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The Earth's Gravity Field and Ocean Dynamics

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ABSTRACT

The spectrum of ocean dynamics lends itself to convenient subdivision into two components in the context of satellite remote sensing. The first is the quasi-stationary constituent, while all features which vary with time during the period of data acquisition, comprise the second. The precision achieved in each case, must be at least ±10 cm through wavelengths of interest.

Data collected at the Earth's surface — gravity, sea surface heights — cannot play a role in the determination of the global gravity field to better than ±0.3 mGal (or ±1 m) unless assumptions are made about the global characteristics of the sea surface topography (SST) — a proposition which is untenable.

Satellite-determined gravity fields are the only source of data on the gravity field which is potentially uncontaminated by data referenced to the ocean surface. However, the resolution obtained is correlated with the noise level of the tracking and through wavelengths which are a function of spacecraft height.

The major problems to be overcome at the present time are the improvement of the precision of the GEM models to the desired levels and extending the resolution of the models to shorter wavelengths of practical significance for ocean dynamic modelling (at least 500 km). An appropriately configured GRAVSAT is one possible means for obtaining the necessary information with wavelengths between 500 km and 2000 km.

The definition of time variations in SST is needed for the synoptic modelling of ocean dynamics. A knowledge of the gravity field is required in this context only for recovering the radial component of altimeter spacecraft position globally to at least ±10 cm as the non-tidal variation of geoid heights with time does not exceed ±5 cm. The gravity field model requirements in this case are less exacting in that constituents with wavelengths shorter than those affecting radial components of orbital position with magnitudes less than the desired precision, need not be known. The effect of the permanent Earth tide on determinations of the quasi-stationary components of the spectrum are also examined.

*On leave of absence from the University of New South Wales, Sydney, Australia.

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THE EARTH'S GRAVITY FIELD AND OCEAN DYNAMICS

1. GRAVITY ANOMALIES AND THE GRAVITY FIELD

In the predictable pursuit for intellectual elegance, physical geodesists have sought to formulate solutions of the geodetic boundary value problem in terms of surface integrals. Practical considerations have constrained the use of such integrals in numerical evaluations. Prior to 1957, it was hoped to solve these integrals from surface gravity data (e.g., Molodenskii, et al., 1962; Heiskanen and Moritz 1967). No serious attempt was made to assess the level of precision attainable even if a global coverage of data were available. Except for a handful of absolute determinations, most gravity values (g) are established using differential determinations. The resulting surface gravity values can be compiled in relation to a world-wide gravity standardization network (IGSN 71) which was established in 1971 with an estimated precision of ±0.2 mGal (Note: 1 Gal = 1 cm s^{-2}), as reported in (Morelli, et al., 1971). The internal precision of modern regional gravity networks is likely to be of equivalent magnitude (e.g., Mather, et al., 1976b).

The same cannot be said with confidence, of gravity anomalies, given by

$$\Delta g = g - \gamma - \frac{2\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma} - 2f\sin^2\phi_g + o\left\{f^2\Delta g\right\}\right). \qquad (1)$$

where ΔW is the difference in geopotential between the general point P (ellipsoidal coordinates ϕ_g, λ on a reference ellipsoid with equatorial radius a and flattening f) at the Earth's surface and the regional datum level surface and not the geoid. γ in Equation (1), is normal gravity computed on the equivalent equipotential ellipsoid rotating with angular velocity ω, using a value adopted for the product of the gravitational constant G and the mass of the Earth M, m being given by

$$m = a^3\omega^2/GM. \qquad (2)$$

If a data bank of gravity anomalies is based on IGSN 71, the precision of the gravity anomaly field in the context of surface integral evaluations, is influenced primarily by the errors in ΔW, established as a network. It has been shown (Mather 1974, p. 102) that the quality of height anomalies ζ' estimated from gravity anomalies using relations of the form

$$\zeta' = K\iint f(\psi)\,\Phi(\Delta g, \Delta W)\,d\sigma, \qquad (3)$$

where Δg is the gravity anomaly and ΔW the difference of geopotential from the global datum level surface at the element of surface area $d\sigma$ which is at a geocentric angular distance ψ from the point of computation, is a function of the wavelength of the errors in the global gravity anomaly data set. Satisfactory results are obtained in practice only when the errors in such a data set decrease rapidly as a function of wavelength. For example, in the case of the gravity anomaly data set for Australia - AUSGAD 76, the long wave error sources are assessed as being the following (Mather, et al., 1976b, p. 79, et seq.):

a. Errors with amplitude 0.15 mGal and wavelength 7000 due to residual errors in the adjusted Australian levelling survey.

b. A constant error of ±0.06 mGal due to the gravity value adopted for the National Base Station at Sydney not being correct.

c. Errors with amplitude 0.2 mGal and wavelength of 7000 km due to residual errors in the Australian National Gravity Network.

A fourth significant source of error when such data banks are used in global solutions, is the effect of the adopted datum level surface not necessarily coinciding with the geoid to better than ±1 m causing systematic effects of ±0.3 mGal in the entire gravity anomaly data bank computed on this datum. This statement presumes that a world-wide definition has been adopted for the geoid with a resolution to at least ±10 cm, as discussed in (Mather 1977, Sec. 1).

The lack of a global coverage of surface gravity data (e.g., Rapp 1977, p. 3) and the questionable quality of oceanic coverage which is widely conceded as being up to an order of magnitude inferior to land based data, result in the use of combination solutions for the geoid. The input data for such solutions are the following:

a. Satellite determined harmonic coefficients $C'_{\alpha nm}$ of the gravity field to some degree n' ($\doteqdot 20$).

b. Several regional gravity field determinations with representation on, say, a 10 km grid - freely available in Europe, North America and Australia; providing surface gravity field coverage within 20° of the point of computation in selected parts of the regions mentioned.

The computation can then be carried out using the truncation function technique first proposed by Molodenskii (Molodenskii, et al., 1962, p. 146) using a relation of the form

$$\zeta' = \zeta_r + K' \sum_{n=0}^{n'} Q_n(\psi_0)\Delta g_n + K \iint_{\psi \leq \psi_0} f(\psi)\, \Phi(\Delta g, \Delta W)\, d\sigma , \qquad (4)$$

where Δg_n is the n-th degree harmonic in Δg, ψ_0 is the outer radius of the cap over which surface integration is undertaken, ζ_r being the residual contribution due to truncating the outer zone effect to representation in terms of a finite set of harmonics to degree n'. Several variants of this method are known (e.g., Mather 1968; Marsh and Chang 1976).

In the past, the ultimate test of the accuracy of such solutions has been obtained on comparison with astro-geodetic determinations. The comparisons should be performed between independent estimates of the height above ellipsoid. In practice, Stokesian solutions are compared against so-called astro-geodetic geoids which can have distortions of up to 2 m in mountainous country. The resulting discrepancies over continental extents have root mean square (rms) values of up to $\pm 1\frac{1}{2}$ m, after allowing for datum transformation (e.g., Mather 1970; Mather 1975a). Obviously, effects which are linearly variant over the region of comparison are not reflected in the rms residuals.

The limited precision of astro-geodetic determinations due to the high cost of such surveys and the local fluctuations in the grades of level surfaces, precludes any possibility of making more exacting tests on the quality of gravimetric determinations by these methods.

2. THE ROLE OF THE GRAVITY FIELD IN OCEAN SURFACE DYNAMICS

A complete treatment of this problem is given in (Mather 1978a). It can be summarized as follows: The dynamics of the surface layer of the oceans are defined by the differential equations

$$\ddot{x}_1 - f\dot{x}_2 = -g\frac{\partial\zeta_s}{\partial x_1} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x_1} + F_1 + o\left\{f\ddot{x}_1\right\}, \tag{5}$$

and

$$\ddot{x}_2 + f\dot{x}_1 = -g\frac{\partial\zeta_s}{\partial x_2} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x_2} + F_2 + o\left\{f\ddot{x}_2\right\}, \tag{6}$$

where f is the Coriolis parameter, given by

$$f = 2\omega\sin\phi , \tag{7}$$

p_a is the atmospheric pressure, $(\ddot{x}_1, \ddot{x}_2)$, $(\dot{x}_1, \dot{x}_2)$ and (F_1, F_2) are components of accelerations, velocities and frictional forces acting on the surface layer of the oceans along the x_1 and x_2 axes oriented east and north respectively in the local horizon.

The major non-equatorial quasi-stationary currents (e.g., the Gulf Stream, the Kuroshio) have velocities in excess of 10^2 cm s^{-1} maintained by gradients of the quasi-stationary dynamic sea surface topography (SST) ζ_s, defined as the height of the sea surface above the geoid. In mid-latitudes, a steady state current of 1 cm s^{-1} is maintained by a quasi-stationary SST gradient of 1.05 cm per 10^2 km, the current being deflected in the direction of the SST contours. The wind velocities and/or atmospheric pressure gradients needed to maintain currents like the Gulf Stream are at least an order of magnitude larger than the strongest measured under extreme conditions at the surface of the Earth as summarized in Figures 1 and 2.

The spectrum of SST is illustrated in Figure 3. The quasi-stationary component in mid-latitude regions where fast flowing steady state currents occur, can be expected to be 4-5 times larger than the time varying constituents, the overall magnitude of ζ_s not exceeding ±1½m.

Remote sensing techniques provide the only plausible means of synoptically monitoring the dynamics of the surface layer of the oceans. An indirect method of doing so can be developed from an analysis of infrared imagery. The primary difficulty is the interpretation of relative measurements to provide an absolute scale. A second limitation is imposed by cloud cover. The greatest apparent strength of infrared imagery is in tracking eddies. However, the temperature structure within an eddy is complex and does not lend itself to straightforward interpretation (e.g., Cheney and Richardson 1976, p. 145), especially in shallow seas.

The radar altimeter provides direct estimates of the position (and hence the height) of the instantaneous sea surface and is unaffected by cloud cover, though refraction corrections to the measured range may be more uncertain under such conditions. If the measurement were reduced to a height (ζ') above a reference surface and if the height of the geoid above this same surface were N, it follows that

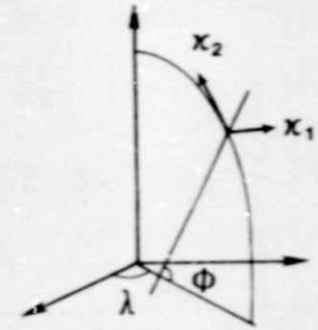
$$\zeta_s = \zeta' - N \tag{8}$$

The role of the gravity field in ocean dynamics modelling is somewhat different from that in the solution of the geodetic boundary value problem. In the latter case, the objective was the geometrical mapping of a surface at which the measurements were made. In the ocean dynamics application, the shape of the

$$\ddot{x}_1 - f\dot{x}_2 = -g\frac{\partial \zeta_S}{\partial x_1} - \frac{1}{\rho_W}\frac{\partial P_a}{\partial x_1} + F_1$$

$$\ddot{x}_2 + f\dot{x}_1 = -g\frac{\partial \zeta_S}{\partial x_2} - \frac{1}{\rho_W}\frac{\partial P_a}{\partial x_2} + F_2$$

ACCELERATION — CORIOLIS PARAMETER — VELOCITY — SST GRADIENT — ATMOSPHERIC PRESSURE GRADIENT — FRICTIONAL FORCES



$$f = 2\omega \,\mathrm{SIN}\Phi = 0\ \{1.46\times10^{-4}\ \mathrm{SIN}\Phi\ S^{-1}\}$$

ORDERS OF MAGNITUDE

TYPE OF CURRENT	$\dot{x}_a$ cm s^{-1}	$\partial\zeta_S/\partial x_a$ ARCSEC	cm PER 10 km
FAST	10^2	3	15
MEDIUM	10^1	0.3	1.5
SLOW	10^0	0.03	0.15

DESIRED RESOLUTION

0.15 cm PER 10 km

Figure 1. Equations of Motion at Ocean Surface

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FRICTIONAL FORCE

$$F_a = \frac{K[\,w_1{}^2 + w_2{}^2\,]^{1/2}\, w_a}{\rho_w H}$$

K = EMPIRICALLY OBTAINED "CONSTANT"

$2 \times 10^{-6} \leqslant K \leqslant 2.6 \times 10^{-6}$ FOR F_a IN cm s^{-2}

H = THICKNESS OF MIXED (EKMAN) LAYER ($H < 10^2$ m)

W_a = WIND SPEED COMPONENTS $0 \leqslant W_a \leqslant 50\ ms^{-1}$ (10^2 mph)

EQUIVALENT MAGNITUDES

WIND SPEED		F_a (cm s^{-2})			H = 100 km	
ms^{-1}	mph	H = 10 m	H = 50 m	H = 100 m	EQUIVALENT CURRENT SPEED cm s^{-1}	EQUIVALENT SST GRADE cm PER 100 km
1	0.2	2×10^{-5}	$\frac{1}{2} \times 10^{-5}$	2×10^{-6}	0.02	0.02
5	10	5×10^{-4}	10^{-4}	5×10^{-5}	0.3	½
10	20	2×10^{-3}	$\frac{1}{2} \times 10^{-3}$	2×10^{-4}	2	2
50	100	5×10^{-2}	10^{-3}	5×10^{-3}	30	45

NOTE: PRECISION GOALS FOR SEASAT SCATTEROMETER: 2ms^{-1} $\pm 20^\circ$
RESULTING ERROR IN $F_a = 0\{4 \times 10^{-5}$ cm $s^{-2}\}$ FOR 10 m s^{-1} WIND SPEED

Figure 2. Effect of Wind

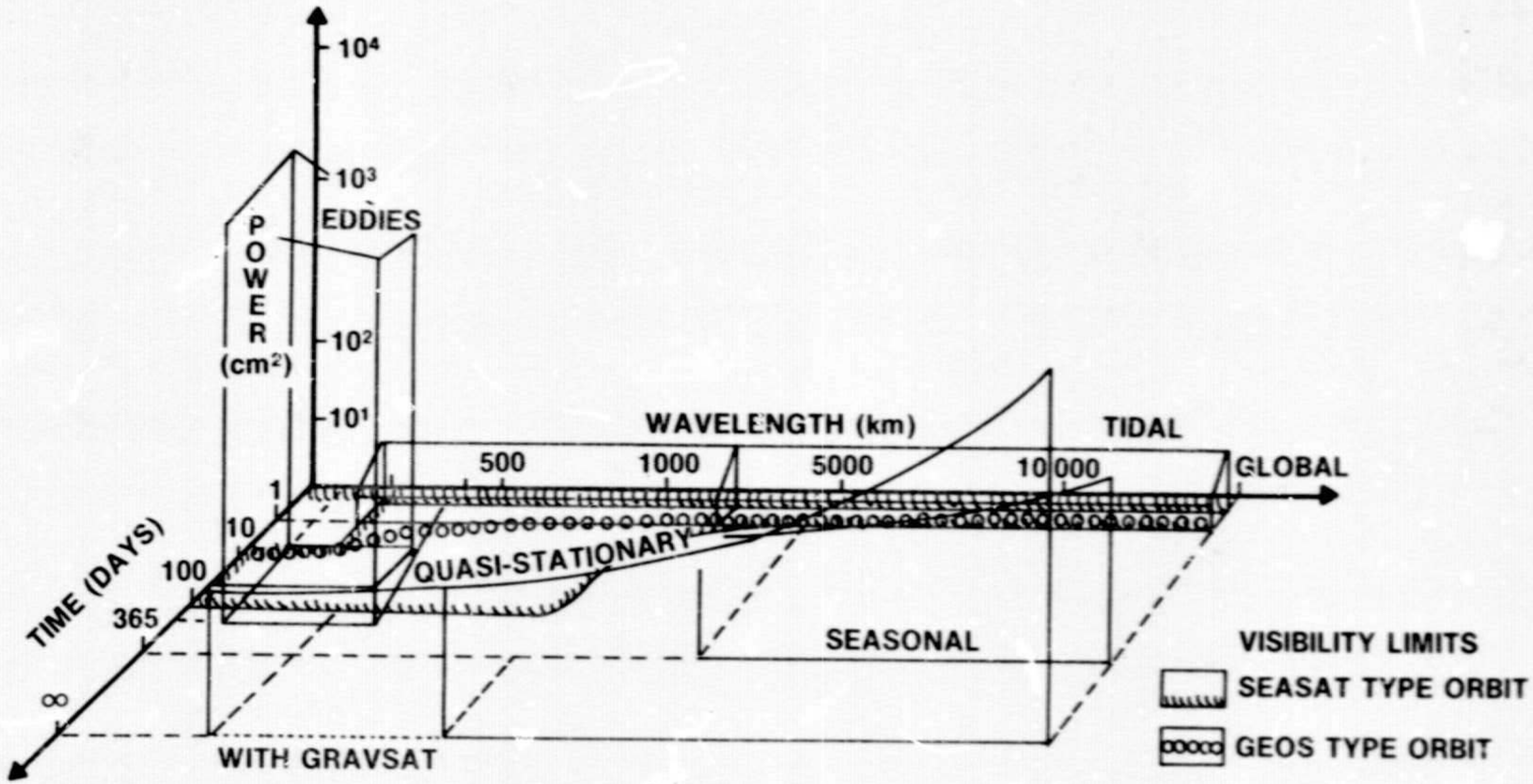

Figure 3. Guestimates of the Space — Time Spectrum of Sea Surface Topography Determinable by Satellite Techniques

bounding surface is known. It is required to geometrically map (in concept) a level surface in ocean areas under circumstances where no measurements have been directly made in relation to it.

Consequently, attempts to find a means for determining SST from a solution of the boundary value problem, without making assumptions about the nature of the SST, have not been successful (Mather, et al., 1976a). In formulating a method for determining ζ_s from satellite altimetry, it is not considered desirable to assume characteristics for the global distribution of ζ_s in processing the data prior to obtaining a solution. Consequently, the only sources of data on the Earth's gravity field which are independent of any relationship to the geometry of the sea surface, are satellite orbital analysis and satellite-to-satellite tracking (e.g., Vonbun, et al., 1977). No reasons exist at present for assuming that gravity field models deduced from a global network of tracking stations to x parts in 10^8 will not achieve a resolution of x parts in 10^8 when transformed into long-wave components of geoid heights through wavelengths which are a function of:

- satellite flying height;
- the network of higher satellites used in satellite-to-satellite tracking; and
- the average distance between stations in the tracking station network,

as can be seen from a study of crossovers of GEOS-3 data (Mather, et al., 1978b, Table 2).

The iteration between purely satellite-determined gravity field models like the odd-numbered Goddard Earth Models - e.g., GEM 9 (Lerch, et al., 1977) and dynamic solutions using a more widespread network of reliable laser ranging systems, can be expected to provide improved estimates of the low degree harmonics $C'_{\alpha nm}$ of the global gravity field through some degree n' (equivalent wavelength of approximately 10^3 km) using global networks of ±10 cm tracking systems. The possibility exists that the minimum wavelength resolved could be decreased to about 500 km if data from satellite-to-satellite tracking were included in the solution, using data collected during a mission of the GRAVSAT type, provided the data was of sufficient precision.

Solution techniques for the recovery of SST from radar altimetry data must reflect the band limited nature of the geoid height signal from satellite determined gravity field models.

The role of purely geodetic techniques in remote sensing ocean dynamics is primarily that of providing values of ζ_s from the radar altimetry data. The significance of this technique lies in its potential to synoptically monitor changes in the shape of the sea surface. The geodetic results have the potential to define a four dimensional frame of reference for sub-surface ocean dynamics, in addition to providing information for modelling the dynamics of the surface layer of the oceans.

3. REMOTE SENSING ζ_s FROM RADAR ALTIMETRY

As discussed by Mather, et al. (1976a), it is not possible to use the boundary value problem approach in determining SST (ζ_s) from sea surface heights ζ' above the reference figure because no data can be unambiguously related to the geoid at the desired level of precision without making unwarranted assumptions about the magnitude and distribution of the SST.

The spectrum of ocean surface dynamics lends itself to convenient sub-division into two components in the context of remote sensing. The first is quasi-stationary in time (ζ_{so}) during the period of data acquisition. The time varying component ζ_{st} is expected to have a magnitude which is about one fifth that of ζ_{so} for reasons given in Figures 1 and 2. Some dominant contributions to ζ_{so} have been recovered from GEOS-3 altimetry and have substantially the same magnitude as obtained from oceanographic surveys. The synoptic variations in the SST are likely to have periods of greater than 2 months through wavelengths greater than 10^3 km.

Consequently, the precision required in monitoring the synoptic variations in ζ_s is almost an order of magnitude greater than that needed to establish a gross model of quasi-stationary SST. Furthermore, there is no necessity for maps of hydrostatically determined quasi-stationary SST (e.g., Levitus and Oort 1977, p. 1283) to agree with satellite determined models to better than ±20 cm as the former are, in essence, averages over long periods of time while the latter represents neo-synoptic monitoring of the phenomenon.

The satellite altimetry data from either of the altimeter-equipped spacecraft GEOS-3 or SEASAT-A, due for launch in 1978, are in the form of profiles. In the former case, the profiles seldom exceed 20 minutes in time due to the absence of on-board recording facilities. SEASAT-A will sweep out a 25° grid every day (Fig. 4). No data is collected outside certain bounding parallels (65° in the case of GEOS-3, 72° in the case of SEASAT-A). The orbital periods of the two satellites are different. GEOS-3 has an orbital period of 101.79 min, which results in a daily offset of 1250 km, an n° x n° grid being generated every 25/n days. The SEASAT-A orbit is planned so that the daily offset is approximately 20 km with the groundtracks repeating themselves every 4 months or so.

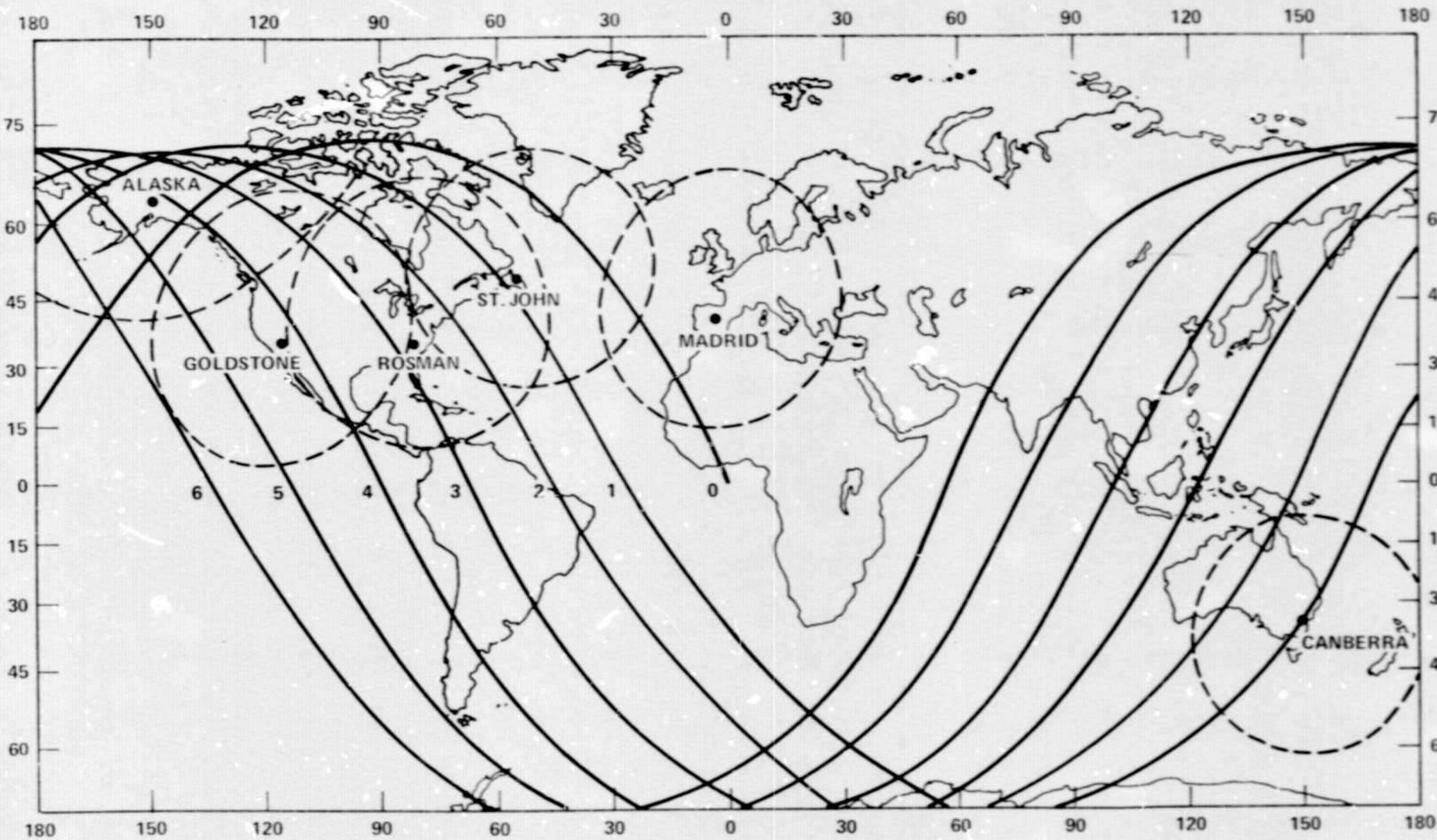

Figure 4. The Ground Track of SEASAT-A With Ground Station Coverage

Consequently, as illustrated in Figure 3, SEASAT-A will provide little information on synoptic variations in ocean circulation with wavelengths between 10^3 km and 2500 km and periods less than 4 months - a potential drawback when attempting to model the energy transfer between the variable wind fields and the surface layer of the oceans, assuming the former to operate with similar constraints in space and time.

4. BASIC RELATIONS

The following is summarized from (Mather 1978a). It is assumed that a harmonic representation of the gravity field of adequate precision is available to some degree n'. It has previously been assumed (e.g., Mather 1974, p. 90) that the desired precision in each coefficient $C'_{\alpha nm}$ defining the gravity field model was about 1 part in 10^8. Table 1 lists the normalized coefficients $\zeta_{s\alpha nm}$ ($n < 5$) in the surface spherical harmonic representation of the hydrostatically determined quasi-stationary SST based on data confined to the oceans lying between the parallels 65°S and 65°N (Mather, et al., 1978b, Sec. 7). Also listed are errors $e_{C\alpha nm}$ in the coefficients $C'_{\alpha nm}$ of GEM 9 (Lerch, et al., 1977, p. 52) with their linear equivalents. A study of the signal-to-noise ratio ($\zeta_{s\alpha nm}/e_{C\alpha nm}$) shows that conditions are favorable only for the recovery of the coefficients ζ_{s111}, ζ_{s120}, ζ_{s130} ζ_{s140} and possibly ζ_{s160}. Table 2 lists the root mean square error per degree in GEM 9, which indicates that there is no possibility of recovering any information on SST from present-day gravity field models with wavelengths less than 10^4 km. The desired precision in the gravity field model for quasi-stationary SST determinations is therefore 0.2 parts in 10^9 through wavelengths greater than 10^3 km, and hopefully, greater than 500 km if satellite-to-satellite tracking methods can provide the necessary precision.

The desired resolution of the gravity field model for synoptic monitoring SST variations, is at least five times more exacting, but only through wavelengths which cause perturbations of ±1 cm in the radial component of orbital position.

The geopotential W exterior to the Earth's atmosphere can be represented in geocentric spherical coordinates (R, ϕ, λ) by the relation

$$W = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C'_{\alpha nm}\, S_{\alpha nm}, \; n \neq 1 , \tag{9}$$

where $C'_{\alpha nm}$ are spherical harmonic coefficients of degree n and order m, $S_{\alpha nm}$ being surface spherical harmonic functions defined by

$$S_{1nm} = P_{nm}(\sin\phi)\cos m\lambda; \qquad S_{2nm} = P_{nm}(\sin\phi)\sin m\lambda . \tag{10}$$

Table 1

Factors Influencing Determinations of Quasi-Stationary Dynamic Sea Surface Topography From Satellite Altimetry — The Signal ($\zeta_{s\alpha nm}$), the Noise per Coefficient ($e_{C\alpha nm}$) for GEM 9, and the Signal-to-Noise Ratio ($\zeta_{s\alpha nm}/e_{C\alpha nm}$)

Degree	Order	The Signal ($\zeta_{s\alpha nm}$)* (cm)		GEM 9 Noise ($e_{C\alpha nm}$) (±kGal cm)		Signal-to-Noise $\lvert \zeta_{s\alpha nm}/e_{C\alpha nm} \rvert$	
		$\alpha = 1$	$\alpha = 2$	$\alpha = 1$	$\alpha = 2$	$\alpha = 1$	$\alpha = 2$
0	0	114.5	—	—	—	—	—
1	0	+6.9	—	—	—	—	—
1	1	-21.8	+2.4	—	—		—
2	0	-46.2	—	0.4	—	115.5	—
2	1	-4.0	+4.4	1.7	1.6	2.4	2.8
2	2	-0.7	-0.2	2.2	2.2	0.3	0.1
3	0	+6.7	—	1.0	—	6.7	—
3	1	-4.1	-5.3	3.4	3.3	1.2	1.6
3	2	-0.7	-2.4	5.1	4.4	0.1	0.6
3	3	-3.0	+1.4	6.1	5.9	0.5	0.2
4	0	-9.5	—	0.8	—	11.9	—
4	1	+2.1	+2.8	3.2	3.0	0.7	0.9
4	2	-0.5	+1.2	3.0	3.1	0.2	0.4
4	3	+1.2	-0.2	2.9	2.7	0.3	0.1
4	4	-1.8	-0.9	4.0	4.0	0.5	0.2
5	0	+0.9	—	1.0	—	0.9	—
5	1	-3.7	-0.8	4.1	4.1	0.9	0.2
5	2	-0.6	+2.4	6.1	5.9	0.1	0.4
5	3	+0.6	+0.2	5.7	6.1	0.1	0.0
5	4	-0.3	+1.1	5.6	5.8	0.1	0.2
5	5	-0.3	-0.8	9.0	9.0	0.0	0.1
6	0	+4.4	—	1.2	—	3.7	—
7	0	-0.5	—	1.4	—	0.4	—
8	0	-0.8	—	1.3	—	0.6	—

*Based on an analysis in ocean areas only between 65°S and 65°N

Table 2

Degree Variances

Degree (n)	Sea Surface Topography* (cm^2)	GEM 9 Error** $(kGal\ cm)^2$
1	528.6	—
2	2170.3	15
3	107.0	138
4	109.7	83
5	23.7	391
6	31.1	266
7	3.9	865
8	4.5	619
9	2.7	1435
10	0.7	1193
11	0.4	2415
12	0.1	1815
13	0.1	2452
14	0.0(4)	2049
15	0.0(7)	2575
16	0.0(1)	2248

*Based on a constrained solution to (16,16) from 5° x 5° area means in ocean areas only fit to data = ±9 cm.

**Based on comparisons with surface gravity (Lerch, et al., 1977). Comparisons with altimetry are a factor of two better on the average.

The quantity a in this development, is the equatorial radius of the ellipsoid of revolution which best fits mean sea level for the epoch of the altimetry. This is obtained by analyzing the heights of the sea surface ζ' in relation to an adopted reference surface in which the ellipsoidal radius (Mather 1974, p. 91, et seq.) is a_0. The change da in a is obtained from (Appendix, Equation (A-12)) by minimizing the residuals

$$v = \zeta' + a\, df \sin^2\phi - da - \sum_{m=0}^{1} \sum_{\alpha=1}^{2} \zeta_{s\alpha 1 m} S_{\alpha 1 m}, \qquad (11)$$

df being the change in the flattening between the sea surface and the value of f obtained from C'_{120}. The question of permanent Earth tide effects on C'_{120} is discussed in Section 10.4. The final set of terms in Equation (11) allows for the first degree harmonic in the quasi-stationary SST.

The value of da obtained in practice (e.g., Mather, et al., 1978b, Sec. 7) is based on a sample which is banded in latitude due to the distribution of the altimetry. The resulting ellipsoid of "best fit" will not be representative of the global oceans. This highlights the difficulty of defining the equatorial radius of an ellipsoid which "best fits the geoid globally." There is no way of sampling the geoid globally. Land areas could be introduced into the definition by using the geopotential differences between tracking stations and the regional mean sea level (MSL) datum. The adoption of such a procedure will no longer define an ellipsoid which best fits the geoid in ocean regions (Mather 1977, Sec. 1). Neither will it provide the additional coverage needed in higher latitudes in the short term.

The approach adopted to date in analyzing GEOS-3 data (Mather, et al., 1978a, Sec. 6.1) for the selection of a level surface as the geoid is the following. The oceans are treated as lying entirely within parallels 65°S and 65°N. All inland seas like the Great Lakes and the Caspian are ignored when sampling ζ' using Equation (11). The resulting level surface is defined in terms of the potential W_0 of the geoid consistent with the value of the product of the gravitational constant G and the mass of the Earth M included in the adopted gravity field model.

The geopotential W in Equation (9) cannot be downward continued through the atmosphere using a spherical harmonic model. If the atmospheric potential V_s is defined at all points exterior to the geocentric sphere of radius R_a enclosing the Earth's atmosphere, by a relation of the form

$$V_s = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R_a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} V_{s\alpha nm} S_{\alpha nm}, \qquad (12)$$

it is possible to define the potential W_e of the solid Earth and oceans but excluding the atmosphere according to the relation

$$W_e = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C''_{\alpha nm} S_{\alpha nm} + U_r , \quad (13)$$

where U_r if the rotational potential and

$$C''_{\alpha nm} = C'_{\alpha nm} - \left(\frac{R_a}{a}\right)^n V_{s\alpha nm} . \quad (14)$$

W_e satisfies Laplace's equation in the space exterior to the Earth's surface (air/sea interface at sea).

It is now possible to consider the disturbing potential T'' in relation to a higher reference model whose potential U in space exterior to the Earth's surface is given by (Mather 1974, p. 91, et seq.)

$$U = \frac{GM}{R} \sum_{n=0}^{n'} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C'_{\alpha nm} S_{\alpha nm} + U_r . \quad (15)$$

Note that the potential W_e will contain non-zero first degree harmonics if the origin of coordinates (R, ϕ, λ) remains at the geocenter. It has been shown that enforcing the condition

$$C''_{110} = C''_{111} = C''_{211} = 0 \quad (16)$$

will introduce errors less than ±5 cm due to the non-coincidence of coordinate origins (Anderson, et al., 1975, p. 33). The height anomaly ζ' on this higher reference model has a magnitude which is one order smaller than the value ζ in relation to a rotating equipotential ellipsoidal reference model, being given in ocean areas by the relationship

$$\zeta' = \zeta - \frac{GM}{R_o \gamma} \sum_{n=2}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \overline{C}_{\alpha nm} S_{\alpha nm} , \quad (17)$$

where

$$\overline{C}_{\alpha nm} = C'_{\alpha nm} \text{ except when } \alpha = 1, m = 0 \text{ and } n = 2, 4 \text{ and } 6. \quad (18)$$

In these three special cases,

$$\overline{C}_{120} = 0, \tag{19}$$

and $\overline{C}_{140}$, $\overline{C}_{160}$ have been corrected for the effect of the ellipsoidal flattening implied in the value of C'_{120} by the relations (Appendix, Equations (A-26))

$$\overline{C}_{140} = C'_{140} - \frac{\sin^4\alpha}{5}\left[1 - \frac{4}{21}\,\frac{m\sin\alpha}{q_2(\alpha)}\right], \tag{20}$$

and

$$\overline{C}_{160} = C'_{160} - \frac{\sin^6\alpha}{7}\left[\frac{2}{9}\,\frac{m\sin\alpha}{q_2(\alpha)} - 1\right], \tag{21}$$

the relationship between C'_{120}, f, α and $q_2(\alpha)$ being given in the Appendix. γ, for all practical purposes, is normal gravity on the implied level ellipsoid implied in the higher reference model. ζ' is related to the disturbing potential T'' in relation to the higher reference model by the relation (Mather 1975, p. 72); Mather 1978a, Equation (19))

$$\begin{aligned} T'' = W_e - U &= \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} C_{\alpha nm}\,S_{\alpha nm} \\ &= (W_0 - U_0) + \gamma\zeta' - \gamma\zeta_s - V, \end{aligned} \tag{22}$$

where W_0 is the potential of the geoid which is determinable from satellite altimetry consistent with the higher reference model (Mather, et al., 1978a, Sec. 6.1), U_0 is the potential on the surface of the equipotential ellipsoid implied in the higher reference system and V is the potential of the atmosphere. The first expression holds in space exterior to the Earth's surface while the second holds at the surface of the Earth. Equations are also given in (Mather 1978a) for the gravity anomaly and solutions of the geodetic boundary value problem which take into account the unknown quasi-stationary SST, the complete zero degree effect and conditions of continuity of the geopotential in the space exterior to the Earth given the existence of the atmosphere.

5. PRACTICAL SOLUTIONS

Numerical results from GEOS-3 data have been reported by Mather, et al., (1978b) where selected low degree harmonics of the quasi-stationary SST have been estimated from a global analysis of the available GEOS-3 altimetry. The following assumptions are made when using Equation (22) in practice.

- The low degree harmonics of the gravity field are known to some degree n'' with an error less than $\pm\sigma_0(=10^{-8}$ in the GEOS-3 study).

Thus

$$C_{\alpha nm} = C''_{\alpha nm} - \overline{C}_{\alpha nm} , \tag{23}$$

where $C''_{\alpha nm}$ and $\overline{C}_{\alpha nm}$ are defiend in Equations (14) and (18) respectively. On adopting surface harmonic models for ζ_s(coefficients $\zeta_{s\alpha nm}$), V (coefficients $V_{g\alpha nm}$) and ζ' (coefficients $\zeta'_{\alpha nm}$), it follows that

$$\zeta_{s\alpha nm} = \zeta'_{\alpha nm} - \left[\frac{GM}{R_0}\left(\frac{a}{R_0}\right)^n [C''_{\alpha nm} - \overline{C}_{\alpha nm}] + V_{g\alpha nm},\right] \tag{24}$$

R_0 being the geocentric distance to the sea surface.

It has been shown (Mather, et al., 1978a, Sec. 6.1) that for all practical purposes, the net effect of the terms within parentheses should be zero if the value of $C'_{\alpha nm}$ used in forming both $C''_{\alpha nm}$ and $\overline{C}_{\alpha nm}$ were free from error. In such a case,

$$\zeta_{s\alpha nm} = \zeta'_{\alpha nm} . \tag{25}$$

Numerical solutions are not so straightforward due to the poor signal-to-noise. This problem is dealt with in depth in (Mather, et al., 1978b, Sec. 7) as are the procedures for modelling the SST (ibid., Sec. 8). As discussed in Section 4, it is judged that only five coefficients can be recovered from a perfect determination of ζ' and the GEM 9 gravity field model. Other coefficients can be estimated but the level of uncertainty is much greater, being a function of

- the uncertainty in the value of the GEM 9 coefficient; and

- the magnitude of the coefficient $\zeta_{s\alpha nm}$ (Tables 1 and 2).

Table 3 lists provisional estimates for the coefficients $\zeta_{s\alpha nm}$ obtained from the analysis of GEOS-3 altimeter data between parallels 65°S and 65°N and the estimated errors. For details of the method of analysis, see (ibid.). These values are based on the following set of constants:

Table 3

Preliminary Estimates of the Dominant Features of Quasi-Stationary Dynamic Sea Surface Topography from the 1977 GEOS-3 Altimeter Data Bank and Goddard Earth Model (GEM 9)

(For Method of Selection, See Table 1)

Coefficient	Normalized Value (cm)*	
	Oceanographic From Steric Anomalies	Geodetic From Altimetry and GEM 9
110	+6.9	-150 ± 15**
111	-21.8	13 ± 15**
211	2.4	20 ± 15**
120	-46.2	-43 ± 6
130	6.7	+7 ± 10
140	-9.5	-18 ± 15
160	4.4	1 ± 15

*Based on an analysis banded between 65°S and 65°N

**GEM 9 does not include a first degree harmonic, system origin being coincident with geocenter. These numbers also include the displacement of the estimated location of the geocenter in GEM 9 tracking station coordinates from the center of the ellipsoid which best fits the sea surface.

$$c = 2.997\,924\,58 \times 10^{10}\,\mathrm{cm\,s^{-1}}$$

$$GM = 3.986\,004\,7 \times 10^{20}\,\mathrm{cm^3\,s^{-2}}, \qquad (26)$$

and the geometrical calibration of the GEOS-3 altimeter performed by Martin and Butler (1977), using the Equinox Data Set (Mather, et. al., 1978b, Sec. 4).

The geopotential of the geoid for the epoch 1976.0 from the GEOS-3 altimetry is

$$W_0 = 6{,}263{,}283.8 \pm 0.4\ \mathrm{kGal\ m}, \qquad (27)$$

The ellipsoid of revolution which best fits the sea surface is defined by the parameters

$$a = 6{,}378{,}139.9 \pm 0.4\ \mathrm{m}; \qquad f = 1/298.237 \pm 0.003, \qquad (28)$$

the latter being consistent with the value of -43 cm for the normalized value of ζ_{s120}. These provisional values are subject to revision with improvements in the orbital ephemeris of GEOS-3. The contribution of the permanent Earth tide has been eliminated as described in Section 10.4.

6. FUTURE TRENDS

(a) Gravity Model Improvement

Further progress in determining quasi-stationary SST is dependent on the resolution with which the global gravity field model is defined. This information must, in the final analysis, come from orbital analysis and satellite-to-satellite tracking as envisaged in Section 2. The greatest limiting factor at the present time is the absence of a global network of high precision tracking systems. In the short term, the satellite altimetry can play a role in gravity model improvement.

As some dominant harmonics of the quasi-stationary SST have been evaluated, it is possible to remove their effect from the data base of ζ' obtained from GEOS-3 altimetry between 65°S and 65°N, assuming that sufficient passes of altimetry are available to eliminate the effect of the tides, as well as mesoscale variations in the SST on the data. The objective is the evaluation of the coefficients of a gravity field model with a resolution of 1 part in 10^8 (i.e., 6 kGal cm) in each harmonic. The best available model at the present time (GEM 9) appears to satisfy this precision requirement to degree 4. Thus, holding these harmonics heavily constrained, it is possible to attempt a gravity model improvement using observation equations of the form (Mather, et al., 1976b, p. 42)

$$\frac{GM}{R_o} \sum_{n=2}^{n_m} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C_{\alpha nm} S_{\alpha nm} - \gamma \zeta_{sr} = v , \qquad (29)$$

on the GEOS-3 altimetry data bank, where

$$\zeta_{sr} = \zeta' - \sum_{n=0}^{\bar{n}} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} + \left[(W_o - U_o) - V\right] / \gamma , \qquad (30)$$

where $\bar{n}$ represents the maximum degree to which the coefficients of the SST are known to ±6 cm.

Further progress is possible only if it is assumed that the magnitude of higher degree coefficients of the quasi-stationary SST are significantly smaller than the errors in GEM 9. This refers to the set of coefficients $C'_{\alpha nm}$ comprising the current gravity field model which has to be updated and is used in defining the higher reference model (Equation (15)). Table 2 sets out the error degree variances in GEM 9 and the degree variances in the oceanographically determined SST to degree 16, showing that considerable scope exists for refining gravity field models from altimetry data corrected for the effect of the dominant features of the SST.

Assuming that the altimetry data bank were of adequate precision, it follows that the resulting gravity field model should also have a resolution of 1 part in 10^8 if there were no aliasing effects due to the data being restricted to ocean areas only (33,902 out of 64,800 equi-angular 1° x 1° squares) and banded in latitude between 65°S and 65°N. This aliasing problem is discussed at length in (Mather, et al., 1978b, Sec. 8).

The goal of complete areal representation can be improved by forming observation equations using gravity anomalies for land areas in the relation (ibid., p. 42)

$$\frac{GM}{R^2} \sum_{n=2}^{n_m} \left(\frac{a}{R}\right)^n (n-1) \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C_{\alpha nm} S_{\alpha nm} - \Delta g_d = v , \qquad (31)$$

where the spherical harmonic series is evaluated at the surface of the Earth, the value of observed gravity (g) is used to compute Δg_d, defined by

$$\Delta g_d = \Delta g - \delta\gamma + \delta g_a - \frac{1}{2} g \zeta_d^2 - \frac{2}{R}(W_0 - U_0) + \frac{2\gamma}{R} \sum_{n=0}^{\bar{n}} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} + o\{fT''/R\} , \tag{32}$$

Δg being defined by Equation (1), $\delta\gamma$ being the correction to Δg when using the higher reference model (Mather 1974, p. 95), δg_a the correction for the atmosphere (Anderson, et al., 1975, p. 25) and ζ_d is the deflection of the vertical. It is assumed that the dominant harmonics of the quasi-stationary SST as determined earlier, adequately model the height of MSL at the regional levelling datum. For some studies of this problem, see (Mather, et al., 1978a, Sec. 7).

It is doubtful whether gravity field models developed in this manner, can play any role in determining higher degree harmonics in the global quasi-stationary SST for obvious reasons. However, the use of such a field as the input model in the re-analysis of orbital data for a further refinement of the gravity field model is the next obvious stage in the process of improving the gravity field to the precision of 2 parts in 10^9 required for ocean dynamic modelling.

(b) Determination of Short Wave Contributions to the Quasi-Stationary SST

These short wave contributions are defined as those with wavelength less than ℓ, where ℓ is the wavelength of the highest full harmonic of the gravity field which perturbs the orbits of near Earth satellites in excess of the noise level of the tracking. For a discussion of this subject, see (Mather 1978a, Sec. 7). This discussion proposed a technique based on the solution of the inverse of the geodetic boundary value problem, formulated to take into account the departures of the sea surface from the geoid (considered known with wavelengths greater than ℓ).

The equations comprising this solution were the following,

$$\Delta g_c + \frac{\gamma N_c''}{\bar{R}} - \left[(W_0 - U_0) - \gamma \sum_{n=2}^{n'} (n+1) \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} - \gamma \sum_{m=0}^{1} \sum_{\alpha=1}^{2} \zeta_{s\alpha 1m} S_{\alpha 1m} \right] \Big/ \bar{R} \tag{33}$$

$$= 0$$

where

$$M_1(\psi) = \sum_{n=2}^{\infty} n(2n+1) P_{n0}(\cos\psi), \tag{34}$$

ψ being the geocentric angular distance of the element of surface area $d\sigma$ from the point of computation P,

$$N_c'' = \zeta' - \frac{1}{\gamma}(V - \delta T''), \tag{35}$$

$$\Delta g_c = \Delta g - \delta\gamma + \delta g_a - \frac{1}{2} g\zeta_d^2 + \delta\Delta g'', \tag{36}$$

$\delta T''$, $\delta\Delta g''$ being the changes in T'' (Equation (22)) and $\Delta g''$ defined by

$$\Delta g'' = \frac{GM}{R^2} \sum_{n=2}^{\infty} (n-1) \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C_{\alpha nm} S_{\alpha nm}, \tag{37}$$

in Equation (31), between the Earth's surface and the minimum geocentric sphere enclosing the Earth's topography (the Brillouin sphere) of radius $\bar{R}$.

The principal difficulty in using this equation in practice is the requirement for appropriate resolution in Δg (2 parts in 10^9) through wavelengths of interest, if it is to play a role in ocean dynamic modelling. The data is required within approximately 500 km of the point of computation, with values of ζ' required everywhere, including land areas. It appears optimistic to expect results from this technique in the foreseeable future as surface gravity data of adequate precision will not be available for the task. Thus surface gravity information does not provide a viable basis for recovering short wave information on ζ_s.

An alternate means, in principle, for obtaining improved definition of the gravity field through wavelengths between 500 km and 2000 km with a precision adequate for ocean dynamic modelling, is by satellite-to-satellite tracking of a low-flying satellite with appropriate precision from a network of spacecraft in synchronous orbit. Research in this area is still at a very early stage. The desired resolution is better than 10^{-5} cm s^{-2} with wavelengths in the range mentioned above. This is almost two orders of magnitude smaller than the resolution achieved to date (Vonbun, et al., 1977; Marsh, et al., 1978). More recent analysis appears to indicate that it is possible to recover accelerations at the low flying satellite altitude to $\pm 2 \times 10^{-4}$ cm s^{-2} from range-rate data with a noise level of ± 0.2 mm s^{-1} (Marsh 1978). These results are based on minimal quantities of data and it would be pessimistic to assume that they cannot be improved upon substantially by averaging over long periods of time for the d sired effects of intermediate wavelength.

7. SYNOPTIC MODELLING

Synoptic modelling of the circulation patterns of the surface layer of the global oceans on a daily basis with wavelengths greater than 2500 km is a very real possibility with the launch of SEASAT-A. The computer requirements are well within the capability of the Goddard network. The principal limitation at the present time is the precision of radial orbital determination using the present tracking network as well as the weakness of current gravity field models through harmonics other than low degree zonals. It is not beyond the realm of possibility that the altimetry data available from the GEOS-3 mission, appropriately corrected for the dominant features of the quasi-stationary SST as determined in Section 5, can be combined with current models and tracking data to produce a gravity field with a resolution of 1 part in 10^8.

The principal difficulty is the aliasing effect produced by the altimetry data being confined to the region between parallels 65°S and 65°N and the lack of data of equivalent precision on land. Another factor which will help resolve this problem is the increase in the density of tracking stations in the high precision global tracking network with the passage of time.

The altimetry data can also be used with local high precision tracking data to study regional circulation patterns with periods greater than 1 month. Studies of eddies and other features with vertical magnitude in excess of 20 cm have been reported from GEOS-3 altimetry (Mather and Coleman 1977; Leitao, et al., 1977; Mather, et al., 1978c). All these studies were done without benefit of precise orbit determination and lose some part of the spectrum of the SST in space and time as a consequence. These evaluations are based on determinations of changes $\delta\zeta_s$ in the SST between epochs (τ = t) and (τ = t + dt), using relations of the form

$$\delta\zeta_s = \zeta'(t + dt) - \zeta'(t), \qquad (38)$$

where ζ' is established directly from the altimetry and the orbits together with the enforcement of crossover constraints to eliminate any unmodelled force field errors in the orbit integration.

It can be stated with confidence that the use of geodetic techniques on radar altimeter data is likely to produce information which will make possible the synoptic monitoring of the dynamics of the surface layer of the oceans.

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10. APPENDIX

10.1 Relations Between Geodetic and Geocentric Parameters to 1 Part in 10^{10}

See Figure A-1. The geocentric latitude, ϕ, the geodetic latitute ϕ_g, the geocentric distance R to the ellipsoid and the radius of curvature ν in the prime vertical, given by

$$\nu = a/(1 - e^2 \sin^2\phi_g)^{1/2} , \qquad \text{(A-1)}$$

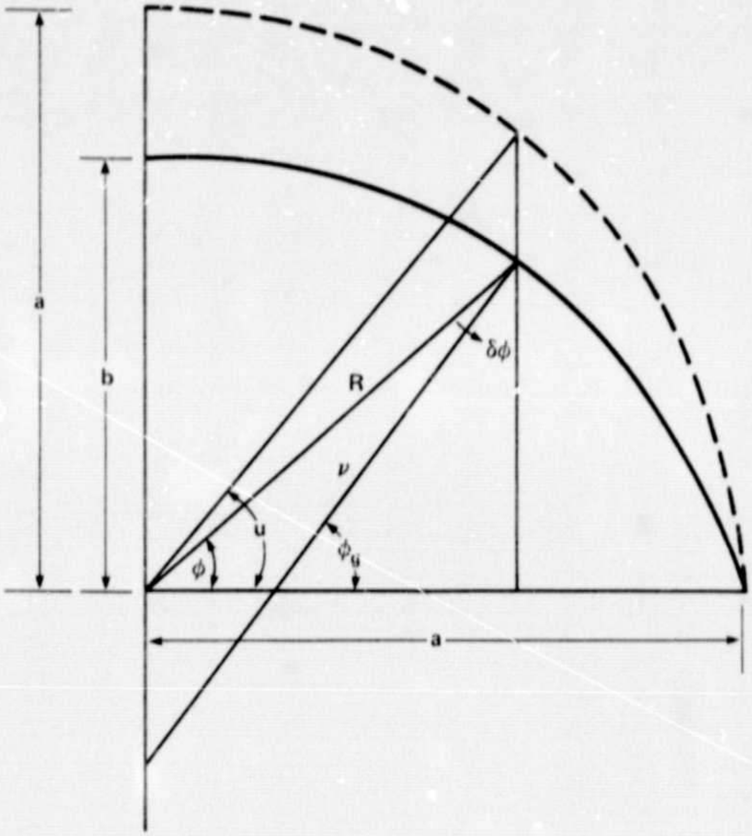

Figure A-1. The Meridian Ellipse

where the meridian ellipse is defined by the equatorial radius a and eccentricity e, by the relations

$$R \sin \phi = \nu \sin \phi_g - \nu e^2 \sin \phi_g = \nu (1 - f)^2 \sin \phi_g , \tag{A-2}$$

and

$$R \cos \phi = \nu \cos \phi_g , \tag{A-3}$$

the flattening f being related to e by the relation

$$e^2 = 2f(1 - f/2) . \tag{A-4}$$

Thus

$$\tan \phi = (1 - f)^2 \tan \phi_g . \tag{A-5}$$

If

$$\delta\phi = \phi_g - \phi = o\left\{f\right\}, \qquad \text{(A-6)}$$

it follows that

$$\tan\delta\phi = \delta\phi + o\left\{\frac{(\delta\phi)^3}{3}\right\} = \frac{\tan\phi_g - \tan\phi}{1 - \tan\phi_g\tan\phi}. \qquad \text{(A-7)}$$

On using Equation (A-5) in Equation (A-7) and expansion using the binomial theorem, it follows that

$$\delta\phi = f\sin 2\phi[1 + f(3/2 - 2\sin^2\phi] + o\left\{f^3\right\}. \qquad \text{(A-8)}$$

On using Taylor's theorem, the following results are obtained:

$$\sin^2\phi_g = (1 + 4f + 10f^2)\sin^2\phi - 4f(1 + 13/2f)\sin^4\phi + 16f^2\sin^6\phi + o\left\{f^3\right\}, \qquad \text{(A-9)}$$

and

$$\sin^4\phi_g = (1 + 8f)\sin^4\phi - 8f\sin^6\phi + o\left\{f^2\right\}. \qquad \text{(A-10)}$$

The geocentric distance to a point on the ellipsoid at latitude ϕ_g is obtained by using Equations (A-1) and (A-4) in the combination of Equations (A-2) and (A-3) to eliminate ϕ, when manipulation using the binomial theorem gives

$$R = a\left[1 - f(1 - \frac{5}{2}f + 2f^2)\sin^2\phi_g - \frac{f^2}{2}(5 - 17f)\sin^4\phi_g - \frac{13}{2}f^3\sin^6\phi_g + o\left\{f^4\right\}\right], \qquad \text{(A-11)}$$

which is the expression for R in geodetic coordinates.

The expression in geocentric coordinates is obtained by substituting Equations (A-9) and (A-10) in (A-11), when the relation

$$R = a\left[1 - f\left(1 + \frac{3}{2}f + 2f^2\right)\sin^2\phi + \frac{3}{2}f^2\left(1 + \frac{41}{3}f\right)\sin^4\phi - \frac{5}{2}f^3\sin^6\phi + o\left\{f^4\right\}\right] \tag{A-12}$$

is obtained.

10.2 The Spherical Harmonic Representation of the Exterior Potential Due to a Rotating Equipotential Ellipsoid to Order f^4

If α were the parameter defined by

$$\cos\alpha = 1 - f \tag{A-13}$$

and

$$\sin\alpha = e = (2f - f^2)^{1/2}, \tag{A-14}$$

and if the equatorial radius a and polar radius b of the equipotential ellipsoid were defined by

$$a = c\operatorname{cosec}\alpha_0; \qquad b = c\cot\alpha_0, \tag{A-15}$$

where α_0 is the value of α on the equipotential ellipsoid, the exterior potential of the rotating equipotential ellipsoid at the general point (α, u) is given by (e.g., Mather 1971, p. 83)

$$U = \frac{GM}{c}\alpha + \frac{a^2\omega^2}{3q_2(\alpha_0)}q_2(\alpha)P_{20}(\sin u) + U_r, \tag{A-16}$$

M being the mass of the ellipsoid, u the reduced latitude (Figure A-1), U_r the rotational potential and $q_2(\alpha)$ is defined by (ibid., pp. 78-9)

$$q_2(\alpha) = \frac{1}{2}\left[\alpha(3\cot^2\alpha + 1) - 3\cot\alpha\right]$$

$$= 2\left[\frac{1}{3.5}\tan^3\alpha - \frac{2}{5.7}\tan^5\alpha + \frac{3}{7.9}\tan^7\alpha - \frac{4}{9.11}\tan^9\alpha + \ldots\right] \tag{A-17}$$

$$= -\sum_{n=1}^{\infty}(-1)^n\frac{2n}{(2n+1)(2n+3)}\tan^{2n+1}\alpha$$

Due to rotational symmetry, the potential in spherical harmonics due to this same rotational equipotential ellipsoid is also given by

$$U = \frac{GM}{R}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a}{R}\right)^{2n} P_{2no}(\sin\phi)\, C_{2no}\right] + U_r . \qquad \text{(A-18)}$$

Expressions (A-16) and (A-18) are identical. The coefficients C_{2no} can be evaluated by comparing the two expressions at the pole of the rotating equipotential ellipsoid when

$$R = b, \; u = \phi = 90^\circ \qquad \text{(A-19)}$$

and all zonal harmonics take the value 1. In such a circumstance,

$$\tan\alpha = \frac{c}{b} = \frac{c}{R} \qquad \text{(A-20)}$$

from Equation (15). From Equation (2),

$$a^2\omega^2 = GMm/a . \qquad \text{(A-21)}$$

Further

$$\alpha = \tan^{-1}(\tan\alpha) = \tan\alpha + \sum_{n=1}^{\infty}(-1)^n \frac{\tan^{2n+1}\alpha}{2n+1} . \qquad \text{(A-22)}$$

Using Equations (A-17), (A-21) and (A-22) in Equation (A-16), simplification after re-arrangement of terms, gives

$$U = \frac{GM}{b}\left[1 + \sum_{n=1}^{\infty}(-1)^n \frac{\tan^{2n}\alpha}{2n+1}\left[1 - \frac{m\sin\alpha}{3q_2(\alpha)}\,\frac{2n}{2n+3}\right] P_{20}(\sin u)\right] + U_r . \qquad \text{(A-23)}$$

Equating coefficients of the same degree in R after imposing the conditions at (A-19) and (A-20) on Equations (A-18) and (A-23) gives

$$C_{2no}a^{2n} = (-1)^n \frac{c^{2n}}{2n+1}\left[1 - \frac{m\sin}{3q_2(\alpha)}\,\frac{2n}{2n+3}\right], \qquad \text{(A-24)}$$

where the subscript o has been suppressed in the value of α which refers to the equipotential ellipsoid. As Equation (A-15) defines

$$\frac{c}{a} = \sin\alpha \tag{A-25}$$

it follows that

$$C_{2n} = \frac{(-1)^n \sin^{2n}\alpha}{2n+1}\left[1 - \frac{m\sin\alpha}{3q_2(\alpha)}\,\frac{2n}{2n+3}\right]. \tag{A-26}$$

10.3 The Flattening f and C_{20}

The flattening f is obtained from the second degree zonal harmonic in the spherical harmonic series by the relations

$$C_{20} = \frac{\sin^2\alpha}{3}\left[\frac{2m\sin\alpha}{15q_2(\alpha)} - 1\right], \tag{A-27}$$

or

$$f = 1 - \left(1 - \left[\frac{2m\sin^3\alpha}{15q_2(\alpha)} - 3C_{20}\right]\right)^{½} \tag{A-28}$$

where $q_2(\alpha)$ is defined by Equation (A-17). This obviou[illegible] has to be solved by iteration with the procedure rather unstable unless sufficient significant digits are carried in computations.

In most computations, a trial value (C_{20t}) is available for C_{20}, corresponding to a value f_t for f. A solution procedure is required for computing df from dC_{20}, given by

$$dC_{20} = C_{20} - C_{20t}, \tag{A-29}$$

using an expression for df evaluated in terms of a rapidly convergent series.

The use of Equation (A-17) enables the complex term in Equation (A-28) to be written as

$$\frac{2m\sin^3\alpha}{15q_2(\alpha)} = m\cos^3\alpha\left[1 + \frac{6}{7}\tan^2\alpha + \frac{1}{49}\tan^4\alpha + o\{f^4\}\right]. \tag{A-30}$$

Changes $d\alpha$ in α are produced from changes dC_{20} in C_{20} according to

$$d\alpha \left[2 \sin\alpha \cos\alpha + 3 m \cos^2\alpha \sin\alpha \left(1 + \frac{6}{7} \tan^2\alpha + \frac{1}{49} \tan^4\alpha \right) - \frac{12}{7} \sin\alpha \right] + 3 dC_{20} = 0 \tag{A-31}$$

which is obtained by differentiating Equation (A-27) after re-arrangement of terms. Then, as

$$df = \sin\alpha \, d\alpha \tag{A-32}$$

$$df = - 3 dC_{20} / \left[2 \cos\alpha + 3m \left\{ \cos^2\alpha \left(1 + \frac{6}{7} \tan^2\alpha + \frac{1}{49} \tan^4\alpha \right) - \frac{4}{7} \right\} \right] , \tag{A-33}$$

and

$$f = f_t + df . \tag{A-34}$$

10.4 The Effect of the Permanent Earth Tide on Ocean Dynamic Modelling

The coefficients of satellite determined gravity field models C'_{nm} referred to in Section 4 are computed after modelling the effect of the Earth tide. The latter has a permanent constituent which causes the second degree zonal harmonic of the true gravity field which influences the quasi-stationary component of ocean circulation to be different from C'_{120}. This problem is discussed in (Mather 1978b). The incorrect pre-processing of the altimeter data for the effect of the Earth tide can cause errors of 13 percent in the value of ζ_{s120} as determined from Equation (24).

The magnitude of the error $d\zeta_{s120}$ is described by the equation

$$d\zeta_{s120} = \frac{a}{\sqrt{5}} M \left\{ \sum_{i=1}^{2} \left(\frac{R}{R_i} \right)^3 \left(\frac{M_i}{M} \right) (1 + k_2 - h_2) P_{20}(\sin \delta_i) \right\} \tag{A-35}$$

where (R_1, R_2) are the distances between the centre of mass of the Earth and those of the sun and moon (masses M_1, M_2; declinations δ_1, δ_2), k_2, h_2 being the second degree Love numbers, $M\{ \}$ referring to the mean value over the period of altimeter data acquisition.

This correction is only necessary when sea surface heights from satellite altimetry have not been corrected dynamically for the effect of the Earth tide. The dynamic correction allows for the change in the instantaneous Earth space position of the geoid, defined as the level surface ($W = W_0$) due to the influence of the Earth tide. It is essential that the correction made by dynamic rather than geometric (i.e., based only on the Love number h_2) when using altimetry data in ocean dynamic modelling.

This will ensure that the satellite altimetry data will be referred to the instantaneous datum level surface rather than some artificial static geoid and thereby ensure that the aliasing effect of the permanent tide is satisfactorily removed in determinations of SST. The value obtained for $d\zeta_{S120}$ is +6.1 cm.

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16. Abstract

The spectrum of ocean dynamics lends itself to convenient subdivision into two components in the context of satellite remote sensing. The first is the quasi-stationary constituent, while all features which vary with time during the period of data acquisition, comprise the second. As data collected at the surface of the oceans is realted to sea level and not the geoid, satellite determined gravity fields are the only source of data on the gravity field which can be used for dynamic sea surface topography determination.

An analysis of the signal-to-noise ratio of the best gravity field available at the present time shows that a basis exists for the recovery of the dominant parameters of the quasi-stationary sea surface topography at the present time. Results obtained from the analysis of GEOS-3 show that it is feasible to recover the quasi-stationary dynamic sea surface topography as a function of wavelength. The gravity field models required for synoptic ocean circulation modeling are less exacting in that constituents affecting radial components of orbital position need not be known through shorter wavelengths.

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