CHAPTER 1

The Economics of Social Networks

Matthew O. Jackson*

1 INTRODUCTION

Social networks are the fabric of many of our interactions. Such networks include the relationships among friends and relatives with whom we share information and favors on a regular basis, and reach as far as influencing decisions by many of the world's companies regarding with whom and how they conduct their business. The many regularities in network structure across applications make a scientific study of social networks a possibility. The deep and pervasive impact that networks have on behavior makes such a study a necessity.

The science of social networks was initiated by sociologists more than a century ago, and has grown to be a central field of sociological study over the past fifty years.[1] Over that same period, a mathematical literature on the structure of random graphs moved steadily along, with intermittent ties to the sociological literature.[2] While economists have occasionally showed interest in networks, an explosion of studies of networks using game-theoretic modeling techniques and economic perspectives has occurred over the last decade.[3]

* HSS 228-77, California Institute of Technology, Pasadena, California 91125, USA, jacksonm@hss.caltech.edu and http://www.hss.caltech.edu/~jacksonm/Jackson.html. I thank the Lee Center for Advanced Networking, The Guggenheim Foundation, and the Center for Advanced Studies in Behavioral Sciences for financial support. I am very grateful to collaborators on networks projects, who have helped me learn about social networks and the fascinating questions they present, and who are inexorably tied to the views I take herein. So I thank (in chronological order), Asher Wolinsky, Alison Watts, Bhaskar Dutta, Anne van den Nouweland, Toni Calvó-Armengol, Francis Bloch, Gary Charness, Alan Kirman, Jernej Copic, Brian Rogers, Dunia Lopez-Pintado, and Leeat Yariv. I also thank Yann Bramoullé, Toni Calvó-Armengol, Yannis Ioannides, Alan Kirman, Dunia Lopez-Pintado, Laurent Mathevet, Torsten Persson, Nicolas Quérou, Alessandro Vespignani, Stanley Wasserman, Duncan Watts, and Leeat Yariv for comments on an earlier draft. I am grateful to Tom Palfrey for an insightful discussion of the paper.

1 See Freeman (2004) for some history of thought of the sociology literature.

2 See Bollobás (2001) for a survey of the random graph literature.

3 The books edited by Dutta and Jackson (2003) and Demange and Wooders (2004) contain surveys.

A recent awakening of an interest in social networks has also occurred in the computer science and statistical physics literatures, mainly over the past five or six years.[4] While these literatures are (slowly) becoming aware of each other, and on occasion drawing from one another, they are still largely distinct in their methods, interests, and approaches. My goal here is to provide some perspective on the research from these literatures, with a focus on the formal modeling of social networks, and to highlight some of the strengths, weaknesses, and potential synergies between the two main approaches.

Given the breadth of these combined literatures, and the fact that there are surveys covering the various literatures,[5] my aim here is not to try to give a comprehensive overview of the literatures, but rather to try to put some of the main contributions and techniques of formal modeling of social networks in context and to relate them to each other. I focus on two main threads of the literatures: the first is models of the formation of networks and the second is models of how social behavior and economic outcomes are influenced by network structure.

In order to provide some context, I start by giving some basic background on social networks and a very cursory look at a few things that have been learned from empirical studies. Next, I turn to discuss models of formation of networks. Here, I distinguish between two different approaches that have been taken. One has its roots in the random graph literature and models formation by specifying either some stochastic process or an algorithmic process through which the links in a network are formed. This literature has mainly deduced properties of large networks. The second approach is game theoretic and stems from the economics literature. It has mainly focused on models where the links are formed at the discretion of the nodes that derive benefits and face costs associated with various links and network configurations. These two approaches lead to complementary insights regarding networks, each of which is adapted to answering different sorts of questions. They also have different strengths and weaknesses that I highlight. Finally, I discuss models that take network structure as a given and study the influence that networks have on social and/or economic outcomes. This last area of study shows why the science of social networks is important for more than just an understanding of the networks themselves.

[4] See Newman (2003).

[5] The sociology literature is too vast for any article to adequately survey, but introductory texts, such as Wasserman and Faust (1994), as well as the recent history of thought book by Freeman (2004), are useful starting points. Concerning the economics literature, see Jackson (2003, 2004) for strategic modeling of networks; van den Nouweland (2004) for graphs and networks in cooperative game theory; Goyal (2004) for learning on networks; Ioannides and Datcher-Loury (2004) for networks in labor economics; Page and Kamat (2004) for farsighted formation of networks; and Bloch (2004) for networks in industrial organization. See Newman (2003, 2004) for surveys covering some of the recent statistical physics and part of the computer science literatures. There are also books that touch on some parts of the physics literature, such as Watts (1999) and Barabasi (2002). A text that bridges some of the modeling from the various literatures is by Jackson (2005).

2 SOME BACKGROUND ON NETWORKS

The systematic study of social networks by sociologists dates from the 1920s and 30s, took root in the 1960s, and has grown rapidly over the past four decades.[6] That literature includes many case studies from which has emerged a rich mosaic of characteristics that are shared by many social networks, as well as a taxonomy for measuring and describing social networks and a broad set of hypotheses and theories about network form and influence. Much of what I discuss in this section is either directly from that literature, or was influenced by it.

Just to get a feeling for one such case study, consider a network analyzed by Padgett and Ansell (1993). It is the network of marriages between the key families in Florence in the 1430s. Figure 1.1 provides the links between the key families in Florence at that time, where a link represents a marriage between members of the two linked families.[7]

As Padgett and Ansell (1993) explain, during this time period the Medici (with Cosimo de' Medici playing the key role) rose in power and largely consolidated control of the business and politics of Florence. Previously Florence had been ruled by an oligarchy of elite families. A key to understanding this, as Padgett and Ansell (1993) detail, can be seen in the network structure. To the extent that marriage relationships were keys to communicating information, business deals, and reaching political decisions, the Medici were much better positioned than other families, at least according to some measures of betweeness or centrality. Padgett and Ansell (1993) point out that, "Medician political control was produced by network disjunctures within the elite, which the Medici alone spanned." It should be emphasized that the Medici came to have such a special position in the network through careful planning. As Padgett and Ansell (1993) say (footnote 13), "The modern reader may need reminding that all of the elite marriages recorded here were arranged by patriarchs (or their equivalents) in the two families. Intraelite marriages were conceived of partially in political alliance terms." Thus, in order to understand how this network, and not some other network, came to arise it is important to have models of strategic network formation, a theme that I return to below.

2.1 Some Notation

Let $N = \{1, 2, \ldots, n\}$ denote a set of *nodes*, which represent the social agents that might be tied in a network of social relationships. In the example above, these are the Florentine families. In the next example these are individual people (researchers), and in other examples they might be firms, web pages, countries, etc.

A *network* g can be represented by an $n \times n$ matrix taking on values 0 or 1. The idea is that if $g_{ij} = 1$, then i is linked to j. In various applications, it might

[6] Again, see Freeman (2004) for some history of thought. Interestingly, while Freeman laments the disconnect between the traditional sociology literature and the emerging physics literature on networks, the gulf between the sociology and economics literatures seems to be equally large.

[7] The data here were originally collected by Kent (1978), but were first coded by Padgett and Ansell (1993) who discuss the network relationships in more detail.

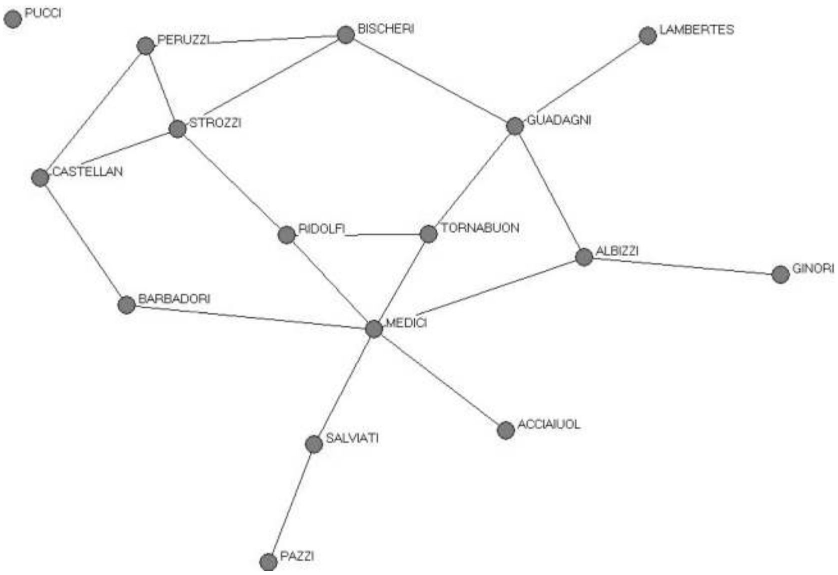

Figure 1.1. 15th-Century Florentine Marriages (Padgett and Ansell (1993))

be that these links are undirected, as in the Florentine families example where marriage is a reciprocal relationship. In such settings $g_{ij} = g_{ji}$ by necessity. In other applications, such as an example where a link represents a citation of one research article by another, the network is naturally directed. In such cases, it is possible that $g_{ij} = 1 \neq 0 = g_{ji}$.[8]

For simplicity, I write ij to represent the *link* between i and j, and also write $ij \in g$ to indicate that i and j are linked under the network g. Shorthand notations for the network obtained by adding or deleting a link ij to or from an existing network g are $g + ij$ and $g - ij$, respectively.

For any network g and agent or node i, let $N_i(g)$ be the *neighborhood* of i in g, that is, the set of agents linked to i in the network g, so that $N_i(g) = \{j \mid ij \in g\}$.

The degree is the most basic characteristic of a node – it represents the number of links that each node has, and is thus simply the cardinality of $N_i(g)$. Generally, there tends to be a wide range of degrees across nodes within a network, and different applications will have very different distributions of degrees across nodes, a topic I return to below.

A *path* in a network $g \in G$ between agents i and j is a sequence of agents $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$, with $i_1 = i$ and $i_K = j$. The *length* of such a path is $K - 1$, the number of links involved.[9]

[8] In some applications, the strength of a link or some other aspect of link may be important, or there may be different types of links that can be simultaneously held between nodes. For the purposes of this article, I stick with the basic model.

[9] In the case of directed networks, one can keep track of directed paths as well as undirected ones. I will be explicit when necessary, and otherwise assume that links are treated as if they are not directed.

A *component* of a network is a maximal connected subgraph. That is, g' is a component of g if: (a) g' is a subnetwork of g (so $ij \in g'$ only if $ij \in g$), (b) $ij \in g'$ and $k\ell \in g'$ implies that there is a path between i and k in g', and (c) $ij \in g'$ and $ik \in g$ implies $ik \in g'$. The network pictured in Figure 1.3 has two components, one consisting of the isolated node 25, and the other consisting of the graph between nodes 1 to 24.

The *distance* between two nodes i and j, denoted $d(i, j)$, is the minimum path length between i and j (and set to be infinite if no such path exists).

The *diameter* of a network g is defined as $\bar{d}(g) = \max_{i,j} d(i, j)$, the maximum distance between any two nodes. If a network is not connected (there are at least two nodes that have no path between them), then the diameter is infinite. As many social networks are not connected, the diameter is often reported for the largest component. For example, in Figure 1.1, the network is not connected as the Pucci are isolated, and the diameter of the largest component is 5 (the distance from the Pazzi to the Lambertes or the Pazzi to the Peruzzi).

Another characteristic of networks is referred to under a variety of names including cliquishness, transitivity, and *clustering*. While there are many variations, the basic idea is to measure how dense the network is on a very local level. Given a node, what fraction of that node's friends or neighbors are friends or neighbors of each other? In particular, if i has links to both j and k, are j and k linked to each other?[10] The percentage of times that the answer is "yes" with regard to a node i is i's clustering coefficient. One can then average across all nodes in the network. Thus the clustering for a node i is[11]

$$C_i(g) = \frac{\#\{jk \in g \mid k \neq j,\ j \in N_i(g),\ k \in N_i(g)\}}{\#\{jk \mid k \neq j,\ j \in N_i(g),\ k \in N_i(g)\}}.$$

In Figure 1.1, the clustering for the Medici is $1/15$, for the Bisteri $1/3$, and for the Guadagni 0. The average clustering coefficient is[12]

$$C^{avg}(g) = \sum_i \frac{C_i(g)}{n}.$$

Example 1 *Erdös Numbers and Co-authorship Networks Among Researchers*

[10] For a directed network, one can either treat links as if they are undirected, or else can look for cycles (when directed links ij and jk are present, one counts the percent of ki's).

[11] If the node i has fewer than two neighbors so that the denominator of $C_i(g)$ is 0, then one can adopt the convention of setting $C_i(g) = 1$. When averaging across i to determine average clustering, such a convention can make a difference and so it makes sense to ignore nodes that have fewer than two neighbors.

[12] Note that this weights the calculations by averaging across nodes rather than links. That is, a node that has just two neighbors is weighted the same as a node that has two hundred neighbors, even though the second node accounts for many more potential triangles in the network. An alternative measure simply examines the number of times the link ik is present over all combinations of pairs of links ij and ik in the network, and divides by the number of pairs of links present in the network. The difference between these two measures can be quite substantial.

Table 1.1. *Co-authorship networks*

	Biology	Economics	Math	Physics
Number of Nodes	1520521	81217	253339	52909
Avg. Degree	15.5	1.7	3.9	9.3
Avg. Path Length	4.9	9.5	7.6	6.2
Diameter	24	29	27	20
Clustering	.09	.16	.15	.45
% Size Largest Component	.92	.41	.82	.85

With some definitions in hand, let us turn to another example. These are networks that keep track of collaboration among researchers. Here a link represents the co-authorship of a paper during some time period covered by the study. The well-known and prolific mathematician Paul Erdös had many co-authors, and as a fun distraction many mathematicians (and economists for that matter) have found the shortest path(s) from themselves to Erdös. These networks are also of scientific interest themselves, as they tell us something about how research is conducted and how information and innovation might be disseminated. Such studies have now been conducted in various fields, including mathematics (Grossman and Ion (1995), de Castro and Grossman (1999)), biology and physics (Newman (2001, 2002)), and economics (Goyal, van der Leij and Moraga-González (2003)). Various statistics from these studies give us some impression of the network structure.[13]

Here we see that despite the noncomparabilities of the networks along many dimensions, average path length and diameters of each of the networks are very comparable. Moreover, these are of an order substantially smaller than the number of nodes in the network. This is an aspect of the "small-world" nature of social networks discussed below.

2.2 The Prevalence of Network Interactions

While the examples in the previous section give us an idea of the variety of networks that have been studied, it is also important for us to have an idea of what role networks might play in a society and how they might influence economic outcomes.

The most obvious and perhaps pervasive role of networks is as a conduit of information, and one of the most extensively documented roles for social networks in economics is that of contacts in labor markets.[14] The magnitude of use of social contacts as a method of matching workers and firms can be seen from various studies. One of the earliest studies, by Myers and Shultz (1951), was

[13] As these networks are not connected (there are many isolated authors), the figures for average path length and diameter are reported for the largest component.

[14] For a recent comprehensive overview of research on networks in labor markets see Ioannides and Loury (2004).

based on interviews with textile workers and found that 62 percent had found their first job through a social contact, in contrast with only 23 percent who applied by direct application, and the remaining 15 percent who found their job through an agency, ads, etc. A study by Rees and Shultz (1970) showed that these numbers were not peculiar to textile workers, but applied very broadly. For instance, the percentage of those interviewed who found their jobs through the use of social contacts as a function of their profession was: typist – 37.3 percent, accountant – 23.5 percent, material handler – 73.8 percent, janitor – 65.5 percent, and electrician – 57.4 percent. Moreover, the prevalent use of social contacts in finding jobs is robust across race and gender (see Corcoran, Datcher, and Duncan (1980)) and across country (see Pellizzari (2004)).

The role of social networks is not unique to labor markets, but it has been documented much more extensively. For example, networks and social interactions play a role in crime,[15] trade,[16] and social insurance,[17] as well as disease transmission, language and culture, and interactions of firms.

2.3 Some Basic Characteristics of Social Networks

Beyond the fact that social networks play a role in many interactions, we also know a great deal about some basic characteristics of social networks.

2.3.1 Small Worlds

One of the most influential studies of social networks was Stanley Milgram's (1967) ingenious "small-worlds" experiment. Milgram gave booklets with instructions to individuals in one place (Nebraska, in the original experiment). The objective was to get the booklet to a geographically distant individual (on the east coast), where the sender is given some information about the target (e.g., the person's name, occupation, and where they live). The key was that the subjects could only send the booklet to an acquaintance. The acquaintance could then forward the letter to another acquaintance, with the same objective of having the booklet eventually reach the target. The experiment collected information regarding the full chain that the booklets followed, including demographic information about each of the acquaintances along the route. One

[15] Reiss (1980, 1988) finds that two thirds of criminals commit crimes with others, and Glaeser, Sacerdote, and Scheinkman (1996) find that social interaction is important in determining criminal activity, especially with respect to petty crime, youth activity in crime, and in areas with less intact households.

[16] Uzzi (1996) finds that relation specific knowledge is critical in the garment industry and that social networks play a key role in that industry. Weisbuch, Kirman, and Herreiner (2000) study repeated interactions in the Marseille fish market and discuss the importance of the network structure.

[17] Fafchamps and Lund (2003) show that social networks are critical to the understanding of risk-sharing in rural Philippines, and De Weerdt (2002) provides similar analyses in Africa.

Table 1.2. *Comparisons across applications*

	WWW	Citations	Co-Author	Ham Radio	Prison	High School Romance
Number of Nodes	325729	396	81217	44	67	572
Randomness: *r*	0.5	.62	3.5	5.0	590	1000
Avg. Degree: *m*	4.5	5	1.7	3.5	2.7	.84
Avg. Clustering	.11	.07	.16	.06	.001	0

remarkable statistic was that roughly a quarter of the booklets reached their destination.[18] Of the chains that were successful, the maximum number of links that a booklet took was 12 and the median was 5! Given that these would generally not have taken the shortest routes from initial sender to target (as the senders are often not fully aware of the most efficient path to the target), these numbers were quite striking.

A simple back-of-the-envelope calculation gives some insight into this. If most individuals in the world have hundreds of acquaintances, then starting from a given individual, the network size (in terms of number of individuals reached) will expand by a factor on the order of a hundred raised to the power of the path length.[19] It will not take very long paths until the network is the size of the whole world's population.

2.3.2 *"High" Clustering*

While it is interesting that social networks exhibit small diameter and average path length, the same is also true of many other networks, including routing networks, power grids, and networks of neurons (e.g., see Watts (1999) and Newman (2003)). What tends to be a more distinguishing feature of social networks is their clustering (recall the definition above). Clustering is a simple but powerful concept that has roots tracing back to the work of Simmel (1908), who first explored triads (relationships between triples of individuals). Social networks tend to have significantly higher clustering coefficients than what would emerge if the links were generated by an independent random process. For example, Adamic (1999) finds a clustering coefficient of .11 for a portion of the www, which would compare with an expected clustering coefficient of .0002 for a (Bernoulli) randomly generated network with the same

[18] Given that twenty to thirty percent is a healthy response rate on a survey, and that having a booklet reach a destination required a chain of subjects to each respond, a twenty-five percent rate of reaching the target is impressive, especially in an unpaid experiment.

[19] This is clearly heuristic and a proper calculation is difficult, as one needs to account for overlap in neighborhoods, among other things. See Bollabás (2001) for some theorems bounding diameters in some classes of random graphs.

number of links. Figures for other networks are reported in Table 1.2 below, where we also see relatively high numbers compared to a benchmark random network. For example, if each link is formed with equal probability and link independently of each other, then the probability of two of node i's neighbors being connected to each other is simply the probability with which links are formed. In the first column of Table 1.2, this would be less than 5/325,000, as each node has an average of fewer than 5 links out of a potential number that is more than 325,000. The observed clustering of .11 is substantially higher.

2.3.3 Degree Distributions

Another easily identified property of a social network is its degree distribution. This gives some idea of the variation in the number of links across different nodes and provides us with some feeling for the shape of a network. Does it have "hub and spoke" like features where there are some very highly connected nodes and others with very few connections, or are connections more evenly distributed? Keeping track of the distribution of degrees in a network can be quite useful. For example, the degrees of the nodes in the Medici marriage network in Figure 1.1 are 0,1,1,1,1,2,3,3,3,3,3,3,4,4,6. From this we see that the Medici had more than twice the average degree (6 compared to 2.53) and twice the median degree.

One of the early studies documenting degree distributions was by Price (1965) who examined networks of citations among research articles. Price noticed that there were more highly connected and lowly connected nodes than what would be expected if links were selected independently and uniformly at random. Much of the recent interest in networks by statistical physicists was sparked by a similar study of Albert, Jeong and Barabasi (1999), which examined the structure of a portion of the www (in the Notre Dame domain). They also found a degree distribution that was distinctly different from what would have been generated by a random process of link formation where all links were equally likely. If links were formed uniformly at random with a link between any two nodes being formed independently of other links and with a probability p, then the degree distribution would approximate a binomial distribution, and would also be well-approximated by a Poisson distribution (see Section 3.1.1). Again, they found that the degree distribution had "fat tails," in that there were many more nodes with very high and very low degrees than would correspond to a binomial or Poisson distribution. In fact, they estimated that the distribution was approximately "scale-free" and followed a "power-law," where the relative frequency of nodes with a degree of k is proportional to $k^{-\gamma}$ for a parameter $\gamma > 1$.[20] The term "power law" clearly

[20] Such distributions date to Pareto (1896), after whom they are named, and have appeared in a wide variety of settings ranging from income distributions, and distribution of city

refers to the fact that the frequency can be expressed as the degree raised to a power. The term "scale-free" refers to the following property. Consider degree k and some other degree ck, for some scalar c. Their relative frequencies are $k^{-\gamma}/(ck^{-\gamma})$ or c^{γ}. Now consider some other degree k' and another degree ck'. Their relative frequencies are also c^{γ}. Thus, regardless of how we have rescaled things, relative frequencies depend only on relative sizes and not on the absolute scale.

An important caution to the literature is in order here. While it is clear that the degree distributions of many observed networks differ significantly from that of a purely random network; it is not clear that they are "scale-free." This is a point first made by Pennock et al. (2002).[21] A standard approach to outlining the degree distribution of many networks has been simply to plot the log(frequency) versus the log(degree) and see whether this "looks" linear. Of course, many things that are far from linear will appear linear on a log-log plot, as most of the data are squeezed into a small portion of the scale on a log-log plot; and such a distribution can be very difficult to distinguish from others, such as a lognormal distribution which can also appear quite linear. Simply fitting a line to the data on a log-log scale does not guarantee that the estimated coefficient means much of anything.

To get a better feeling for the shape of degree distributions, and whether most social networks exhibit features that are close to scale-free, it is possible to consider families of distributions and see which one best fits a given social network. We can do this with a family of degree distributions that have, at one extreme, networks whose links are generated uniformly at random, and at the other extreme, networks with scale-free distributions. Jackson and Rogers (2004) examine a family of degree distributions where the probability that a given node has degree k is given by $P(k) = c(k + rm)^{-(2+r)}$, where c is a constant (ensuring a sum to 1 across k's), m is the average degree, and r is a parameter which varies between 0 and ∞. More specifically, the model is one where new nodes are born over time and can attach to existing nodes either by choosing one uniformly at random or through a search process that makes the likelihood of meeting a given node proportional to the number of links the node already has. r represents the ratio of how many links are formed uniformly at random compared to how many are formed proportionally to the number of links existing nodes already have. As r approaches 0, the distribution converges to be scale-free, while as m tends to infinity the distribution converges to a negative exponential distribution, which corresponds to the degree distribution of a purely uniform and independent link formation process on a network that grows over time.

populations, to the usage of words in a language. For an informative overview, see Mitzenmacher (127).

21 See Eeckhout (2004) for a similar point regarding Zipf's law as applied to city sizes, and also Ioannides (2004) for a similar point.