Contents

Preface xv
Acknowledgments xix

I General Theory 1

1 The Expected Utility Model 3
  1.1 Simple and Compound Lotteries 3
  1.2 Axioms on Preferences under Uncertainty 4
  1.3 The Expected Utility Theorem 6
  1.4 Critics of the Expected Utility Model 9
    1.4.1 The Allais Paradox 10
    1.4.2 The Allais Paradox and Time Consistency 11
  1.5 Concluding Remark 14
  1.6 Exercises and Extensions 14

2 Risk Aversion 17
  2.1 Characterization of Risk Aversion 17
  2.2 Comparative Risk Aversion 18
  2.3 Certainty Equivalent and Risk Premium 20
  2.4 The Arrow-Pratt Approximation 21
  2.5 Decreasing Absolute Risk Aversion 24
  2.6 Some Classical Utility Functions 25
  2.7 Test for Your Own Degree of Risk Aversion 29
  2.8 An Application: The Cost of Macroeconomic Risks 32
  2.9 Conclusion 34
  2.10 Exercises and Extensions 35
3 Change in Risk 39
  3.1 The Extremal Approach 40
  3.2 Second-Order Stochastic Dominance 42
  3.3 Diversification 45
  3.4 First-Order Stochastic Dominance 46
  3.5 Concluding Remark 47
  3.6 Exercises and Extensions 48

II The Standard Portfolio Problem 51

4 The Standard Portfolio Problem 53
  4.1 The Model and Its Basic Properties 53
  4.2 The Case of a Small Risk 55
  4.3 The Case of HARA Functions 57
  4.4 The Impact of Risk Aversion 58
  4.5 The Impact of a Change in Risk 59
  4.6 Concluding Remark 61
  4.7 Exercises and Extensions 62

5 The Equilibrium Price of Risk 65
  5.1 A Simple Equilibrium Model for Financial Markets 65
  5.2 The Equity Premium Puzzle 68
  5.3 The Equity Premium with Limited Participation 71
  5.4 The Equity Premium and the Integration of International Financial Markets 73
  5.5 Conclusion 75
  5.6 Exercises 76

III Some Technical Tools and Their Applications 79

6 A Hyperplane Separation Theorem 81
  6.1 The Diffidence Theorem 81
  6.2 Link with the Jensen’s Inequality 88
  6.3 Applications of the Diffidence Theorem 89
    6.3.1 Diffidence 89
    6.3.2 Comparative Diffidence 90
    6.3.3 Central Risk Aversion 91
    6.3.4 Central Riskiness 92
6.4 The Covariance Rule 94
6.5 Conclusion 95
6.6 Exercises and Extensions 96

7 Log-Supermodularity 99
7.1 Definition 99
7.2 Log-Supermodularity and Single Crossing 102
  7.2.1 A Theoretical Result 102
  7.2.2 Applications to the Standard Portfolio Problem 103
  7.2.3 Jewitt's Preference Orders 104
7.3 Expectation of a Log-Supermodular Function 105
  7.3.1 A Theoretical Result 105
  7.3.2 Two Applications 106
7.4 Concluding Remark 107
7.5 Exercises and Extensions 107
7.6 Appendix 108

IV Multiple Risks 111

8 Risk Aversion with Background Risk 113
8.1 Preservation of DARA 114
8.2 The Comparative Risk Aversion Is Not Preserved 117
8.3 Extensions with Dependent Background Risk 119
  8.3.1 Affiliated Background Risk 119
  8.3.2 The Comparative Risk Aversion in the Sense of Ross 121
8.4 Conclusion 123
8.5 Exercises and Extensions 124

9 The Tempering Effect of Background Risk 125
9.1 Risk Vulnerability 126
9.2 Risk Vulnerability and Increase in Risk 130
  9.2.1 Increase in Background Risk 130
  9.2.2 Increase in the Endogenous Risk 130
9.3 Risk Vulnerability and the Equity Premium Puzzle 131
9.4 Generalized Risk Vulnerability 132
9.5 Standardness 135
9.6 Conclusion 138
9.7 Exercises and Extensions 139

10 Taking Multiple Risks 141
10.1 The Interaction between Asset Demand and Small Gambles 142
10.2 Are Independent Assets Substitutes? 144
   10.2.1 The i.i.d. Case 144
   10.2.2 The General Case 150
10.3 Conclusion 153
10.4 Exercises and Extensions 153

11 The Dynamic Investment Problem 155
11.1 Static versus Dynamic Optimization 157
11.2 The Standard Portfolio Problem 158
   11.2.1 The Model 158
   11.2.2 The HARA Case 160
   11.2.3 A Sufficient Condition for Younger People to Be More Risk-Averse 161
11.3 Discussion of the Results 165
   11.3.1 Nonlinear Risk Tolerance 165
   11.3.2 Nondifferentiable Marginal Utility 166
11.4 Background Risk and Time Horizon 168
   11.4.1 Investors Bear a Background Risk at Retirement 168
   11.4.2 Stationary Income Process 171
11.5 Final Remark 172
11.6 Exercises and Extensions 173

12 Special Topics in Dynamic Finance 175
12.1 The Length of Periods between Trade 175
12.2 Dynamic Discrete Choice 179
12.3 Constraints on Feasible Strategies 183
12.4 The Effect of a Leverage Constraint 185
   12.4.1 The Case of a Lower Bound on the Investment in the Risky Asset 185
   12.4.2 The Case of an Upper Bound on the Investment in the Risky Asset 187
12.5 Concluding Remarks 190
12.6 Exercises and Extensions 190
V The Arrow-Debreu Portfolio Problem 193

13 The Demand for Contingent Claims 195
13.1 The Model 196
13.2 Characterization of the Optimal Portfolio 197
13.3 The Impact of Risk Aversion 200
13.4 Conclusion 201
13.5 Exercises and Extensions 202

14 Risk on Wealth 205
14.1 The Marginal Propensity to Consume in State $\pi$ 206
14.2 The Preservation of DARA and IARA 208
14.3 The Marginal Value of Wealth 210
14.4 Aversion to Risk on Wealth 211
14.5 Concluding Remark 212
14.6 Exercises and Extensions 212

VI Consumption and Saving 215

15 Consumption under Certainty 217
15.1 Time Separability 217
15.2 Exponential Discounting 218
15.3 Consumption Smoothing under Certainty 219
15.4 Analogy with the Portfolio Problem 221
15.5 The Social Cost of Volatility 224
15.6 The Marginal Propensity to Consume 226
15.7 Time Diversification and Self-Insurance 227
15.8 Concluding Remark 232
15.9 Exercises and Extensions 232

16 Precautionary Saving and Prudence 235
16.1 Prudence 235
16.2 The Demand for Saving 239
16.3 The Marginal Propensity to Consume under Uncertainty 239
16.3.1 Does Uncertainty Increase the MPC? 240
16.3.2 Does Uncertainty Make the MPC Decreasing in Wealth? 241
16.4 More Than Two Periods 242
16.4.1 The Euler Equation 242
16.4.2 Multiperiod Precautionary Saving 244
16.5 Illiquid Saving under Uncertainty 246
16.6 Conclusion 247
16.7 Exercises and Extensions 248

17 The Equilibrium Price of Time 249
17.1 Description of the Economy 250
17.2 The Determinants of the Interest Rate 252
17.2.1 The Interest Rate in the Absence of Growth 252
17.2.2 The Effect of a Sure Growth 253
17.2.3 The Effect of Uncertainty 254
17.3 The Risk-Free Rate Puzzle 256
17.4 The Yield Curve 258
17.4.1 The Pricing Formula 258
17.4.2 The Yield Curve with HARA Utility Functions 260
17.4.3 A Result When There Is No Risk of Recession 261
17.4.4 Exploring the Slope of the Yield Curve When There Is a Risk of Recession 264
17.5 Concluding Remark 267
17.6 Exercises and Extensions 268

18 The Liquidity Constraint 269
18.1 Saving as a Buffer Stock 270
18.2 The Liquidity Constraint Raises Risk Aversion 272
18.3 The Liquidity Constraint and the Shape of Absolute Risk Tolerance 273
18.4 Numerical Simulations 277
18.5 Conclusion 279
18.6 Exercises and Extensions 281

19 The Saving-Portfolio Problem 285
19.1 Precautionary Saving with an Endogenous Risk 285
19.1.1 The Case of Complete Markets 285
19.1.2 The Case of the Standard Portfolio Problem 287
19.1.3 Discussion of the Results 288
### Contents

19.2 Optimal Portfolio Strategy with Consumption 290
19.3 The Merton-Samuelson Model 291
19.4 Concluding Remark 295
19.5 Exercises and Extensions 295

20 Disentangling Risk and Time 297
20.1 The Model of Kreps and Porteus 298
20.2 Preferences for an Early Resolution of Uncertainty 299
20.3 Prudence with Kreps-Porteus Preferences 300
20.4 Conclusion 302
20.5 Exercises and Extensions 303

VII Equilibrium Prices of Risk and Time 305

21 Efficient Risk Sharing 307
21.1 The Case of a Static Exchange Economy 307
21.2 The Mutuality Principle 309
21.3 The Sharing of the Social Risk 311
21.3.1 Decomposition of the Problem 311
21.3.2 The Veil of Ignorance 312
21.3.3 Efficient Sharing Rules of the Macro Risk 312
21.3.4 A Two-Fund Separation Theorem 314
21.3.5 The Case of Small Risk per Capita 315
21.4 Group’s Attitude toward Risk 316
21.4.1 The Representative Agent 316
21.4.2 Arrow-Lind Theorem 317
21.4.3 Group Decision and Individual Choice 317
21.5 Introducing Time and Investment 319
21.6 A Final Remark: The Concavity of the Certainty Equivalent Functional 321
21.7 Conclusion 323
21.8 Exercises and Extensions 323
21.9 Appendix 325

22 The Equilibrium Price of Risk and Time 327
22.1 An Arrow-Debreu Economy 327
22.2 Application of the First Theorem of Welfare Economics 328
22.3 Pricing Arrow-Debreu Securities 329