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THE ECONOMETRICS OF DAMAGE CONTROL: WHY SPECIFICATION MATTERS

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THE ECONOMETRICS OF DAMAGE CONTROL:
WHY SPECIFICATION MATTERS

One of the most important classes of factors of production is that consisting of damage control agents. Unlike standard factors of production (land, labor, and capital), these inputs do not increase (they may, in fact, decrease) potential output. Instead, their distinctive contribution lies in their ability to increase the share of potential output that producers realize by reducing damage due to both natural and human causes. Many of the innovations in agriculture over the past few decades have involved the introduction of damage control agents, e.g., pesticides, windbreaks, sprinklers for frost protection, immunizations and antibiotics in feedlot operations, etc. Advances in storage technology (for instance, the fumigation of stored grains) have hinged on improvements in damage control agents. Other important examples of damage control agents include the use of smoke alarms and sprinkler systems to reduce fire damage, antitheft/antivandalism measures (in fact, the prevention of crimes against property is essentially an exercise in damage control), etc.

The use of damage control agents also tends to subject producers to certain difficulties which do not arise in connection with the use of standard inputs. The most important problem is that, in many cases, the damaging agents involved (be they human, insect, or weed) adapt to the damage control measures taken as time passes, rendering the latter increasingly ineffective. This problem of growing resistance to damage control has important economic ramifications.

In many situations, notably those involving natural systems (e.g., pest control, immunization, etc.), the details of optimal damage control strategies are best explored using simulation models. In positive studies, however, where the aim is to explain observed behavior and to estimate behavioral or physical parameters, econometric methods are generally required. The computational complexities and data requirements of econometric methods restrict them to specifications that are simpler and less detailed than the ones used for simulations. Thus, econometric methods are inevitably confined to a lower level of precision than simulation models. The key to maximizing the accuracy and information content of econometric models lies in incorporating as much as possible the critical elements of the available scientific knowledge without fatally compromising their generality and estimatability.

To date, econometric investigations of damage control have ignored the specific contributions of the scientific community in quantifying production relationships, relying instead on generic econometric models to specify the relevant functional forms. Specification errors arising in this way may generate biases of considerable size in estimates of productivity and, hence, faulty conclusions about efficient input usage.

Economic analysis of agricultural pesticide use is a prime example of this phenomenon. Theoretical (Feder and Regev; Regev, Shalit, and Gutierrez) and normative empirical (Shoemaker; Regev, Gutierrez, and Feder; Talpaz and Borosh; Regev, Shalit, and Gutierrez) models of pest management at the farm or regional level have incorporated the available entomological knowledge in their model specifications and have derived optimal management patterns and policy recommendations on this basis. By contrast, econometric measurements of pesticide productivity have been derived from standard production theory models, notably using Cobb-Douglas specifications. It will be shown that productivity estimates are flawed conceptually and, as a result, contain significant statistical biases.

Presented first is a general discussion of the role of damaged abatement in the production process. It is argued that damage control inputs should be incorporated into production analysis in a different manner than regular inputs; in fact, the theoretical and normative empirical literature on pest management has led to the use of the types of specifications suggested by the nature of the biological processes involved.

This approach to the specification of the role of damage control agents in production has two important implications for theoretical and, especially, empirical work. First, it is shown that the types of production function specifications used most commonly to estimate factor productivity overestimate the productivity of damage control inputs even in large samples. The source of this upward bias is a misspecification of the shape of the marginal factor productivity curve of damage control inputs which decrease more rapidly in the economic range than standard specifications impose.

The kind of specification proposed for incorporating damage control agents into production analysis produces empirical models in which factor productivity can be estimated easily from existing data in a number of important instances. Specifications will be derived and estimation procedures will be discussed for several cases of special interest with respect to pesticides.

The second important characteristic of this specification is the way it handles changes in damage control agent productivity over time. In the case of pesticides, for example, the spread of resistance through a pest population is an important problem. Treating a damage control agent, such as a pesticide, in the same way as an ordinary factor of production has led economists to predict behavior contrary to observed fact. In a standard production function, decreasing factor effectiveness is reflected in decreasing marginal factor productivity and, thus, in reduced levels of factor use. In the specification, decreasing effectiveness may increase factor demand; this is precisely the phenomenon observed in pesticide use trends.

A Model of Damage Control

Damage control agents do not enhance productivity directly as do the standard types of production factors. To the contrary, such inputs may even impede productivity somewhat. The application of a pesticide, for example, may be harmful to crop plants to a certain extent. The contribution to production made by these inputs come from their function as damage control agents. Damage control agents increase final output by limiting damage. The role of such inputs is thus to lessen the difference between potential output (by which is meant the maximum level of output attainable from a given combination of directly productive inputs) and actual output.

This characterization of the productive services provided by damage control agents suggests that the proper way to specify their role in production is through the use of a damage abatement function defined as the proportion of damage avoided by application of any given amount of control agent. Damage abatement functions naturally have the same characteristics as a probability distribution. For the case of pesticides, for example, the damage abatement function is usually called the pesticide effectiveness function or the kill function. It measures the proportion of the target pest population killed by the application of any given amount of pesticide.

Production functions that incorporate such damage control agents thus have the following general form: output, Q, is a function of regular inputs, Z, and a damage control agent, X, through the damage abatement function G(X):[1]

(1) $$Q = F[Z, G(X)].$$

As noted, the damage abatement function, G(X), has the usual properties of a probability distribution defined on the interval $(0, \infty)$ or, possibly, on the interval $(0, X_m)$ for some finite X_m; the control agent effectiveness (or marginal productivity) function is simply the density of G(X).

What is the impact of such a specification on production analysis? To see how damage control input use differs from regular input use, consider first the profit-maximizing behavior of a producer using a regular input, Z, and damage abatement, G, treated as a regular input as well. The relevant maximization problem is

(2) $$\max_{Z,G} \Pi = pF(Z, G) - rZ - sG,$$

where p, r, and s are the prices of Q, Z, and G, respectively. Assuming an interior maximum, the necessary conditions for maximization are given by:

(3) $$pF_Z = r$$
$$pF_G = s.$$

Sufficiency is assured by the negative semidefiniteness of the Hessian matrix which implies:

(4) $$F_{ZZ} \leq 0$$
$$F_{GG} \leq 0$$
$$F_{ZZ}F_{GG} - F_{GZ}^2 \geq 0.$$

The elasticity of demand for damage abatement, G, found by differentiation of (3) with respect to s, is:

$$\text{(5)} \qquad \frac{s}{G}\frac{\partial G}{\partial s} = \frac{1}{\frac{F_{GG}G}{F_G} - \frac{F_{GZ}^2 G}{F_G F_{ZZ}}} = \varepsilon_G.$$

The assumed concavity of the production function in damage abatement means that the marginal productivity of damage abatement will be decreasing everywhere. The profit-maximizing quantity will be found by the intersection of the value of marginal product and marginal factor cost curves.

When damage abatement is a function of damage control inputs, the picture changes. The relevant profit-maximization problem becomes:

$$\text{(6)} \qquad \max_{Z,X} \Pi = pF[Z, G(X)] - wX - rZ,$$

where w is the price of X. Letting $G_X = g$, the necessary conditions become:

$$\text{(7)} \qquad \begin{aligned} pF_Z &= r \\ pF_G g &= w. \end{aligned}$$

Sufficiency is ensured by the negative semidefiniteness of the Hessian matrix which implies:

$$\text{(8)} \qquad \begin{aligned} &F_{ZZ} \leq 0 \\ &F_{GG} \cdot g^2 + F_G \cdot g' \leq 0 \\ &F_{ZZ}(F_{GG} \cdot g^2 + F_G \cdot g') - F_{ZG} \cdot g^2 \geq 0; \end{aligned}$$

both the marginal productivity of damage abatement, F_G, and the marginal effectiveness of the damage control input, g, must be declining to ensure that a maximum has been attained. The elasticity of demand for the damage control input is:

$$\text{(9)} \qquad \varepsilon_X = \frac{w}{X}\frac{\partial X}{\partial w} = \frac{1}{\frac{\eta_G}{\varepsilon_G} + \eta_g},$$

where $\eta_G = gX/G$ is the elasticity of the damage abatement function, and $\eta_g = g'X/g$ (the elasticity of the marginal effectiveness of the damage control input) measures the curvature of the damage abatement function.

Evaluation of the expression on the right-hand side of (9) allows one to draw several conclusions about the qualitative characteristics of demand for damage control agents.

First, because g(X) has the properties of a probability density, it is reasonable to assume that $|\eta_g| > 1$, that is, that the marginal effectiveness curve is always elastic. The existence of a finite damage abatement function (probability distribution), G(X), defined on $0 \leq X \leq \infty$ is assured if the marginal effectiveness curve (density function), g(X), is declining faster than 1/X (since $\int 1/X = \ln X$, which does not converge as $X \to \infty$) which implies that $g'(X)\, X|g(X) < -1|g(X) < -1$, a property which is easily verified for any of the commonly used distributions (normal, gamma, etc.). As a result of this property of g(X), it is obvious from (9) that $|\varepsilon_X| < 1$; that is, the demand for damage control inputs is everywhere inelastic in all practical instances.

Second, it is evident from (9) that the demand for damage abatement (represented by its elasticity ε_G) influences the demand for damage control inputs. However, the extent of this influence varies considerably.

Consider first the case where $\varepsilon_G = 0$, that is, where the demand for damage abatement is perfectly inelastic. Rearrangement of (9) produces the relation $\varepsilon_X = \varepsilon_G/(\eta_G + \eta_g \varepsilon_G)$, from which it is evident that $\varepsilon_X = 0$ whenever $\varepsilon_G = 0$; that is, that the demand for damage control agents is perfectly inelastic whenever the demand for damage abatement is perfectly inelastic. Whenever the demand for damage abatement is perfectly inelastic, then the demand for damage control agents will be dominated by the demand for damage abatement.

One situation where this may occur is when the relevant production function exhibits fixed proportions with respect to damage abatement. Another is the case where damage abatement exhibits threshold effects, that is, where some positive proportion of damage abated is equivalent to no abatement at all. For example, U. S. Food and Drug Administration regulations prohibit the sale of shipments of apples in which more than 5 percent have been found to be wormy; here damage abatement of 94 percent is equivalent to none, while 95 percent passes muster. (Similar regulations govern the sale of most produce.) In this case, all that matters to the grower is that worm infestations affect no more than 5 percent of the crop; hence, the demand for pesticides to control this problem will be perfectly inelastic at the 5 percent damage abatement level.

In the case where the demand for damage abatement is not perfectly inelastic--where $|\varepsilon_G| > 0$--it is easy to verify that $\partial\varepsilon_X/\partial\varepsilon_G = [1/(1 + \eta_g \varepsilon_G/\eta_G)]^2 > 0$, i.e., that the elasticity of the demand for damage abatement has a positive effect on the elasticity of demand for damage control inputs. Therefore, the more elastic the demand for damage abatement is, the more inelastic the demand for damage control inputs will be.

As the level of damage control agent use rises, however, η_G declines (since $\partial\eta_G/\partial X = [1 - \eta_G + \eta_g]\ \eta_G/X$) and, hence, the influence of the demand for damage abatement, ε_G, on the demand for damage control inputs, ε_X, tends to diminish. In fact, as X gets sufficiently large that G(X) approaches 1, η_G tends to vanish; as a result, ε_X approaches $1/\eta_g$. Since in most observed cases damage control agents tend to be used at close to full effectiveness, one can conclude that, whenever $|\varepsilon_G| > 0$, the elasticity of demand for damage control inputs is the reciprocal of the elasticity of the marginal effectiveness curve.

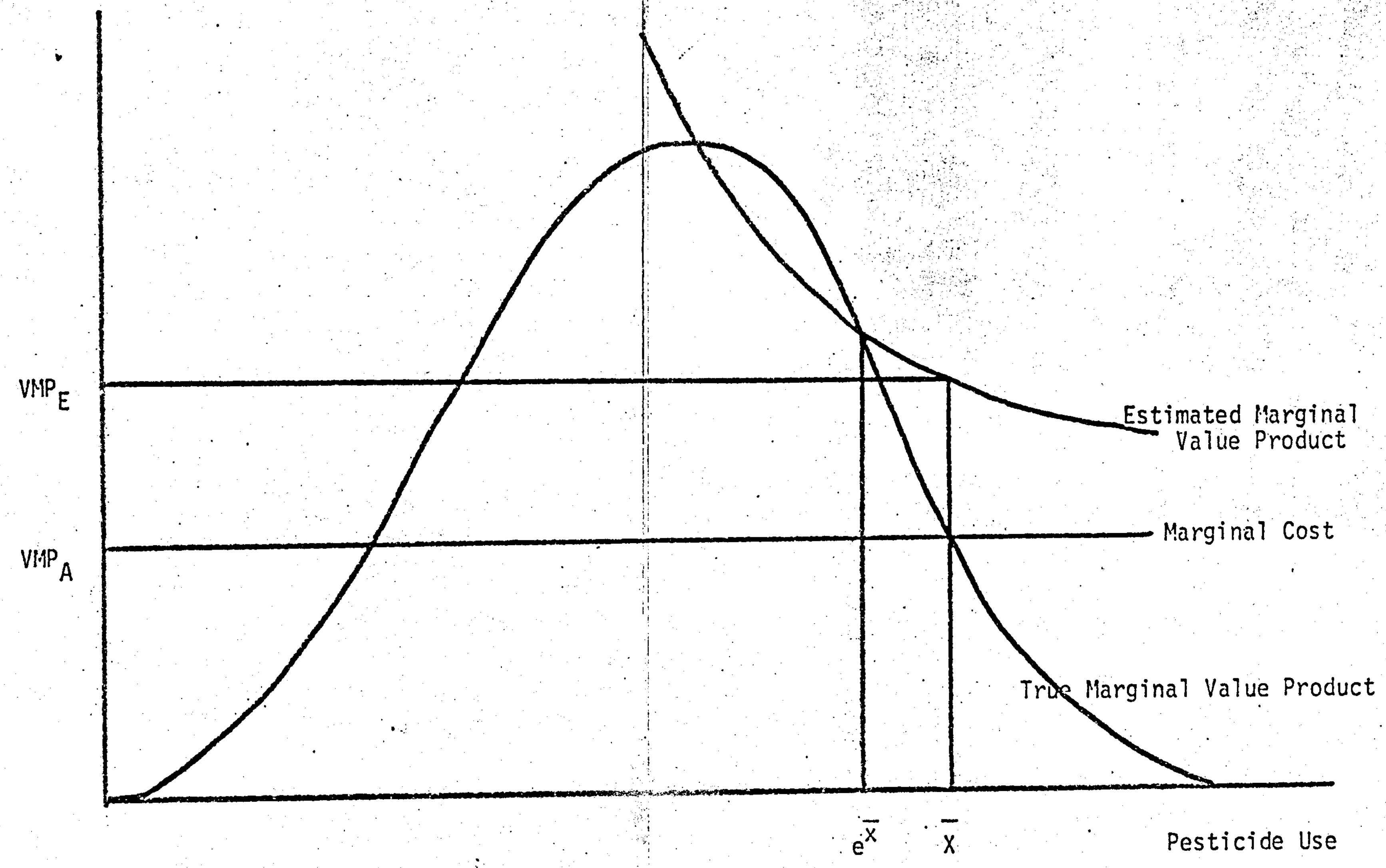
Econometric Implications of the Specification

What happens when a standard production function specification, such as a Cobb-Douglas, is used to estimate the marginal productivity of damage control? The result of such a misspecification can be seen in figure 1 which compares a standard Cobb-Douglas marginal productivity curve with one derived from the damage control specification proposed above. It is easily seen that any specification which restricts the rate at which the marginal effectiveness curve declines will tend to produce overestimates of the marginal productivity of damage control agents and, at the same time, to produce underestimates of the productivity of natural factors. Moreover, these biases will occur even when the specification used is a good approximation of the true model in every respect but the incorporation of the damage control input.

More formally, assume that the output elasticities of damage abatement, G, and of all other inputs, Z, are constant and that the elasticities of

FIGURE 1

THE IMPACT OF MISSPECIFICATION ON DAMAGE CONTROL AGENT PRODUCTIVITY ESTIMATES



substitution differ only negligibly from one over the relevant range; then the Cobb-Douglas specification,

(10) $$Q = e^{\alpha} Z^{\beta} [G(X)]^{\gamma} e^{u},$$

represents the underlying production function and the random error associated with it. It will be convenient to use the logarithmic form of the model. Letting the lower case letters, g, z, and x, represent the natural logarithms of Q, Z, and X, respectively, the model can be rewritten:

(11) $$q = \alpha + z\beta + \gamma \ln G(X) + u.$$

Now suppose that, instead of the model given by (10) and (11), a Cobb-Douglas specification using the damage control agent, X, instead of damage abatement, G(X), is used to estimate this; that is, that the estimating model used is:

(12) $$q = \alpha + z\beta + \gamma x + v.$$

As we show formally in the Appendix, the ordinary least-squares (OLS) estimation of α, $\hat{\alpha}$ converges in probability to a number less than α; specifically,

(13) $$\operatorname*{plim}_{n \to \infty} \hat{\alpha} = \alpha + \gamma \, [\ln G(e^{\bar{x}}) - \eta_G(e^{\bar{x}}) \, \bar{x}] < \alpha,$$

where $\bar{x}$ is the mean value of ln(X). At the same time, the OLS estimation of γ, $\hat{\gamma}$, converges in probability to a number which is greater than the measure of damage control agent productivity at mean usage level $\bar{X}$, $\eta_G(\bar{X}) \, \gamma$:

$$\text{(14)} \qquad \underset{n\to\infty}{\text{plim}}\ \hat{\gamma} = \eta_G(e^{\bar{X}})\ \gamma > \eta_G(\bar{X})\ \gamma.$$

The implication of these findings is that the use of a standard Cobb-Douglas specification to estimate damage control agent (pesticide) productivity leads to overestimation of the marginal productivity of the damage control agent and underestimation of the marginal productivity of natural and omitted factors even when the Cobb-Douglas specification is good for damage abatement.

The intuition behind these results can be grasped easily upon examination of figure 1. The specification of damage control agent productivity proposed here suggests that the marginal product (marginal effectiveness) curves of the damage control agent will decline at an increasing rate in the economic region. The reason for this increasingly rapid decline lies in the specification of marginal effectiveness as a probability density: To converge, $g(X)$ must decline faster than $1/X$ and, hence, must decrease more rapidly as X gets larger. As a result, the elasticity of the marginal effectiveness curve also grows as X increases. A specification like the Cobb-Douglas cannot match this behavior. Instead, a standard Cobb-Douglas specification will produce a marginal effectiveness curve whose elasticity is constant and, hence, which declines more slowly than the true marginal effectiveness curve. The implications of this fact can be seen easily in figure 1. The standard Cobb-Douglas specification will produce consistent estimates of the damage control agent productivity parameter $\eta_G\gamma$ at a point $e^{\bar{X}}$ which necessarily lies to the left of the average level of damage control agent use $\bar{X}$. Since the true parameter tends to decline quite rapidly, the estimated marginal product curve will lie above the true curve for levels of control agent use greater than $e^{\bar{x}}$. At average use levels then, the estimated value of marginal damage control agent productivity (VMP_E) will be greater (conceivably substantially greater) than the true value of marginal damage control agent productivity (VMP_A) and will appear to be greater than marginal control agent cost (MC).

This result explains one of the most perplexing findings of the econometric literature on pesticide use: that marginal pesticide productivity has been well above marginal application cost. Perhaps the clearest example is the work of Campbell, who applied a Cobb-Douglas production function to data on output, pesticide use, and other factors in Canadian apple orchards and found marginal pesticide productivities that were about 12 times marginal cost. The implication, of course, is that pesticides are greatly underutilized. In light of the biological and behavioral literature on pesticide use, such a conclusion is astounding, to say the least. The overwhelming consensus opinion of the theoretical, normative empirical, and casual empirical studies performed concerning pesticide use is that pesticides are overused rather than underutilized as the econometric literature suggests. Consideration of such factors as the potential growth of resistance, common stock externalities, informational and human capital problems, and the like suggests that marginal pesticide productivity lies below marginal cost at common usage levels.

The analysis of econometric method presented above indicates that the source of this contradiction is the incorrect methodology employed in these studies. Estimation of the production function using the damage control agent (pesticide) instead of a damage abatement (kill) function produces an upward bias in the estimates of damage control agent (pesticide) which, in turn, implies the productivity underutilization of the damage control agent.

Estimating Damage Control: Some Sample Specifications

Because the damage abatement function can be represented quite naturally by a cumulative distribution, it is not difficult to specify empirical models for estimating the productivity of damage control agents. In this section we give some examples of possible specifications derived from distributions that have been used in the pest management literature. It turns out that, in a number of important cases, estimation of the parameters of these models is remarkably simple so that use of damage abatement functions in empirical work entails little or no additional cost.

Since the Cobb-Douglas specification is used so commonly, assume that the modified Cobb-Douglas form given by (10) represents the production function well. Under this assumption and the assumption of a specific form for the damage abatement function G(X), it becomes possible to derive production function and damage control agent demand function specifications for use in econometric work. Table 1 shows the production functions (in log form) and damage control agent demand functions implied by four specifications of G(X): the Pareto distribution, the exponential distribution, the logistic distribution, and the Weibull distribution. The latter three specifications are of particular interest because of their use in this capacity in normative empirical models of pest management. The Pareto, on the other hand, is of interest primarily because of its econometric implications.

Consider, first, the case of the Pareto damage abatement function. In the form given in table 1, both the production function and damage control agent demand relation are quite intractable for linear estimation and must be approached by nonlinear means. But in the special case where $\gamma = 1$, that

Table 1. Alternative Econometric Specifications

Distribution	G(X)	Production function	Damage control agent demand
Pareto	$1 - K^{\lambda} X^{-\lambda}$	$q = \alpha + z\beta + \gamma \ln [1 - K^{\lambda} X^{-\lambda}]$	$K^{\lambda} X^{-\lambda} [w + \lambda\gamma pQX^{-1}] = w$
Exponential	$1 - e^{-\lambda X}$	$q = \alpha + z\beta + \gamma \ln [1 - e^{-\lambda X}]$	$X = \frac{1}{\lambda} \ln \left[1 + \frac{\lambda\gamma pQ}{w}\right]$
Logistic	$[1 + \exp\{\mu - \sigma X\}]^{-1}$	$q = \alpha + z\beta - \gamma \ln [1 + \exp\{\mu - \sigma X\}]^{-1}$	$X = \frac{\mu}{\sigma} + \frac{1}{\sigma} \ln \left[\frac{pQ}{w} - \frac{1}{\gamma\sigma}\right]$
Weibull	$1 - \exp\{-X^{c}\}$	$q = \alpha + z\beta + \gamma \ln [1 - \exp\{-X^{c}\}]$	$X = \frac{1}{c} \ln \left[c\gamma X^{c-1} \frac{pQ}{w} + 1\right]$

is, damage abatement is proportional to potential output, it can be shown that the supply function can be expressed as

(15) $$Q = a_0 \, p^{a_1} \, r^{a_2} \, w^{a_3}$$

where $a_0 = [e^{2\alpha+\lambda} \, \beta^{\beta/(1+\lambda)} \, \lambda K^{\lambda}]^{1/[1+\lambda-\beta(2+\lambda)]}$; $a_1 = [1 + \beta(2 + \lambda)]/[1 + \lambda - \beta(2 + \lambda)]$; $a_2 = -[1 + \beta(2 + \lambda)]/[1 + \lambda - \beta(2 + \lambda)]$; and $a_3 = -1/[1 + \lambda - \beta(2 + \lambda)]$. Similarly, demand for the damage control agent can be expressed as:

(16) $$X = a_0 \, p^{a_1} \, r^{a_2} \, w^{a_3} \, Q^{a_4}$$

where $a_0 = [\lambda e^{\alpha} \, K^{\lambda}]^{1/(1+\lambda)}$, $a_1 = (1 + \beta)/(1 + \lambda)$, $a_2 = -\beta/(1 + \lambda)$, $a_3 = -1/(1 + \lambda)$, and $a_4 = \beta/(1 + \lambda)$. In short, a Pareto damage abatement function, together with the assumption of damage abatement proportional to potential output, yields standard Cobb-Douglas specifications for the supply function and for damage control agent demand. It turns out that this result arises from the fact that the Pareto distribution, like the Cobb-Douglas, possesses a marginal curve (density) elasticity of which is constant. For the form of the Pareto distribution given here, for instance, it is readily apparent that the marginal effectiveness curve, $g(X) = \lambda K^{\lambda} X^{-(\lambda+1)}$ has an elasticity of $-(\lambda + 1)$ for X. The Pareto distribution is exceptional in this regard: The density elasticities of most distributions increase relatively rapidly. In fact, the Pareto distribution can be considered a limiting case for probability distributions in this respect.

While this demonstration shows that the standard Cobb-Douglas specification may be valid for examining the role of damage control agents in production under some conditions, it turns out that these conditions are so restrictive as to be unimportant practically. The validity of a standard Cobb-Douglas specification depends on two conditions: (1) that damage abatement be proportional to potential output and (2) that damage abatement be well represented by a Pareto distribution. Condition (1) is certainly a good description of damage abatement in many situations; it should be recognized, however, that there are also many situations where it does not characterize the role of damage abatement well. Condition (2) may also hold in some cases. But, by and large, Pareto distributions have not been found to characterize damage abatement very well precisely because of their slow rates of change. (The distributions used for pesticide effectiveness, for example, are discussed in detail below.)

Now consider the case where the damage abatement function is assumed to be exponential. (In the pesticide literature, this specification was used by Regev, Gutierrez, and Feder in their study of alfalfa weevil control.) The production function is nonlinear in λ. It can be estimated, of course, by nonlinear methods or, since λ should lie between zero and one, the parameters of the model can be estimated by linear techniques combined with a grid search for λ.

Alternatively, consider the demand for the damage control agent. A slight rearrangement of the relation given in table 1 yields a function of the form:

$$e^{X} = a_0 + a_1\left(\frac{pQ}{w}\right), \tag{17}$$

where $a_0 = 1 + e^{1/\lambda}$ and $a_1 = \lambda\gamma$. This relation is estimated easily using OLS methods because the right-hand side is a simple linear function of revenue and pesticide price, data for both of which, it is important to note, are generally available. The production function parameters of particular interest, γ and λ, are recovered easily from the estimated coefficients, a_0 and a_1.

Alternatively, assume that the damage abatement function can be represented by a logistic distribution as was done by Shoemaker in her study of flour moth control. As is evident from table 1, the demand function for the damage control agent (pesticide) can be expressed as

$$X = a_0 + a_1 \ln\left[\frac{pQ}{w} - \frac{1}{\gamma\sigma}\right], \tag{18}$$

where $a_0 = \mu/\sigma + 1/\sigma \cdot \ln \gamma\sigma$ and $a_1 = 1/\sigma$. If $1/\gamma\sigma$ is sufficiently small, $\ln [pQ/w]$ can be used as a proxy for $\ln [pQ/w - 1/\gamma\sigma]$ at a cost of a negligible reduction in efficiency. In this case, use of a logistic damage abatement function implies that the proper specification of damage control agent demand is as a linear function of $\ln [pQ/w]$.

This approximate demand relation can be estimated easily using OLS, and the parameter σ can be recovered from the estimate of a_1. In general, it will not be possible to recover estimates of the two remaining parameters γ and μ. If, however, there is reason to believe that damage is strictly proportional to potential output, i.e., we believe that $\gamma = 1$, then estimation of both of the parameters of the damage abatement function can be estimated using the damage control agent demand function.

As a final example, consider the case where the damage abatement function can be represented by a Weibull distribution as Talpaz and Borosh assume in their study of pest control in cotton. The demand of the damage control agent is shown in table 1. In general, $c\gamma X^{c-1} (pQ/w)$ will be large enough to be a very close approximation to $c\gamma X^{c-1} (pQ/w) + 1$ so that the relation

$$X = \frac{1}{c} \ln \left[c\gamma X^{c-1} \left(\frac{pQ}{w} \right) \right] \tag{19}$$

will be a good approximation to the demand function given in table 1. The relation in (19) can be rearranged to yield the demand function,

$$X + \frac{c - 1}{c} \ln X = a_0 + a_1 \ln \left(\frac{pQ}{w} \right), \tag{20}$$

where $a_0 = (\ln c\gamma)/c$ and $a_1 = 1/c$. By and large, then, a Weibull damage abatement function implies that demand should be specified as the function in (20).

This demand relation is nonlinear in the parameters; hence, the parameters cannot be estimated by straightforward linear regression. It does seem, however, that a fairly simple iterative procedure could be used. The first stage of such a procedure would involve an OLS regression of $X + \ln X$ on a constant and $\ln [pQ/w]$; for reasonable values of c, $(c - 1)/c$ will be quite close to one so that $X + \ln X$ will be a good approximation for the left-hand side of (20). Estimates of c and γ can be derived easily from the estimates of a_0 and a_1. The approximation error can be reduced by using the estimate of c obtained from such a regression to recalculate the left-hand side and redoing the OLS regression using the recalculated value of the dependent variable, a step which can be repeated as many times as may seem desirable.

Changes in Damage Control Agent Productivity

Damage control agents differ from normal inputs in a second important way, namely, in the manner in which their utilization responds to environmentally induced changes in their productivity. Consider what happens to demand for a normal factor of production when its productivity decreases because of some change in the productive environment. Decreased factor productivity means that total output will be less than it was previously for every level of input use. If the production function is a standard neoclassical one (specifically, if output is zero when use of any input is zero, if the marginal productivity of any factor is quite large at a zero level of utilization, and if marginal productivity is monotonically decreasing in factor use, i.e., the production function is concave), then this decline in factor productivity means that the marginal productivity of the factor will decrease at every level of factor use and, hence, that the level of utilization of that factor will also decline. In short, an environmentally induced decrease in productivity of a factor will decrease demand for it.

This line of argument was put forward for the case of pesticides by Carlson in his empirical study of the impact of resistance on pesticide use. Carlson argued that the development of resistance implied decreasing marginal pesticide productivity over time and, thus, that the demand should fall for pesticides to which resistance was developing. The standard characterization of this phenomenon, however, is that farmers' typical short-run response to the development of resistance to some pesticides is to increase usage levels as compensation for the decrease in pesticide productivity. Use of the affected pesticide decreases only when productivity is so low that alternative pesticides become more efficient. This pattern has been observed in every case in which resistance has eroded pesticide productivity over time. In fact, it is further borne out by the results of Carlson's study. In his investigation of pesticide demand, he found that resistance measures were positively correlated with demand for organophosphates (to which resistance had emerged only recently) while they were negatively correlated with demand for DDT--a chemical to which resistance was quite extensive.

Similar phenomena occur in most types of damage abatement. For example, bacteria populations typically develop resistance to antibiotics, necessitating the use of larger doses to achieve satisfactory control of infections. Dams and other flood control devices are subjected to water erosion, gradually weakening their ability to prevent floods and necessitating additional investment in repair and renovation. Criminals tend to find ways of coping with each improvement in prevention technology making further improvements a continual necessity. In short, because damage abatement typically involves natural systems in which damaging agents tend to adapt to abatement efforts, declines in damage control agent productivity tend to be the rule rather than the exception.

Treating damage control agents as normal inputs implies that farmers, medical practitioners, crime prevention experts, and others respond irrationally to environmentally induced changes in damage control agent productivity. By contrast, analyzing damage control agents in the context of a damage abatement function supports fully the rationality of their behavior in such situations. The optimality of increased damage control agent usage is shown easily under short-run profit maximization using the model of damage abatement introduced above.

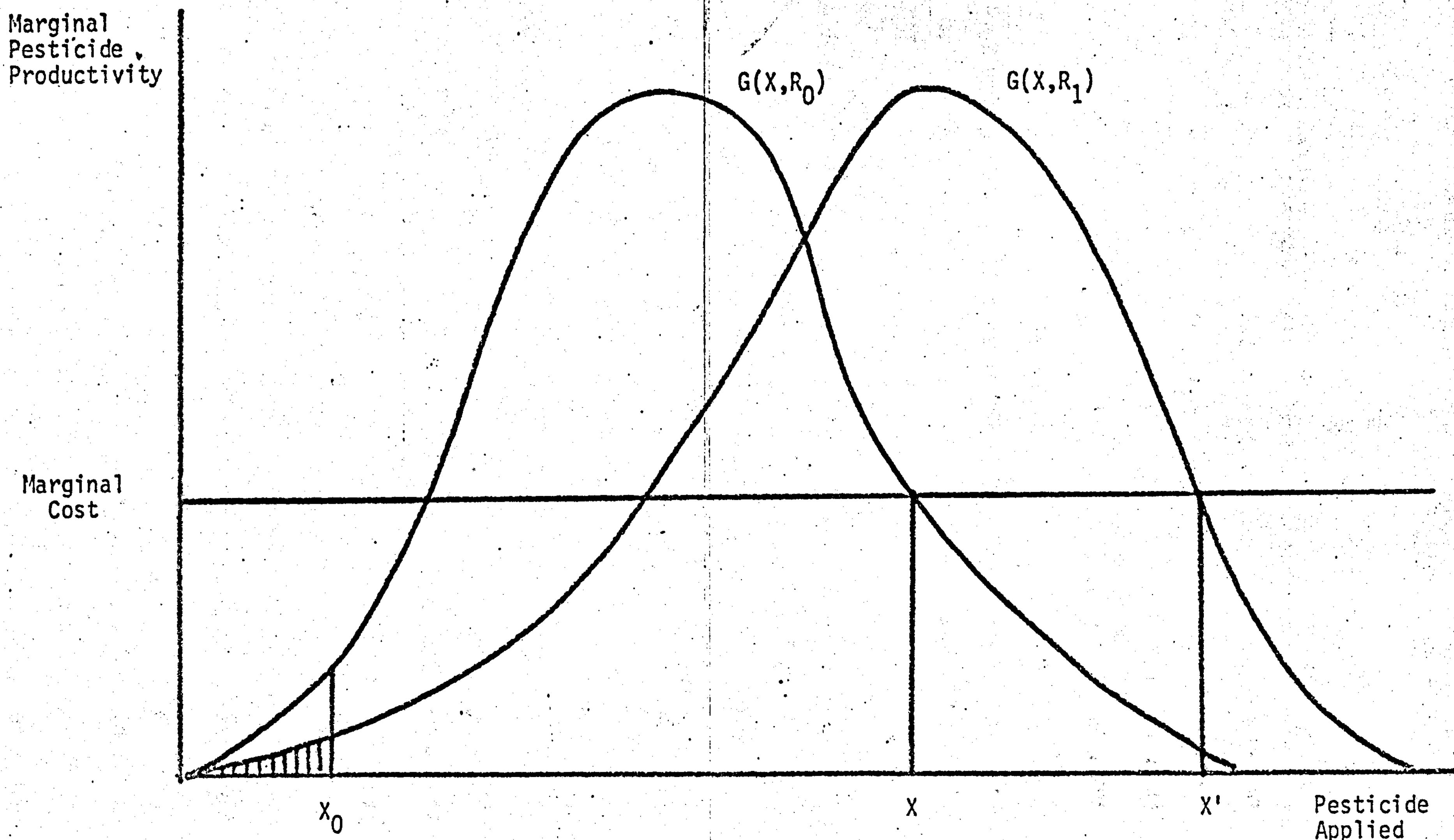
The types of changes discussed above have the effect of reducing the effectiveness of any given level of damage control agent applied. Any given amount of damage control agent will thus abate smaller proportion of damage than before; in other words, more damage control agent is required to achieve any given level of abatement.

To capture this effect, we redefine the damage abatement function, $G(\cdot)$, as a function of the amount of damage control agent applied, X, and the level of resistance, R, where $G(X, R_1) \leq G(X, R_0)$ for $R_0 < R_1$ and for all X; in other words, $G_R \leq 0$ for all X. In fact, we will define R such that the strict inequality holds everywhere but at the minimal and maximal levels where increases in resistance may have no effect.

For ease of analysis, we will impose two additional restrictions. First, we will consider the effect of resistance only for the case of unimodal damage abatement functions which are the only ones used for empirical purposes. Second, we assume that the curves representing marginal effectiveness for two different levels of resistance cross only once; in essence, this assumption merely says that increased levels of resistance do not distort the shape of the marginal effectiveness function too much. Together, these imply that $G_{XR} > 0$--that increased resistance increases the marginal effectiveness of the damage control agent--in the economic region. The reason for this is simple. For small values of X, increased resistance implies that marginal effectiveness must decrease. As shown in figure 2, only when $G_X(X_0, R_1) < G_X(X_0, R_0)$ will $G(X_0, R_1)$, the area under the new marginal effectiveness curve, be less than $G(X, R_0)$. Formally, $\int_0^{X_0} G_X(X, R_1)\, dX < \int_0^{X_0} G_X(X, R_0)\, dX$ implies that $G_{XR} < 0$ for small values of X. Since the two functions must both attain a value of 1 at the maximal dose level, however, G_{XR} must

FIGURE 2

IMPACT OF INCREASED RESISTANCE ON MARGINAL PESTICIDE PRODUCTIVITY



be positive for at least some X; the single crossing assumption ensures that this condition will not be reversed once it is attained.

The impact of increased resistance on damage control agent demand can be analyzed formally via total differentiation of the first-order conditions given by (3), amended to include the shifter R in the damage abatement function. Rearrangement of the resulting equation yields:

$$\frac{\partial X}{\partial R} = -\frac{\varepsilon_G}{G}\left[G_{XR} + \frac{G_X G_R}{G\varepsilon_G}\right]. \tag{21}$$

The expression on the right-hand side of (21) is positive whenever $G_{XR} > 0$; for the cases we are considering, the latter is true everywhere in the economic region and, thus, $\partial X/\partial R > 0$ for all the relevant application levels.

Why this is so can be seen easily in figure 2. The condition $G_{XR} > 0$ means that the economic region of the marginal effectiveness curve shifts to the right for all X's (as does the marginal value product curve) as shown by the change from $G(X_1\ R_0)$ to $G(X_1\ R_1)$. For any given level of marginal cost, the new level of demand, X', is then necessarily greater than the old level X.

Under the most commonly encountered conditions, then, an environmentally induced reduction in damage control agent productivity has effects on a damage control agent that are completely the opposite of the effect it would have on a normal input. Increased resistance <u>increases</u> marginal effectiveness and, hence, marginal productivity. The optimal profit-maximizing response, obviously, is to increase damage control agent use precisely as has been observed in such situations.

Conclusions

This paper demonstrates the importance of incorporating correct specification of damage abatement processes in the estimation of production functions and input productivity. First, it shows that the use of traditional specifications (e.g., the Cobb-Douglas) leads to overestimation of the productivity of damage-control inputs and underestimation of the productivity of other inputs. When and if such estimates are used in policy determination, the resulting errors can be quite serious. In the case of pesticides, for instance, a policymaker guided by the econometric studies available would be led to encourage more extensive and intensive use of pesticides--at a time when pesticides were extremely overutilized.

The paper also shows that traditional specifications produce misleading predictions when damage control agent productivity is changing over time. Traditional specifications suggest that the spread of resistance will lead to the reductions in the use of a damage control agent. In contrast, the specification proposed here captures the phenomenon that actually occurs, namely, that the use of a damage control agent will increase in response to resistance and that it will decrease only when resistance is so widespread that alternative measures are more cost effective.

Finally, the paper shows that a more sophisticated approach to damage abatement in production (like the one proposed here) can be incorporated into econometric work at little or no extra computational cost. Many of the distributions especially relevant in this context yield easily estimated damage control agent demand relations from which most or all of the structural parameters can be recovered. The general availability of nonlinear estimation packages removes much, if not all, of the remaining difficulties associated with direct estimation of production. In sum, the specification of damage abatement proposed here adds considerable sophistication and accuracy to the analysis of the role of damage control agents in production without making estimation any more difficult. It should thus prove to be quite useful for improving quantitative decision-making in all areas in which damage abatement is an important factor.

APPENDIX

"Proof of the Upward Bias in Cobb-Douglas Estimates of Damage Control Agent Productivity"

To investigate the impact of using the standard Cobb-Douglas form (12) in place of the true model given in (11), consider the standard Cobb-Douglas form as an approximation to the true model. Specifically, consider the Taylor series expansion of ln G(X) around $\bar{x}$, the mean value of ln X. Then, X becomes a function of x: $X(x) = e^x$ since $X = e^{\ln x}$; we thus have $\partial X/\partial x = \partial^2 X/\partial x^2 = e^x = X$.

The approximation is

$$\ln G(X) = \ln G(e^{\bar{x}}) + \eta_G(e^{\bar{x}})\,(x - \bar{x})$$

(A1)

$$+ \frac{1}{2} \cdot \frac{\eta_G(e^{\bar{x}})}{e^{\bar{x}}}\,[1 + \eta_g(e^{\bar{x}}) - \eta_G(e^{\bar{x}})]\,(x - \bar{x})^2 + \ldots$$

The approximated model is

(A2) $$q = \alpha + z\beta + \gamma \ln G(e^{\bar{x}}) + \gamma\eta_G(e^{\bar{x}})\,(x - \bar{x}) + v$$

where v, as we see from (13), is the sum of the higher order terms of the Taylor expansion and of the white-noise random variable u. The model given by (A2) is more conveniently written

(A3) $$q = \tilde{\alpha} + z\beta + x\tilde{\gamma} + v$$

where $\tilde{\alpha} = \alpha + \gamma[\ln G(e^{\bar{x}}) - \eta_G(e^{\bar{x}})\, x]$ and $\tilde{\gamma} = \gamma\, \eta_G(e^{\bar{x}})$. Since the error term v contains terms in $(x - \bar{x})^2$, $(x - \bar{x})^3$, and so on, which are undoubtedly correlated with x and may well be correlated with z, the OLS estimators $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ will give biased estimates of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$. Assuming, however, that the coefficients of these terms are suitably small, it is easy to show that the OLS estimators will be consistent for $\hat{\alpha}$, β, and $\hat{\gamma}$.[2]

Letting $y = [1\ z]$ and $\delta = \begin{bmatrix} \tilde{\alpha} \\ \beta \end{bmatrix}$, rewrite the model as

$$q = y\delta + x\gamma + v. \tag{A4}$$

Let $M_x = I - x(x'x)^{-1} x'$ and $M_y = I - y(y'y)^{-1} y'$. Then the OLS estimators, $\hat{\delta}$ and $\hat{\gamma}$, are

$$\begin{aligned} \hat{\delta} &= \delta + \left[\frac{y'y}{n} - \left(\frac{y'x}{n}\right)\left(\frac{x'x}{n}\right)^{-1}\left(\frac{x'y}{n}\right)\right]^{-1}\left[\frac{y'v}{n} - \left(\frac{y'x}{n}\right)\left(\frac{x'x}{n}\right)^{-1}\left(\frac{x'v}{n}\right)\right] \\ \hat{\gamma} &= \tilde{\gamma} + \left[\frac{x'x}{n} - \left(\frac{x'y}{n}\right)\left(\frac{y'y}{n}\right)^{-1}\left(\frac{y'x}{n}\right)\right]^{-1}\left[\frac{x'v}{n} - \left(\frac{x'y}{n}\right)\left(\frac{y'y}{n}\right)^{-1}\left(\frac{y'v}{n}\right)\right]. \end{aligned} \tag{A5}$$

As long as $(x'x)/n$, $(y'y)/n$, and $(x'y)/n$ converge to finite numbers as the sample size gets large and as long as the coefficients of the terms in v are of order smaller than the sample size, the estimators $\hat{\delta}$ and $\hat{\gamma}$ will converge to δ and $\tilde{\gamma}$, respectively, as the sample size gets large.

However, $\tilde{\alpha}$ and $\tilde{\gamma}$ are biased measures of α (the productivity of natural factors and omitted variables) and $\gamma\eta_G$ (the productivity of the damage control agent), respectively.

Consider first the case of $\tilde{\alpha}$. As we saw above,

$$\tilde{\alpha} = \alpha + \gamma\, [\ln G(e^{\bar{x}}) - \eta_G(e^{\bar{x}})\, \bar{x}]. \tag{A6}$$

The term in the square brackets is negative since $\ln G(e^{\bar{x}}) \leq 0$ and $\eta_G(e^{\bar{x}})$ $\bar{x} \geq 0$. As a result, $\tilde{\alpha} < \alpha$: The OLS estimator from the standard Cobb-Douglas specification underestimates the productivity of natural factors and omitted variables.

The bias in $\tilde{\gamma}$ is more subtle. The measure of marginal factor productivity generally derived from econometric studies is the marginal productivity of a factor evaluated at the mean levels of output and all the relevant inputs; this, for instance, is the measure used in the pesticide studies conducted by Headley; Campbell; and Carlson. For the case of the damage control agent X, this is

(A7) $$\widehat{\frac{\partial Q}{\partial X}} = \frac{\gamma\, \bar{Q}\, \eta_G(\bar{X})}{\bar{X}} \, .$$

The estimate derived from the standard specification is

(A8) $$\frac{\partial Q}{\partial X} = \frac{\widehat{\gamma\, \eta_G(e^{\bar{x}})}\, \bar{Q}}{\bar{X}} \, .$$

Now, ln X is a concave function of X; hence, by Jensen's inequality, $E \ln X < \ln EX$, i.e., $\bar{x} < \ln \bar{X}$. Since e^X is monotonically increasing in X, $e^{\bar{x}} < e^{\ln\bar{X}} = \bar{X}$.

Next, consider the behavior of η_g in the economic region. It is straightforward to show that

(A9) $$\frac{\partial \eta_G}{\partial X} = \frac{\eta_G}{X}\left[1 + \eta_g - \eta_G\right] .$$

Since $\lambda\ \eta_g < -1$ because $g(X)$ is a probability density and since $\eta_G > 0$ for the same reason, it is evident from (A9) that $\partial\eta_G/\partial X < 0$, i.e., $\eta_G(X)$ is monotonically decreasing in the range of economic use. This fact implies that $\eta_G(\overline{e^X}) > \eta_G(\overline{X})$ and, hence, that the estimate of marginal productivity derived from the standard specification is biased upward.

Moreover, as X gets larger, the rate of decrease of η_G increases and, therefore, the difference between $\eta_G(e^X)$ and $\eta_G(X)$ increases also. Since damage control agents tend to be used at close to maximum effectiveness, i.e., X tends to be quite large, this bias will also tend to be quite substantial in practical examples.

Footnotes

[1]The damage abatement function, G(X), may be a function of other variables besides the damage control agent X. In the case of pesticides, for instance, G(•) may also be a function of pest population and other indicators of crop ecosystem status. In the case of immunization, G(•) may also be a function of age and other indicators of health status. Since the concern here is with input selection, these other variables are treated as parameters and, hence, G(•) is treated as a single-valued function of X.

[2]Briefly, the specific condition for the consistency of the OLS estimators is that the coefficients of the higher order terms of the Taylor expansion of ln G(X) be of an order of magnitude smaller than n, the number of observations in the sample.

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