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The Dynamics of Poverty, Drug Addiction and Snatching In Sylhet, Bangladesh

Islam, M. A.[1], Sakib, M. A.[2], Shahrear, P.[3*], Rahman, S. M. S.[4]
[1] *Professor, Department of Mathematics, Shahjalal University of Science and Technology, Sylhet-, Bangladesh*
[2]*M.S. (Thesis), Department of Mathematics, Shahjalal University of Science and Technology, Sylhet-, Bangladesh*
[3]*Associate Professor, Department of Mathematics, Shahjalal University of Science and Technology, Sylhet-, Bangladesh*
[4]*M.S. (Thesis), Department of Mathematics, Shahjalal University of Science and Technology, Sylhet-, Bangladesh*
3* Corresponding Author

Abstract: *A compartmental mathematical model is established to study the dynamics of poverty, drug addiction and snatching. In the model, we have compartments, which allow for an approach of government and non-government interventions. The stability analysis in this model holds for an addiction and snatching free equilibrium. We constituted that, the equilibrium is locally asymptotically stable when the reproduction number is less than 1 and unstable when it is greater than 1. Numerical simulations of the systems have been presented to show the variations of the population in different situations. We figured out that, the high rate of interventions will reduce poverty, drug addiction and snatching faster towards their barest minimum. Data that have been used for simulations are based on the addiction and snatching happens in the district of Sylhet in Bangladesh. But we believe that, our model is applicable for the whole country and even for the whole world.*
Keywords: *Compartmental Model, Drug addiction, Intervention, Poverty, Reproduction Number, Snatching, Stability Analysis, ODE.*

I. Introduction

Sylhet is an illustrious district of Bangladesh where the rate of drug addiction is increasing day by day. On the other hand, there is an increasing rate of snatching as well. Majority of the snatching committed because of addiction or because of scarcity of money. Also there is a significant rate 24.1% [15] of poverty in Sylhet which is another very important factor for the increase of addiction and snatching. So it is a modern day challenge for us to have a strong fight against poverty, addiction and snatching in Sylhet for optimal control. Poverty, drug addiction and snatching are some strongly connected major problems of our modern life. It is seen that, poor people are addicted to drugs because some of the reasons of addiction [5] are directly or indirectly related to poverty like:- ignorance, unhealthy social environment, inability to deal with life and stress, unavailability of social and psychological help, family and social damages, family troubles and strained relationship, inability to complete education etc. On the other hand, the people who are living in non-impoverished class (rich or middle class), also addicted to drugs in large numbers. It mainly happens because of availability of drugs, curiosity of enjoying drugs, influence of the friends, inability to deal with life and stress, family troubles and strained relationship [5] etc. Many of the addicted people (no matter in which class they belong to) are committing crimes, especially property crimes (more briefly snatching) for bearing the expenses of the drugs and because of mental disorder caused by addiction. This rate of committing snatching is very alarming especially in the urban region of developing countries like:- Sylhet district in Bangladesh.

In previous works [1] and [2] the respective epidemiologists have developed the mathematical models on Poverty and Crime as well as Poverty and Prostitution. Their works were based on the works by Kelly, 2000 [4]; Block and Heineke, 1975. These have inspiration from Becker's economic theory of crime (1968), described in [3]. In Becker's paper [3], he used statistical and economic analysis to determine the optimal control of crime. In the papers [1] and [2] the respective epidemiologists used a system of ODEs to try and get more realistic, dynamical solution based on the correlation between poverty, crime, prostitution etc. In the paper [2], the respective epidemiologists considered prostitution as a crime and use a mathematical model to understand the dynamics of the system and how both poverty and prostitution can be reduced to the barest minimum, just like in the paper [1], the respective epidemiologists use a mathematical model to understand the dynamics of the system and how both poverty and crime can be reduced to the barest minimum. We also previously developed a model (described in the paper [17]) based on the dynamics of poverty and addiction which is very important for our study in this paper. Moreover, the dynamics of snatching can be modelled from the model described in the paper [1] by considering snatching as the crime.

In this paper, our target is to develop a compartmental mathematical model (can be shortly named as PDS model) which examines the dynamics of poverty, drug addiction and snatching at a time. We use a system of ODEs to try and obtain a more powerful and realistic, dynamical solution of the system. Our model includes compartments that allow for an approach of government and non-government interventions. The basic difference between the models described in [1], [2] and [17] with our model in this paper is that, we consider the fact, people living in non-poverty/non-impoverished class (rich or middle class) of any gender (male, female or third) may also get addicted to drugs. Also the respected epidemiologists in their paper only used one epidemic or crime at a time, but in this model we used two at a time. We are also aware of the fact that, addiction can cause snatching and snatchers can also be addicted to drugs. So our model is much improved and up-to-date one comparing to the models described in [1], [2] and [17]. Moreover, we try to obtain a useful approach at minimizing the drug addiction and snatching while minimizing the poverty in Sylhet, Bangladesh.

II. Model Formulation

We know that, it is a difficult task to stop drug addiction and snatching totally from our society. But it is possible to enhance interventions in order to control addiction and snatching reasonably and the cost of intervention is minimal. Also addiction can cause snatching and snatchers can also be addicted to drugs. This model deals with these facts as well. In this model, the population is divided into seven compartments; the non-impoverished class *N*, the poverty/impoverished class *P*, the drug addicted class *D*, the snatchers class *S*, the rehabilitation class *I*, the captured by police (can be jailed or locked-up) class *J* and the recovery class *R* (from the poverty class, rehabilitation class or captured class). All the variables are functions of time *t*. α denotes the rate of flow from the non-impoverished class to the poverty class/impoverished class, which is dependent upon unemployment and underemployment rate, normally is directly related to poverty. β denotes the rate at which the individuals in the poverty/impoverished class get into the drug addicted class. Similarly, φ denotes the rate at which the individuals in the non-impoverished class get into to the drug addicted class. A person in the *P* class will resort to addiction after coming in contact with an addict over a given period of time. On the other hand, a person in the *N* class will also resort to addiction after coming in contact with an addict over a given period of time. The term $\frac{\varphi ND}{T}$ represents the conversion rate from the *N* class to the *D* class, where φ is the transmission rate. Similarly the term $\frac{\beta PD}{T}$ represents the conversion rate from the *N* class to the *D* class, where β is the transmission rate. An addict can be involved to snatching at a rate *k* whereas a snatcher can be addicted to drugs at a rate *u*. The term $\frac{kDS}{T}$ represents the conversion rate from the *D* class to the *S* class, where k is the transmission rate. Similarly the term $\frac{uSD}{T}$ represents the conversion rate from the *S* class to the *D* class, where u is the transmission rate. A recovered individual may also involve to addiction again but at reduced rates $\frac{\beta\sigma RD}{T}$, $\frac{\varphi\sigma RD}{T}$ and $\frac{u\sigma RD}{T}$,where $0 \le \sigma \le 1$ is the reduction fraction that account for the phenomena. v denotes the rate at which the individuals in the poverty/impoverished class get into the snatchers class. η is the rate at which a snatcher has been captured by police. A person in the *P* class will resort to snatching after coming in contact with a snatcher over a given period of time. The term $\frac{vPS}{T}$ represents the conversion rate from the *P* class to the *S* class, where the transmission rate is v . A recovered individual may also involve to snatching again but at a reduced rates $\frac{mvRS}{T}$ and $\frac{mkRS}{T}$ where $0 \le m \le 1$ is the reduction fraction that account for the phenomena.

The addicts, already been rehabilitated immediately move to the class *R* at a rate δ and then due to contact with other addicts may return back to addiction at some reduced rates $\beta\sigma$, $\varphi\sigma$ and $u\sigma$. Also The snatchers, already been captured move to the class *R* at a rate r and then due to contact with other snatchers may return back to snatching again at some reduced rates mv and mk . Then for *S* class to *D* class conversion, we have some reduced rates βu , φu and σu . For *D* class to *S* class conversion, we have some reduced rates vk and mk . So the terms $\frac{\varphi uSD}{T}$, $\frac{\beta uSD}{T}$, $\frac{\sigma uSD}{T}$, $\frac{vkDS}{T}$ and $\frac{mkDS}{T}$ have significant meanings in our system. There are also disease induced deaths due to infection in different classes, but the death rates are very low. We assume that, the total population T remains constant ($\frac{dT}{dt} = 0$) i.e. the per capita death rate is equal in magnitude to the per capita birth rate ($\mu = \lambda$). All parameters are assumed to be non-negative.

Now the relationship between poverty, drug addiction and snatching can be governed by the following system of ODEs:

$$\begin{aligned}
\frac{dN}{dt} &= \mu T - (\mu + \alpha) N - \frac{\varphi N D}{T} \\
\frac{dP}{dt} &= \alpha N - (\mu + \rho) P - \frac{\beta P D}{T} - \frac{v P S}{T} \\
\frac{dD}{dt} &= \frac{\beta P D}{T} + \frac{\varphi N D}{T} + (\varphi + \beta + u)\frac{\sigma R D}{T} + (\varphi + \beta + \sigma)\frac{u S D}{T} - (v + m)\frac{k D S}{T} - (\mu + k_1 + \gamma) D \\
\frac{dS}{dt} &= \frac{v P S}{T} + (v + m)\frac{k D S}{T} + (v + k)\frac{m R S}{T} - (\mu + k_3 + \eta) S - (\varphi + \beta + \sigma)\frac{u S D}{T} \qquad (1) \\
\frac{dI}{dt} &= \gamma D - (\mu + k_2 + \delta) I \\
\frac{dJ}{dt} &= \eta S - (\mu + k_4 + r) J \\
\frac{dR}{dt} &= \delta I + \rho P + r J - \mu R - (v + k)\frac{m R S}{T} - (\varphi + \beta + u)\frac{\sigma R D}{T} \\
T &= N + P + D + S + I + J + R
\end{aligned}$$

In the following tables, the variables and parameters used in the model are defined:

Table 1 : Model Variables

Variables	Description
N(t)	Non-impoverished class
P(t)	Poverty/impoverished class
D(t)	Drug addicted class
S(t)	Snatchers class
I(t)	Rehabilitation class
J(t)	Captured (can be jailed or locked-up) class
R(t)	Recovery class
T(t)	Total Population

Table 2 : Model Parameters

Parameters	Description
α	The rate of flow from the non-impoverished class *N* to the poverty/impoverished class *P*
β	The rate at which the individuals in the poverty/impoverished class *P* get into the drug addicted class *D*
$\mu = \lambda$	Per capita death rate = Per capita birth rate
φ	The rate at which the individuals in the non-impoverished class *N* get into to the drug addicted class *D*
γ	The rate at which an addict are recruited into the rehabilitation class *I*
ρ	The conversion rate from the poverty class *P* to the recovery class *R* due to intervention
δ	The conversion rate from the rehabilitation class *I* to the recovery class *R*
σ	The rate of *R* to *D* transmission $(0 \le \sigma \le 1)$
v	The rate at which the individuals in the poverty/impoverished class *P* get into the snatchers class *S*
η	The rate at which a snatcher has been captured
r	The rate at which captured snatchers are recruited into the recovery class *R*
m	The rate of *R* to *S* transmission $(0 \le m \le 1)$
k	The rate at which the individuals in the drug addicted class *D* get into the snatchers class *S*
u	The rate at which the individuals in the snatchers class *S* get into the drug

	addicted class D
k_1	Disease induced death rate due to infection in D
k_2	Disease induced death rate due to infection in I
k_3	Disease induced death rate due to infection in S
k_4	Disease induced death rate due to infection in J

The flow diagram for the system is given below:

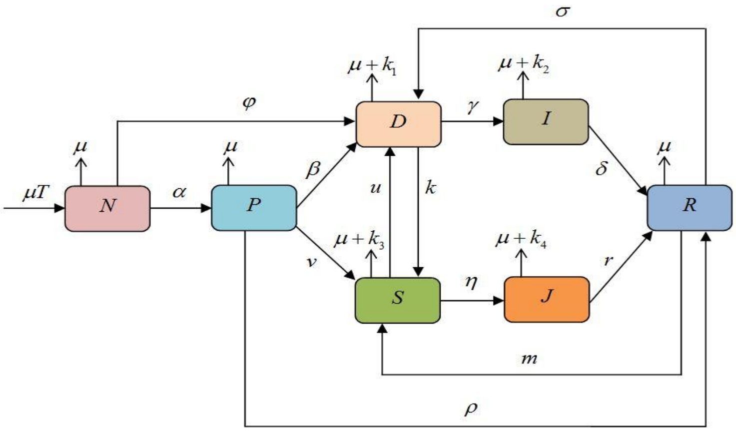

Fig.1: Flow diagram for the system (1)

III. Mathematical Analysis Of The Model

In this section, we discuss the existence and uniqueness of the Addiction and Snatching Free Equilibrium (ASFE) of the model and its stability analysis. It can be obtained by considering the different compartments as proportions and setting the LHS of the system (1) as zero and then solving the system simultaneously.

But first of all we consider an equilibrium named as Addiction and Snatching Equilibrium (ASE) as,

$$E^* = \left(\frac{N^*}{T}, \frac{P^*}{T}, \frac{D^*}{T}, \frac{S^*}{T}, \frac{I^*}{T}, \frac{J^*}{T}, \frac{R^*}{T}\right) \text{ with } D^* > 0,\ S^* > 0 \tag{2}$$

Now considering $\frac{D^*}{T} = x \Rightarrow D^* = xT$, $\frac{S^*}{T} = y \Rightarrow S^* = yT$ and LHS of the system (1) as zero and then solving the system simultaneously, we have obtained,

$$\frac{N^*}{T} = \frac{\mu}{[(\mu+\alpha)+\varphi x]}, \quad \frac{P^*}{T} = \frac{\alpha\mu}{[(\mu+\alpha)+\varphi x][(\mu+\rho)+\beta x+vy]}, \quad \frac{I^*}{T} = \frac{\gamma x}{(\mu+k_2+\delta)},$$

$$\frac{J^*}{T} = \frac{\eta y}{(\mu+k_4+r)} \text{ and } \frac{R^*}{T} = \frac{\dfrac{\delta\gamma x}{(\mu+k_2+\delta)} + \dfrac{\alpha\rho\mu}{[(\mu+\alpha)+\varphi x][(\mu+\rho)+\beta x+vy]} + \dfrac{r\eta y}{(\mu+k_4+r)}}{[\mu+(\beta+\varphi+u)\sigma x+(v+k)my]} \tag{3}$$

Then considering $\frac{D^*}{T} = x = 0 \Rightarrow D^* = 0$ and $\frac{S^*}{T} = y = 0 \Rightarrow S^* = 0$ we have $\frac{I^*}{T} = 0 \Rightarrow I^* = 0$ and $\frac{J^*}{T} = 0 \Rightarrow J^* = 0$. Eventually, we have obtained the Addiction and Snatching Free Equilibrium (ASFE) which is given by,

$$E_0 = \left(\frac{N_0}{T}, \frac{P_0}{T}, \frac{D_0}{T}, \frac{S_0}{T}, \frac{I_0}{T}, \frac{J_0}{T}, \frac{R_0}{T}\right) = \left(\frac{\mu}{(\mu+\alpha)}, \frac{\alpha\mu}{(\mu+\alpha)(\mu+\rho)}, 0, 0, 0, 0, \frac{\alpha\rho}{(\mu+\alpha)(\mu+\rho)}\right) \tag{4}$$

When $D^* > 0 \Rightarrow \frac{D^*}{T} > 0 \Rightarrow x > 0$ and $S^* > 0 \Rightarrow \frac{S^*}{T} > 0 \Rightarrow y > 0$ then $\frac{I^*}{T} > 0 \Rightarrow I^* > 0$ and $\frac{J^*}{T} > 0 \Rightarrow J^* > 0$ since I^* and J^* are directly depending on x and y respectively. Then for E^*, considering the LHS as zero, the third equation of the system (1) can be written as:

$$0 = \frac{\beta P^*}{T}x + \frac{\varphi N^*}{T}x + (\varphi + \beta + u)\frac{\sigma R^*}{T}x + (\varphi + \beta + \sigma)uyx - (v + m)kxy - (\mu + k_1 + \gamma)x$$

implies

$$\left[\frac{\beta P^*}{T} + \frac{\varphi N^*}{T} + (\varphi + \beta + u)\frac{\sigma R^*}{T} + (\varphi + \beta + \sigma)uy - (v + m)ky - (\mu + k_1 + \gamma)\right]x = 0$$

Since $x > 0$ so clearly,

$$\left[\frac{\beta P^*}{T} + \frac{\varphi N^*}{T} + (\varphi + \beta + u)\frac{\sigma R^*}{T} + (\varphi + \beta + \sigma)uy - (v + m)ky - (\mu + k_1 + \gamma)\right] = 0$$

$$\Rightarrow \left[\frac{\beta P^*}{T} + \frac{\varphi N^*}{T} + (\varphi + \beta + u)\frac{\sigma R^*}{T} + (\varphi + \beta + \sigma)uy\right] = (\mu + k_1 + \gamma) + (v + m)ky \tag{5}$$

Again, since $x > 0,\ y > 0$ and all the parameters are non-negative, we can conclude from (3) and (5) that,

$$\frac{P^*}{T} > 0 \Rightarrow P^* > 0,\ \frac{N^*}{T} > 0 \Rightarrow N^* > 0 \text{ and } \frac{R^*}{T} > 0 \Rightarrow R^* > 0$$

For a similar approach in the fourth equation of the system (1) we can also show the very last findings.

So there exists a unique ASE i.e. E^* when $D^* > 0, I^* > 0$ and $S^* > 0, J^* > 0$. We assumed that, it is only exists when the ASFE i.e. E_0 is unstable.

Now using the approach of *Next Generation Matrix Operator* described in [6], [7], [8], [9], [10] and [11], we obtained the *reproduction number*.

By using the approach we obtained,

$$F = \begin{pmatrix} \frac{\beta\alpha\mu + \varphi\mu(\mu + \rho) + (\beta + \varphi + u)\sigma\alpha\rho}{(\mu + \alpha)(\mu + \rho)} & 0 & 0 & 0 \\ 0 & \frac{v\alpha\mu + (v + k)m\alpha\rho}{(\mu + \alpha)(\mu + \rho)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

and

$$V = \begin{pmatrix} (\mu + k_1 + \gamma) & 0 & 0 & 0 \\ 0 & (\mu + k_3 + \eta) & 0 & 0 \\ -\gamma & 0 & (\mu + k_2 + \delta) & 0 \\ 0 & -\eta & 0 & (\mu + k_4 + r) \end{pmatrix} \tag{7}$$

Then, the next generation matrix is,

$$FV^{-1} = \begin{pmatrix} \dfrac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi+u)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)} & 0 & 0 & 0 \\ 0 & \dfrac{v\alpha\mu + (v+k)m\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_3+\eta)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

Since we do not identify which of the two non-zero terms in the above matrix is greater than the other. So the *reproduction number* can be termed as,

$$\Re = \max\{\Re_1, \Re_2\} \tag{9}$$

where,

$$\Re_1 = \frac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi+u)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)} \tag{10}$$

and

$$\Re_2 = \frac{v\alpha\mu + (v+k)m\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_3+\eta)} \tag{11}$$

We move forward to show the stability of the equilibria. To study the behaviour of the system (1) around the ASFE, we refer the linearlized stability principals. We evaluate the partial derivatives of the system (1) at ASFE i.e. E_0 to get the *Jacobian matrix* $J_{[ASFE_0]} = J_{E_0}$ for ASFE which is given by,

$$\begin{pmatrix} -(\mu+\gamma) & 0 & \dfrac{-\varphi N_0}{T} & 0 & 0 & 0 & 0 \\ \alpha & -(\mu+\rho) & \dfrac{-\beta P_0}{T} & \dfrac{-vP_0}{T} & 0 & 0 & 0 \\ 0 & 0 & \varphi\dfrac{N_0}{T} + \beta\dfrac{P_0}{T} + (\varphi+\beta+u)\sigma\dfrac{R_0}{T} - (\mu+k_1+\gamma) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v\dfrac{P_0}{T} + (v+k)m\dfrac{R_0}{T} - (\mu+k_3+\eta) & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & -(\mu+k_2+\delta) & 0 & 0 \\ 0 & 0 & 0 & \eta & 0 & -(\mu+k_4+r) & 0 \\ 0 & \rho & -(\varphi+\beta+u)\dfrac{\sigma R_0}{T} & -(v+k)\dfrac{mR_0}{T} & \delta & r & -\mu \end{pmatrix} \tag{12}$$

where

$$\frac{N_0}{T} = \frac{\mu}{(\mu+\alpha)},\quad \frac{P_0}{T} = \frac{\alpha\mu}{(\mu+\alpha)(\mu+\rho)} \quad \text{and} \quad \frac{R_0}{T} = \frac{\alpha\rho}{(\mu+\alpha)(\mu+\rho)} \tag{13}$$

Now we try to calculate the eigenvalues of J_{E_0} by finding the characteristic equation.

Let the eigenvalues be defined as θ. By proceeding forward we have the characteristic equation,

$$\left[\theta - \left(\varphi\frac{N_0}{T} + \beta\frac{P_0}{T} + (\varphi+\beta+u)\sigma\frac{R_0}{T}\right) + (\mu+k_1+\gamma)\right]\left[\theta - \left(v\frac{P_0}{T} + (v+k)m\frac{R_0}{T}\right) + (\mu+k_3+\eta)\right]\left[\theta+\mu\right]$$
$$\left[\theta + (\mu+k_2+\delta)\right]\left[\theta + (\mu+k_4+r)\right]\left[\theta + (\mu+\alpha)\right]\left[\theta + (\mu+\rho)\right] = 0 \tag{14}$$

From the characteristic equation we obtain the eigenvalues,

$\left(\varphi \frac{N_0}{T} + \beta \frac{P_0}{T} + (\varphi + \beta + u)\sigma \frac{R_0}{T}\right) - (\mu + k_1 + \gamma)$, $\left(v \frac{P_0}{T} + (v + k)m \frac{R_0}{T}\right) - (\mu + k_3 + \eta)$, $-\mu$, $-(\mu + k_2 + \delta)$, $-(\mu + k_4 + r)$, $-(\mu + \alpha)$ and $-(\mu + \rho)$.

Then, E_0 is only exists when all the eigenvalues of J_{E_0} are non-positive [13]. Also E_0 is stable if all the eigenvalues of J_{E_0} has negative real parts [12].

These holds only if, $\left(\varphi \frac{N_0}{T} + \beta \frac{P_0}{T} + (\varphi + \beta + u)\sigma \frac{R_0}{T}\right) - (\mu + k_1 + \gamma) < 0$

$$\Rightarrow \left(\varphi \frac{N_0}{T} + \beta \frac{P_0}{T} + (\varphi + \beta + u)\sigma \frac{R_0}{T}\right) < (\mu + k_1 + \gamma)$$

$$\Rightarrow \frac{\beta\alpha\mu + \varphi\mu(\mu + \rho) + (\beta + \varphi + u)\sigma\alpha\rho}{(\mu + \alpha)(\mu + \rho)(\mu + k_1 + \gamma)} < 1$$

Hence, $\Re_1 < 1$

and $\left(v \frac{P_0}{T} + (v + k)m \frac{R_0}{T}\right) - (\mu + k_3 + \eta) < 0$

$$\Rightarrow \left(v \frac{P_0}{T} + (v + k)m \frac{R_0}{T}\right) < (\mu + k_3 + \eta)$$

$$\Rightarrow \frac{v\alpha\mu + (v + k)m\alpha\rho}{(\mu + \alpha)(\mu + \rho)(\mu + k_3 + \eta)} < 1$$

Hence, $\Re_2 < 1$

Again, E_0 is unstable if at least one of the eigenvalues of J_{E_0} has positive real part [12].

These holds only if, $\left(\varphi \frac{N_0}{T} + \beta \frac{P_0}{T} + (\varphi + \beta + u)\sigma \frac{R_0}{T}\right) - (\mu + k_1 + \gamma) > 0$

$$\Rightarrow \left(\varphi \frac{N_0}{T} + \beta \frac{P_0}{T} + (\varphi + \beta + u)\sigma \frac{R_0}{T}\right) > (\mu + k_1 + \gamma)$$

$$\Rightarrow \frac{\beta\alpha\mu + \varphi\mu(\mu + \rho) + (\beta + \varphi + u)\sigma\alpha\rho}{(\mu + \alpha)(\mu + \rho)(\mu + k_1 + \gamma)} > 1$$

Hence, $\Re_1 > 1$

or $\left(v \frac{P_0}{T} + (v + k)m \frac{R_0}{T}\right) - (\mu + k_3 + \eta) > 0$

$$\Rightarrow \left(v \frac{P_0}{T} + (v + k)m \frac{R_0}{T}\right) > (\mu + k_3 + \eta)$$

$$\Rightarrow \frac{v\alpha\mu + (v + k)m\alpha\rho}{(\mu + \alpha)(\mu + \rho)(\mu + k_3 + \eta)} > 1$$

Hence, $\Re_2 > 1$

These two eigenvalues can be either both positive or one of them can be negative when the instability holds. But that's for sure, the negative one is the smaller one between the two.

On the other hand, a unique E^* is only exists if E_0 is unstable. So a unique E^* is only exists if $\Re_1 > 1$ or $\Re_2 > 1$.

Again, from (9) we know, $\Re = \max\{\Re_1, \Re_2\}$

So our findings then can be concluded as the following lemma:

Lemma 1

For the system (1), the ASFE i.e. E_0 is locally asymptotically stable if $\Re < 1$ and unstable if $\Re > 1$. Also the system has only the ASFE i.e. E_0 if $\Re < 1$ and an unique ASE i.e. E^* if $\Re > 1$.

IV. Numerical Simulation Of The Model

We are going to discuss the numerical simulation and results from PDS model in this section. We solved the system (1) by using MATLAB. The baseline variables and used parameter estimated for Sylhet, Bangladesh are shown in the following tables. We found that, our estimations are actually correct enough to present the model properly.

Table 3 : Estimation of the Baseline Variables

Variables	Values	Sources
T	3567138	[14] and [17]
N(0)	2707458	Estimated on the basis of [14], [15] and [17]
P(0)	859680	Estimated on the basis of [14], [15] and [17]
D(0)	86890	Estimated from the survey results
S(0)	27376	Estimated from the survey results
I(0)	25689	Estimated from the survey results
J(0)	8722	Estimated from the survey results
R(0)	15921	Estimated from the survey results

Table 4 : Estimation of the Parameters used

Parameters	Values	Sources
α	0.08	Estimated from the survey results
β	0.28	Estimated from the survey results
$\mu = \lambda$	0.01413	Estimated on the basis of [16] and [17]
φ	0.47	Estimated from the survey results
γ	0.37	Estimated from the survey results
ρ	0.12	Estimated from the survey results
δ	0.61	Estimated from the survey results
σ	0.40	Estimated from the survey results
v	0.31	Estimated from the survey results
η	0.23	Estimated from the survey results
r	0.37	Estimated from the survey results
m	0.45	Estimated from the survey results
k	0.54	Estimated from the survey results
u	0.34	Estimated from the survey results
k_1	0.02	Estimated from the survey results
k_2	0.01	Estimated from the survey results
k_3	0.01	Estimated from the survey results
k_4	0.005	Estimated from the survey results

V. Results And Discussion

For the system (1), the parameters γ, η and ρ can be considered as the *intervention parameters*. First we will analyze the situation of the different population classes with respect to time without implementing any kind of government and non-government interventions considering the values of all the intervention parameters as zero i.e. $\gamma = 0,\ \eta = 0$ and $\rho = 0$. Then we will analyze the situation of the different population classes with respect to time with government and non-government interventions considering the values of all the intervention parameters as our estimation in Table 4. i.e. $\gamma = 0.37,\ \eta = 0.23$ and $\rho = 0.12$. Finally we will analyze the situation of the different population classes with respect to time with high and low government and non-government interventions considering the values intervention parameters as $\gamma = 0.40,\ \eta = 0.28,\ \rho = 0.15$ (for high intervention) and $\gamma = 0.34,\ \eta = 0.18,\ \rho = 0.09$ (for low intervention).

When there are no government interventions i.e. $\gamma = 0,\ \eta = 0$ and $\rho = 0$, we have plotted the following graphs in MATLAB:

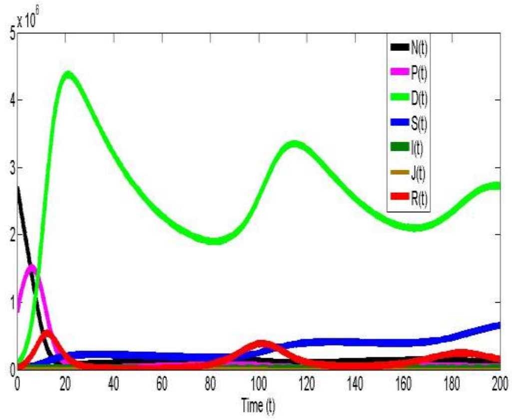

Fig. 2 : Situations of the different population classes with respect to *t* without interventions for PDS model

The plots in the Fig. 2 show that, when there are no interventions, the population classes have the following situations:

- The population in the class *N(t)* decreases with respect to time *t* and after a certain period of time it becomes unstable.
- The population in the class *P(t)* increases with respect to time *t* till a small amount of time. After that it decreases with respect to time *t* and after another certain period of time it becomes unstable.
- The population in the class *D(t)* increases with respect to time *t* till a certain period of time. After that it started to decrease and then after another certain period of time becomes unstable.
- The population in the class *S(t)* continuously grows higher.
- The population in the class *I(t)* decreases towards zero with respect to time *t*.
- The population in the class *J(t)* decreases towards zero with respect to time *t*.
- The population in the class *R(t)* becomes unstable with respect to time *t*.

We see that, when there are no interventions the control of poverty, addiction and snatching becomes impossible. The existence of government and non-government approach of interventions is a must while controlling poverty, drug addiction and snatching. When there are interventions (taking intervention parameters as our estimation in Table 4), we have plotted the following graphs in MATLAB:

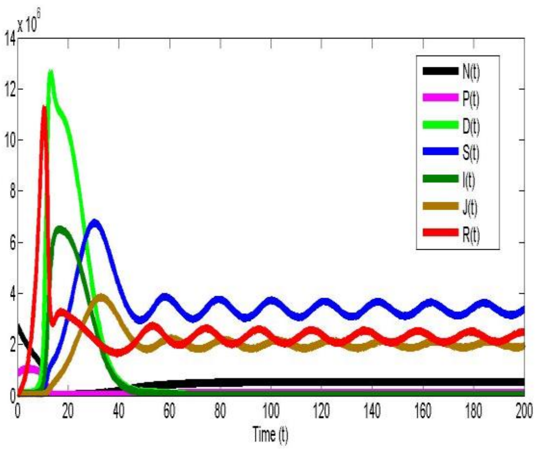

Fig. 3: Situations of the different population classes with respect to *t* with interventions for PDS model

The plots in the Fig. 3 show that, when there are existence of interventions, the population in the class *N(t)* decreases with respect to time *t* till a certain period of time. After that it started to increase again and after another small period of time it becomes constant. The population in the class *P(t)* increases with respect to time *t* till a small period of time. After that it started to decrease again and after another small period of time it increases to become constant. The population in the classes *D(t)* and *I(t)* increase with respect to time *t* from zero till a small period of time and after that it started to decrease towards zero. The population in the classes *S(t)*, *J(t)* and *R(t)* almost become oscillatory in some region after an increase and another small decrease.

So we can conclude that, poverty, addiction and snatching can be reduced to a controlled and expectable situation when government and non-government interventions exist.

For high and low interventions, taking intervention parameters as $\gamma = 0.40,\ \ \eta = 0.28,\ \ \rho = 0.15$ (for high intervention) and $\gamma = 0.34,\ \ \eta = 0.18,\ \ \rho = 0.09$ (for low intervention) we have plotted the following graphs in MATLAB:

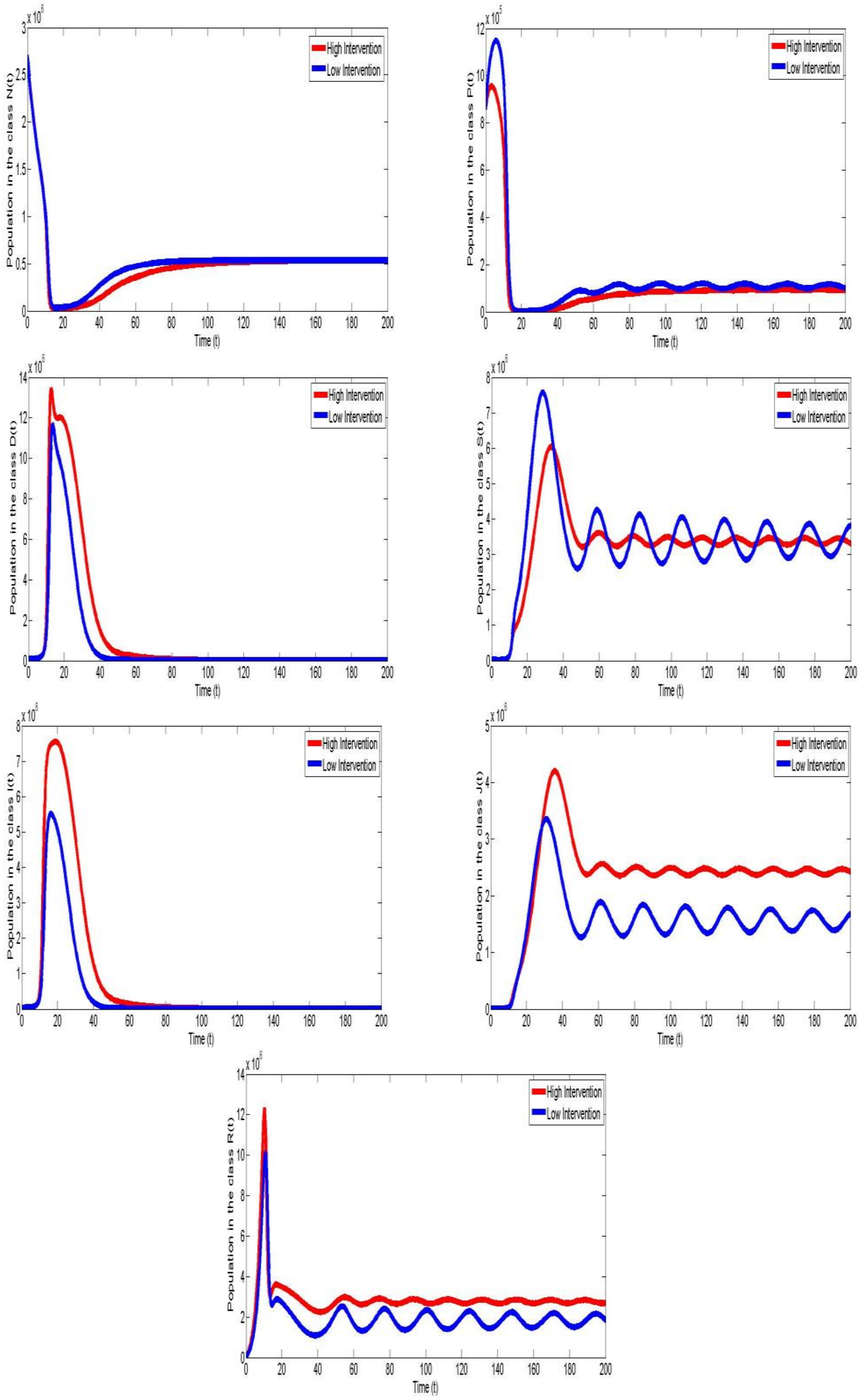

Fig. 4 : Situations of the different population classes with respect to *t* with high vs low interventions for PDS model

The plots in the Fig. 4 show the effects on the population classes with respect to time *t* for high and low approach of government and non-government interventions. The population in the class *N(t)* and *P(t)* becomes constant in a region more quicker for the high rate of interventions than the low rate. The population in the classes *S(t)*, *J(t)* and *R(t)* becomes almost become oscillatory in some region after the reduction of snatchers. High intervention rate gives the best graphs for the situations rather than the low rate. Although the population in the classes *D(t)* and *I(t)* becomes zero faster for low interventions, but the situation of the classes *P(t)* and *S(t)* shows better graphs for high interventions. That's why for overall control over the poverty, addiction and snatching can be achieved faster for high rate of interventions.

So we can say that, the high rate approach of government and non-government interventions will reduce poverty, addiction and snatching faster towards barest minimum.By intervention for controlling the poverty we mean policies, strategies and programmes like:- skill development, soft loan, job opportunities, educational opportunities, entrepreneurship opportunities, self employment, raising awareness, food for work, good governance etc. which are resulting empowerment of the poverty class. By intervention for controlling the addiction we mean policies, strategies and programmes which are resulting reduction of drug addiction such as:- raising awareness, rehabilitate the addicts, increase of rehabilitation centres, counselling, controlling the availability of drugs, financial assistance, raising religious values, strictness of law and order, free treatment etc. On the other hand, by intervention for controlling the snatching we indicate the policies, strategies and programmes which are resulting reduction of snatching including raising awareness, jail as reformatory, lock-up as reformatory, counselling, fine, raising religious values, strictness of law and order etc. Our research deals with all the types of interventions towards the people of Sylhet of all genders. We also think that, the high intervention policies, strategies and programmes should be closely monitored for effectiveness if poverty, addiction and snatching are to be reduced to their barest minimum.

VI. Conclusion

Modern epidemiologists like to model the dynamics of practical and social problems. In this paper, we have provided a useful but simple compartmental mathematical model (can be named as PDS model) that helps us to understand the dynamics of some modern day crucial problems:- poverty, addiction and snatching in Sylhet, Bangladesh. We introduced compartments that focus on recovering addicts and snatchers from all genders. The model introduced in this paper, is obviously more up-to-date than other models related to poverty and crime. The eradication and control of poverty, addiction and snatching should be the focus of good governance as can be seen in the result of the model. We established in this model that the addiction and snatching free equilibrium is locally asymptotically stable when the reproduction number $\Re < 1$ and unstable when $\Re > 1$. Numerical simulations of the system have been presented to show the variations of the population in different situations. Also we find out that, the high rate of government and non-government interventions will reduce poverty, addiction and snatching faster towards their barest minimum. Data that have been used for the simulations are based on Sylhet, Bangladesh. We believe that, our approximations are actually correct enough to present the PDS model accurately and the model is useful to study the dynamics of poverty, drug addiction and snatching not only in Sylhet, but also across the country and even all over the world. The eradication of those modern-day problems from our society is now a great challenge. We believe the challenge can be overcome by help the study in this paper. In further research we will try to involve other probable facts like:- drug smuggling, Drug Induced Diseases (DIDs) infections, reformation programmes by NGOs, beating by public etc. to make the model more attractive and useful.

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