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The dynamics of minority opinions in democratic debate

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Abstract

A model for the dynamics of opinion forming in democratic public debate is presented. Using concepts and techniques from the physics of disorder the dynamics of social refusal spreading is studied within a perfect world, where the minority holds neither better arguments nor lobbying backing. The one-person-one-vote rule, together with local majority rules, are used to determine the outcome of local group discussions. In case of a local tie, the group decides on keeping the Status Quo. The geometry of social life shaped by offices, houses, bars, and restaurants is shown to determine the distribution size of these discussion groups. It is found to yield very asymmetric unstable thresholds to the total spreading of one opinion at the benefit of the refusal one. The associated dynamics is rather quick and completed within few days. This democratic paradox of public debate driven majority opinion reversal is discussed in light of some European construction issues. The model may apply to rumor and fear propagation.

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While dealing with a reform proposal the first difficulty, which arises a priori, is on how to negotiate with the various lobbies, which are taking advantage of the current situation. Even if the reform proposal aims at a real improvement for the majority of the people, the minority at stake, there is always at least one, is very efficient in organizing the opposition to the project. Indeed, the prospect of losing definite advantages is much

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more energizing than the hypothetical gain of a reform making the 'fight' for the reform very unbalanced at the cost of the majority interest.

Accordingly, most research has concentrated on analyzing the complicated psycho-sociological mechanisms involved in the process of opinion forming. In particular, focusing on those by which a huge majority of people give up to an initial minority view [1,2]. The main feature being that a minority is rather active with more aggressiveness and persuasive power than a majority inclined to be more passive by its own size. Such a view could lead to consider reforms possible only using social violence or authoritarian top leadership decisions. It thus raises the fundamental question whether or not a reform can be decided democratically at least in principle.

However, in this paper, we argue that it is not only this strong asymmetry in the motivation to fight for or against a reform which is responsible for most of the reform rejection [3]. We claim that there exists an intrinsic mechanism inherent to the nature of public debate itself, which strongly favors the rejection whatever the reform is. To demonstrate this view we consider a perfect world in which there exist no lobbying, no advantage to both opinion and no correlation among people. We use the democratic principle of one-person-one-vote to determine the outcome of the very complicated psycho-sociological dynamics of people's mind change while discussing in-groups.

More precisely, we consider a population divided initially between a minority against a reform proposal and a majority which supports it. Then a public debate is initiated to come up with some final collective choice. The opinion dynamics is monitored through a series of repeated local discussions within groups of various sizes among the same population. While the same people meet again and again randomly in the same cluster configuration, at each new encounter, they discuss locally the issue at stake and change their mind according to a local majority rule. In case of a tie, the group doubts and thus decides to preserve the current situation which means to reject the reform proposal. Following the time evolution of the reform support we found out that there exist very asymmetric unstable thresholds to the total spreading of one opinion at the benefit of the initial hostile minority view. Moreover, the associated dynamics is rather quick (few days).

Our simple model of opinion forming is rooted in the physics of disorder [4]. A diffusion reaction model is implemented on a space of random geometry of groups with various sizes. We start from a population with N individuals. At some time $t=0$ prior to the discussion, the reform proposal has a support from $N_+(t)$ individuals while $N_-(t)$ are against it. Assuming that each person has an opinion, we have $N_+(t)+N_-(t)=N$. The respective associated individual probabilities to be in favor or against the reform proposal are at time $t=0$,

$$P_\pm \equiv \frac{N_\pm(t)}{N} \tag{1}$$

with $P_-(t)+P_+(t)=1$. From this initial configuration, people start discussing the project. However, it does not happen continuously and with all the people together at once. They gather in small groups at some intervals of time and then discuss the issue at stake. The geometry of social life via the space organization of offices, houses, bars and restaurants determines the distribution of group sizes. It varies from one to

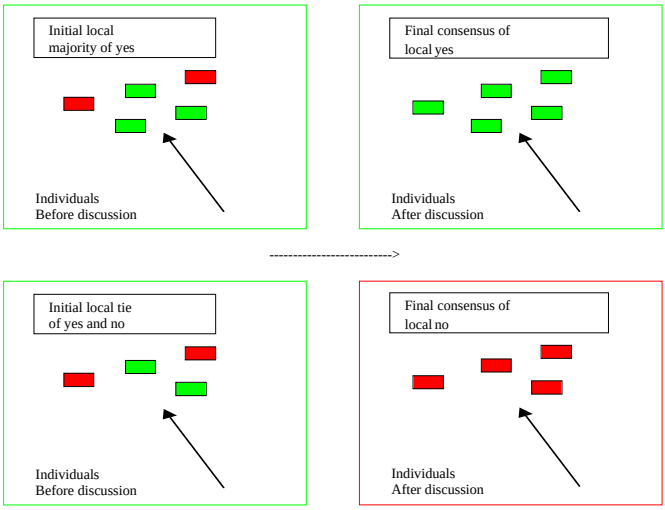

Fig. 1. A local discussion process. Upper figure shows a group with five individuals. Before discussion (left side) there are three light opinions and two dark ones. After discussion everyone in the group has a light opinion. Lower figure shows a group of four at a tie (left side). After discussion the four have turned dark (right side).

just a few. A given social space thus yields a well-defined distribution of group size gathering $\{a_i\}$ which satisfy the constraint,

$$\sum_{i=1}^{L} a_i = 1, \tag{2}$$

where $i = 1, 2, \ldots, L$ stands for respective sizes $1, \ldots, L$ with L being the larger group.

In the course of time, the same people meet again and again randomly in the same cluster configuration determined by the $\{a_i\}$ probabilities. The social daily meetings like lunch, dinner, happy hour and late drink monitor these discussions. Each new cycle of multi-size discussions is marked by a time increment $+1$. We suppose that all individuals are simultaneously involved in some group gathering. It means a given person is, on an average, taking part in a group of size i with probability a_i. The existence of one-person groups makes this assumption realistic.

At each new encounter people discuss locally the issue at stake. They change their mind according to a local majority rule. A one-person-one-argument principle is used to implement the psychological process of collective mind update. A perfect world is considered with no advantage whatsoever to the minority. People align within each group along the local initial majority there.

However, in case of an even group this rule of one person–one argument leaves the local possibility of a temporary absence of a collective majority. The group is then at a tie within a non-decisional state (see Fig. 1). It doubts. And from the fundamental psychological asymmetry that exists between what is known and what is hypothetical we assume that a local majority of at least one voice is necessary to go along a change.

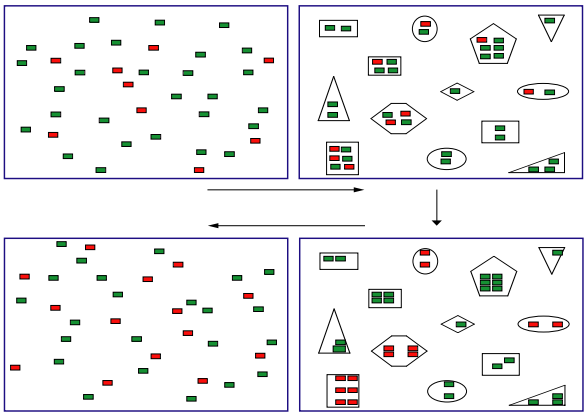

Fig. 2. One cycle of discussion with 37 persons, 28 light opinions and nine dark ones (upper left). One random distribution in several small group (upper right). After discussion, all local groups become homogeneous (lower right). People split (lower left) with now 13 light and 14 dark. The dark opinion has jumped from minority to majority.

In terms of our model, at a tie the group turns against the reform proposal to preserve the existing situation. It is worth stressing that this is not an advantage given to the minority in terms of being more convincing. It is a collective outcome that results from a state of doubt. According to the above rules of local decision making, starting from $P_+(t)$ at time t yields at time $t+1$ to the expression $P_+(t+1)$,

$$P_+(t+1) = \sum_{k=1}^{L} a_k \sum_{j=N[k/2+1]}^{k} C_j^k P_+(t)^j P_-(t)^{(k-j)}, \tag{3}$$

which is a binomial distribution, where $C_j^k \equiv k!/((k-j)!j!)$ and $N[k/2+1] \equiv$ IntegerPart$[k/2+1]$. Simultaneously we have $P_-(t+1) = 1 - P_+(t+1)$.

As a function of time, the same people will meet again and again but at each time, are randomly distributed in the same cluster configuration. At each new clustering, they discuss locally the issue at stake and change their mind according to the above majority rule. Fig. 2 shows an example of a sequence with a geometry of two groups of one, six groups of two, one group of three, two groups of four and two groups of six totalling 37 persons. The first initial configuration is shown (upper left) with 28 persons in favor of the reform in light color and nine against it in dark color. They go to dinner as shown in the upper right part of Fig. 2. Once the dinner is over, people have all the same opinion within each group as seen in the lower right part of Fig. 2. They split with the new proportion of 23 persons in favor of the reform and now 14 against it (lower right). Repeating this process a few more times yields all 37 persons against the reform.

More generally, to follow the time evolution of the reform support Eq. (4) is iterated until a stable value is reached. A monotonic flow is obtained towards either one of the two stable fixed points $P_N = 0$ and $P_Y = 1$. The flow and its direction are produced by an unstable fixed point P_F located in between $P_N = 0$ and $P_Y = 1$. Its value depends

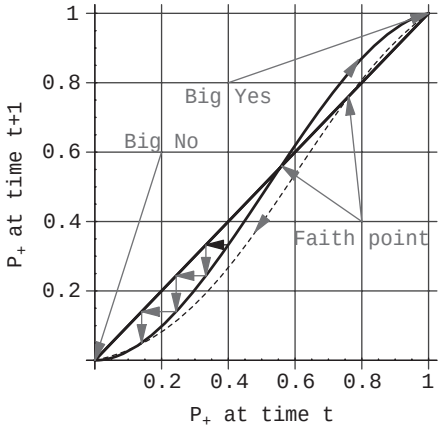

Fig. 3. Variation of $P_+(t+1)$ as function of $P_+(t)$. The dashed line is for the set $a_1 = a_2 = a_3 = a_4 = 0.2$ and $a_5 = a_6 = 0.1$, where $L = 6$. The plain line is for the set $a_1 = 0$, $a_2 = 0.1$, $a_3 = 0.9$ with $L = 3$ and $P_F = 0.56$. Arrows show the direction of the flow.

on both the $\{a_i\}$ and L. We denote it the Faith point. For $P_+(t) \prec P_F$ there exists a number n such that $P_+(t+m) = P_Y = 0$ while for $P_+(t) \succ P_F$ it is another number m, which yields $P_+(t+m) = P_Y = 1$. Both n and m measure the required time at reaching a stable and final opinion. It is either a "Big Yes" to the reform at $P_Y = 1$ or a "Big No" at $P_N = 0$. Their respective values depend on the $\{a_i\}$, L and the initial value $P_+(t)$.

Repeated successive local discussions thus drive the whole population to a full polarization at a "Big Yes" to the reform project or at a "Big No". Accordingly, public opinion is not volatile. It stabilizes rather quickly (n and m are usually small numbers) to a clear stand. Fig. 3 shows the variation of $P_+(t+1)$ as a function of $P_+(t)$ for two particular sets of the $\{a_i\}$. The first is $a_1 = a_2 = a_3 = a_4 = 0.2$ and $a_5 = a_6 = 0.1$ where $L = 6$. There $P_F = 0.74$ which puts the required initial support to the reform success at a very high value of more than 74%. Simultaneously, an initial minority above 26% is enough to produce a final total refusal. The second set is $a_1 = 0, a_2 = 0.1, a_3 = 0.9$ with $L = 3$ and $P_F = 0.56$. There the situation is much milder but also unrealistic since always pair discussions are much more numerous than just 10%.

To make a quantitative illustration of the dynamics refusal let us consider the above first setting with an initial $P_+(t) = 0.70$ at time t. The associated series in time is $P_+(t+1) = 0.68$, $P_+(t+2) = 0.66$, $P_+(t+3) = 0.63$, $P_+(t+4) = 0.58$, $P_+(t+5) = 0.51$, $P_+(t+6) = 0.41$, $P_+(t+7) = 0.27$, $P_+(t+8) = 0.14$, $P_+(t+9) = 0.05$, $P_+(t+10) = 0.01$ and eventually $P_+(t+11) = p_N = 0$. Eleven cycles of discussion make all 70% of reform supporters turn against it by merging with the initial 30% of reform opponents. On the basis of one discussion a day on average, less than two weeks is enough for a total crystallization of the 'No' against the reform proposal. Moreover, a majority against the reform is obtained already within 6 days (see Fig. 4).

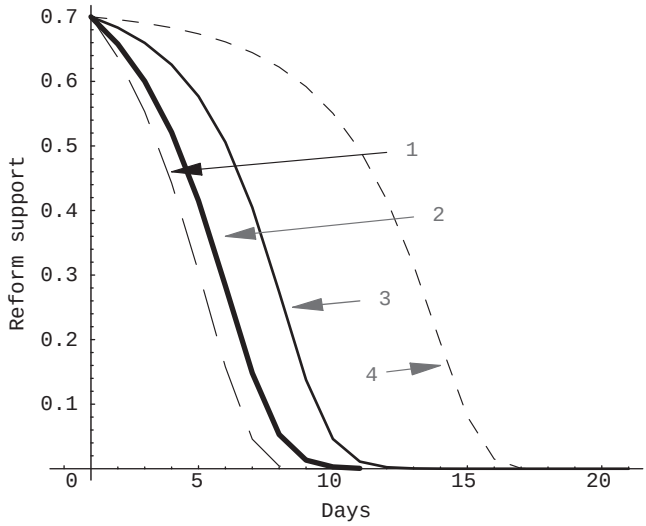

Fig. 4. Variation of $P_+(t)$ as a function of successive days with $L = 6$. The initial value at $t = 1$ is $P_+(t) = 0.70$. Long dashed line (1): $a_1 = 0,\ a_2 = 0.5,\ a_3 = 0.5,\ a_4 = 0,\ a_5 = 0,\ a_6 = 0$ with $P_F = 1$. Heavy thick line (2): $a_1 = 0.2,\ a_2 = 0.3,\ a_3 = 0.2,\ a_4 = 0.2,\ a_5 = 0.1,\ a_6 = 0$ with $P_F = 0.85$. Other line (3): $a_1 = 0.2,\ a_2 = 0.2,\ a_3 = 0.2,\ a_4 = 0.2,\ a_5 = 0.1,\ a_6 = 0.1$ with $P_F = 74$. Dashed line (4): $a_1 = 0,\ a_2 = 0.3,\ a_3 = 0.7,\ a_4 = 0,\ a_5 = 0,\ a_6 = 0$ with $P_F = 0.71$.

Table 1
Values of the various fixed points for each group size from two to six

Group size	Stable fixed point total no P_N	Unstable fixed point P_F	Stable fixed point total yes P_Y
2	0	1	1
3	0	$\frac{1}{2}$	1
4	0	$\frac{1+\sqrt{13}}{6} \approx 0.77$	1
5	0	$\frac{1}{2}$	1
6	0	≈ 0.65	1

It is the existence of an unstable fixed point between the two stable ones, which produces the whole polarization dynamics. The stable ones are constant and independent of the $\{a_i\}$ but the unstable one varies with both, sizes and the $\{a_i\}$ distribution. To single out the specific contribution of each gathering size to the aggregation effect we now determine the associated unstable fixed point for groups from two to six. The values are shown in Table 1. The flow landscape is identical for all odd sizes with an unstable fixed point at $P_F = \frac{1}{2}$. On the contrary, for even sizes the unstable fixed point starts at $P_F = 1$ for size two and decreases to $P_F = 0.65$ at size six, via $P_F = 0.77$ at size four.

A large number of combinations of the $\{a_i\}$ is possible. However, for whatever combinations chosen, temporary local doubts are preserved and they ultimately produce a strong polarization towards social refusal. Of course, in real life many people are not open to change. Accordingly, some fractions of the population keep their opinion unchanged even after several cycles of discussion. Including this effect in the model does not change qualitatively the results. It only makes the polarization process not total with the two stable fixed points shifted towards, respectively, larger and smaller values than zero and one. In this sense, it is more realistic since the total polarization of a whole population is a very rare event.

Applying our results to the European Union leads to the conclusion that it would be rather misleading to initiate large public debates in most of the involved countries. Indeed, even starting from a huge initial majority of people in favor of the European Union, an open and free debate would lead to the creation of huge majority hostile to the European Union. This provides a strong ground to legitimize the on-going reluctance of most European governments to hold referendum on associated issues. This danger of reversing support has been proved several years ago when French president Mitterand decided to run a referendum to accept the Maastricht agreement [5]. While the Yes won massively the vote, it obtained just a bit beyond the required 50%. The more the people were discussing it, the less support there was for the proposal. On this basis it is possible to conjuncture that an additional 2 weeks extension of the public debate would have made the No win. The very recent Irish No [6], which also came as a blow to all analysts may obey the same logic. The difference with the French case was certainly a weaker initial support.

To conclude, our model is not aimed at an exact description of reality. But rather, by doing some crude approximations, it showed how the holding of free public debate leads almost systematically to the total spreading of an initial hostile minority view even against a huge majority initially favorable to the project. The associated dynamics was found to result from the existence of asymmetric unstable thresholds [3,4] that are produced by the random temporary local doubts which occur during the local free and open discussions. Some recent nation-wide issues with respect to European construction have thus been revisited. The model may generalize to a large spectrum of social, and political phenomena that involve propagation effects. In particular, it could shed new light on both processes of fear propagation and rumor spreading.

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