for having referred their results to absolute insulations only, and for not having computed them as resistivities referred to the cubic centimetre. For had they stated their results in the form of resistivity, we might have been inveigled into the belief that such was a real physical constant of the material; whereas it was probably a function of a number of local conditions any variation in which might have led to a different numerical result.

The following paper on "The Effect of Armature Induction upon the Electromotive Force Curves of an Alternator" was then read by Prof. W. E. Goldsborough.

A paper presented at the 14th General Meeting of the American Institute of Electrical Engineers, Eliot, Me., July 27th, 1897, Vice-President Steinmetz in the Chair.

THE EFFECT OF ARMATURE INDUCTANCE UPON THE ELECTROMOTIVE FORCE CURVES OF AN ALTERNATOR.

BY W. E. GOLDSBOROUGH.

The subject of the regulation of alternating dynamo-electric machines, considered as a function of the inductance of their armature coils, has been treated by a number of writers, and during the last few years has attracted much attention.

In looking up the bibliography of the subject, however, I find that but few records have been published of the actual value of the inductance of the generating coils of these machines, and that even less data are available regarding the real nature of the periodic fluctuations that take place in this quantity.

We are indebted to Hopkinson, Ayrton, Kapp, Sumpner, Duncan, Tobey and Walbridge, Reid, Steinmetz, Fleming, Rothert, Roessler and many others for much valuable information touching upon the theory and practice of the design and handling of machines of this type, but these writers, in-so-far as I am informed, have failed to furnish us with a record of the actual internal relations existing between the factors involved, freed of conventional and restricting assumptions.

With the assistance of Mr. W. N. Motter and Mr. S. R. Fox, I have carried on a series of experiments in the electrical laboratories of Purdue University for the purpose of investigating the subject.

APPARATUS AND METHOD EMPLOYED.

The experiments which I shall describe were made upon a three-light Brush arc machine, fitted with the necessary exploring coils, collector rings and revolving contact-making device. The machine was selected on account of the peculiarities of its design. These were particularly valuable, as the end sought was to obtain results of an exaggerated nature in order that the factors entering into the problem could be brought into bold relief and thereby lend themselves to more ready investigation.

The core of the armature is of the ring type and is built up of laminated iron stampings, held together by laminated iron bands passing around the core between the successive layers of stampings. Each stamping, therefore, forms a portion of the surface of a cylinder having its axis coincident with the axis of rotation. The stampings are shaped so as to make sixteen large teeth of trapezoidal section, which project laterally from the core, there being eight on each side. The armature winding is composed of eight coils or bobbins of 286 turns each. The bobbins on opposite sides of the armature are connected in series, and their free ends brought out to one pair of the eight commutator segments. Previous to making the experiments, all of the copper commutator segments were removed, and two cast-iron collector rings substituted for them. Fig. 1 shows the machine with these in position, and the collecting brushes in place. Each ring was connected electrically to one of a pair of commutator segment terminals, to which the free ends of one pair of the armature bobbins were fastened. The rings were then carefully insulated from contact with all the other commutator terminals. As the

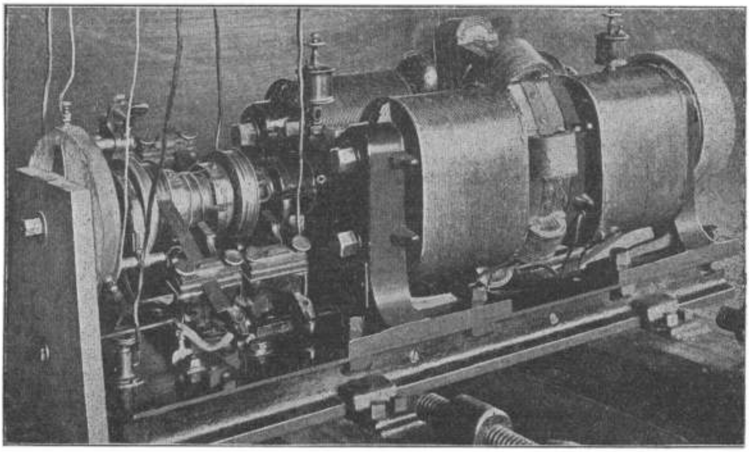

FIG. 1.

armature is of the open coil type, each commutator segment is attached to the terminal of but one coil, and therefore, by the arrangement described, the arc machine was converted into a two-coil alternator. Throughout the test, the remaining six coils were practically dead, being cut out and left on open circuit.

Owing to the fact that the armature bobbins are wound between large projecting teeth, and since the clearance spaces between the teeth and the pole-faces are relatively small,[1] the inductance of the coils is high. The coils are practically buried in iron, and the leakage of the magnetic lines of force set up in them is slight. There are in all, 16 teeth on the armature core, or 4 to each pole face. As the armature revolves, there are alternatively 3 and then 4 teeth opposite each pole-face The latter condition occurs when two of the coils take up positions midway between the pole corners, as for instance, coils 1 and 2 in Fig. 2. The former when four of the teeth reach positions midway between the pole corners N, S, and N′, S′. The reluctance of the path of the magnetic flux through the field frame and the armature core is therefore a variable quantity, and pulsates between its maximum and minimum values eight times during each revolution of the armature.

The four field exciting coils are connected in series in such a way as to form like magnetic poles facing one another, on the same side of the armature shaft. By this arrangement the lines of force entering the armature core from abutting pole-pieces repel one another and, dividing, penetrate the armature core above and below the shaft in a direction perpendicular to a vertical plane passed through the centre of rotation and parallel to the shaft. When the armature core is at rest, therefore, and the field coils separately excited, the field flux passing through any pair of armature coils will be a maximum when they are in positions 1, 2, Fig. 2, and will be zero when they are in positions 5, 6.

The method employed in making determinations of the self-induction of the armature was as follows:

An exploring coil of 42 turns of No. 36 B & S copper wire was wound over armature coil 1, and connected in series with the field coils of a high-resistance Nalder ballistic galvanometer and an adjustable non-inductive resistance. A constant exciting current of 10 amperes was kept flowing through the field circuit at all times. During the time of taking any one set of readings, a direct current of definite value was maintained in coils 1 and 2 except when the deflections were made. In circuit with the two armature coils and the source of power, were connected a Weston ammeter, the primary of the calibrating coil of the ballistic galvanometer and a snap switch. The snap switch being actuated by a spring, gave exactly the same form of mechanical "make" and "break" of the current at each reading. Readings were taken when making and when breaking the current in the armature coils, and the average of four observations made a record for a given angular position of the coils.

The angular position of the coils relative to the poles was indicated by the graduated disk of the contact-making device by adjusting it relatively to a point on the contact wheel at the time of taking each reading. Knowing the deflections of the galvanometer and having complete data covering the calibration of the instrument and the construction of the armature, the inductance of the coils at any given point was very readily calculated.

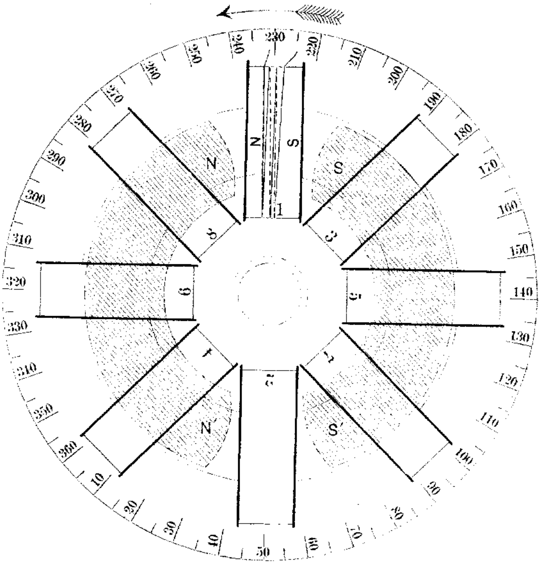

FIG. 2.—Showing coils No. 1 and 2 at "Zero Position" between the Pole Tips N, S, and N′, S′.

In determining the form of the electromotive force waves induced in the coils and appearing at the brushes of the machine, a form of contact-making device shown in Fig. 1 was used in connection with an improvised form of electro-dynamometer. The contact-making device possesses no new features. The galvanometer is of the Wiedemann type. The bell-shaped magnet and copper damper were removed from the instrument, and in place of these a wooden ball carrying a coil of fine wire imbedded in a deep narrow groove cut into its surface was substituted. Two pieces of copper wire were mounted upon the ball or needle in the plane of the coil, and at the extremities of a diameter to form an axis of rotation. The upper end of this axis made one terminal of the coil, it carried a small plane mirror and was connected at its extremity to a piece of hard-drawn brass wire. This wire being also attached to the torsion head, both sustained the weight of the ball and balanced by its torsion the deflecting force of the current impulses passed through the needle. The lower end of the axis formed the other terminal of the coil and carried a small paddle. It dipped into a cup containing mercury and glycerine: the glycerine sufficing to dampen the needle and the mercury making electric contact with it. When in use, the field coils of the galvanometer were excited with a constant current. The ball or needle was connected directly in series with the rotary contact, and with from two to three incandescent lamps, according to the value of voltage to be measured. The instrument was calibrated by connecting a constant current machine in series with the needle, lamps and rotary contact, and comparing the deflections of the needle taken while the contact was rotating at normal speed, with the readings of a voltmeter connected to the terminals of the source of constant E. M. F. During all the experiments, the galvanometer gave deflections as constant and as dead beat as those of a Weston ammeter. The arrangement of this apparatus is analogous to that described by Duncan[1] in 1892.

ARMATURE INDUCTANCE.

In determining the coefficient of self-induction of coils 1 and 2, coil 1 of the armature was placed at the 230° position, as indicated in Fig. 2, since at this point the magnetizing power of the coils was least, owing to the large magnetic reluctance of the circuit traversed by the lines of force emanating from the coils. The high reluctance is due to the fact that the iron of the core surrounded by the coils, and the iron of the pole tips N, N′ and S, S′, is all highly saturated by the field flux, and to the fact that the lines of force set up by the coils must either span an air-gap

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[1] Duncan, "Note on some Experiments with Alternating Currents." TRANSACTIONS, vol. ix, p. 79.

many times larger than the clearance space, or else pass through the entire field circuit of the machine.

When in the 230° position and with a constant current of ten amperes flowing in the field circuit, and a constant current of 4.3 amperes flowing in coils 1 and 2, the ballistic galvanometer gave a deflection of 25 both at the "make" and at the "break" of the current. The current in the coils at this time was flowing in the same direction as that in which the alternating current is flowing when the coils are passing out from under the pole tips, and the armature is rotating in the normal counter-clockwise direction. From position 230° the coils were moved in a clockwise direction, as viewed from the commutator end. This is opposite to the direction of rotation of the armature when running, but in the same direction as that in which the rotary contact brush was moved when the E. M. F. curves were taken. At the 220° position, "make" and "break" readings both equal to 24 were obtained. At position 210° the readings gave 25 divisions again. At 200° they were all 30 divisions, and so on through one-half of a revolution of the armature. When plotted, these readings gave the curve A of Plate 1.

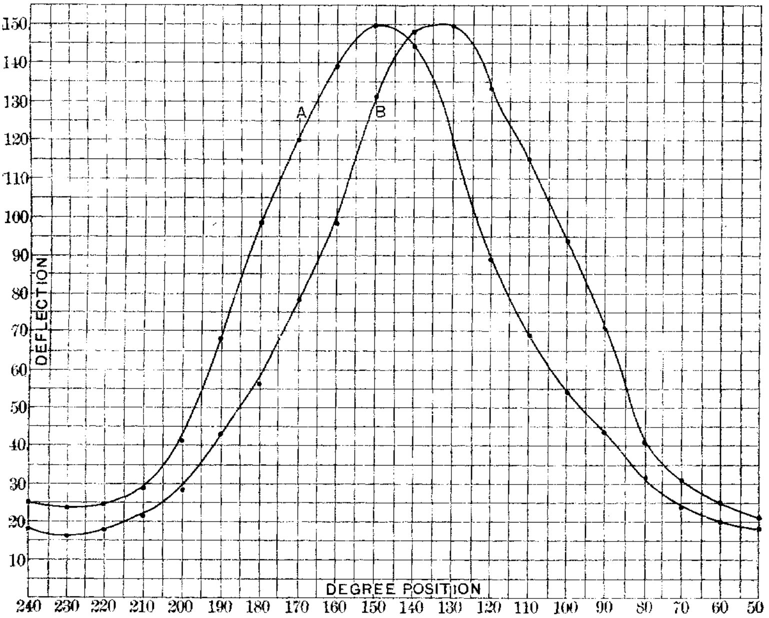

Plate 1.

An inspection of this curve shows that it is distorted in the direction of the normal rotation of the armature. Thinking that its unsymmetrical form might not be entirely due to a magnetic reaction between the armature and fields, and that possibly it was caused, in part at least, by a variation in the width of the air-gap space, owing to the core of the armature not being quite true, the curve was continued through the complete revolution. The result of the test, however, gave another curve exactly like curve A. This indicated conclusively that lack of symmetry in the mechanical construction of the machine played no part in causing the distortion. The current in the coils 1 and 2 was then reversed in its direction while being kept at the same volume. The exploration resulted this time in giving the curve B, of Plate 1. As before, the "make" and "break" gave the same deflection for any given position, but the curve was found to be drawn over towards pole tips S′ and N, Fig. 2, instead of towards S and N′, as in the case of curve A.

The distortion of these curves, the ordinates of which are proportional to the inductance of the coils in various angular positions, is entirely due to the variation of the permeability of the iron composing the armature core and pole-faces, as the energized armature coils are moved through the half revolution. When curve A, Plate 1, was taken, the current passing through the coils tended to produce a magnetic field in opposition to that induced by the field circuit when the coils were in the 185° position, and one aiding the field flux when they were in the 95° position. In other words, poles were generated in the coils as indicated by the small N and S on coil 1 of Fig. 2. With the coils in the 95° position, when the circuit was made, a number of ampere turns were brought into action, which increased the induction in the surrounding masses of iron and at the same time decreased their permeability. With the coils in the 185° position, making the circuit through the armature, decreased the magnetic density in the surrounding iron and thereby increased its permeability. Breaking the armature circuit in either case brought the induction back to the same initial value, since the two positions assumed are symmetrically located relatively to the pole-faces. The introduction of the same magnetizing force into the coils, however, did not cause the same variation in the induction threading the coils. Since the variation in the value of the permeability is in opposite directions in the two cases, the variation in the flux in the 185° position is greater than that which occurs in the 95° position, and therefore the self-induction of the coils in the former is greater than in the latter, under the assumed conditions.

Magnetic hysteresis also plays a part in the variability of the armature inductance. As the armature revolves, the iron of the core passes through a complete hysteretic cycle once every revolution, and in making determinations of the armature inductance, account must be taken of the fact that the magnetization of the iron of the core follows the perimeter of the loop of hysteresis and not the "B and H curve," as the permeance of the core changes.

To determine the extent of the error introduced by neglecting to adhere to the true magnetic changes through which the iron passes, as is the case in the method so far described, a rather laborious process had to be resorted to of carrying the iron in the core completely through the hysteretic cycle before taking each reading. By this means, owing to the fact that when a coil is between the pole tips, the laminated core of the coil is highly saturated, it was possible to get a very fair estimate of the influence of hysteresis.

Suppose we take the hysteresis loop of Fig. 3 to represent the cycle through which the magnetization of the iron passes during a revolution of the armature. Let *a* represent the magnetic density in the core of coil 1 when it is in 230° position, and no current flowing in the coil. As the armature is revolved in a counter-clock-wise direction, the intensity of the magnetization in the core will decrease until at the 320° position it will have a value equal to *o*, *b*. At some point farther on, say at 330°, there will be a sufficient negative force to reduce the residual magnetism to zero, as at *c*, Fig. 3; and finally, when coil 1 reaches the 50° position, the magnetism will have been brought up to a negative maximum within the coil, as at *d*. If at this point the circuit is made through the armature coils, and a constant current sent through them in the direction which aids the field flux, the density in the core will be increased to a still higher value, and rise up to *e*. Suppose now that the armature is revolved still further round, for instance, to the 120° position. The flux penetrating the coil will diminish to *f*. If at this point the armature current is broken by the snap switch, a current impulse will pass to the galvanometer needle from the exploring coil, due to the decrease in the induction threading the coil from *f* to *g*. If, without changing the position of the coil, the current is made again, the density will rise to *h*, but will not come up to the original value at *f*. In other words, any number of "makes" and "breaks" can be made after the first "break" with the same result, but the first "break" will give a larger reading than

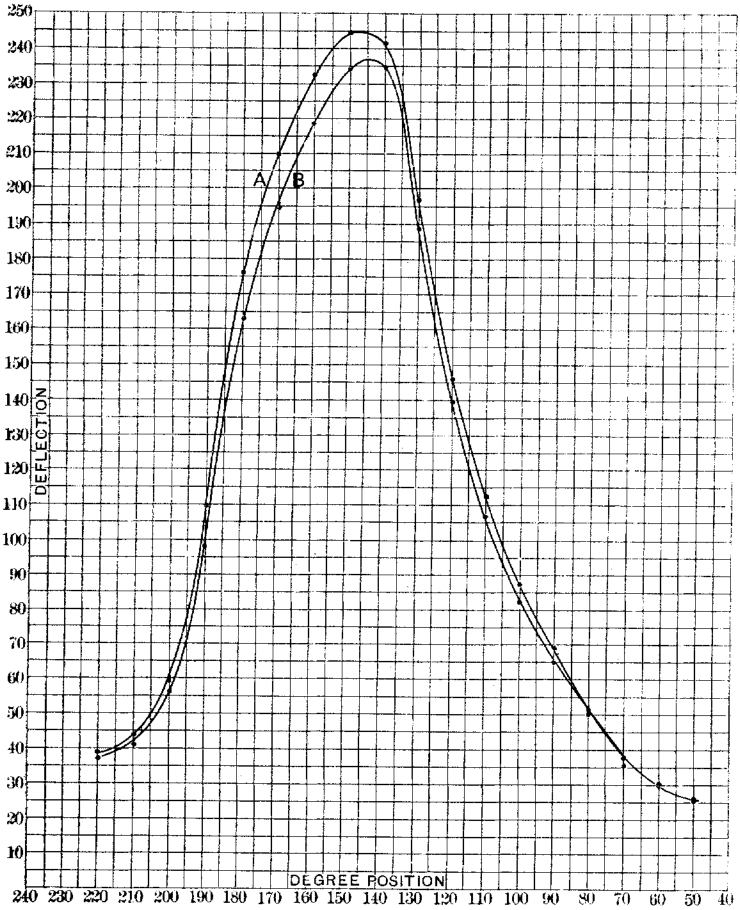

PLATE 2.

the others. The first "break" gives the most accurate result. By this method, however, coil 1 has to be brought back to the 230° position before taking each reading, then revolved counter-clock-wise to the 50° position, and, after the current is made, has finally to be moved to a position between s′ and s, to take the reading at the "break."

Following out this method for one set of readings, curve A, of Plate 2, was obtained. Curve B is a curve plotted from an average of the readings due to the second and third "breaks" and their corresponding "makes." It is identical with a curve taken by the method used in obtaining curves A and B, of Plate 1, but with the same armature current as that used in obtaining the curve A of Plate 2. An inspection of the curves of Plate 2 shows that the maximum percentage variation occurs at the 190° position, where it amounts to 12 per cent., and that the maximum actual variation occurs at the 170° position, where it amounts to 15 divisions, or 7.8 per cent. of the deflection at this point. This represents an actual difference of .0117 henrys at the 170° position.

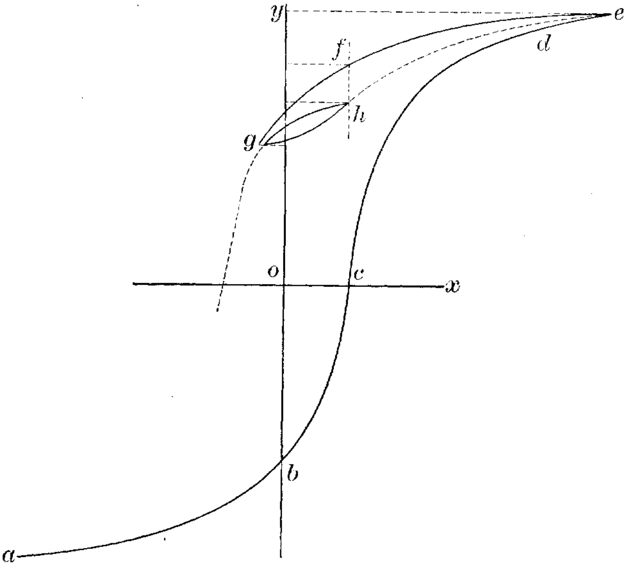

FIG. 3.

As was to be expected, from the considerations outlined above, the curve A of Plate 2 shows a greater distortion in the direction of rotation than does the curve B, taken by the less exact method. It reaches a maximum later in the cycle, and although of uniformly greater amplitude than curve B, the differences are most marked in the last half of the cycle, counting time from right to left.

Curves A and B are, however, exactly similar in form and so nearly alike, that it was not deemed necessary to follow out the more elaborate and exact method in succeeding determinations of the variable inductance, as the end in view was to determine the character of the changes occurring, rather than the absolute value of them.

Plate 3 exhibits a series of these inductance curves. They represent five explorations, made with direct currents of different intensities flowing through coils 1 and 2, while the field exciting current was maintained constant at 10 amperes, as usual. The readings have been reduced to henrys; the calculation of the values being based upon the supposition that the inductance of the two coils at any point is proportional to the change in the number of lines of force existing in the coils at that point, divided by the current flowing in the coils and producing the change. The plotted values, therefore, represent the inductances of the two coils in series, proper allowance having been made for the fact that the exploring coil was only wound about one of them.

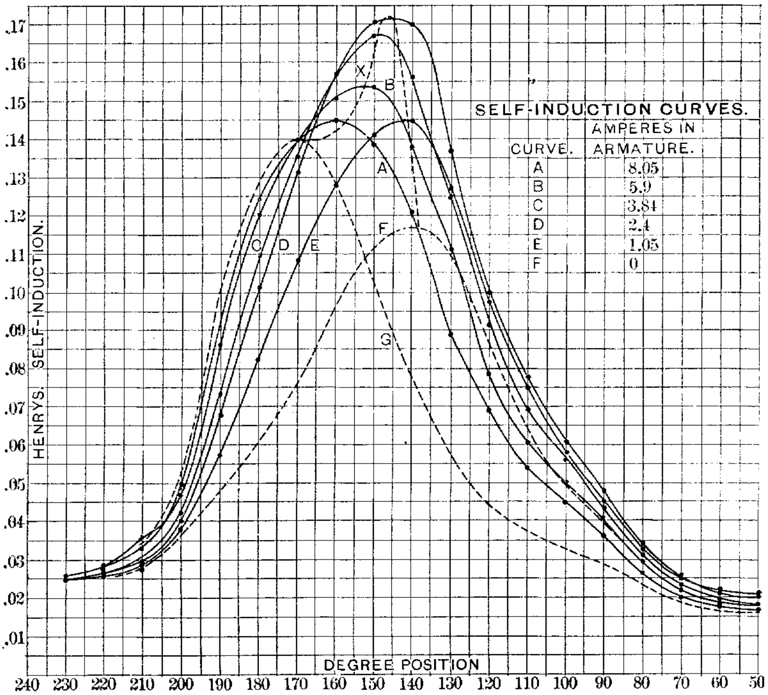

PLATE 3.

The curves of Plates 4 and 5 have been derived from those of

Plate 3. They show the relation between the direct currents flowing in the coils and the inductance of the coils, and are useful when it is desired to determine the curve of instantaneous inductance that corresponds to a given alternating current wave.

The curves of Plates 3, 4 and 5 bring to light some very interesting facts. They show that the maximum coefficient of self-induction occurs when the coils are in the 145° position and the current flowing in them has a value of 2.5 amperes; under these conditions their inductance is over .172 henrys. They indicate that as the armature current is varied from zero up to its maximum safe value, the inductance of the coils undergoes very marked and relatively rapid changes. The dotted curve F has been plotted from the ordinates of the curves on Plates 4 and 5 that correspond to the zero value of current. It represents the initial inductance cycle, and is the basic curve to either side of which the inductance curves for the various currents fluctuate. Curve F is practically symmetrical relatively to the pole-faces. Its crest occurs as the coils pass the centre of the pole-faces, and it slopes equally on either side. The line x, drawn from its crest through the crests of the other curves, indicates the successive positions and values assumed by the maximum inductance cor-

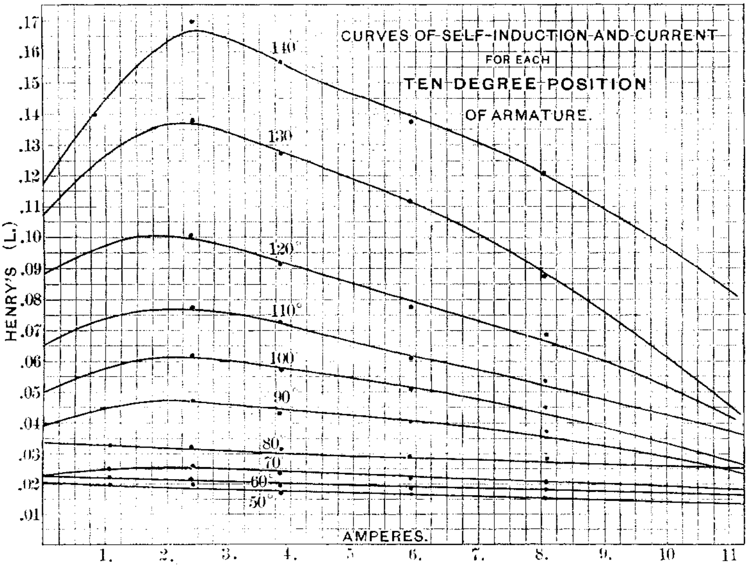

Plate 4.

responding to different constant currents, as the volume of these currents is increased from zero. The maximum current-carrying capacity of the armature is 15 amperes, but the inductance is changed from its maximum initial value of .117 henrys to its maximum possible value of .172 henrys when the armature current reaches 2.5 amperes. Any further increase in the current causes the amplitude of the inductance wave to diminish, and at 8 amperes it is only 85 per cent. of what it is at 2.5 amperes.

Besides varying in amplitude, the inductance waves undergo a lateral shifting in the direction of the rotation of the armature.

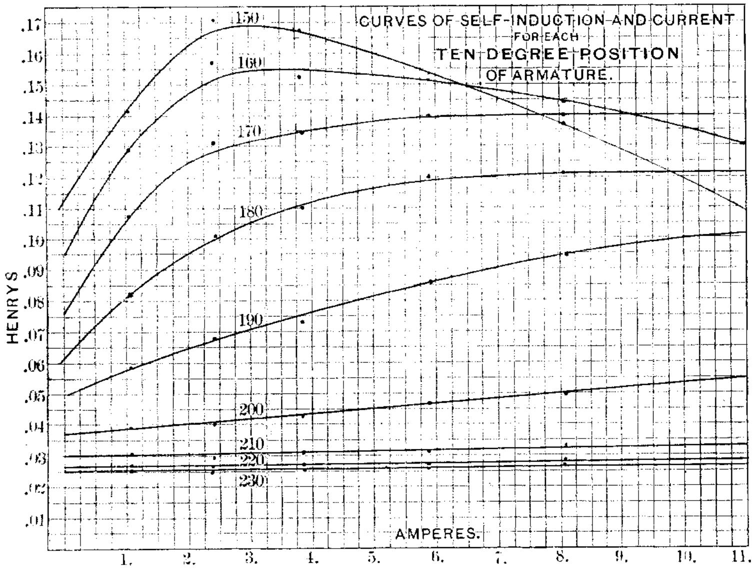

Plate 5.

The dotted curve G has been taken from Plates 4 and 5, and represents the inductance curve corresponding to 11 amperes. An inspection of Plate 5 shows that this curve practically marks the limit of the variability of the inductance. For a further increase in the current, the 170° curve of Plate 5, tends to remain horizontal, and the 180° curve exhibits no disposition to rise appreciably beyond the 11-ampere point, there will therefore be practically no further shifting of the crest, or change in the amplitude of the induction curve.

To better appreciate the extent of the distortion that occurs, it is well to note that at the 170° position there is a difference of about .065 of a henry between curves A and F of Plate 3, showing that the inductance of the coils at this point has double the value at full load that it has at no load. This fact is brought out strongly in Fig. 4, which shows the differences between the ordinates of curves D, F and G. In this figure, the amounts by which the ordinates of curves D and G of Plate 3 differ from those of curve F have been plotted as ordinates relatively to curve F reduced to a horizontal base line.

It is also interesting to note that between no load and full load, the crest of the induction curve moves through an angle of 30°, or over one fourth of the width of the pole-faces.

The fluctuations which take place in the armature inductance, are due to the changes occurring in the reluctance of the path traversed by the flux induced by the armature current. The action of the armature current is to intensify the induction in the armature core between the 60° and 130° positions of coil 1. The initial induction in the core was over 24,000 gausses when the coils were between the pole tips. The armature current acting with the field excitation maintained this saturation during the first part of the revolution and prevented any great variation in the induction in the first quadrant. But during the second and fourth quarters of each revolution, when the coils were between the 170° and the 200° positions, the armature current was reacting against the field excitation, and thereby, by diminishing the flux density, was increasing the permeance of the iron and

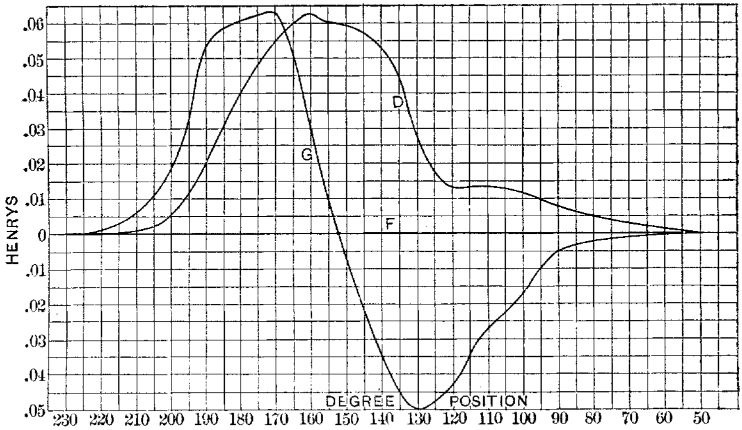

FIG. 4.

the inductance of the coils for these positions. The magnetizing power of the coils is considerable, as with 8 amperes flowing through them, they set up an induction of 18,000 gausses in the core when in the 160° position.

The great amplitude of the induction wave for 2.4 amperes in the coils is in the same way due to the fact that the permeance of the path is least when the magnetizing force is that due to this current. If we plot the values of the 140° curve of Plate 4 in terms of the armature current and the induction set up in the core by the current, we obtain the curve shown in Fig. 5. This is "B and H" curve for the magnetic circuit through the armature core in this position, and the point T which corresponds to the maximum ordinate of the 140° curve, is the point where a tangent to the curve makes the greatest angle with the base line.

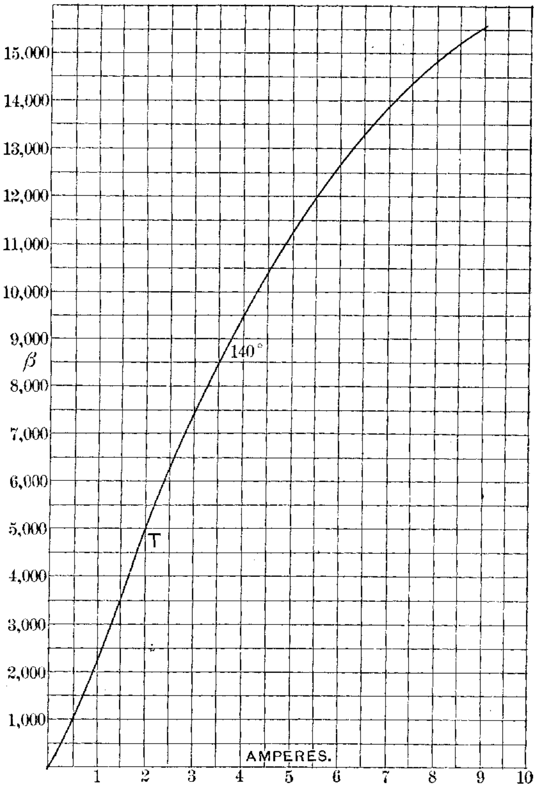

FIG. 5.

The large teeth of the armature core do not seem to have the effect of causing any marked irregularities in the inductance curves. It is noticeable, however, that there is a slight hump in all of them at the 90° position. This is caused when the teeth immediately behind the coils pass the pole tips. The same effect is produced by the other teeth, but it is too slight to appreciably affect the readings.

ELECTROMOTIVE FORCE CURVES.

A series of electromotive force curves taken from the machine are shown in Plate 6. These illustrate the gradual change in the

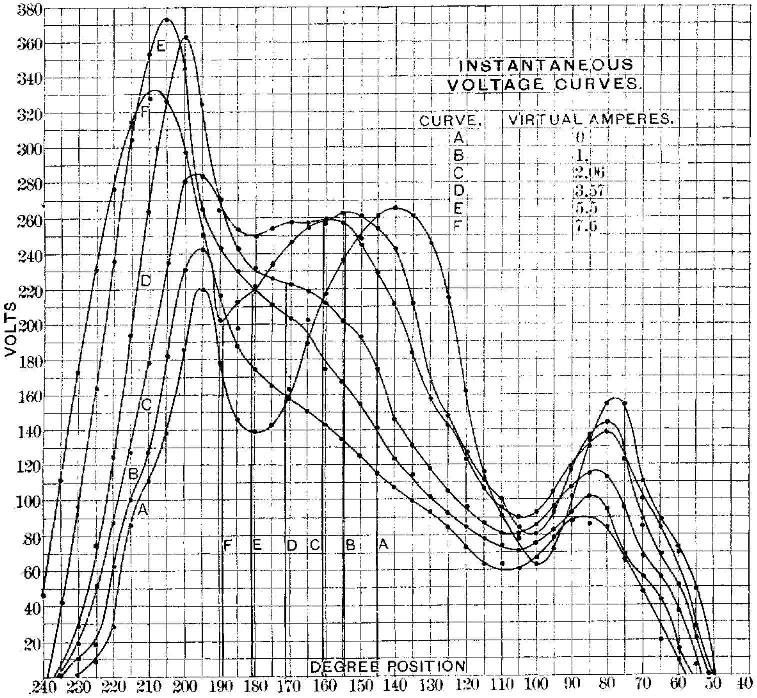

PLATE 6.

wave form of the effective electromotive force as the load increases. By the effective electromotive force is meant the electromotive force that is in phase with the current and overcomes the ohmic resistance.

There is quite an appreciable lag in the effective electromotive force for the higher loads, owing to the armature inductance. This can hardly be regarded in the light of a simple phase displacement, as the lag partakes largely of the nature of a transformation of the fundamental wave A, that is due to the phase positions of the higher harmonics being shifted relatively to the fundamental harmonic, and not to any material change in the phase position of the fundamental harmonic itself. The extent of the shifting is best appreciated by noting the positions of the ordinates A, B, C, D, E and F. These lines bisect the areas of the electromotive force curves designated by corresponding letters, and indicate the points at which the induction threading the coils passes through zero and changes sign.

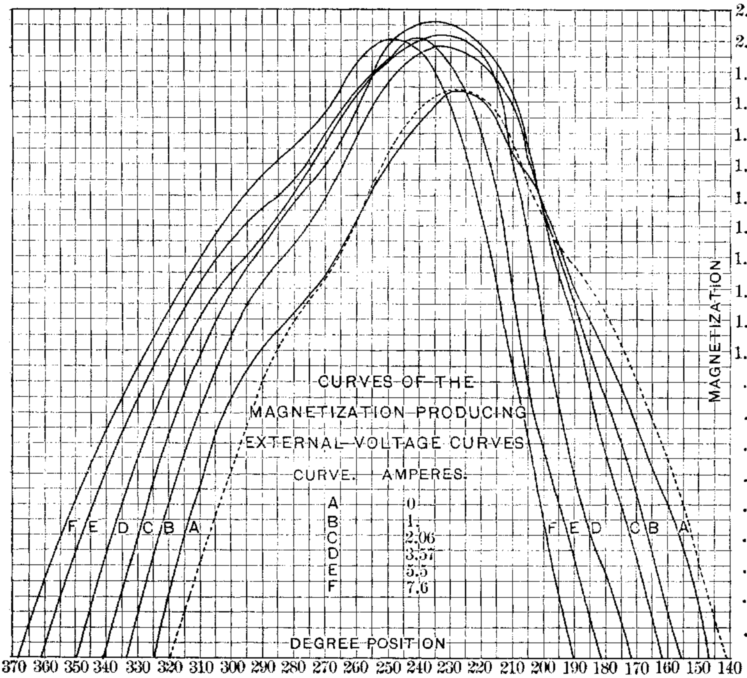

PLATE 7.

The curves of Plate 7 have been determined from those of Plate 6 by integrating the areas of the latter. They illustrate the waves of magnetism producing the effective electromotive force curves, and give a better idea of the extent of the lag that takes place. The scale of ordinates of Plate 7 expresses the value of the integral,

$$\int e\, dt = -N, \tag{1}$$

as determined from the curves of Plate 6, and must be multiplied by the constant 12206 to express the value of the induction per sq. cm. in the iron of the armature core. The maximum induction occurring in the core is therefore a little over 25,000 gausses.[1] In determining the value of the constant, an allowance of 20 per cent. was made for the lamination of the iron, 80 per cent. of the gross area of the core being used as the equivalent of the iron.

Curve A of Plate 6 was obtained with zero current in the armature. This is the "fundamental" or internal electromotive force wave of the machine. It will be noticed that it is not symmetrical about the 140° ordinate. It is an irregular curve having three prominent peaks; the right hand one slightly depressed, the left hand one raised somewhat, and the central one practically symmetrical about the centre line of the pole-faces. The ear peaks are due to higher harmonics being superimposed upon the fundamental wave by the magnetic disturbances in the air-gap that are caused by the large teeth. Steinmetz[2] has noted this effect and treated the subject of the distorting influence of armature teeth in a masterly manner. His results indicate comparatively symmetrical waves for no-load conditions. In the case under consideration, a marked shifting of the electromotive force curve in direction of rotation is caused by the teeth immediately on each side of the coils. As the teeth approach the pole tips, the lines of force do not appear to reach out to receive them to any extent. There is a sluggishness apparent. On the other hand, when the teeth are leaving the trailing pole tips, the lines of force seem to hold on with tenacity, and when they do let go, fly back with a snap as would extended strands of elastic. In the extreme cases, this action does not appear to occur until the tooth is over an inch away from the pole tips. The rapid cutting of the lines necessarily augments the potential at this point.

This explanation has its bearing upon the case, but the distortion is perhaps better explained by looking at it in the light of the part played by hysteresis and eddy currents. The eddy currents induced in the pole tips tend to oppose any change that

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[1] Some elaborate tests were made on Brush dynamos in 1889, by Mr. Murry. The values of B attained were about 4,800 gausses in the field, and 27,000 in the armature cores. See S. P. Thompson's "Dynamo Electric Machinery," p. 464, 1892 edition.

[2] C. P. Steinmetz, TRANSACTIONS, vol. xii., p. 470.

takes place in the field distribution of the induction. They help to maintain a high induction in the trailing pole tips, and keep down the induction in the leading pole tips. The iron in the core of the armature is being carried through a complete hysteretic cycle at each revolution. The iron in the pole tips is carried through an hysteretic loop with the passage of each tooth across the pole-face. At the leading pole tips, the magnetism in the core iron is always working on the descending portion of the curve of hysteresis, and the reluctance of the magnetic circuit by the leading pole tips is increased accordingly. At the trailing pole tips the iron is worked on the ascending side of the hysteretic curve, and the reluctance of the path by the trailing pole tips is thereby reduced. The same magnetizing force, therefore, acting upon the two paths, induces a distorted field when the armature is running, and a symmetrical field when the armature is at rest, as shown by curve A and the dotted curve of Plate 7. The latter curve was obtained by exploring the air-gap distribution of the flux by the ballistic method, using the exploring coil wound over coil 1, and breaking the field circuit.

The activity displayed by the eddy currents in heating up the pole tips is quite remarkable. The curves B and A of Plate 8 indicate the rise in temperature of the leading and trailing tips, respectively, throughout a two hours' run. The readings were taken from thermometers fastened directly to the pole-pieces. The curve C is a curve of differences derived from curves A and B, and it indicates how very much more rapidly than the temperature of the leading pole tips the temperature of the trailing pole tips rises during the first twenty minutes of the run. Their differences in temperature after that time is practically constant. The curves plainly indicate that much greater magnetic disturbances occur in the leading pole tips than at other points in the pole-faces. The thermometers had to be fastened to the back of the pole tips during the test; had it been possible to place them in contact with the air-gap face, more rapid changes would doubtless have been recorded. During the test the field coils were excited with a current of 10 amperes, and the armature was loaded to about 5 amperes. At the end of the run, the temperature of the armature was 42° C.

As the armature core has sixteen teeth, or four per pole face, we may expect harmonics as high as the ninth to play a prominent part in the wave structure.[1] The general form of the internal

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[1] C. P. Steinmetz, TRANSACTIONS, Vol. xii, p. 475.

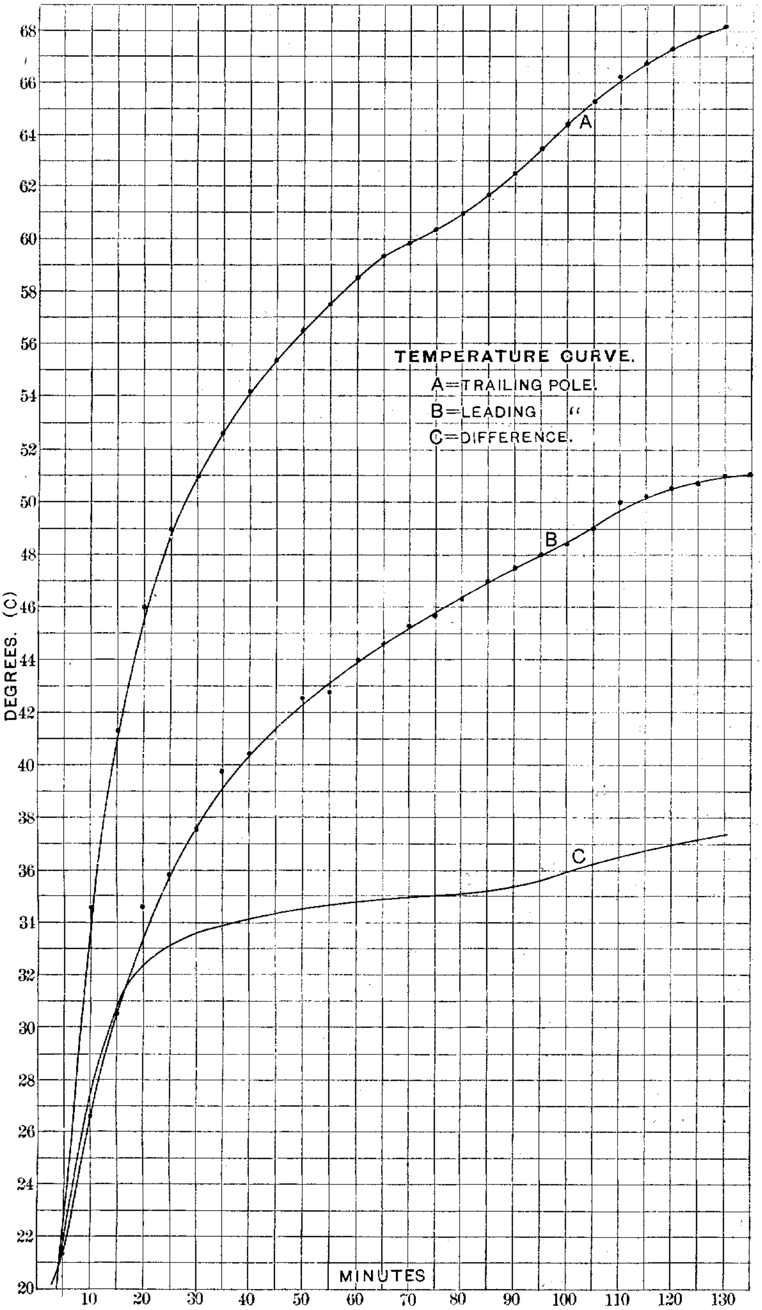
TEMPERATURE CURVE.
A=TRAILING POLE.
B=LEADING "
C=DIFFERENCE.
A
B
C
DEGREES. (C)
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
31
32
30
28
26
24
22
20
MINUTES
10
20
30
40
50
60
70
80
90
100
110
120
130

PLATE 8.

electromotive force curve A is indicative of the fifth harmonic; the side peaks have been sharpened, however, by the upper harmonics, and the central one somewhat broadened by the third.

The wave form of the electromotive force induced in the armature when the machine is loaded depends upon the character of the inductance curve of the armature coils. As already explained, the effect of the teeth upon the inductance curves is to introduce harmonics and cause irregularities in their contours. Practically the same number of harmonics are prominent in all the electromotive force curves, although the armature inductance has the effect of smoothing out the irregularities by giving the third harmonic greater prominence, and diminishing the effect of the higher harmonics. As the current rises and the magnetizing power of the armature becomes apparent, the harmonics introduced into the circuit by the variable inductance grow in amplitude. They combine with the harmonics of like order of the fundamental wave, and an electromotive force curve results that is compounded of a series of harmonics that depend upon the value of the armature current for their amplitude and phase positions.

A very small amount if any of the change in the form of the effective electromotive force waves is due to any departure of the original no-load wave of field flux from its initial wave form. In fact, to all practical intents and purposes the pulsations that occur in the magnetic reluctance of the field circuit are sensibly of the same intensity at full load as they are at no load. In other words the changes that occur in the permeance of the core of the armature as the load varies do not appreciably alter the permeance of the field circuit. The fundamental wave of magnetism and of electromotive force may therefore be regarded as always being present, if not always tangible.

Where alternating current generators are operated on circuits having a constant resistance for constant conditions of load, the disturbances that modify the form of the electromotive force and current waves can be attributed to the cyclic variations in the inductance of the system, and when these are known within a fair degree of approximation the character of the wave modifications that will result can be determined.

In the development of the experimental results contained in this paper the dynamo was loaded by inserting sets of incandescent lamps in the external circuit. A special point was made of keeping the external circuit entirely free of any inductance. The current curves are therefore necessarily proportional to the electromotive force curves. However, to make their relative value and form more easy of appreciation, they have been plotted and

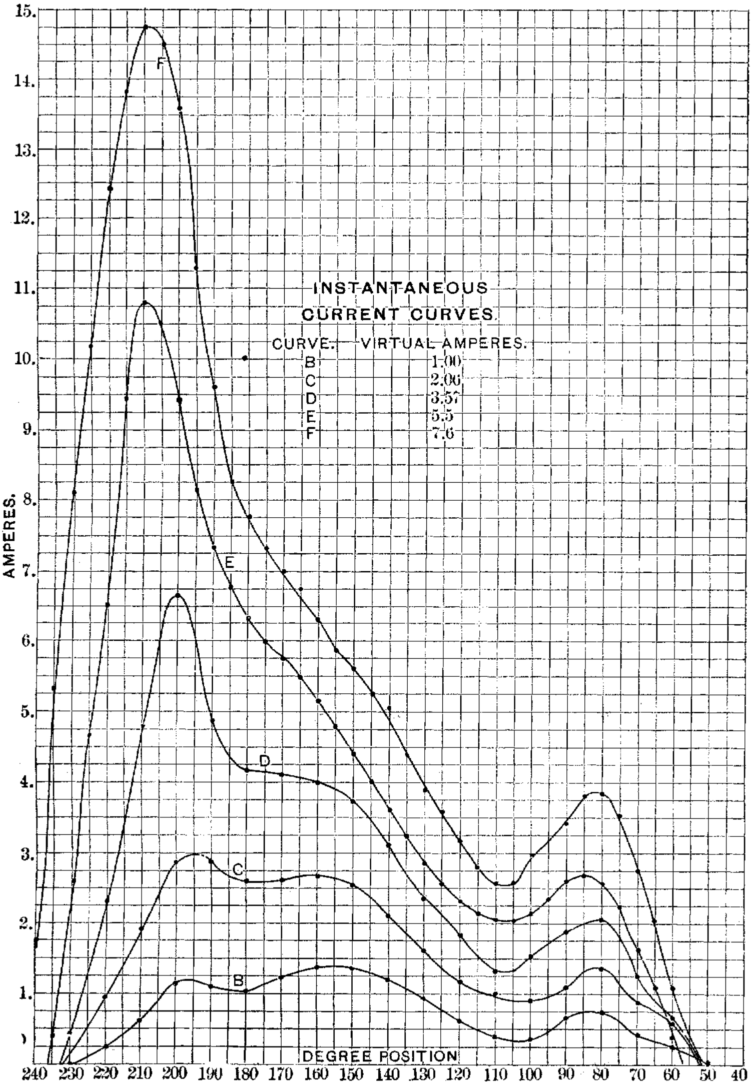

PLATE 9.

are shown in Plate 9. The inductance curves of the coils that correspond to the alternating current curves are given in Plate 10. These curves were determined by selecting from the curves of

Plates 4 and 5, inductance values corresponding to the instantaneous values of the current curves and plotting them to the same degree positions.

It is noticeable, owing to the different scales used in plotting the curves, that the current curves seem more even and smoother than the electromotive force curves.

The inductance curves also are more nearly alike, both in phase and in amplitude, than the curves given in Plate 3. This is owing to the fact that the alternating currents all start from, and end at zero values when the coils are in positions of slight inductance, while they attain their maximum values at points of large inductance. For this reason the average inductance per cycle of the coils, is greater with an alternating current flowing in them than with a direct current of the same effective value. The alternating currents, for analogous reasons, force the induction curves up to the saturation limit marked by curve G of Plate 3, more rapidly than do the direct currents, and therefore for the same variation in effective current strength a less marked variation is caused in the inductance curves with these currents. The curves of Plate 3 certainly lead us to expect a greater variation, relatively to current intensity, in the armature inductance when the armature is in actual operation than that which is depicted in Plate 10.

It is a curious coincidence that the alternating current inductance curves all cross at a common point: namely, at the 164 ° position.

COUNTER ELECTROMOTIVE FORCE[1] CURVES.

Having the instantaneous current and inductance curves corresponding to a series of loads on the machine, it was a comparatively easy matter to determine the curves of the counter electromotive forces developed in the armature. To obtain a curve showing the cyclic variation of the induction set up in the coils by the armature current it is only necessary to plot the products of the ordinates of corresponding points on corresponding current and inductance curves. Such a set of curves obtained from a combination of the curves of Plates 9 and 10 are given in Plate 11.

If N represents the total flux that is induced in both coils by the armature current, divided by 10^8; L represents the inductance

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[1] In this paper the term "counter E. M. F." is used to indicate the E. M. F. which added to the effective E. M. F. will equal the fundamental E. M. F. The counter E. M. F. is therefore equal and opposite to the inductance E. M. F., or the E. M F. of self-induction.

of the coils connected in series in henrys; s represents the total number of turns (572) of wire in both armature coils; and A represents 80 per cent. of the gross area of the armature core inside the coils, or 11 sq. cms.

Then the induction per sq. cm. in the coils due to the armature current will equal,

$$B = \frac{N \times 10^8}{S\,A} = 15892 \times N. \qquad (2)$$

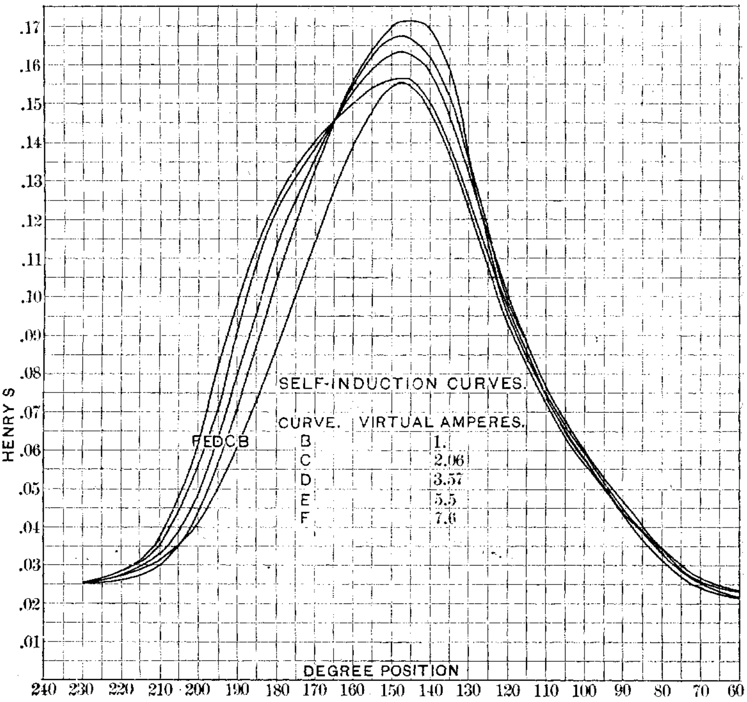

PLATE 10.

This equation gives the constant (15892) that must be applied to the scale of ordinates in Plates 11, 12 and 13 to determine the actual core densities induced by the armature currents.

Again, if i represents the instantaneous value of the armature current,

$$N = L\,i. \qquad (3)$$

and

$$d\,N = L \times d\,i + i \times d\,L, \qquad (4)$$

since L is a variable. Therefore, the counter electromotive force developed in the coils equals

$$e = -\left(-\frac{d\,N}{d\,t}\right) = \left(L\frac{d\,i}{d\,t} + i\frac{d\,L}{d\,t}\right). \tag{5}$$

In Plates 9, 10 and 11, we have the necessary curves for determining the instantaneous values of the counter electromotive force corresponding to any degree position by applying the principles underlying either of the above expressions. Both methods were used and were found to check within narrow limits. In Plates 12 and 13, the curves marked A have been taken from Plate 11. The ordinates of the curves marked B are equal to the

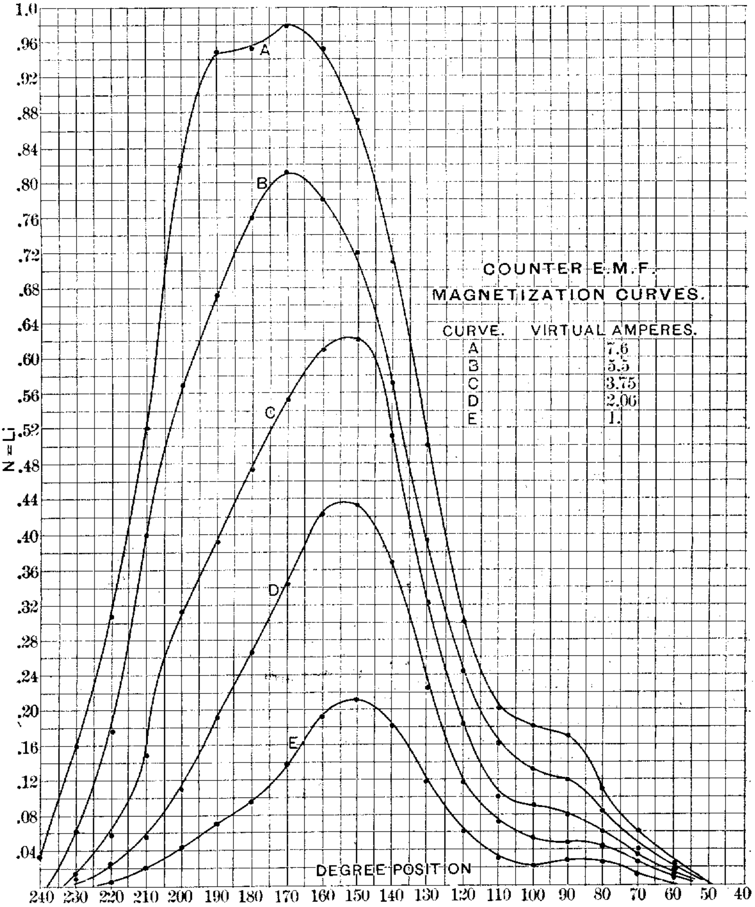

PLATE 11.

tangents of the angles made with the horizontal axis by lines drawn tangent to the A curves at the extremities of corresponding ordinates. The B curves are the calculated curves of counter electromotive force developed in the armature. The dotted curves were obtained by subtracting curves D and E of Plate 6 from curve A of Plate 6. On the assumption that the fundamental electromotive force wave of the machine does not change with the load, the dotted curve should also represent the counter electromotive force curve of the armature, and, in fact, the two curves should coincide. Plate 12 represents the poorest, and

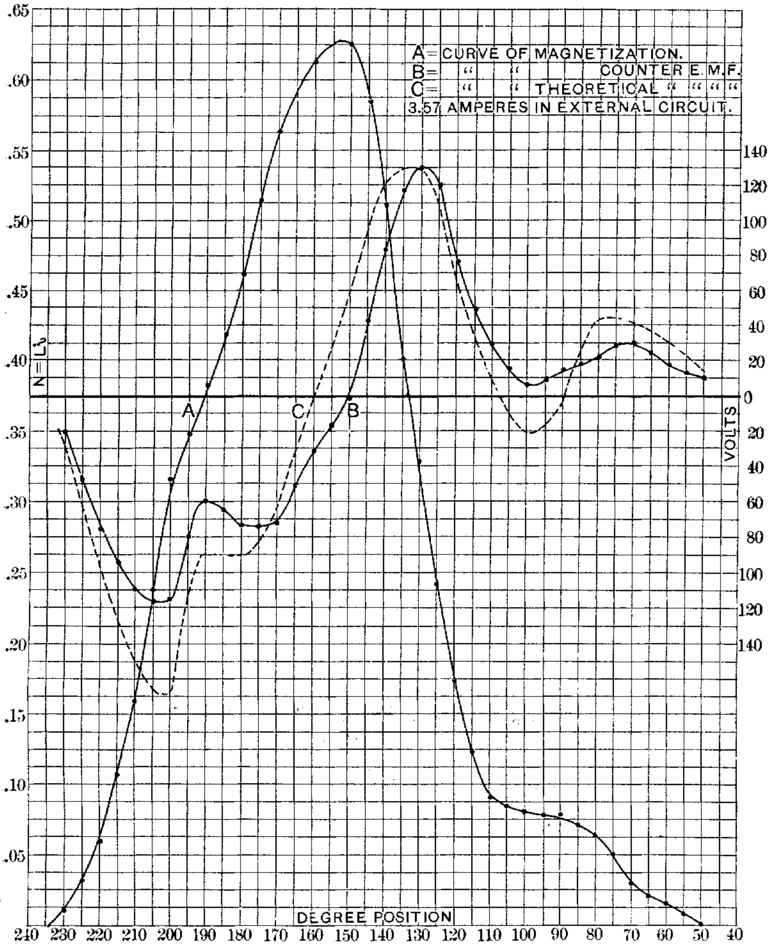

PLATE 12.

Plate 13 one of the best results obtained from the application of this construction to each of the five sets of curves taken from the dynamo. When the fact is taken into account that the inductance of the coils was determined when the armature was at rest, the likeness between the dotted and the full curves is quite remarkable, and apparently justifies the assumptions that have been made. The counter electromotive force curves are highly irregular in form. They oscillate from the positive to the negative value twice in a period instead of once, and are generally useful in filling up gaps. As shown, the curves represent the halves of one of the positive and of one of the negative loops; in other words, the half period of the curves lies between the 350° and 170° positions, and not between the 50° and 230° posi-

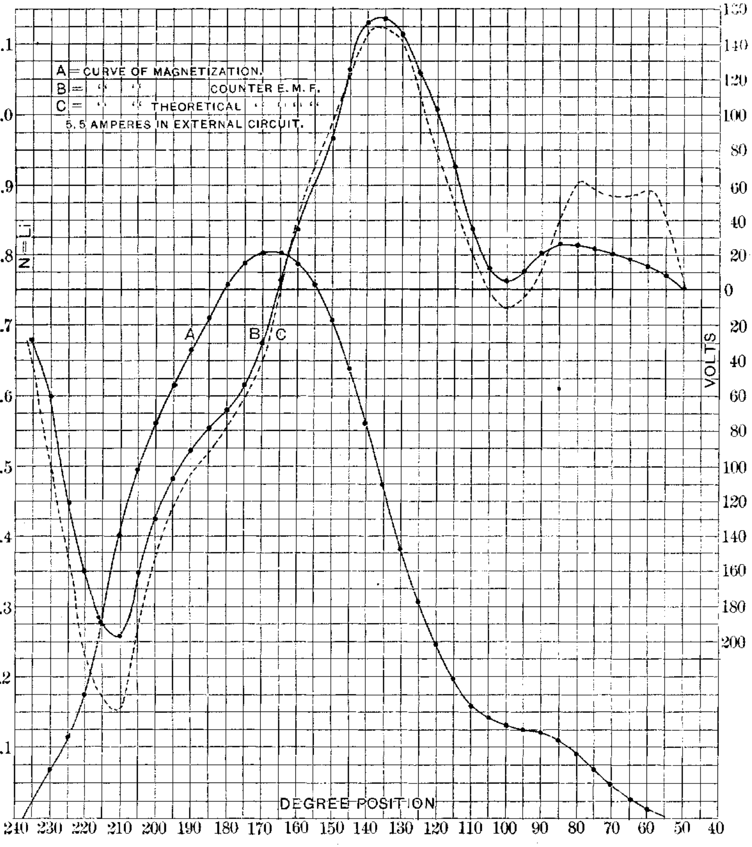

PLATE 13.

tions. This will be made more evident by a reference to curve c of Plate 18.

The whole series of inductance curves, as obtained by the subtraction method, are plotted in Plate 14. This assemblage shows that the curves follow one another in regular order in spite of their *lack of symmetry*, and it is interesting to note the receding of one and the building up of another "hump" in these curves, as illustrated at the lower left hand edge of the sheet.

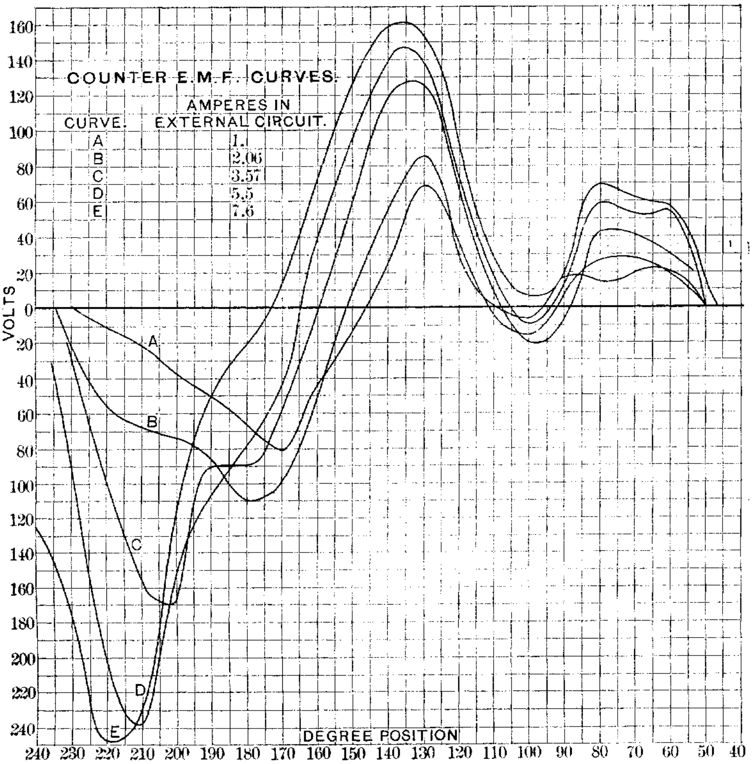

PLATE 14.

Plate 15 represents another phase of the subject. The curves traced in the fine black lines are the results of adding the calculated electromotive force curves determined by the means illustrated in Plates 12 and 13, to the corresponding effective or external electromotive force curves of Plate 6. They show the result of an attempt to work back from the effective electromotive force curves, or from the wave form of electromotive force appearing at the collector rings when the machine is loaded, to the fundamental electromotive force wave or the electromotive force wave appearing at the collector rings when the machine is running on open circuit.

The curve A, reproduced on this plate for comparison with the curves just mentioned, is the same as curve A of Plate 6. The agreement between the curves is marked. The derived curves show the greatest departure from the fundamental curve early in the cycle, between the 70° and the 110° positions. This is due to the fact that the initial "hump" at the 90° position of the induction curves was not developed with sufficient care. A very slight change in the contour of the induction curves at this point makes a great difference in the form of the counter electromotive force curves, owing to its effect in altering the direction of the tangents drawn to the magnetization curves of Plate 11. On the whole the likeness between the curves is well within the limits of the errors of observation.

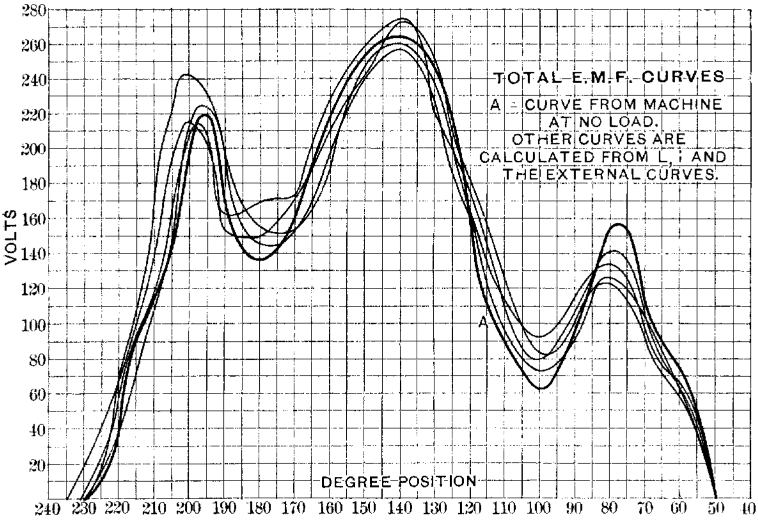

PLATE 15.

DERIVATION OF CURRENT CURVES.

Another interesting set of curves is shown in Plate 16. The success attained in working out the counter electromotive force curves, led to a series of calculations to determine to what extent the form of the current curves was influenced by the shifting of the phase of the inductance curves with the load. To this end an average of the inductance waves A, B, C, D and E of Plate 3 was taken to represent the average inductance wave of the armature for all loads. This curve is shown as curve B of plate 16. With this curve and the fundamental electromotive force wave A, forming a basis for calculation, the current that will flow with any given resistance in the circuit can be readily determined, provided that the curve B is taken to represent all of the variations that occur in the armature inductance.[1] The relation existing between the impressed electromotive force, the current and inductance in any alternating current circuit similar to the one under discussion is expressed by the equation

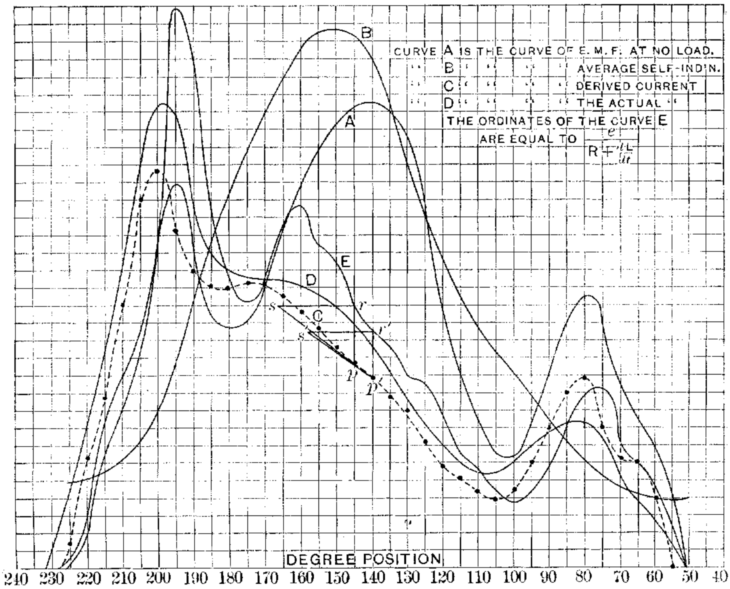

PLATE 16.

$$e = R\, i + \frac{d\, N}{d\, t}, \qquad (6)$$

There e, i and $\left(\frac{2\ V \times 10^{8}}{S}\right)$ are, respectively, the instantaneous

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[1] Steinmetz, "On the Law of Hysteresis." TRANSACTIONS, vol. xi., p 574. In this article the author calls attention to the solution of a problem that is analogous to this one, differing from it only to the extent that he works from an assumed sinusoidal inductance-electromotive force and a circuit of negligible resistance.

values of the impressed potential, current, and the magnetic flux induced by the current, and R is the resistance of the circuit.

Since $N = L\,i$, as above, (see equation 1), in the present case we have:

$$e = R\,i + L\,\frac{d\,i}{d\,t} + i\,\frac{d\,L}{d\,t}. \tag{7}$$

Then,

$$i\left(R + \frac{d\,L}{d\,t}\right) = e - L\,\frac{d\,i}{d\,t}, \tag{8}$$

and

$$i = \frac{e}{\left(R + \frac{d\,L}{d\,t}\right)} - \frac{L}{\left(R + \frac{d\,L}{d\,t}\right)} \times \frac{d\,i}{d\,t}. \tag{9}$$

By applying this formula to the curves of Plate 16, and following out a graphical construction, the successive instantaneous values of the current were finally determined, although in accomplishing this result the current curve E, which was the outcome of the process, had to be carried through successive cycles until it repeated itself.

The values of $\left(\frac{d\,L}{d\,t}\right)$ were taken from curve B. The ordinates of the curve A were then divided by the corresponding values of $R + \frac{d\,L}{d\,t}$, and these quotients, when plotted, gave the curve E.

Next, referring still to Plate 16, a point, as for instance p', was taken, that was thought to lie near the current curve, and the line $(p'\,r')$ drawn. From (r') the line

$$(r'\,s') = \frac{L}{R + \frac{d\,L}{d\,t}} \tag{10}$$

was laid off and $(s'\,p')$ drawn. Now from the assumed position of p',

$$\frac{p'\,r'}{r'\,s'} = \frac{p'\,r'}{\left[\frac{L}{R + \frac{d\,L}{d\,t}}\right]} = \frac{d\,i}{d\,t}, \tag{11}$$

and $(p'\,s')$ established the direction of the current curve, since from equation (9),

$$\left[\frac{L}{R + \frac{d\,L}{d\,t}}\right]\frac{d\,i}{d\,t} = \frac{e}{R + \frac{d\,L}{d\,t}} - i, \tag{12}$$

and

$$\frac{d\,i}{d\,t} = \frac{\dfrac{e}{R + \dfrac{d\,L}{d\,t}} - i}{\left[\dfrac{L}{R + \dfrac{d\,L}{d\,t}}\right]} \tag{13}$$

and therefore,

$$(p'\,r') \text{ approximated the value of } \frac{e}{R + \dfrac{d\,L}{d\,t}} - i. \tag{14}$$

By taking another point p on p' s′, and continuing the construction, a chain made up of short lengths $p'\,p$ was obtained which ultimately developed into the periodic wave C, for which the values of

$$p'\,r' = \frac{e}{R + \dfrac{d\,L}{d\,t}} - i. \tag{15}$$

The graphical construction used is not new. It is an adaptation of one of Dr. Sumpner's[1] unique methods of treating alternating current problems, and I have developed it here simply as a matter of interest in connection with the discussion.

Returning to the curves, it is noticeable that the dotted curve C resembles the current curves of Plate 9 very closely. Comparing it with curve D, which is a current curve taken from the machine, with 55 ohms resistance in the complete circuit, the chief difference noticed is that the right hand peak of the dotted curve is a little high, and the left hand peak a little low, and that the curve as a whole is somewhat depressed below the curve D. The differences between the curves are, however, not very marked. They result largely from the curve B having a less amplitude and smaller slope than the alternating current inductance curves of Plate 8, since it is derived from the direct current inductance curves of Plate 3.

The construction indicates a method that can be employed with success in determining before the machines are built, the wave forms that will be developed by alternators. It is possible to predetermine the fundamental electromotive force and inductance waves of an alternator, by the application of modern

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[1] Dr. W. E. Sumpner, *Philosophical Magazine*, June, 1887, p. 470.

methods of design, and from these the load curves and armature reactions can be ascertained by processes analogous to those shown here.

If in the present case it had been desired to calculate the actual current curve with great exactness, an extension of the method used could have been employed. This elaboration necessitates taking the inductance values from Plates 4 and 5, as each point on the current curve is fixed. The process represents a refinement, however, that is hardly warranted in view of the good approximations resulting from the use of the less tedious plan.

OVER-COMPOUNDING.

It was noticed during the test that the armature reaction caused an increase in the total effective potential for certain ranges of load.

This phenomenon has been noted by other writers, and in his paper before the INSTITUTE, already referred to, Steinmetz has pointed out an explanation for it. Curve A of Plate 17 is the external characteristic of the dynamo, with constant field excitation, and coils 1 and 2 alone in action. The curve was plotted from the readings of a Weston voltmeter and a Siemens dynamometer connected in the external circuit. The load was a non-inductive one, and the speed of the dynamo averaged 1,450 r. p. m., which is equivalent to a frequency of a little over 24 periods per second.

The line R represents by its slope the resistance of the two coils, and curve B, which was obtained from the sum of the ordinates of the curves A and R, is the total effective E. M. F. characteristic of the coil. It will be noticed that the curve B shows an over-compounding effect up to a load of 7 amperes, and that the total effective E. M. F. was a maximum for a current of 4 amperes in the coils.

The term "armature reaction," as used above, is not intended to convey the idea that the current in the coils has any effect in varying or modifying the intensity of the flux passing through the field spools. The influence of the M. M. F. of the coils is purely a local one, and the paths traversed by the flux which it induces are confined to the air-gaps, poles and armature core. The influence of the armature inductance upon the flux density in the cores of the field spools is negligible, as a careful exploration established the fact that the field current was perfectly constant.

The compounding action of the armature currents in augmenting the effective voltage is a matter with which you are doubtless familiar. As a matter of interest, however, and in order to present a better picture of the wave forms involved, a series of curves has been plotted, that includes the power curves, and enables us to more readily appreciate the cycle of events that combine to produce this result. In Plate 18, curve A is the fundamental E. M. F. wave; curve B is the total effective E. M. F. wave and also represents the instantaneous values of the current; and curve C is the counter E. M. F. wave. Curve D is the power curve of the machine determined by taking the product of the corresponding in-

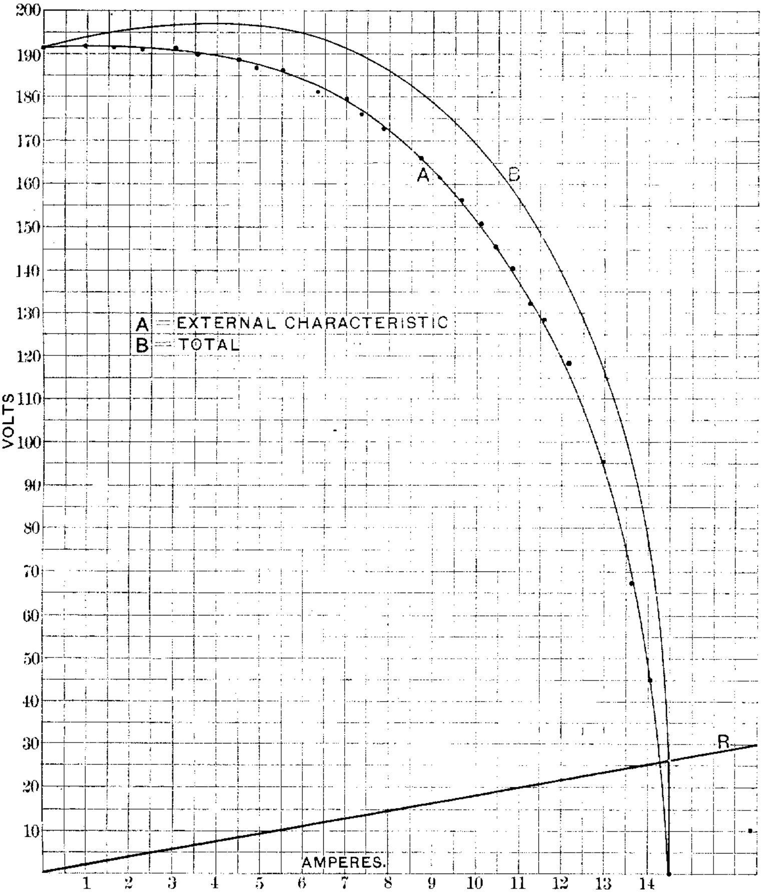

PLATE 17.

stantaneous values of the current and the fundamental E. M. F. curve. Curve E is a power curve obtained by taking the product of the corresponding values of the current and the counter E. M. F. The power curve of the total power developed by the machine, and which is proportional to the product of the corresponding values of the current and effective E. M. F. curves, is not plotted, as its ordinates are proportional to the square of those of curve B, and equal to the vertical distances intercepted between the power curves D and E.

It will be noticed that the power area inclosed between the curve E and the base line changes sign four times in a complete period, owing to the irregular form of the curve C, and that the sum of its positive and negative areas is not zero, but equal to a negative quantity.

Positive work done by the counter E. M. F. indicates power absorbed from the circuit. The energy consumed in overcoming the hysteresis and eddy current losses in transformer cores is a familiar example of this fact. Negative work performed by the counter E. M. F. in the same way represents power given to the circuit. In the present instance the negative area is due to a peculiar combination of irregular wave forms, involving the inductance curve, the current curve, the curve of the magnetism inducing the counter E. M. F., and the curve of counter E. M. F. The peaks of all the curves are shifted more and more in the direction of rotation as the load comes on, but some are given a greater displacement than others. The peak of the current curve, for instance, lags behind the peak of the inductance curve. In other words, the current is a maximum when the inductance is least. This results in the production of a wave of M. M. F of self-induction that is in advance of the current, and that produces a wave of magnetism in the coils which reaches a maximum midway between the peaks of the inductance and current curves. These phase relationships can be followed out by comparing Plates 10, 9 and 11. The wave of inductance magnetism (curve A, Plate 11) induces a curve of inductance E. M. F. that has an effective phase displacement behind the current wave of less than 90°. This makes the effective phase displacement of the counter E. M. F. curve (curve C, Plate 12, and curve C, Plate 18) more than 90° in advance of the current curve, and therefore the product of the instantaneous values of the counter E. M. F. and current curves gives a power curve having an excess negative area. It is rather difficult to estimate the phase position of a wave that has an outline as irregular as that of the curve C of Plate 18, but it is best approximated by noting the location of its greatest positive area, and this, in the case in hand, is early in each half period.

CONCLUSIONS.

An inspection of curves B and D of Plate 18 shows that any decrease in the maximum slant of the curves of Plate 11 will decrease the amplitude of the 200° peaks of the counter E. M. F. curves, and thereby decrease their form factors, and at the same time decrease the form factors of the effective E. M. F. curves. This is what occurs in the case of curve D of Plate 11, owing to the great amplitude of the inductance loop for a value of the current of two amperes, which also accounts for the drop in the "form factor curve" of Fig. 6. This curve represents the form factors of the E. M. F. curves of Plate 6, and of the current curves of Plate 9, plotted relatively to the virtual value of the current flowing in the coils at the time the curves were taken. The form factors have been calculated according to Fleming's[1] definition, which is that the form factor equals the ratio of the square root of the mean square of the ordinates of the wave to the true mean ordinate of the wave.

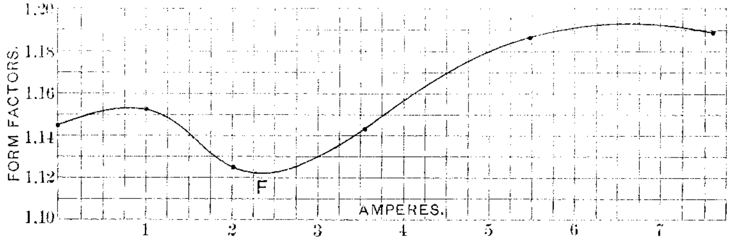

FIG. 6.

The influence of the inductance of the coils upon the form factor can best be appreciated by looking at the curves of Plate 10. They all have the same width of base, but vary in amplitude, and necessarily other things being equal, produce counter E. M. F. curves having the smallest form factors when currents which are favorable to a large armature inductance are flowing in the coils.

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[1] J. A. Fleming, "The Form Factor of Alternate Current Curves." London *Electrician*, vol. xxxvi., p. 338.

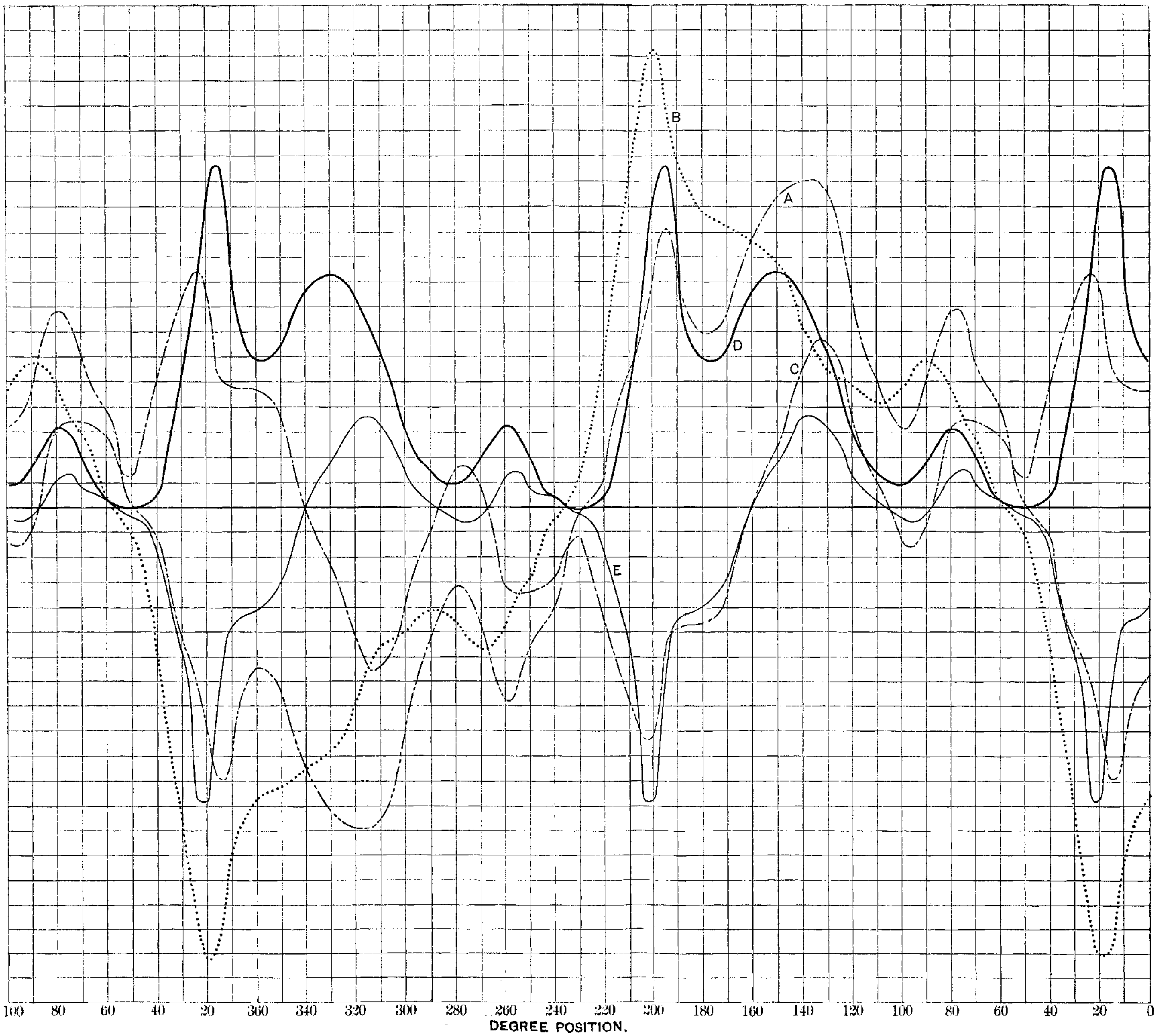

Plate 18.

A = Fundamental E. M. F. wave.
B = Total effective E. M. F. curve and current curve.
C = Counter E. M. F. curve.
D = Fundamental E. M. F. power curve.
E = Counter E. M. F. power curve.

The total power developed is represented by vertical distances between the curves D and E.

It is noticeable that the right sides of the curves of Plate 10 all have the same slant. They therefore produce counter E. M. F. loops, having similar shapes (see Plate 14). On the other hand, the left sides of these curves vary in shape, owing to variations in the reluctance at this part of the cycle. Greater variations, therefore, occur in the instantaneous values of the counter E. M. F. induced in this part of the cycle than in the former part, with the result of increasing the form factor where the induction curves corresponding to the current curves are relatively low and broad, and of decreasing the form factor of the counter E. M. F. curves where the inductance curves are narrow and high.

Again, since the maximum peak of the effective E. M. F. curve is opposite in sign and position to the leading peak of the corresponding counter E. M. F. curve, and since the sum of these peaks must not be greater than the adjacent peak of the fundamental E. M. F. wave, we see that an increase in the form factor of a counter E. M. F. curve causes an increase in the form factor of the corresponding effective E. M. F. curve, and vice versa. That the changes in the form factor of the effective E. M. F. curves are approximately directly proportional to the mean width of the inductance loops, and inversely proportional to the amplitude of the inductance loops. These conclusions are amply sustained by the data presented.

The variations that take place in the value of the armature inductance when the load is increased, lead us to expect that there is a proportionally greater increase in the value of the counter E. M. F. as the current passes 2 amperes in value. An accelerated increase in the counter E. M. F. would have the effect of making the compounding proportionally greater, for it would add a proportionately greater amount of energy to the circuit, owing to the fact that although the amplitude of the counter E. M. F. is increased, its phase position remains practically the same. That a positive acceleration does occur in the rate of increase of the counter E. M. F. is true, and also, that this is followed by a negative acceleration. The value of the counter E. M. F. depends upon the intensity of the current, and increases as the current increases. It, however, does not increase at a rate that is proportional to the rate of increase of the average intensity of the current, for its value also depends upon the armature inductance which first increases to a maximum and then decreases, as is shown by the curves of Plate 10. We, therefore, find that as the average intensity of the armature current is uniformly increased, the counter E. M. F. experiences a proportionally greater rate of increase up to a certain point and that after this point is passed, its rate of increase is diminished.

This means that as the load comes on, the value of the effective electromotive force increases above the value of the fundamental wave for a time. For from the relative positions of the highest peaks of the curves B and C of Plate 11, it will be evident that any increase in the virtual value of the counter E. M. F. will cause the virtual value of the effective E. M. F. to increase also, and any increase in the virtual value of the effective E. M. F. above the virtual value of the fundamental wave of E. M. F. raises the potential at the brushes. The fact that the virtual value of the effective E. M. F. finally falls below that of the open circuit E. M. F. is due to the ultimate gradual decline in the armature inductance wave and a consequent change in the form of the counter E. M. F. magnetization curve, which reduce the amplitude of the leading peaks and augment the amplitudes of the trailing peaks of the counter E. M. F. curves. This effect is apparent in curves D and E of Plate 14.

As exemplified in the curves of Plate 7, we find that the maximum induction threading the armature coils occurs when the form factor is least, and the armature inductance greatest. Again, the analysis of the forces involved shows us that the point F on the form factor curve of Fig. 6, and the point T on the "B & H curve" of Fig. 5, are intimately connected.

The double rôle played by the permeability of the iron of the armature core and pole-pieces, and the importance that may be attached to the permeability of the iron under favorable conditions is forcibly brought to our minds. We are enabled to see a little more clearly the complex relations of the interlinked forces that act and react upon one another in the conflict of the air-gaps, and there is established a basis for the analysis of a problem that has been more or less obscure.

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