t} = \kappa\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\Big(\omega(c^M_{t+j}+g^M_{t+j}) + \omega(\theta-\eta)p^M_{H,t+j} - \omega\theta p^M_{h,t} + \omega\theta\sum_{\tau=t+1}^{t+j}\pi^M_\tau$$
$$+\sigma^{-1}c^M_{t+j} + \phi_F\sigma^{-1}c^R_{t+j} + \phi^M_{H,t+j} - \tilde{a}_{t+j} - \hat{\theta}_{t+j}\Big) + \frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j} \quad (55)$$

Notice that

$$\sum_{j=0}^{\infty}(\alpha\beta)^j\sum_{\tau=t+1}^{t+j}\pi^M_\tau = \frac{1}{1-\alpha\beta}\sum_{j=1}^{\infty}(\alpha\beta)^j\pi^M_{t+j}.$$

Using this and equations (42), (47) and (55) may be written

$$(1+\omega\theta)\left(\pi_{H,t}+\frac{1-\alpha}{\alpha}p_{H,t}\right) - \phi_F\omega\theta\left(\pi^R_{H,t}+\frac{1-\alpha}{\alpha}p^R_{H,t}\right) = \kappa\sum_{j=0}^{\infty}(\alpha\beta)^j(\omega+\sigma^{-1})E_t c^M_{t+j}$$
$$+\kappa\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\Big(\omega(\theta-\eta)p^M_{H,t+j} + \phi_F(q_{t+j}+\epsilon^R_{t+j}) + \phi^M_{H,t+j} - \tilde{a}_{t+j} + \omega g^M_{t+j} - \hat{\theta}_{t+j}\Big)$$
$$+(1+\omega\theta)\frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j} - \phi_F\omega\theta\frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi^R_{t+j}$$

Now, using the fact that $p_{H,t} - p_{H,t-1} = \pi_{H,t} - \pi_t$ and defining

$$\kappa = \tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad \text{and} \quad \zeta = \tfrac{\omega+\sigma^{-1}}{1+\omega\theta},$$

this equation can be rewritten as

$$\pi_{H,t} - \beta E_t\pi_{H,t+1} + \kappa p_{H,t} - \phi_F\frac{\omega\theta}{1+\omega\theta}\left(\pi^R_{H,t} - \beta E_t\pi^R_{H,t+1} + \kappa p^R_{H,t}\right)$$
$$= \kappa\zeta c^M_t + \kappa\frac{\omega(\theta-\eta)}{1+\omega\theta}p^M_{H,t} + \kappa\frac{\phi_F}{1+\omega\theta}(q_t+\epsilon^R_t) - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t).$$

A similar set of manipulations involving $\pi^*_{H,t}$ yields

$$\pi^*_{H,t} - \beta E_t\pi^*_{H,t+1} + \kappa p^*_{H,t} + \phi_H\frac{\omega\theta}{1+\omega\theta}\left(\pi^R_{H,t} - \beta E_t\pi^R_{H,t+1} + \kappa p^R_{H,t}\right)$$
$$= \kappa\zeta c^M_t + \kappa\frac{\omega(\theta-\eta)}{1+\omega\theta}p^M_{H,t} - \kappa\frac{1-\phi_F}{1+\omega\theta}(q_t+\epsilon^R_t) - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t).$$

Combining the last two equations yields

$$\pi^R_{H,t} = \beta E_t\pi^R_{H,t+1} + \kappa q_t - \kappa p^R_{H,t},$$

$$\pi^M_{H,t} = \beta E_t\pi^M_{H,t+1} + \kappa\zeta c^M_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^M_{H,t} + \kappa\frac{2\phi_H\phi_F}{1+\omega\theta}q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t),$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa\zeta c_t^M - \kappa \frac{1+\omega\eta}{1+\omega\theta} p_{H,t}^M - \kappa\phi_F p_{H,t}^R + \kappa\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g_t^M - \phi_{H,t+j}^M + \hat{\theta}_t),$$

$$\pi_{H,t}^* = \beta E_t \pi_{H,t+1}^* + \kappa\zeta c_t^M - \kappa \frac{1+\omega\eta}{1+\omega\theta} p_{H,t}^M + \kappa\phi_H p_{H,t}^R - \kappa\phi_H q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g_t^M - \phi_{H,t+j}^M + \hat{\theta}_t).$$

And a similar set of manipulations involving $\pi_{F,t}$ and $\pi_{F,t}^*$ yields

$$\pi_{F,t}^R = \beta E_t \pi_{F,t+1}^R + \kappa q_t - \kappa p_{F,t}^R - \kappa \hat{\theta}_t^R,$$

$$\pi_{F,t}^{M*} = \beta E_t \pi_{F,t+1}^{M*} + \kappa\zeta c_t^{M*} - \kappa \frac{1+\omega\eta}{1+\omega\theta} p_{F,t}^{M*} - \kappa \frac{2\phi_F\phi_H}{1+\omega\theta} q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^* - \omega g_t^{M*} + \phi_{H,t+j}^M + \hat{\theta}_t^*),$$

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \kappa\zeta c_t^{M*} - \kappa \frac{1+\omega\eta}{1+\omega\theta} p_{F,t}^{M*} - \kappa\phi_H p_{F,t}^R + \kappa\phi_H q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^* - \omega g_t^{M*} + \phi_{H,t+j}^M + \hat{\theta}_t^*),$$

$$\pi_{F,t}^* = \beta E_t \pi_{F,t+1}^* + \kappa\zeta c_t^{M*} - \kappa \frac{1+\omega\eta}{1+\omega\theta} p_{F,t}^{M*} + \kappa\phi_F p_{F,t}^R - \kappa\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^* - \omega g_t^{M*} + \phi_{H,t+j}^M + \hat{\theta}_t^*).$$

These equations along with equations (45) and (46) imply that

$$\begin{aligned}\pi_t = \beta E_t \pi_{t+1} &+ \kappa\zeta(\phi_H c_t^M + \phi_F c_t^{M*}) - \kappa \frac{1+\omega\eta}{1+\omega\theta}(\phi_H p_{H,t}^M + \phi_F p_{F,t}^{M*}) - \kappa\phi_H\phi_F(p_{H,t}^R + p_{F,t}^R) \\ &+ \kappa 2\phi_H\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^M - \omega(\phi_H g_t^M + \phi_F g_t^{M*}) - (\phi_H - \phi_F)\phi_{H,t}^M + \theta_t^M),\end{aligned}$$

$$\begin{aligned}\pi_t^* = \beta E_t \pi_{t+1}^* &+ \kappa\zeta(\phi_F c_t^M + \phi_H c_t^{M*}) - \kappa \frac{1+\omega\eta}{1+\omega\theta}(\phi_F p_{H,t}^M + \phi_H p_{F,t}^{M*}) + \kappa\phi_H\phi_F(p_{H,t}^R + p_{F,t}^R) \\ &- \kappa 2\phi_H\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^{M*} - \omega(\phi_F g_t^M + \phi_H g_t^{M*}) - (\phi_F - \phi_H)\phi_{H,t}^M + \theta_t^{M*}).\end{aligned}$$

Using equations (43) and (44), these equations may be simplified:

$$\begin{aligned}\pi_t = \beta E_t \pi_{t+1} + \kappa\zeta(\phi_H c_t^M + \phi_F c_t^{M*}) &- \kappa \frac{(\phi_H - \phi_F)\omega(\theta - \eta)}{1+\omega\theta} p_{F,t}^{M*} + \kappa 2\phi_H\phi_F q_t \\ &- \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^M - \omega(\phi_H g_t^M + \phi_F g_t^{M*}) - (\phi_H - \phi_F)\phi_{H,t}^M + \theta_t^M),\end{aligned}$$

$$\begin{aligned}\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa\zeta(\phi_F c_t^M + \phi_H c_t^{M*}) &+ \kappa \frac{(\phi_H - \phi_F)\omega(\theta - \eta)}{1+\omega\theta} p_{F,t}^{M*} - \kappa 2\phi_H\phi_F q_t \\ &- \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^{M*} - \omega(\phi_F g_t^M + \phi_H g_t^{M*}) - (\phi_F - \phi_H)\phi_{H,t}^M + \theta_t^{M*}).\end{aligned}$$

Notice, furthermore, that if $\theta = \eta$ the $p_{F,t}^{M*}$ terms drop out of these equations.

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Table 1: Empirical Properties of the Trade Weighted U.S. Real Exchange Rate

Panel A: Point and Interval Estimation

Statistic	MU point estimate	90 % Confidence Interval
α	0.954	[0.879, 1.000]
Half-life	4.46	[2.05, ∞]
Up-life	2.37	[0.00, ∞]
Quarter-life	6.36	[2.85, ∞]
UL/HL	0.53	[0.00, 0.74]
QL - HL	1.91	[0.64, 14.45]
2HL - QL	2.55	[0.01, 7.14]
$\rho_{1,hp}$	0.78	[0.64, 0.85]
St.Dev(Q)/St.Dev(C)	5.51	

Panel B: Hypothesis Testing

Hypothesis	P-value
$\alpha = 1$	0.05
UL/HL = 0	0.15
HL $<$ QL - HL	0.05

An AR(5) model was estimated for the trade weighted log real exchange rate for each country. α denotes the sum of the AR coefficients (see equation (1)). HL, UL, and QL, denote the half-life, up-life and quarter-life of the real exchange rate, respectively. These statistics are measured in years. $\rho_{1,hp}$ denotes the first order autocorrelation of the HP-filtered real exchange rate. St.Dev(Q)/St.Dev(C) denotes the ratio of the standard deviation of the HP-filtered real exchange rate to HP-filtered consumption. For each statistic, I report a point estimate, a 90% confidence interval and a P-value. Median unbiased point estimates for the parameters in equation (1) were calculated using the grid-bootstrap method of Hansen (1999) with parameters G = 80 and B = 249. Confidence intervals and P-values were calculated using a conventional bootstrap with sample size 1000. Confidence intervals for UL/HL and 2HL-QL were calculated conditional on there statistics being defined.

Table 2: Empirical Properties of Trade Weighted Real Exchange Rates

Panel A: Point Estimates

	HL	QL-HL	UL/HL	$\rho_{1,hp}$	$\frac{\text{St.Dev}(Q)}{\text{St.Dev}(C)}$
Canada	7.44	3.58	0.54	0.83	3.83
Euro Area	2.69	1.22	0.53	0.80	
France	3.23	2.66	0.35	0.79	1.89
Germany	3.84	2.20	0.44	0.77	2.72
Italy	3.76	3.57	0.00	0.73	2.38
Japan	3.69	1.92	0.46	0.80	6.01
Switzerland	1.59	0.85	0.37	0.76	2.82
UK	2.02	1.40	0.28	0.76	3.92
US	4.46	1.91	0.53	0.78	5.51

Panel B: Hypothesis Testing

	$\alpha = 1$	UL/HL=0	HL<QL-HL
Canada	0.15	0.03	0.01
Euro Area	0.02	0.12	0.08
France	0.06	0.18	0.24
Germany	0.06	0.22	0.08
Italy	0.12	0.60	0.38
Japan	0.05	0.04	0.08
Switzerland	0.00	0.29	0.15
UK	0.01	0.34	0.24
US	0.05	0.15	0.05

An AR(5) model was estimated for the trade weighted log real exchange rate for each country. α denotes the sum of the AR coefficients (see equation (1)). HL, UL, and QL, denote the half-life, up-life and quarter-life of the real exchange rate, respectively. These statistics are measured in years. $\rho_{1,hp}$ denotes the first order autocorrelation of the HP-filtered real exchange rate. St.Dev(Q)/St.Dev(C) denotes the ratio of the standard deviation of the HP-filtered real exchange rate to HP-filtered consumption. For each statistic, I report a point estimate, a 90% confidence interval and a P-value. Median unbiased point estimates for the parameters in equation (1) were calculated using the grid-bootstrap method of Hansen (1999) with parameters G = 80 and B = 249. Confidence intervals and P-values were calculated using a conventional bootstrap with sample size 1000. Confidence intervals for UL/HL and 2HL-QL were calculated conditional on there statistics being defined.

Table 3: Parameter Values

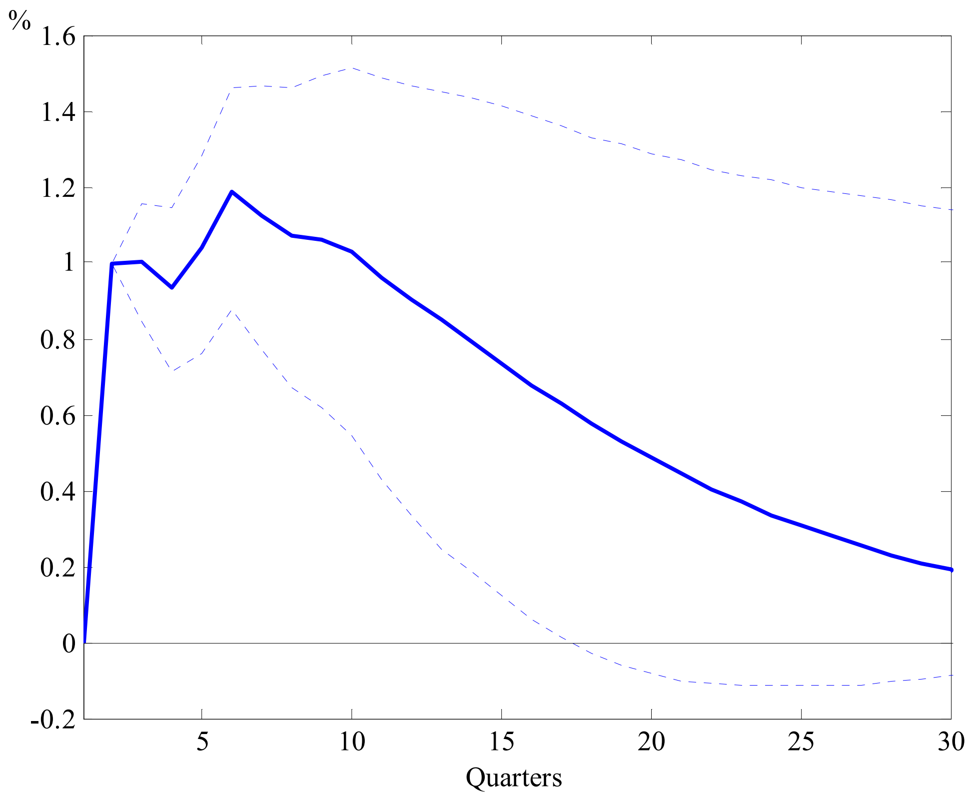
Benchmark Calibration:	
Discount factor	$\beta = 0.99$
Elast. of intertemporal substitution	$\sigma = 1/5$
Marginal cost elasticity	$\omega = 3$
Elasticity of demand	$\eta = \theta = 10$
Fraction of firms that change prices	$1 - \alpha = 0.25$
Home bias parameters	$\phi_H = 0.94$, $\phi_F = 0.06$
Taylor rule paremeters	$\rho_i = 0.85$, $\psi_c = 0.5$, $\psi_\pi = 2$
Monetary policy shocks	$\rho_i = 0.9$, $\text{corr}(\nu_t, \nu_t^*) = 0.5$
Phillips curve shocks	$\rho_\eta = 0.9$, $\text{corr}(\nu_{a,t}, \nu_{a,t}^*) = 0$
Composite parameters:	
$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} = 0.086$	$\gamma_q = 2\phi_H\phi_F = 0.113$
$\zeta_{homog.} = \omega + \sigma^{-1} = 8$	$\zeta_{heterog.} = \frac{\omega+\sigma^{-1}}{1+\omega\theta} = 0.26$

Table 4: Behavior the Real Exchange Rate in the Model

		HL	UL/HL	QL - HL	$\rho_{1,hp}$	$\frac{\text{st.dev}(q_t)}{\text{st.dev}(c_t)}$
1.	Median Empirical Value for 9 Countries	3.7	0.44	1.9	0.78	3.3
2.	Homog. Labor Market Money Supply Shocks	0.6	0.00	0.7	0.49	5.1
3.	Heterog. Labor Market Money Supply Shocks	1.3	0.00	1.3	0.64	3.7
4.	Extreme Model Money Supply Shocks	1.4	0.00	1.4	0.65	3.3
5.	Homog. Labor Market Phillips Curve Shocks	3.3	0.41	2.1	0.82	6.9
6.	Heterog. Labor Market Phillips Curve Shocks	4.1	0.40	2.6	0.84	4.2

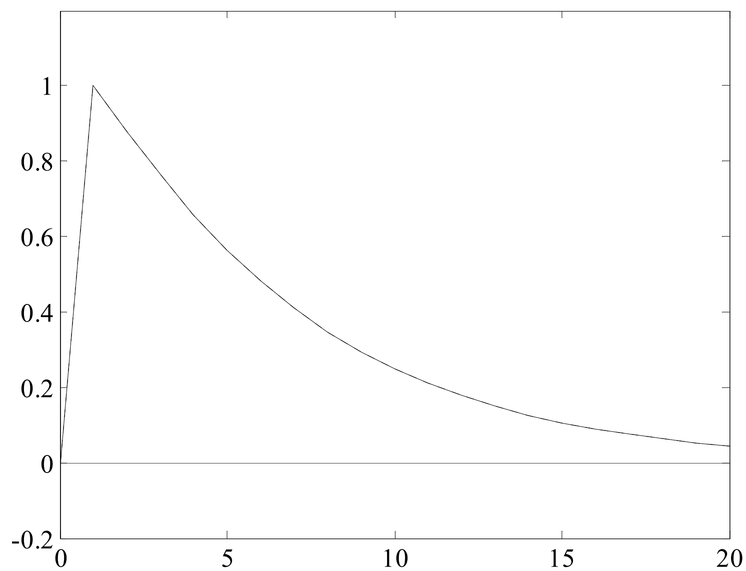
The table reports median unbiased estimates and 95% confidence intervals. HL denotes half-life (measured in years), UL/HL denotes up-life divided by half-life, $\rho_{1,hp}$ denotes the first order autocorrelation of the HP-filtered series and st.dev(q_t)/st.dev(c_t) denotes the standard deviation of HP-filtered q_t divided by the standard deviation of HP-filtered c_t. Point estimates of HL, UL/HL and QL - HL were calculated by estimating equation (1) with $p = 5$ using the grid-bootstrap method described in Hansen (1999) with parameters G = 80 and B = 249. The point estimates for $\rho_{1,hp}$ and st.dev(q_t)/st.dev(c_t) were calculated by simulating 1000 data series from each model—each of length 127 (corresponding to the length of my data set). The point estimate is the median value of the resulting distribution.

Figure 1: Impulse Response of the U.S. Real Exchange Rate



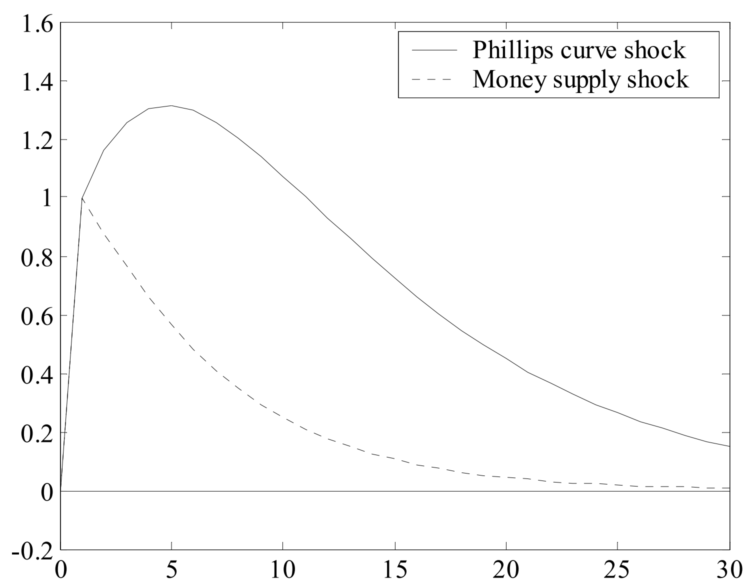
The figure plots an estimated impulse response function for the trade weighted log U.S. real exchange rate. The impulse response is based on median unbiased estimation of an AR(5) model on quarterly data from the period 1975:1-2006:3. The dotted lines denote a 90% bootstrap confidence interval.

Figure 2: Response of the Real Exchange Rate to a Monetary Shock



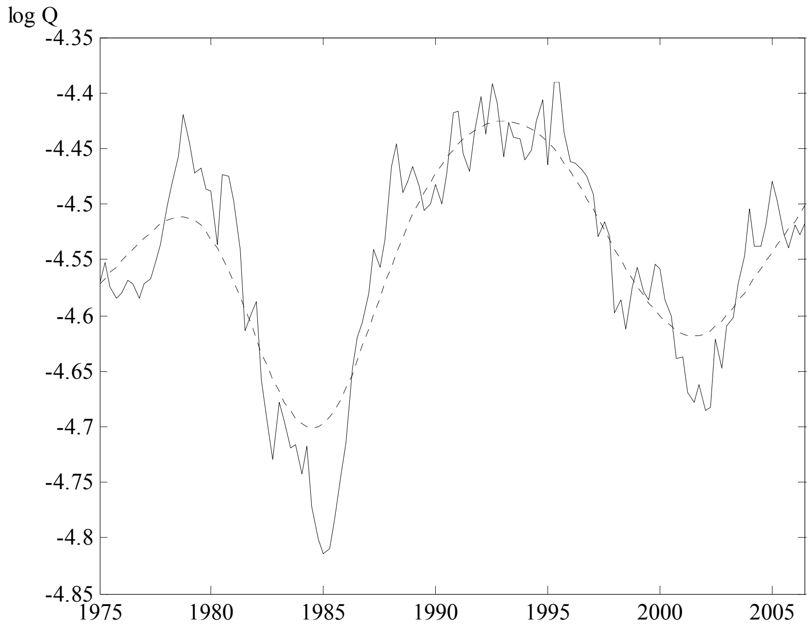
This figure plots the response of the real exchange rate to a home monetary policy shock in the model with heterogeneous labor markets ($\zeta = 0.26$).

Figure 3: Response of the Real Exchange Rate to a Phillips curve shock



This figure plots the response of the real exchange rate to a shock to the home Phillips curve in the model with heterogeneous labor markets ($\zeta = 0.26$). Also reported is the response of the real exchange rate to a home monetary policy shock.

Figure 4: The U.S. Real Exchange Rate and its HP-Filter Trend



This figure plots the log of the trade weighted U.S. real exchange rate and a trend line from the HP-filter with bandwidth 1600.

Figure 5: Desired Path of the Real Interest Rate

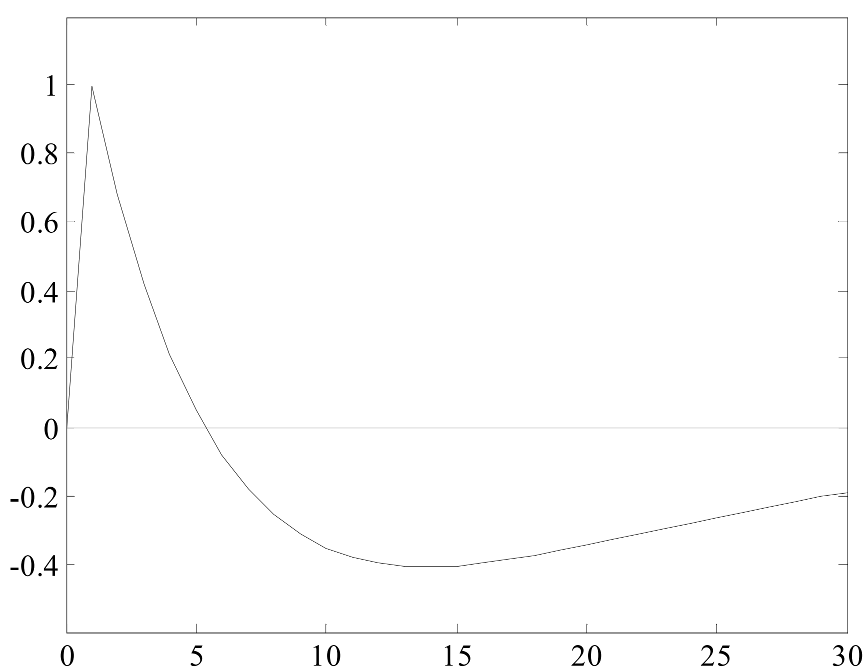


Figure 6: Response of Consumption and Inflation to a Monetary Policy Shock

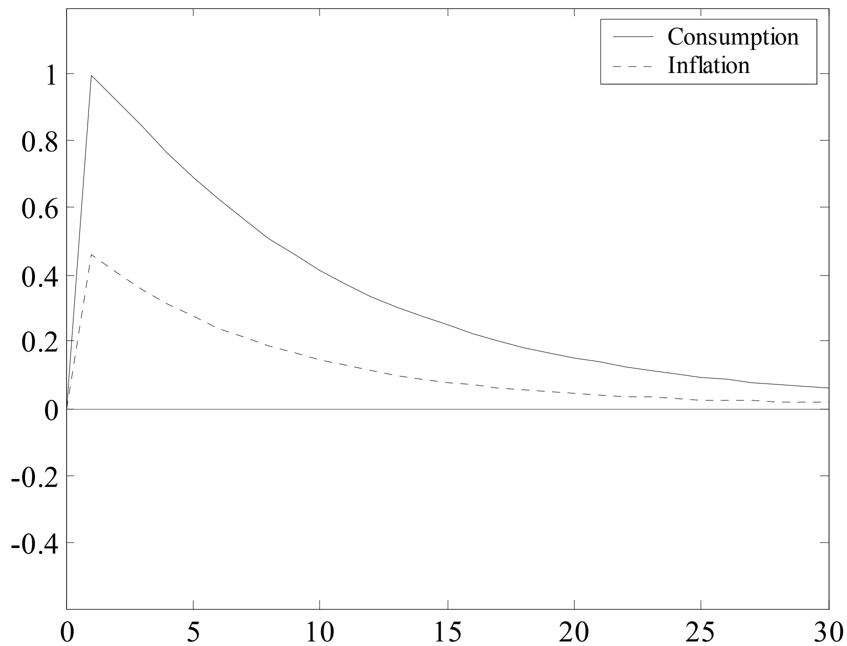


Figure 7: Response of Consumption and Inflation to a Phillips Curve Shock