NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2345

THE EFFECT OF AN ARBITRARY SURFACE-TEMPERATURE VARIATION ALONG A FLAT PLATE ON THE CONVECTIVE HEAT TRANSFER IN AN INCOMPRESSIBLE TURBULENT BOUNDARY LAYER

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SUMMARY

An analysis is performed to determine the effects on the rates of convective heat transfer produced by variations of the surface temperature with distance along a flat plate. The analysis is confined to the case of a low-speed turbulent boundary layer on a flat plate oriented parallel to the free-stream velocity. All the physical properties of the fluid are considered to be constant. The temperature distribution in the boundary layer and the local heat-transfer rate at the surface are obtained in the form of definite integrals, with the integrands containing the prescribed surface-temperature variation or its derivative with respect to the distance along the plate. Numerical evaluation of these integrals permits application of the results to any type of surface temperature distribution that may occur physically.

The basic solution of this analysis, one applying to a stepwise-discontinuous surface temperature in which the temperature is constant on either side of the discontinuity, is correlated with a set of existing experimental data. This work is extended to show that the existence of a stepwise-discontinuous surface temperature upstream of a plug-type heat meter makes the results obtained from this type of an instrument subject to large errors.

For convenience, algebraic equations are presented for the surface temperature, the surface temperature gradient with respect to the distance along the plate, and the local convective heat-transfer rate. These equations are in the form of power series, with positive exponents, in terms of the distance along the plate. When one of the quantities, surface temperature or local heat transfer, is specified and the coefficients and exponents of the particular power series are evaluated, the other quantities, together with the total heat-transfer rate, can be determined directly by substituting these coefficients and exponents into the respective power series.

INTRODUCTION

Several experiments and analyses have been performed which indicate that a variable surface-temperature distribution can produce either a marked increase or decrease in the local and average convective heat-transfer rates to or from a surface. In reference 1 an approximate analysis is described in which the effects on the convective heat-transfer rates produced by a stepwise-discontinuous surface temperature on a flat plate were determined. These effects were determined from solutions of the integral form of the boundary-layer-energy equations for both laminar and turbulent boundary layers. Further, these solutions were restricted to the assumption of constant physical properties for the fluids. It was found that extremely large effects occur in the region directly downstream of the discontinuity in the surface temperature.

In reference 2, there is described an analysis in which the analytical solution of the boundary-layer equations was determined for the compressible, laminar boundary layer on a flat plate. The surface temperature in this analysis was represented by an arbitrary polynomial in terms of distance along the plate. It was found that even a continuous surface-temperature variation could produce large effects on the convective heat transfer in the laminar boundary layer.

In reference 3, there is described an analysis in which the solution of the boundary-layer energy equation was determined for laminar flow on the surface of a wedge. When the velocity distribution in the boundary layer was assumed to be linear with the distance normal to the surface and all the physical properties of the fluid were assumed to be constant, a solution of the energy equation was obtained analytically for a stepwise-discontinuous surface temperature. For the case of a flat plate this solution gives results in agreement with the results of the solution for the laminar boundary layer in reference 1. This solution (reference 3) for the case of a stepwise-discontinuous surface temperature was then extended to an arbitrary surface temperature through the employment of an integral solution similar in idea to that of Duhamel (reference 4). Recently another analysis (reference 5) was performed determining the heat transfer from a body on which the velocity at the edge of the boundary layer and the surface temperature vary. This analysis required the same limiting assumptions as in reference 3 although the mathematical details were somewhat different. Again it was found from these analyses that the surface-temperature variation influenced the convective heat-transfer rates considerably.

It has also been shown experimentally that a variation in the surface temperature with distance along the surface produces large effects on the convective heat transfer. In reference 6, an experiment is described in which the effect of unheated starting sections on the average heat transfer was determined from a cylindrical probe having a constant surface temperature in the heated region and with its axis parallel to the free-stream velocity. It was found that the length of the unheated starting section does influence the average rate of heat transfer on the remainder of the probe.

In the experiment described in reference 7, it was found that values of the local heat-transfer rates measured on a cone where severe surface-temperature variations occurred deviated from predictions based on theories in which the surface temperature is assumed constant. These results were obtained for both laminar and turbulent boundary layers.

Recently an experiment was performed to determine the effect of a stepwise-discontinuous surface temperature on the local heat transfer in the turbulent boundary layer of a flat plate, reference 8.[1] These data show a marked effect on the local heat transfer, and will be correlated with the results of the analysis of this paper.

A quantitative determination, either experimental or analytical, is necessary to reveal the effect of a continuous variation of surface temperature with distance on the convective heat transfer in the turbulent boundary layer. It is the purpose of this paper, therefore, to make this determination in an approximate analytical manner subject to the limiting assumptions of a flat plate, of constant physical properties of the fluid, and of no frictional dissipation of energy within the boundary layer.

SYMBOLS

a_n coefficients of power series, $^{\circ}$F per foot^{-n}

A symbol defined by equation (22), foot$^{\frac{39+28m}{140}}$

A_n coefficients of power series, dimensionless

B symbol defined by equation (42), foot$^{-1/35}$

C coefficient defined by equation (25), dimensionless

c_p specific heat at constant pressure, Btu per slug, $^{\circ}$F

F coefficient in equation (37), dimensionless

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[1]The author wishes to acknowledge his indebtedness to Mr. Steve Scesa and the Department of Mechanical Engineering, University of California, Berkeley, for their kind permission to use hitherto unpublished data in this report.

G coefficient in equation (37), dimensionless

h local heat-transfer coefficient, Btu per second, square foot, °F

h^* local heat-transfer coefficient defined by equation (66), Btu per second, square foot, °F

$\bar{h}$ average heat-transfer coefficient, Btu per second, square foot, °F

H coefficient defined by equation (40), dimensionless

k summation index, dimensionless

L distance along flat plate to point of the stepwise discontinuity of the surface temperature, feet

m parameter introduced in equation (20), dimensionless

n summation index, dimensionless

Nu Nusselt number (hx/λ), dimensionless

P function defined by equation (34), dimensionless

Pr Prandtl number $(\mu c_p/\lambda)$, dimensionless

q local heat-transfer rate per unit area, Btu per second, square foot

Q total heat-transfer rate per unit width, Btu per second, foot

Re Reynolds number $(u_o x/\nu)$, dimensionless

s total length of heated portion of plate, feet

t temperature of fluid in the boundary layer, °F

u local velocity in the boundary layer parallel to plate, feet per second

u_o free-stream velocity, feet per second

U upper limit of integral defined in equation (48), feet

v local velocity in the boundary layer normal to plate, feet per second

W distance from leading edge of flat plate to rear of heated section following a surface-temperature discontinuity, feet

x distance along plate from leading edge, feet

y distance normal to plate surface, feet

Y_n function defined by equation (65)

z temperature ratio defined by equation (38), dimensionless

δ thickness of the flow boundary layer, feet

Δ thickness of the thermal boundary layer, feet

θ exact temperature distribution in the boundary layer for a plate at constant temperature preceded by an unheated starting section, dimensionless

θ' approximate temperature distribution in the boundary layer for a plate at constant temperature preceded by an unheated starting section, dimensionless

λ thermal conductivity of fluid, Btu per second, square foot, $^{\circ}F$ per foot

μ absolute viscosity of fluid, pound-seconds per square foot

ν kinematic viscosity of fluid, square feet per second

ρ mass density of fluid, pound-seconds^2 per foot^4

τ local shear stress, pounds per square foot

Subscripts

e effective property for turbulent flow

o free-stream condition

w surface condition

p referring to surface of heat-meter plug

s referring to surface of surrounding material

ANALYSIS

General Expressions for the Local Heat-Transfer Rate and the Temperature Distribution in the Boundary Layer

The method of analysis in this paper will be to determine an approximate solution of the boundary-layer energy equation in integral form for a stepwise-discontinuous surface temperature, and then to extend this solution to apply to an arbitrary surface temperature by employing an integral solution similar in idea to that of Duhamel (reference 4).

Boundary-layer temperature distribution.- The energy equation for the turbulent boundary layer on a flat plate, neglecting viscous dissipation, can be expressed as

$$\rho c_p \left(u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} \right) = \frac{\partial}{\partial y} \left(\lambda_e \frac{\partial t}{\partial y} \right) \tag{1}$$

where λ_e, ρ, and c_p are assumed to be independent of temperature. Suppose an exact solution of this equation exists for the case of a plate which is unheated and at the free-stream temperature t_o in the region $x \leq L$, and is heated and at the temperature t_w in the region $x > L$. For the region $x > L$

$$\frac{t-t_w}{t_o-t_w} = \theta(x,y,L) \tag{2}$$

where

$$\theta(x,0,L) = 0$$

$$\theta(L+,y,L) = 1$$

$$\theta(x,\infty,L) = 1$$

Because of the linearity in temperature t of equation (1), various solutions may be added to satisfy desired boundary conditions. For instance, the temperature in the boundary layer on a flat plate which is maintained at t_{w_1}, different from t_o, for $x \leq L$ and t_{w_2}, different from t_o, for $x > L$ is given by

$$t = t_{w_2} + (t_o - t_{w_1})\theta(x,y,0) + (t_{w_1} - t_{w_2})\theta(x,y,L) \tag{3}$$

in the region $x > L$. Each term of equation (3) satisfies the energy equation and the sum of the terms satisfies the desired boundary conditions; thus, the temperature represented by equation (3) can be considered as the formal solution of the problem. The idea underlying equation (3) may be extended to a large number of temperature discontinuities by

$$t = t_{w_n} + \sum_{k=1}^{n} \left[t_{w_{(k-1)}} - t_{w_k} \right] \theta(x,y,L_{k-1}) \tag{4}$$

where $x > L_{k-1}$. Equation (4) may be rewritten as

$$t = t_o\,\theta(x,y,0) + \sum_{k=1}^{n} t_{w_k}\left[\theta(x,y,L_k) - \theta(x,y,L_{k-1})\right] \tag{5}$$

Equation (5) is restricted to $x = L_n$. If n is allowed to increase indefinitely, the interval $(L_k - L_{k-1})$ becomes small and equation (5) may be rewritten in integral form as

$$t(x,y) = t_o\,\theta(x,y,0) + \int_0^x t(L,0)\,\frac{\partial\theta}{\partial L}\,dL \tag{6}$$

Equation (6) constitutes the general expression for the temperature distribution in the boundary layer for an arbitrary surface-temperature distribution.

<u>Local heat-transfer rate.</u>- The convective heat-transfer rate per unit area from a surface, where $\lambda = \lambda_e$, is given by

$$q = -\lambda \left.\frac{\partial t}{\partial y}\right|_{y=0} \tag{7}$$

In order to introduce a temperature potential in the heat-transfer expression, equation (6) is integrated by parts which yields

$$t(x,y) = t(x,0) + \left[t_o - t(0,0)\right]\theta(x,y,0) - \int_0^x \frac{\partial t(L,0)}{\partial L}\,\theta\,dL \tag{8}$$

after use of the boundary conditions of equation (2). The local rate of heat transfer obtained by differentiating and substituting equation (8) into equation (7) is

$$q(x) = -\lambda[t_o - t(0,0)] \left.\frac{\partial\theta(x,y,0)}{\partial y}\right|_{y=0} + \lambda \int_o^x \frac{\partial t(L,0)}{\partial L} \left.\frac{\partial\theta}{\partial y}\right|_{y=0} dL \quad (9)$$

When the local heat-transfer coefficient is defined by

$$h(x,L) = \lambda \left.\frac{\partial\theta(x,y,L)}{\partial y}\right|_{y=0} \quad (10)$$

equation (9) becomes

$$q(x) = h(x,0)\ [t(0,0) - t_o] + \int_o^x \frac{\partial t(L,0)}{\partial L} h(x,L)\ dL \quad (11)$$

Equation (11) constitutes the general expression for the local heat-transfer rate per unit area. Note that $h(x,0)$ represents the heat-transfer coefficient on a plate having a constant surface temperature. It is interesting to note that the local heat-transfer rate per unit area is comprised of the local heat-transfer rate on a plate of constant temperature, equal to the leading-edge temperature of the plate, plus corrective terms introduced by the variation of the surface temperature.

Approximate Solution for a Stepwise-Discontinuous Surface Temperature

General solution.- The purpose of this section is to obtain the value of $h(x,L)$ to be used in equation (11). Because of the complexity of the problem, an exact solution θ, or indirectly $h(x,L)$, cannot be determined at present and an approximate approach is indicated. Thus, the analysis presented in reference 1 will be repeated herein in a more general fashion.

The equation which will be solved is the boundary-layer energy equation

$$h = \frac{\rho c_p}{t_w - t_o} \frac{d}{dx} \int_o^\Delta u(t - t_o)\ dy \quad (12)$$

The method of solving this equation is to transform it to an ordinary differential equation in terms of x as the independent variable and Δ as the dependent variable. The resulting differential equation is then integrated.

Suppose the velocity term u in the right member of equation (12) is given by

$$\frac{u}{u_o} = \left(\frac{y}{\delta}\right)^{1/7} \tag{13}$$

where

$$\delta = 0.37\, u_o^{-1/5}\, \nu^{1/5}\, x^{4/5} \tag{14}$$

These are the well-known 1/7-power-law relationships (reference 9). In addition, suppose

$$\theta' = \frac{t-t_w}{t_o-t_w} = \left(\frac{y}{\Delta}\right)^{1/7} \tag{15}$$

even though δ and Δ may not be of the same order of magnitude. When the velocity and temperature expressed by equations (13) and (15) are substituted into the right member of equation (12), there is obtained after simplification

$$\frac{h}{\lambda} = \frac{7}{72} \Pr \left(\frac{u_o}{\nu}\right) \frac{d}{dx} \left(\frac{\Delta^{8/7}}{\delta^{1/7}}\right) \tag{16}$$

when Δ is less than δ. It should be noted that for the case of a constant plate temperature in air Δ equals 1.2δ. When the integration of equation (12) is performed with these limits on the thicknesses of the boundary layers it is found that the results of the heat-transfer coefficient differ from those of equation (16) by the order of 0.1 percent. Therefore, even though Δ is greater than δ for some cases, the results of the following analysis are believed to be valid over the entire plate to the order of 0.1 percent for air.

In order to transform equation (16) to an ordinary differential equation, it is necessary to express the left member of equation (16) in terms of the local characteristics of the boundary layer. Many alternative expressions may be obtained in this manner. This avoids imposing a prescribed variation of the heat-transfer coefficient h in terms of the distance along the plate x, which would make the problem trivial. To determine which of the alternative solutions is applicable, correlation of a general solution depending on a single parameter will be made with the data of reference 8. The value of the parameter which achieves the best correlation of the data and the analysis will then be used in the remainder of the report.

In order to obtain these alternative expressions for h in terms of local characteristics, it is necessary to restrict the analysis, for the moment, to the case of a constant surface temperature. For the case of a constant surface temperature, the Colburn analogy between local heat-transfer rate and skin friction (reference 10) is

$$\frac{h}{\rho u_o c_p} (Pr)^{2/3} = \frac{\tau}{\rho u_o^2} \tag{17}$$

The right member of equation (17) is expressed, from 1/7-power-law considerations, as

$$\frac{\tau}{\rho u_o^2} = 0.0225\ u_o^{-1/4}\ \nu^{1/4}\ \delta^{-1/4} \tag{18}$$

Further, it is shown in appendix A that for the case of a constant surface temperature the ratio of the thickness of the thermal boundary layer to the flow boundary layer is given by

$$\frac{\Delta}{\delta} = (Pr)^{-7/12} \tag{19}$$

When equations (17), (18), and (19) are combined so as to express h/λ in terms of Pr, δ, and Δ, the variables which appear in equation (16), there results

$$\frac{h}{\lambda} = 0.0225 \left(\frac{u_o}{\nu}\right)^{3/4} Pr^{\frac{(9-28m)}{48}}\ \Delta^{-\frac{1+4m}{4}}\ \delta^m \tag{20}$$

The thermal boundary-layer thickness was introduced into this equation by raising both sides of equation (19) to the m-th power. Equation (20) constitutes a general expression for the local heat-transfer coefficient in terms of the local boundary-layer characteristics and the Prandtl number. Although this equation was derived for heat transfer from a plate at a constant surface temperature, it will be assumed to apply to the case of a flat plate with the region $x < L$ unheated and at the free-stream temperature. It is interesting to note that there are limits on the possible variation of the parameter m. For instance, in equation (20) when m is less than $-1/4$ it is found that the heat-transfer coefficient increases with an increasing thermal boundary-layer thickness. When the parameter m is greater than $9/28$, it is found that the heat-transfer coefficient increases with a decreasing Prandtl number. These variations are in contradiction with existing knowledge concerning convective heat transfer. The values of the parameter m must, therefore, lie in the interval $-1/4 \leq m \leq 9/28$.

When equations (16) and (20) are equated, there results after simplification

$$\frac{d\Delta}{dx} - \frac{1}{10x}\Delta = Ax^{\frac{28m+4}{35}}\Delta^{-\frac{11+28m}{28}} \tag{21}$$

where

$$A = (0.2025)(0.37)^{\frac{7m+1}{7}}\left(\frac{u_o}{\nu}\right)^{-\frac{39+28m}{140}}\mathrm{Pr}^{-\frac{39+28m}{48}} \tag{22}$$

Equation (21) is of the Bernoulli type which can be integrated to yield

$$\Delta = \left[\frac{10}{7}A\left(x^{\frac{224m+312}{280}} - L^{\frac{196m+273}{280}}x^{\frac{39+28m}{280}}\right)\right]^{\frac{28}{39+28m}} \tag{23}$$

when the boundary condition $\Delta = 0$ at $x = L$ is imposed. Equation (23), therefore, gives the thermal boundary-layer thickness on a plate which is unheated and at the free-stream temperature t_o for $x \leq L$ and is heated and at the temperature t_w for $x > L$. When equation (23) is substituted into equation (20) there results

$$\frac{h(x,L)x}{\lambda} = C(m)\left(\frac{u_o x}{\nu}\right)^{4/5}\mathrm{Pr}^{1/3}\left[1 - \left(\frac{L}{x}\right)^{\frac{28m+39}{40}}\right]^{-\frac{7+28m}{39+28m}} \tag{24}$$

The coefficient in equation (24) is given by

$$C(m) = (0.0225)^{\frac{32}{39+28m}}(0.37)^{\frac{28m-1}{39+28m}}(0.0778)^{\frac{7+28m}{39+28m}} \tag{25}$$

In the range $-1/4 \leq m \leq 9/28$ the value of $C(m)$ is essentially constant and equal to $\overline{0.0288}$. It is apparent from equation (24) that when m is greater than $-1/4$ and when x approaches L the effect is to increase the local heat-transfer coefficient many fold. When L equals zero this equation degenerates to the customary heat-transfer expression based on the 1/7-power laws for velocity and temperature distribution (reference 11).

Correlation with experiment.- To determine which value of m makes equation (24) more nearly conform to the measurements, comparison is made of this solution with the data of Scesa in reference 8. The data of reference 8 were determined in a well-controlled experiment on a flat plate oriented parallel to the free-stream velocity. Free-stream speeds of 50 to 70 feet per second were employed. The plate had an unheated starting section which was followed by a heated section in which there was an additional temperature discontinuity. All the basic data except those from heater elements which can be considered guard heaters are plotted in figure 1. It is observed that the data lie consistently above the theoretical line for a plate at a constant temperature.

The temperatures in the heated region of the plate are defined as t_{w_1} and t_{w_2} upstream and downstream from the temperature discontinuity, and the temperature of the unheated starting section is t_o, the free-stream temperature. The length of the unheated starting section is L_1, and the distance from the leading edge of the plate to the temperature discontinuity in the heated portion of the plate is L_2. The local heat-transfer rates per unit area are then given, respectively, by

$$q = h(x,L_1)(t_{w_1}-t_o) \tag{26}$$

for $L_1 < x \leq L_2$, and

$$q = h(x,L_1)(t_{w_1}-t_o) + h(x,L_2)(t_{w_2}-t_{w_1}) \tag{27}$$

for $x > L_2$. The heat-transfer coefficients in this experiment were defined in terms of the local temperature difference between the surface and the free stream. The Nusselt numbers corresponding to those measured in each of these regions are given by equation (24) as

$$Nu = (Nu)_{L=0} \left[1 - \left(\frac{L_1}{x}\right)^{\frac{28m+39}{40}}\right]^{-\frac{7+28m}{39+28m}} \tag{28}$$

and

$$Nu = (Nu)_{L=0} \left\{ \frac{t_{w_1}-t_o}{t_{w_2}-t_o} \left[1 - \left(\frac{L_1}{x} \right)^{\frac{28m+39}{40}} \right]^{-\frac{7+28m}{39+28m}} + \right.$$

$$\left. \frac{t_{w_2}-t_{w_1}}{t_{w_2}-t_o} \left[1 - \left(\frac{L_2}{x} \right)^{\frac{28m+39}{40}} \right]^{-\frac{7+28m}{39+28m}} \right\} \tag{29}$$

If the general solution, equation (24), yields results which conform to experiment, then the experimental Nusselt numbers divided by the bracketed quantities should yield experimental Nusselt numbers which correspond to $(Nu)_{L=0}$. These adjusted data are shown for two values of the parameter m, $m = 0$, and $m = 9/28$, in figures 2 and 3, respectively. It should be noted that $m = -1/4$ corresponds to the data of figure 1, that is, the bracketed quantities are equal to unity. The value $m = 0$ apparently correlates the data very well. This corresponds to the results of reference 1, that is, the local heat-transfer coefficient is dependent on the thermal boundary-layer thickness and the Prandtl number and is independent of the flow boundary-layer thickness. For $m = 0$ equation (24) becomes

$$\frac{h(x,L)x}{\lambda} = 0.0288 \left(\frac{u_o x}{\nu} \right)^{4/5} Pr^{1/3} \left[1 - \left(\frac{L}{x} \right)^{39/40} \right]^{-7/39} \tag{30}$$

Equation (30) will be used in the remainder of the analysis.

Average heat-transfer coefficient determined from particular solution.- In many cases it is desirable to know the average heat-transfer coefficient over a region at constant temperature preceded by an unheated starting section. The total heat transferred in the region per unit width is given by

$$Q = \int_L^W q(x)dx = \int_L^W h(x,L)(t_w-t_o)\, dx \tag{31}$$

The symbol W represents the distance from the leading edge of the plate to the rear of the region considered. The average heat-transfer coefficient over this region is defined as

$$\bar{h} = \frac{Q}{(t_w-t_o)(W-L)} \tag{32}$$

When equations (30), (31), and (32) are combined there results

$$\bar{h}(W,L) = h(W,0)\,\frac{40}{39}\,\frac{(L/W)^{4/5}}{1-(L/W)}\,P\left(\frac{L}{W}\right) \tag{33}$$

where

$$P\left(\frac{L}{W}\right) = \int_{\left(\frac{L}{W}\right)^{39/40}}^{1} \eta^{-71/39}\,(1-\eta)^{-7/39}\,d\eta \tag{34}$$

Equation (34) was evaluated numerically and the results are shown in figure 4.

The experimental data obtained by Jakob and Dow (reference 6) on the average heat transferred from a probe in axial air flow preceded by unheated starting sections corresponds, in form, to the results obtained from equations (33) and (34). It cannot be assumed tacitly, however, that the data obtained from probes should compare identically with theory based on a flat plate. In particular, the data obtained on the probe for a constant surface temperature were about 13 percent lower than corresponding theoretical or experimental results on a flat plate. As any theoretical comparison of probes and flat plates indicates that the rate of heat transfer from the probe should be the greater, there appears to be some unexplained reason for the low values of the data. In view of this, no extensive comparison of the data and the results of the present analysis are made. A single comparison for the case of an unheated starting section reveals that $\bar{h}/h(W,0) = 1.37$ from the Jakob and Dow results, while the corresponding result from the present analysis is $\bar{h}/h(W,0) = 1.45$. This comparison is favorable when consideration is made of the questions expressed previously concerning the comparison of results from probes and flat plates.

<u>Particular solution applied to general single surface-temperature discontinuity.</u>- When the region of the plate preceding the temperature discontinuity is at a temperature t_{w_1} other than the free-stream temperature, the local and average heat-transfer coefficients in the region $L < x \leq W$ at a temperature t_{w_2} are given by

$$h(x,L) = h(x,0)\left\{\frac{t_{w_1}-t_o}{t_{w_2}-t_o} + \frac{t_{w_2}-t_{w_1}}{t_{w_2}-t_o}\left[1-\left(\frac{L}{x}\right)^{39/40}\right]^{-7/39}\right\} \tag{35}$$

$$\bar{h}(W,L) = h(W,0)\left\{\frac{5}{4}\frac{\left[1-(L/W)^{4/5}\right]}{1-(L/W)}\frac{t_{w_1}-t_o}{t_{w_2}-t_o} + \frac{40}{39}\frac{(L/W)^{4/5}\,P(L/W)}{1-(L/W)}\frac{t_{w_2}-t_{w_1}}{t_{w_2}-t_o}\right\} \tag{36}$$

For certain applications in a small region (W–L) it is interesting to know the ratio of the average heat-transfer coefficient to the local heat-transfer coefficient which would prevail at the center of the region if the plate were at a constant temperature. This ratio is obtained by dividing equation (36) by $h\left(\frac{W+L}{2},0\right)$

$$\frac{\bar{h}(W,L)}{h\left(\frac{W+L}{2},0\right)} = F\left(\frac{L}{W}\right)\frac{t_{w_1}-t_o}{t_{w_2}-t_o} + G\left(\frac{L}{W}\right)\frac{t_{w_2}-t_{w_1}}{t_{w_2}-t_o} \tag{37}$$

where F(L/W) and G(L/W) were evaluated numerically. If

$$z = \frac{t_{w_2}-t_{w_1}}{t_{w_2}-t_o} \tag{38}$$

equation (37) may be rewritten as

$$\frac{\bar{h}(W,L)}{h\left(\frac{W+L}{2},0\right)} = F(L/W) + H(L/W)z \tag{39}$$

where

$$H(L/W) = G(L/W) - F(L/W) \tag{40}$$

The numerical values of F(L/W) and H(L/W) are plotted in figure 5.

Extension of Approximate Solution to the Problem of an Arbitrary Surface Temperature

Boundary-layer temperature distribution.- The approximate temperature distribution determined from equations (15) and (23) with $m = 0$ is

$$\theta' = \frac{By^{1/7}}{\left(x^{312/280} - L^{273/280}\ x^{39/280}\right)^{41/39}} \tag{41}$$

where

$$B = 1.15\ u_o^{1/35}\ \nu^{-1/35}\ Pr^{1/12} \tag{42}$$

At the outer edge of the boundary layer θ' becomes equal to unity; therefore, equation (41) is valid only in the region

$$y \leq \left(\frac{1}{B}\right)^7 \left(x^{312/280} - L^{273/280}\ x^{39/280}\right)^{28/39} \tag{43}$$

For values of y greater than given by the inequality (43)

$$\theta' = 1 \tag{44}$$

For the range of y indicated by the inequality (43)

$$\frac{\partial\theta'}{\partial L} = \frac{B}{10} \frac{y^{1/7}\ x^{39/280}\ L^{-1/40}}{\left(x^{312/280} - L^{273/280}\ x^{39/280}\right)^{43/39}} \tag{45}$$

For greater y

$$\frac{\partial\theta'}{\partial L} = 0 \tag{46}$$

When equations (41), (45), and (46) are substituted into equation (6) there results

$$t(x,y) = By^{1/7}\left\{\frac{t_o}{x^{4/35}} + \frac{1}{10}\int_o^U \frac{t(L,0)\ x^{39/280}\ L^{-1/40} dL}{\left(x^{312/280} - L^{273/280}\ x^{39/280}\right)^{43/39}}\right\} \tag{47}$$

where

$$U = \left(x^{273/280} - B^{39/4}\, y^{39/28}\, x^{-39/280}\right)^{280/273} \tag{48}$$

When $By^{1/7} = x^{4/35}$ it is apparent from equation (48) that $U = 0$, and consequently from equation (47) $t(x,y) = t_o$. It is not obvious, however, that the left member of equation (47) equals the prescribed surface temperature at $y = 0$. It is shown in appendix B that equation (47) does satisfy the boundary conditions at $y = 0$.

Local heat-transfer date.- The general expression for the local heat-transfer rate is obtained by substituting the local heat-transfer coefficient given by equation (30) into equation (11)

$$q = h(x,0)\left\{t(0,0) - t_o + \int_0^x \frac{\partial t(L,0)}{\partial L}\left[1 - \left(\frac{L}{x}\right)^{39/40}\right]^{-7/39} dL\right\} \tag{49}$$

For an arbitrary surface-temperature distribution it is necessary to evaluate the integrals in equations (47) and (49) numerically. There is, however, a type of surface distribution for which the local heat-transfer rate can be determined easily by analytical treatment. This will be shown in the next section.

Surface-temperature distribution represented by power series.- Let the surface temperature be represented by

$$t(x,0) - t_o = \sum_n a_n x^n \qquad n \geq 0 \tag{50}$$

where n is not necessarily an integer. When equation (50) is differentiated with respect to x there results

$$\frac{\partial t(x,0)}{\partial x} = \sum_n n a_n x^n \tag{51}$$

The substitution of equation (51) at $x = L$ into equation (49) yields

$$q = h(x,0)\left\{a_o + \int_0^x \sum_n n a_n L^{n-1}\left[1 - \left(\frac{L}{x}\right)^{39/40}\right]^{-7/39} dL\right\} \tag{52}$$

This equation may be rewritten as

$$q = h(x,0)\left\{a_o + \sum_n n a_n \int_0^x L^{n-1}\left[1 - \left(\frac{L}{x}\right)^{39/40}\right]^{-7/39} dL\right\} \tag{53}$$

Letting $\alpha = (L/x)^{39/40}$ allows transformation of the integral terms of equation (53) so that the equation reads

$$q = h(x,0) \left[a_o + \frac{40}{39} \sum_n n a_n x^n \int_0^1 \alpha^{\frac{40}{39}n-1} (1-\alpha)^{-7/39} d\alpha \right] \qquad (54)$$

The integral terms of this equation are Eulerian integrals of the first kind (reference 12), the values of which are given by the Beta function which may be expressed in Gamma functions as

$$\int_0^1 \alpha^{\frac{40}{39}n-1} (1-\alpha)^{-7/39} d\alpha = \frac{\Gamma\left(\frac{40n}{39}\right)\Gamma\left(\frac{32}{39}\right)}{\Gamma\left(\frac{40n}{39} + \frac{32}{39}\right)} \qquad (55)$$

where n is greater than or equal to zero. The values of the Gamma functions may be obtained from tables in reference 13. In view of equation (55), equation (54) becomes

$$q = h(x,0) \left[a_o + \frac{40}{39} \sum_n n a_n x^n \frac{\Gamma\left(\frac{40n}{39}\right)\Gamma\left(\frac{32}{39}\right)}{\Gamma\left(\frac{40n}{39} + \frac{32}{39}\right)} \right] \qquad (56)$$

When the recursion equation

$$\Gamma(y+1) = y\Gamma(y) \qquad (57)$$

is used, equation (56) may be rewritten as

$$q = h(x,0) \sum_n a_n x^n \frac{\Gamma\left(\frac{40}{39}n+1\right)\Gamma\left(\frac{32}{39}\right)}{\Gamma\left(\frac{40n}{39} + \frac{32}{39}\right)} \qquad (58)$$

It is apparent that once the type of surface-temperature distribution is specified, that is, values of a_n specified, the distribution of the local heat-transfer rate per unit area can be determined directly from equation (58). The total rate of heat transfer per unit width Q is obtained by integrating equation (58) from $x = 0$ to $x = s$

$$Q = \bar{h}(s,0) \sum_n \frac{a_n}{\frac{5}{4} n+1} s^{n+1} \frac{\Gamma\left(\frac{40}{39} n+1\right) \Gamma\left(\frac{32}{39}\right)}{\Gamma\left(\frac{40n}{39} + \frac{32}{39}\right)} \tag{59}$$

The term $\bar{h}(s,0)$ is the average heat-transfer coefficient for the case of a constant surface temperature determined from the expression

$$\frac{\bar{h}(s,0)s}{\lambda} = 0.036\left(\frac{u_o s}{\nu}\right)^{4/5} Pr^{1/3} \tag{60}$$

For convenience, equations (51), (52), (58), and (59) are put in the dimensionless form

$$\frac{t(x,0)}{t_o} = 1 + \sum_n A_n\left(\frac{x}{s}\right)^n \tag{61}$$

$$\frac{\partial\left(\frac{t(x,0)}{t_o}\right)}{\partial\left(\frac{x}{s}\right)} = \sum_n nA_n\left(\frac{x}{s}\right)^{n-1} \tag{62}$$

$$\frac{q}{t_o\ h(x,0)} = \sum_n A_n Y_n\left(\frac{x}{s}\right)^n \tag{63}$$

$$\frac{Q}{t_o s\bar{h}(s,0)} = \sum_n \frac{A_n}{\left(\frac{5}{4} n+1\right)} Y_n \tag{64}$$

In these equations

$$Y_n = \frac{\Gamma\left(\frac{40}{39} n+1\right) \Gamma\left(\frac{32}{39}\right)}{\Gamma\left(\frac{40n}{39} + \frac{32}{39}\right)} \tag{65}$$

The term Y_n is plotted as a function of n in figure 6. If the heat-transfer coefficient for the case of a variable surface temperature is defined in terms of the local temperature difference

$$h^* = \frac{q}{t(x,0) - t_o} \tag{66}$$

then

$$\frac{h^*}{h(x,0)} = \frac{\sum_n A_n Y_n \left(\frac{x}{s}\right)^n}{\sum_n A_n \left(\frac{x}{s}\right)^n} \tag{67}$$

Inspection of the right member of equation (67) reveals several interesting facts. For a problem in which the denominator has a local value of zero, the numerator will be finite in all but a very exceptional case; thus $h^*/h(x,0)$ will be infinite at this point. For another case the numerator may be zero locally while the denominator will probably be finite; thus $h^*/h(x,0)$ will be zero at this point. It is seen, therefore, that $h^*/h(x,0)$ can attain any numerical value, depending on the problem considered.

In summary, it is noted from equations (61), (62), and (63) that the values of the coefficient A_n may be determined by a prescribed distribution of surface temperature, a prescribed distribution of the surface temperature gradient, or a prescribed distribution of the local heat-transfer rate per unit area. Once the coefficients A_n are evaluated from the prescribed quantity the other two quantities, together with the total heat-transfer rate, can be determined directly.

EXAMPLES

Plug-Type Heat Meter

Convective heat-transfer rates at a surface are often measured conveniently by determining the heat-transfer rate to or from a plug which is thermally isolated from the surrounding material. The plug can be used as a steady-state device in which heat is generated electrically and the temperature of the plug maintained constant. The usual practice, however, is to use the plug as a transient device by measuring the temperature-time history of the plug. The local heat-transfer rate through the surface of the plug is represented by the product of the rate of change of temperature and the thermal capacity of the plug. Regardless of the convenience of using such a device, the present analysis indicates that the measurements are subject to certain inherent errors in addition to those which may arise from imperfect thermal isolation.

It is not possible to design a simple plug-type heat meter which will always maintain a temperature identical to the temperature of the surrounding material. If an extremely thin insulating material is used between the plug and the surrounding material, a surface-temperature discontinuity will occur at the seam of the plug and the surrounding material. If the instantaneous surface temperature of the plug is defined as t_p, while the surface temperature of the surrounding material to the leading edge of the plate is denoted t_s, it is apparent that equations (36) through (40) are applicable to this problem, where $t_p = t_{w_2}$ and $t_s = t_{w_1}$.

It is desirable to know how well a plug-type heat meter measures the local heat-transfer coefficient which would exist at the position of the center of the plug if the plate were at a constant temperature. Equation (37) is directly applicable to this problem. For example, suppose the instantaneous temperatures of a heat-meter plug, the surrounding material, and the free stream are 125° F, 100° F, and 200° F, respectively. The dimensions of the plug are $L = 24$ inches and $W = 25$ inches. It is found from equation (38) that $z = -0.33$, and $L/W = 0.96$. From equation (39) and figure 5

$$\frac{\bar{h}(25,24)}{h(24.5,0)} = 0.62 \tag{68}$$

Thus, the plug-type heat meter measures a heat-transfer coefficient which is only 62 percent of the local heat-transfer coefficient which would have existed at the position of the center of the plug had the plate been at a constant surface temperature.

Although this example is arbitrary, the conditions are by no means implausible. It should be noted from equation (39) that more severe surface-temperature conditions can result in negative values of

$\dfrac{\bar{h}(W,L)}{h(W+L/2,0)}$, which means that measurement of heat transfer by a plug would be in a direction opposite to that which would normally prevail on a plate having a constant surface temperature. These results indicate that extreme caution should be exercised in the interpretation of the data obtained by plug-type heat meters.

Constant Heat-Transfer Rate

Suppose it is desired to maintain a surface-temperature distribution such that the local heat-transfer rate per unit area is constant along a plate. From equation (30) it can be seen that $h(x,0)$ is inversely proportional to the 1/5-power of x. Equation (63) can therefore be written as

$$q = t_o\, h(s,0) \left(\frac{x}{s}\right)^{-1/5} \sum_n A_n\, Y_n \left(\frac{x}{s}\right)^n \tag{69}$$

It is apparent that to maintain q constant, n must have the value 1/5 only. Equation (69) becomes

$$q = t_o\, h(s,0)\, A_{1/5}\, Y_{1/5} \tag{70}$$

Therefore,

$$A_{1/5} = \frac{q}{t_o\, h(s,0)\, Y_{1/5}} \tag{71}$$

The required surface-temperature variation as given by equation (61) is

$$\frac{t(x,0)}{t_o} = 1 + \frac{q}{t_o\, h(s,0)\, Y_{1/5}} \left(\frac{x}{s}\right)^{1/5} \tag{72}$$

Further

$$\frac{h^*}{h(x,0)} = Y_{1/5} = 1.06 \tag{73}$$

This latter equation indicates, from the values of figure 6, that the local heat-transfer coefficient on a plate having a constant heat-transfer rate differs from that on a plate at a constant temperature by only 6 percent.

Surface Temperature Represented by Polynomial

Suppose the temperature distribution on a flat plate is given by

$$\frac{t(x,0)}{t_o} = 1.5 + 0.5\left(\frac{x}{s}\right) \tag{74}$$

Then in equation (61), $n = 0$ and $n = 1$, and $A_o = 0.5$ and $A_1 = 0.5$. From equation (63) and figure 6, the local heat-transfer rates are given by

$$\frac{q}{t_o\ h(x,0)} = 0.5 + 0.612\left(\frac{x}{s}\right) \tag{75}$$

The ratio of the local heat-transfer coefficient to the heat-transfer coefficient which would exist if the plate were at a constant surface temperature is

$$\frac{h^*}{h(x,0)} = \frac{0.5 + 0.612(x/s)}{0.5 + 0.5(x/s)} \tag{76}$$

The values of the members of equations (74) and (75) are plotted in figure 7. It can be observed that although the temperature variation along the plate increased the over-all temperature difference between the plate and the free stream by 100 percent, the local heat-transfer coefficient differs from that on a constant-temperature plate by a maximum of about 11 percent.

CONCLUSIONS

The conclusions of this report are subject to certain limitations inherent in the assumptions of the analysis. The following assumptions have been used:

1. All physical properties of the fluid are constant.

2. Frictional dissipation of energy within the boundary layer is negligible.

3. The velocity distribution in the boundary layer is of the 1/7-power form.

4. The temperature distribution in the boundary layer is of the 1/7-power form even when the thermal boundary-layer thickness differs considerably from the flow boundary-layer thickness.

5. The local heat-transfer coefficient determined on a plate having a constant surface temperature applies to a plate having a variable surface temperature when it is expressed by an equation based on the local flow and thermal boundary-layer thicknesses instead of the distance along the plate.

Although the analysis based on these assumptions is correlated with experimental data, this is not a verification of the individual assumptions of the analysis, but rather, is a justification of the use of the end results.

The foregoing analysis, together with the examples cited, has indicated that a variation in the surface temperature with distance along a flat plate influences the local convective heat transfer to an extent which depends on the type of variation. In general, a sudden surface-temperature jump, or discontinuity, produces extremely large increases or decreases in the convective heat transfer directly downstream of the position of the discontinuity. From this it can be concluded that measurements by a heat meter of the plug type can, in most instances, deviate considerably from the heat transfer which would normally exist at the location of the instrument. A continuously variable surface temperature was shown, in general, to have a smaller effect on the convective heat transfer than a sudden temperature discontinuity. For instance, it was shown that the local heat-transfer coefficient on a plate having a constant heat-transfer rate along the surface, with the necessary 1/5-power variation of the surface temperature, differs from that on a plate with a constant surface temperature by only 6 percent. For the case of continuously varying surface temperatures, large effects on the local heat-transfer coefficient are expected only where the surface temperature approaches that of the free stream at a point other than the leading edge of the plate.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Feb. 9, 1951.

APPENDIX A

RATIO OF BOUNDARY-LAYER THICKNESSES ON PLATE WITH CONSTANT SURFACE TEMPERATURE

If the 1/7-power-law velocity distribution

$$\frac{u}{u_o} = \left(\frac{y}{\delta}\right)^{1/7} \tag{A1}$$

is substituted into the von Kármán momentum integral for a flat plate

$$\frac{\tau}{\rho} = \frac{d}{dx}\int_o^\delta u(u_o-u)\,dy \tag{A2}$$

there results

$$\frac{\tau}{\rho} = \frac{d}{dx}\int_o^\delta u_o\left(\frac{y}{\delta}\right)^{1/7}\left[u_o - u_o\left(\frac{y}{\delta}\right)^{1/7}\right]dy \tag{A3}$$

When the indicated operations are performed, there results

$$\frac{\tau}{\rho u_o^2} = \frac{7}{72}\frac{d\delta}{dx} \tag{A4}$$

Similarly, if the 1/7-power temperature distribution

$$\frac{t-t_w}{t_o-t_w} = \left(\frac{y}{\Delta}\right)^{1/7} \tag{A5}$$

and equation (A1) are inserted into the integral form of the energy equation

$$h = \frac{\rho c_p}{t_w-t_o}\frac{d}{dx}\int_o^\Delta u(t-t_o)\,dy \tag{A6}$$

there results

$$h = \rho c_p\, u_o \frac{d}{dx}\left(\frac{7}{72}\frac{\Delta^{8/7}}{\delta^{1/7}}\right) \tag{A7}$$

when t_w is constant and it is postulated that δ is greater than Δ.

For a constant surface temperature the Colburn modification of the Reynolds analogy is expressed as

$$\frac{h}{\rho c_p u_o} (Pr)^{2/3} = \frac{\tau}{\rho u_o^2} \tag{A8}$$

When equations (A4) and (A7) are substituted into equation (A8), there is obtained

$$\frac{d}{dx} \left[(Pr)^{2/3} \frac{\Delta^{8/7}}{\delta^{1/7}} \right] = \frac{d}{dx} (\delta) \tag{A9}$$

Therefore

$$(Pr)^{2/3} \frac{\Delta^{8/7}}{\delta^{1/7}} = \delta + C \tag{A10}$$

For the case of a constant surface temperature, both the thickness of the thermal and flow boundary layers are zero at the leading edge of the plate; thus, C equals zero in equation (A10) and

$$\frac{\Delta}{\delta} = (Pr)^{-7/12} \tag{A11}$$

APPENDIX B

VERIFICATION OF BOUNDARY CONDITION

Equation (47) in the text is

$$t(x,y) = By^{1/7}\left\{\frac{t_o}{x^{4/35}} + \frac{1}{10}\int_o^U \frac{t(L,0)\ x^{39/280}\ L^{-1/40}\ dL}{\left(x^{312/280} - L^{273/280}\ x^{39/280}\right)^{43/39}}\right\} \tag{B1}$$

where

$$U = \left(x^{273/280} - B^{39/4}\ y^{39/28}\ x^{-39/280}\right)^{280/273} \tag{B2}$$

To simplify the integration let

$$\varphi = \left(x^{312/280} - L^{273/280}\ x^{39/280}\right)\ y^n \tag{B3}$$

or

$$d\varphi = -\frac{273}{280}\, y^n\ x^{39/280}\ L^{-1/40} dL \tag{B4}$$

When the terms of equation (B4) are transposed

$$x^{39/280}\ L^{-1/40} dL = -\frac{40}{39}\ y^{-n}\ d\varphi \tag{B5}$$

From equation (B3) it can be found that

$$L = \left(x^{39/40} - x^{-39/280} y^{-n}\ \varphi\right)^{40/39} \tag{B6}$$

With this transformation of the variable, equation (B1) becomes

$$t(x,y) = \frac{By^{1/7}\, t_o}{x^{4/35}} - \frac{4B}{39}\int_{x^{312/280}y^{n}}^{B^{39/4}y^{n+39/28}} \frac{t\left[\left(x^{39/40} - x^{-39/280}y^{-n}\varphi\right)^{40/39}, 0\right] y^{-n+1/7} d\varphi}{\varphi^{43/39}\, y^{-43n/39}} \tag{B7}$$

When $n = -39/28$, equation (B7) reduces to

$$t(x,y) = \frac{By^{1/7}\, t_o}{x^{4/35}} - \frac{4B}{39}\int_{\frac{x^{312/280}}{y^{39/28}}}^{B^{39/4}} \frac{t\left[\left(x^{39/40} - x^{-39/280}y^{39/28}\varphi\right)^{40/39}, 0\right] d\varphi}{\varphi^{43/39}} \tag{B8}$$

At $y = 0$, equation (B8) becomes

$$t(x,0) = -\frac{4B}{39}\int_{\infty}^{B^{39/4}} \frac{t(x,0)\, d\varphi}{\varphi^{43/39}} \tag{B9}$$

On integration, equation (B9) becomes

$$t(x,0) = \frac{4B}{39}\, t(x,0)\left(\frac{39}{4}\, \varphi^{-4/39}\Big|_{\infty}^{B^{39/4}}\right) \tag{B10}$$

which reduces to the identity

$$t(x,0) = t(x,0) \tag{B11}$$

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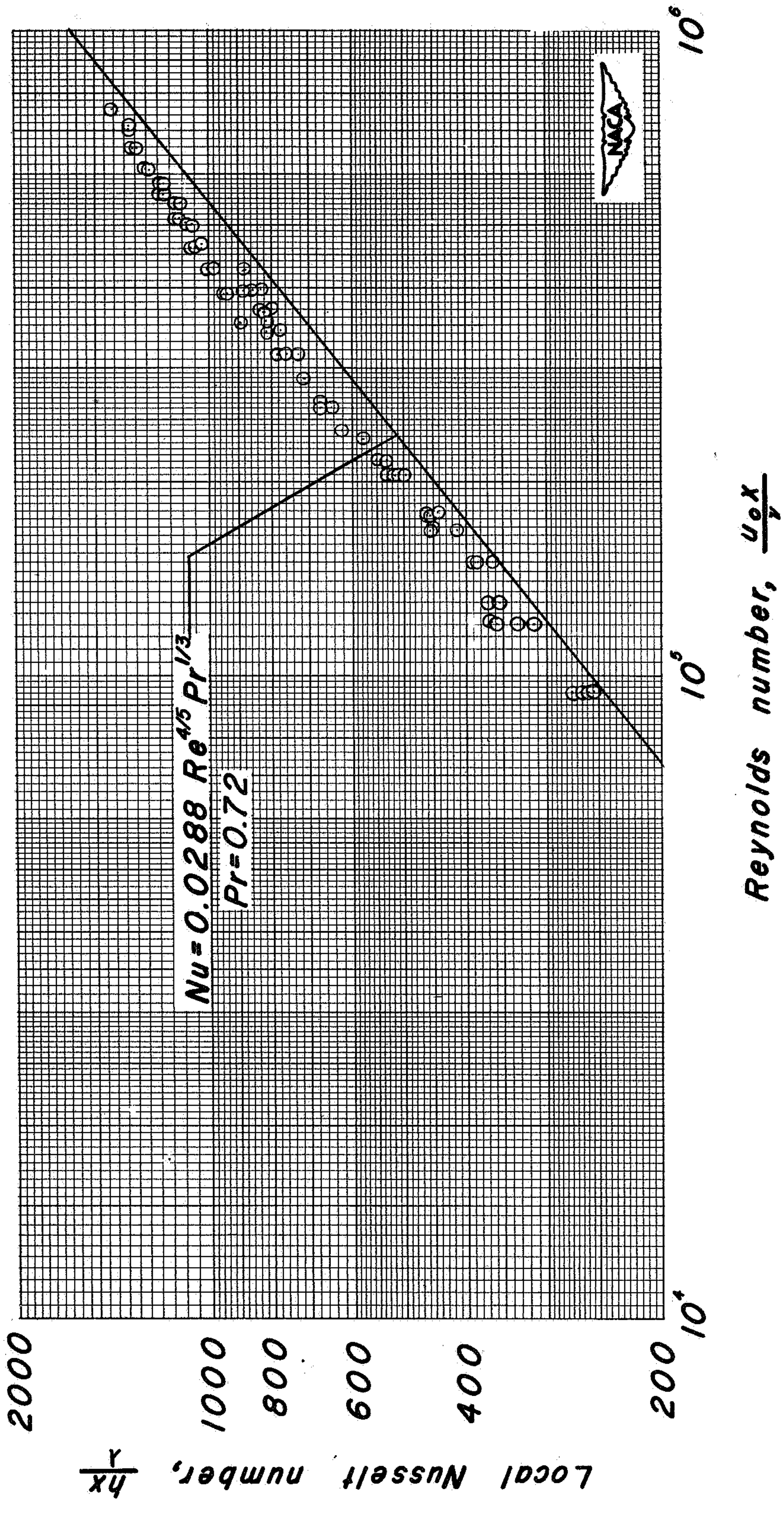

Figure 1.- Scesa's original data, reference 8.

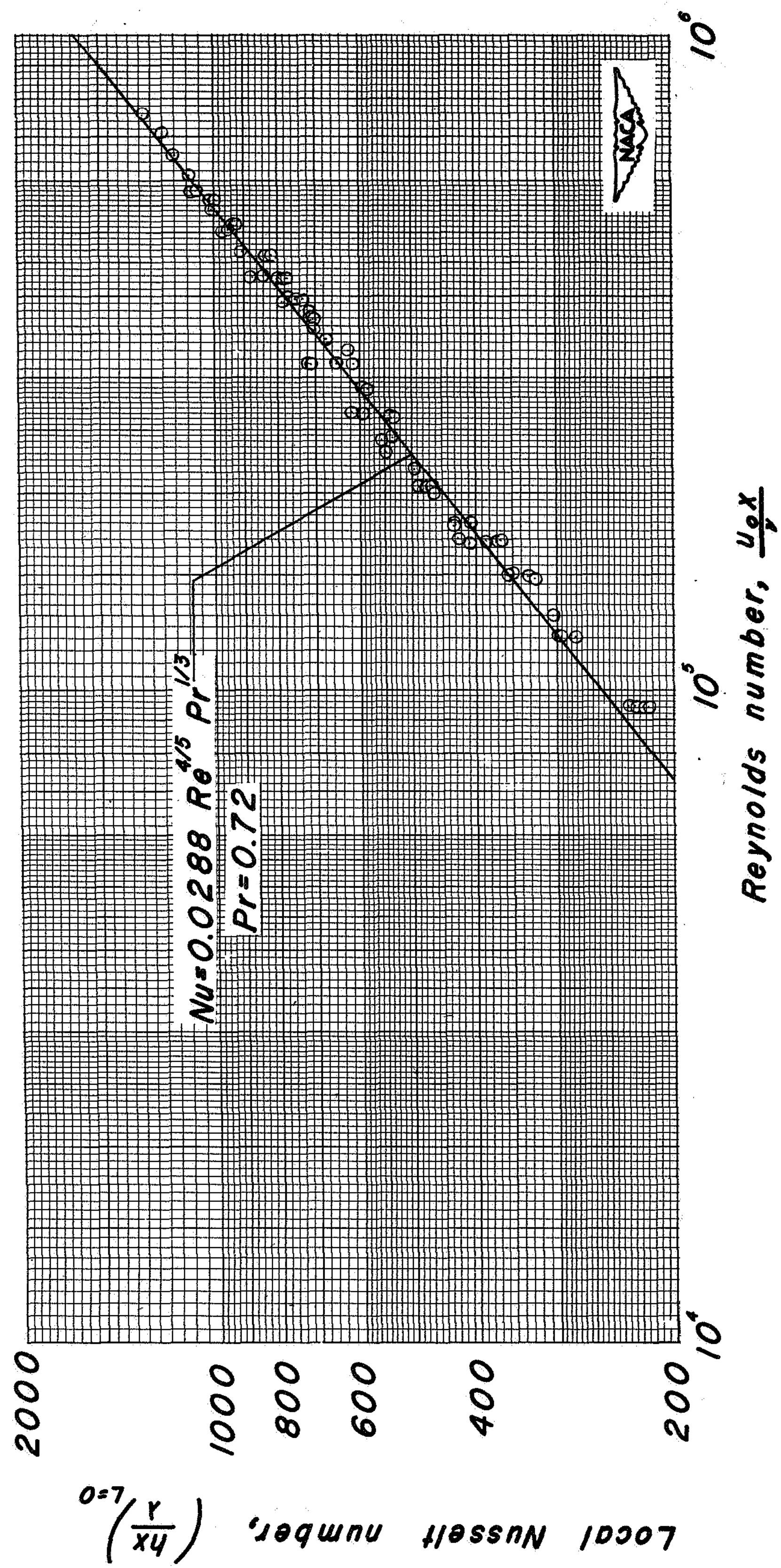

Figure 2.- Scesa's data altered by approximate theory with m=0.

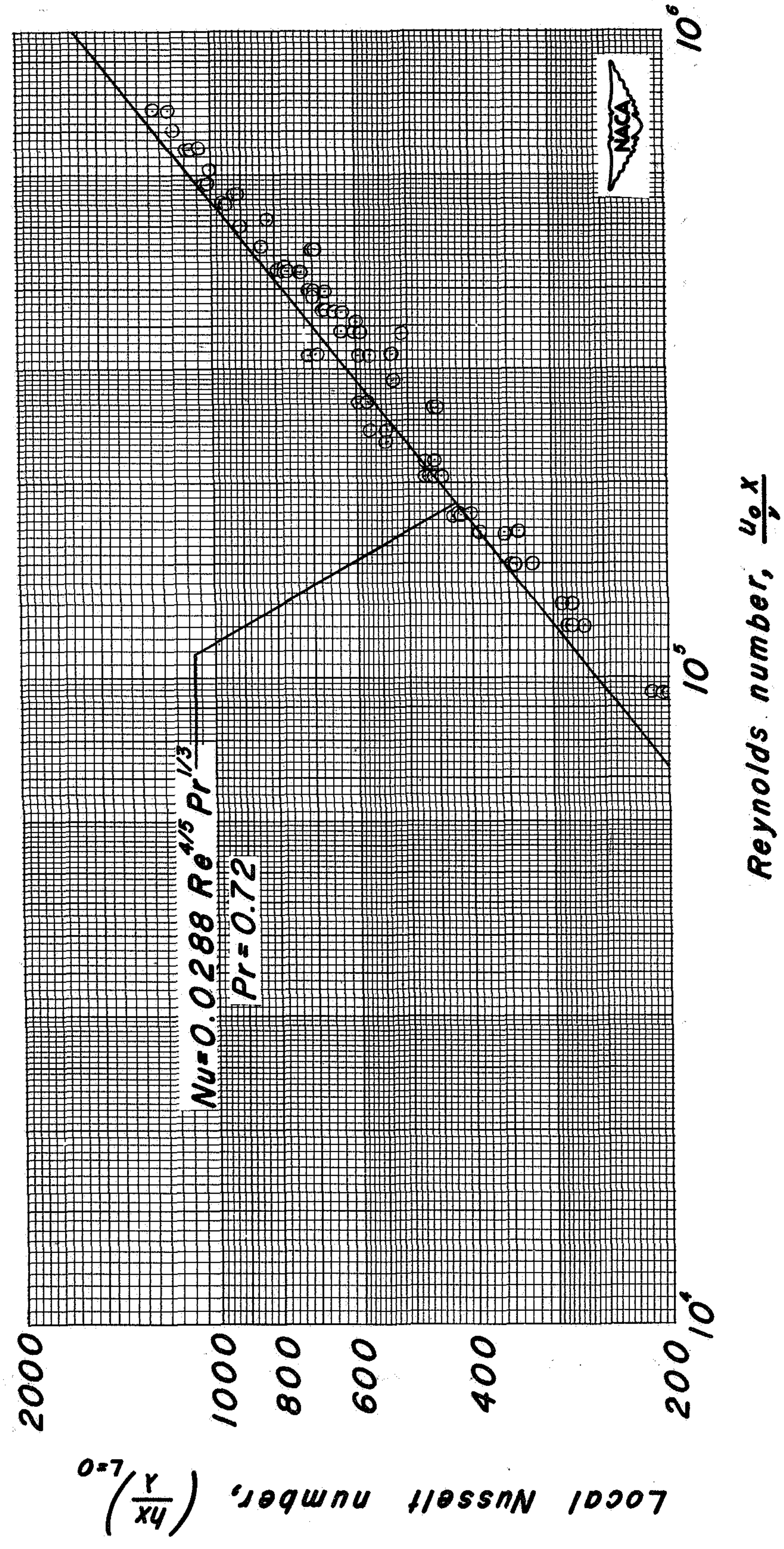

Figure 3. – Scesa's data altered by approximate theory with $m = \frac{9}{28}$.

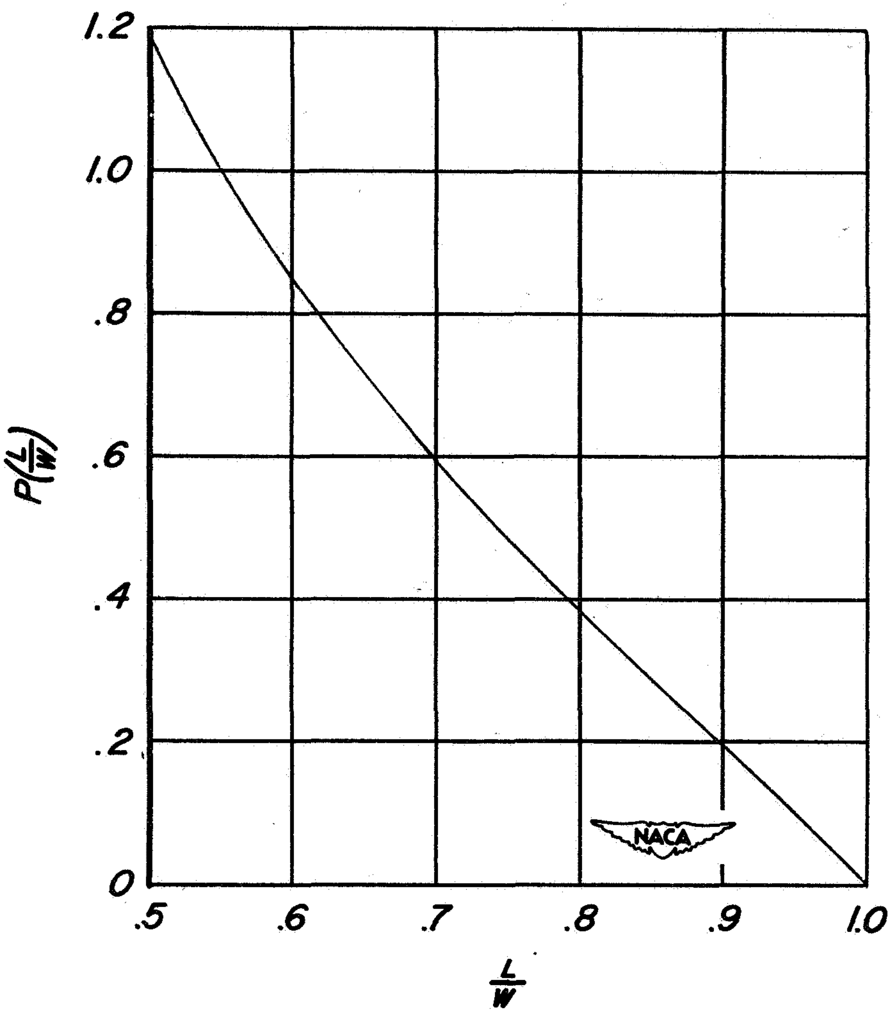

Figure 4.– Values of $P\left(\frac{L}{W}\right)$ defined by equation (34).

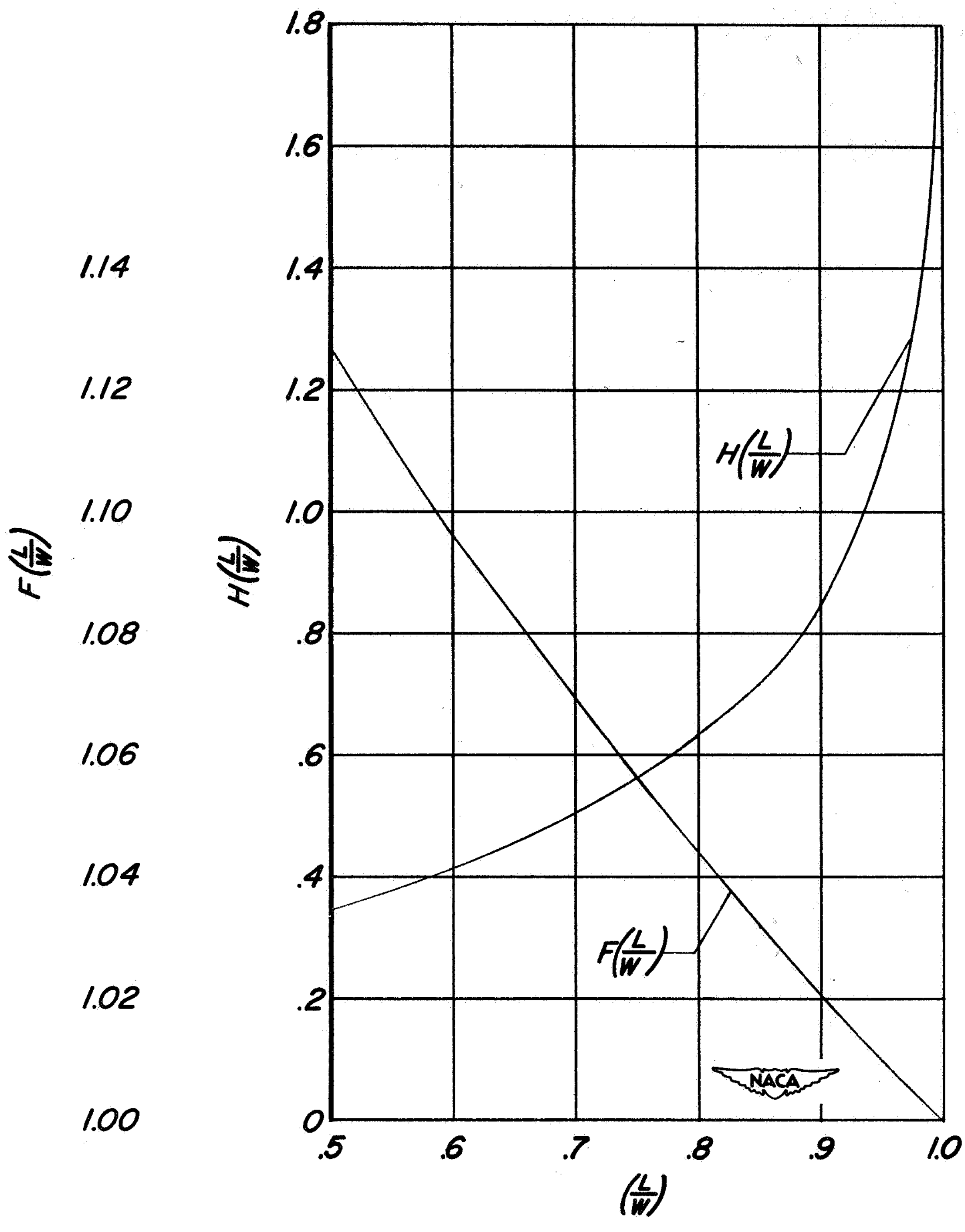

Figure 5.– Numerical values of coefficients in equation (39).

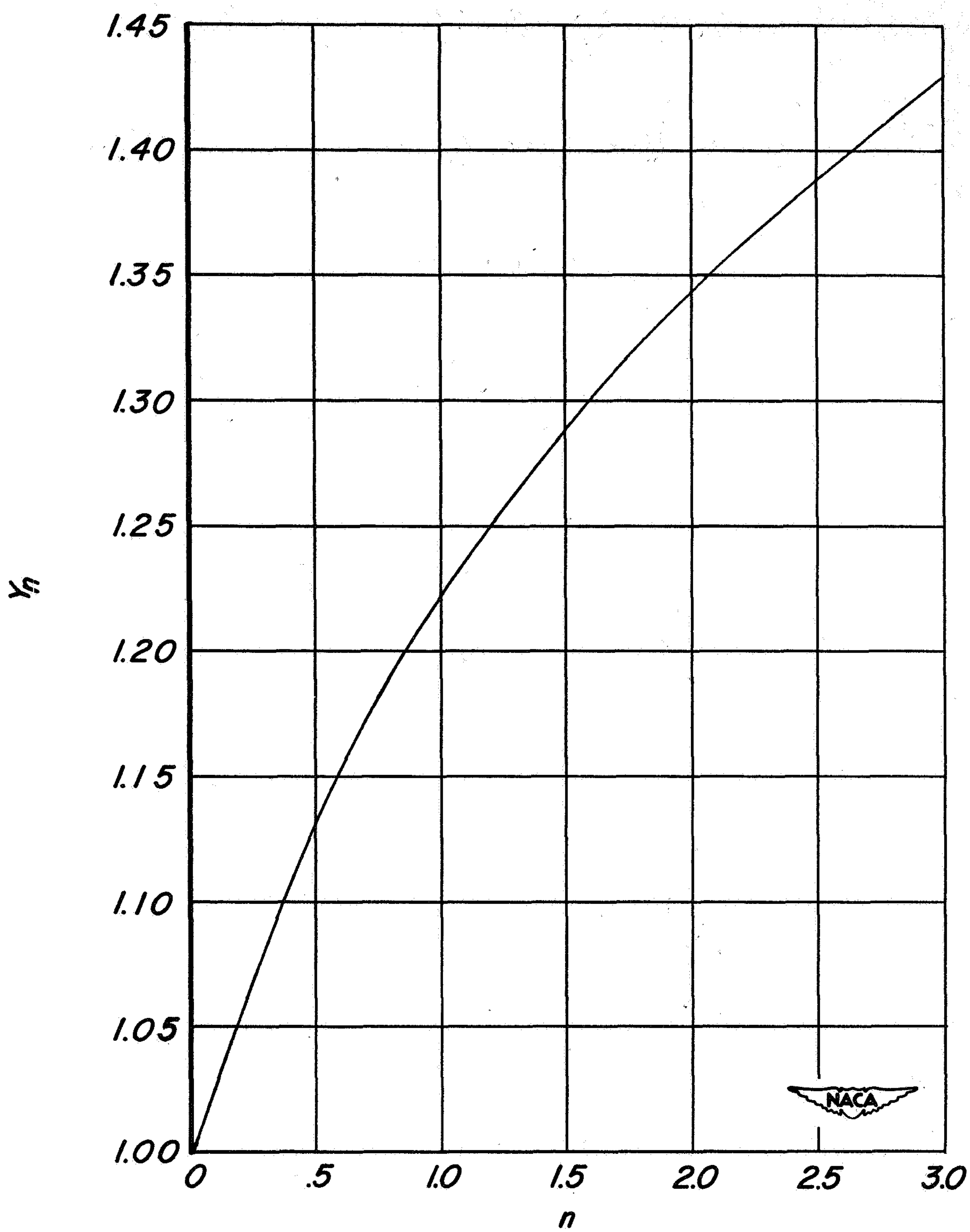

Figure 6.- Numerical values of Y_n defined by equation (65).

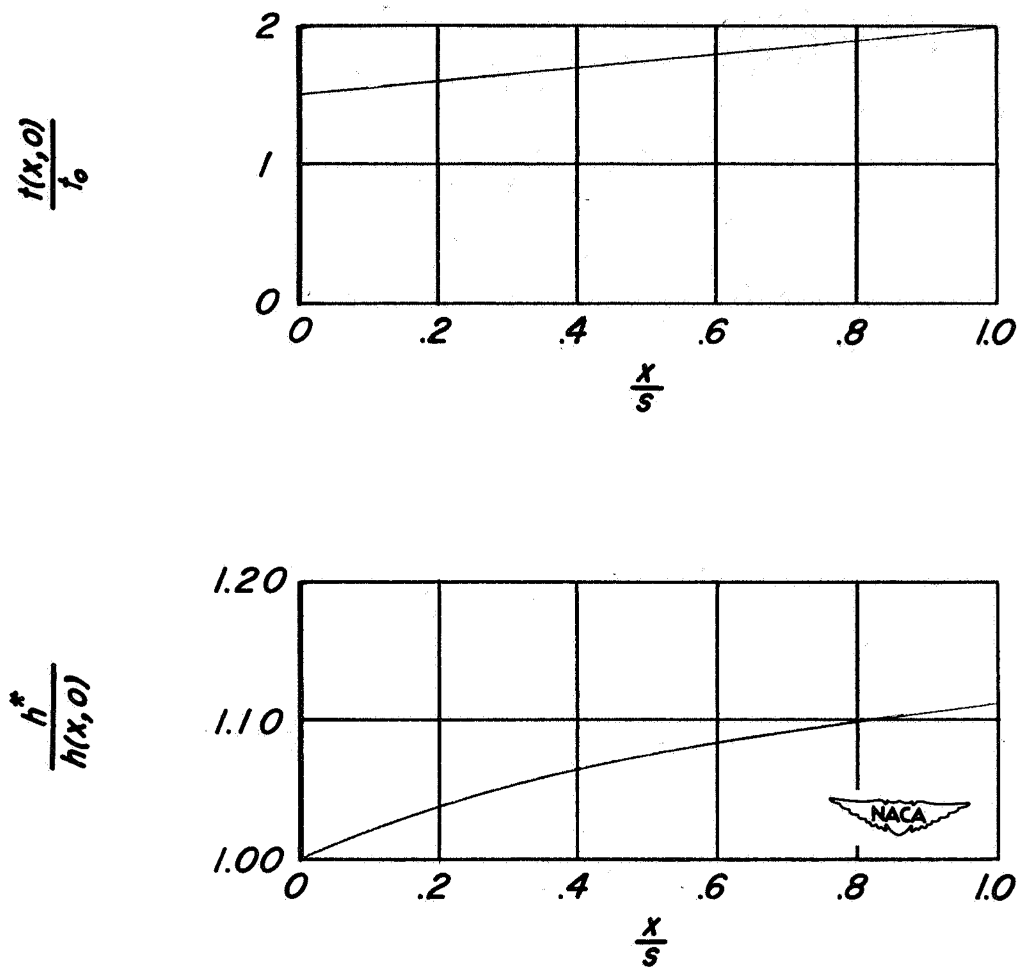

Figure 7. – Effect of variable surface temperature on the heat-transfer characteristics in example problem.