THE EFFECT OF DEFENSE SPENDING ON US OUTPUT: A FACTOR AUGMENTED VECTOR AUTOREGRESSION (FAVAR) APPROACH

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Abstract

Empirical evidence on the effect of defense spending on US output is at best mixed. Against this backdrop, this paper assesses the impact of a positive defense spending shock on the growth rate of real GNP using a FAVAR model estimated with 116 variables spanning the quarterly period of 1976:01 to 2005:02. Overall, the results show that a positive shock to the growth rate of the real defense spending translates to a positive and long lasting effect on the growth rate of real GNP, but the effect is significant only for two quarters. In addition, we indicate that the mixed empirical evidence could be a result of small information sets, by showing the sensitivity of the results to sample size using a small-scale VAR typically used in the literature to analyze the effect of defense spending on output. Finally, given that the FAVAR model was found outperform the VAR in forecasting the growth rate of real GNP, we concluded that the FAVAR framework is superior and should be relied upon more for the analysis of the impact of defense spending on US output.

Keywords: Defense Spending; Output; FAVAR.
JEL Classification Codes: C32; C52; E27; H41.

1.   Introduction

Evidence on the effect of defense spending on US output, measured in level or growth rate, is at best mixed.[1] While studies like Atesoglu and Mueller (1990), Ireland and Otrok (1992), Mueller and Atesoglu (1993), Atesoglu (2002, 2009) concludes of a positive effect of defense spending on a measure of output, empirical evidence provided by Ward and Davis (1992) and Smith and Tuttle (2008) suggests of a negative relationship between defense spending and output. Huang and Mintz (1990) reported no effect and Brauer (2007), based on rolling regressions, found unstable results in the form of no effect and negative effect at different points of time. Cuaresma and Reitschuler (2003)

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[1] Refer to Atesoglu (2009) for an excellent literature review on the effect of defense expenditure on output.

obtained level dependent effect of defense spending on growth, i.e., defense spending was found to have a positive effect at low levels of growth, while the direction changed at higher levels. Using a non-linear approach, Ocal and Brauer (2007) found results that were similar to those of Cuaresma and Reitschuler (2003), in the sense they indicated that the effect of defense spending depends on whether the economy is in a low or a high growth period, with a large negative effect during a high growth period.

Against this backdrop, this paper assesses the impact of defense spending on the growth rate of real Gross National Product (GNP) of the US economy by exploiting a data-rich environment which includes 116 quarterly series over the period 1976:01 to 2005:02. For this purpose, the framework used in this paper is a factor-augmented vector autoregressive (FAVAR) model, as proposed by Bernanke et al. (2005). Even though Atesoglu (2009) suggests that no definitive statements can be accepted for the effect of defense spending on output, and one needs to use up-to-date data and revisit the question time and again, we believe that the mixed empirical evidence on the effect of defense spending on output could be a result of small information sets, and, hence, the motivation to use a FAVAR approach. To put it differently, all the above mentioned studies are based on either, one (or two) equation(s) macro models, reduced-form Vector Autoregressive (VAR) models or Vector Error Correction Models (VECMs), which, in turn, limits them to at the most 6 to 12 variables to conserve the degrees of freedom. And as large number of variables affects output, besides defense spending, not including them could often lead to results that might not be capturing the true relationship and dynamics of the data (Walsh, 2000). Moreover, in these studies, the authors often arbitrarily accept specific variables as the counterparts of theoretical constructs, which, may not be perfectly represented by the selected variables. Given its econometric construct, the FAVAR model solves these problems.

To the best of our knowledge, this is the first attempt to analyze the impact of defense spending on output using a wide information set of 116 variables that accounts for the various sectors of the economy. The remainder of the paper is organized as follows: Section 2 briefly discusses the FAVAR framework, while, Section 3 discusses the data and the identification structure. Section 4 reports and analyzes the impulse response function of output following a defense shock, and Section 5 concludes.

2.   The FAVAR[2]

Let Y_t be a $M \times 1$ vector of observable economic variable assumed to drive the dynamics of the economy, in our case, this happens to be the real defense spending only. Assume that F_t is a $K \times 1$ vector of unobserved factors that summarizes additional important information, such as potential output not fully captured by Y_t. Note F_t can also represent theoretical concepts such as price pressures, credit conditions, or even economic activity that are a combination of economic variables which cannot be represented by one particular series.

Assume that the joint dynamics of (F_t', Y_t') are given by the following equation:

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[2] This paper follows the econometric framework of the FAVAR model described in Bernanke et al. (2005).

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \tag{1}$$

where $\Phi(L)$ is a conformable lag polynomial of finite order p and v_t is the error term with zero mean and a covariance matrix Q.

Equation 1 is a standard VAR. However, the difficulty here, compared to standard VARs, is that the vector of factors F_t is unobserved, which means that the model cannot be estimated based on standard econometric techniques, such as the ordinary least squares (OLS). The proper estimation of the model entails the use of factor analysis, as proposed by Stock and Watson (1998). For this purpose, we assume that the factors summarize information contained in a large panel of economic time series. Let X_t be a $N \times 1$ vector of informational variables, where N is large, such that $N > K + M$. Assume X_t is related to both the observed variables Y_t and unobserved factors F_t as follows:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \tag{2}$$

where Λ^f is a $N \times K$ matrix of factor loadings, Λ^y is $N \times M$, and e_t is a $N \times 1$ vector of the error term, which, in turn, is weakly correlated with mean zero. In essence Y_t and F_t are common forces that drive the dynamics of X_t. Note, it is not restrictive to assume in principle, that X_t is dependent on current value of F_t, as factors can always capture arbitrary lags of some fundamental factors. Excluding the observable factors from Equation 2, we have what Stock and Watson (1998) refer as a dynamic factor model. In this paper, we assume that the only observable variable to the econometrician is the real defense expenditure, i.e., $Y_t = DEF_t$.

The estimation procedure consists of a two-step approach proposed by Bernanke et al. (2005), which, in turn, provides a way of uncovering the common space spanned by the factors of X_t, $C(F_t, Y_t)$. In the first step, the space spanned by the factors is estimated using the first $K + M$ principal components of X_t, $\hat{C}(F_t, Y_t)$. Stock and Watson (2002) demonstrates that with a large N, and if the number of principal components is at least as large as the number of factors, the principal component recover the space spanned by both F_t and Y_t. However, $\hat{F}_t$ is obtained as the part of $\hat{C}(F_t, Y_t)$, which is not spanned by Y_t. In the second step, the FAVAR model is estimated by a standard VAR method with F_t replaced by $\hat{F}_t$. As in standard a VAR, defense spending is ordered last with the assumption that unobserved factors do not react to defense spending shocks contemporaneously, which, in turn, produces orthogonal residuals. The reduced form VAR, based on Equation 1, then has the following structural form:

$$\Gamma(L) \begin{bmatrix} \hat{F}_t \\ Y_t \end{bmatrix} = u_t \tag{3}$$

where $\Gamma(L)$ is a conformable lag polynomial of finite order p and u_t is a vector of structural innovations. Given this, we compute the IRFs of $\hat{F}_t$ and Y_t as follows:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Psi(L)u_t \tag{4}$$

where $\Psi(L)$ is a lag polynomial of order h and $\Psi(L) = \Gamma(L)^{-1}$.

Given that X_t is estimated by $\hat{X}_t = \hat{\Lambda}^f \hat{F}_t + \hat{\Lambda}^y Y_t + e_t$, based on Equation 2, the IRFs of $\hat{X}_t$ are given by:

$$\hat{X}_t = \begin{bmatrix} \hat{\Lambda}^f & \hat{\Lambda}^y \end{bmatrix} \begin{bmatrix} \hat{F}_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \hat{\Lambda}^f & \hat{\Lambda}^y \end{bmatrix} \Psi(L)u_t \tag{5}$$

3.   Data

The data set contains 111 quarterly macroeconomic series of the US economy used by Boivin et al. (2008), and covers the period of 1976:01 to 2005:02. The data set includes measures of industrial production, several price indices, interest rates, employment as well as other key macroeconomic and financial variables. To this data set, following Ireland and Otrok (1992) and Atesoglu (2009), we added real defense spending, real debt, the implicit price deflator for GNP, real GNP (nominal GNP divided by the GNP deflator) and real non-defense government spending (including federal and state government spending),[3] making it a dataset of 116 variables.[4] Besides the above five variables, Ireland and Otrok (1992) used M2 and the nominal six month commercial paper rate interest rate measure, namely, while Atesoglu (2009) used Moody's Seasoned AAA Corporate Bond Yield less the rate of change in Gross Domestic Product Deflator: Chain-type Price Index (Seasonally Adjusted), as the interest rate measure. M2 is already in the data set, while, the Federal funds rate is chosen as the measure of short-term interest rate, also already available in the data. All series are seasonally adjusted and transformed to induce stationarity. As in Bernanke et al. (2005), we include five common factors in the estimation of the FAVAR with a lag length (p) of 4. Similar to these authors, we find that increasing the number of factors further does not change the results substantially. To account for uncertainty in the estimation of the factors, a bootstrap

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[3] Given that only quarterly data is available for the two key variables of interest, namely GNP and defense spending, the 111 monthly macroeconomic variables taken from the Boivin et al. (2008) data set were converted to their quarterly values by calculating averages of the monthly data.

[4] Please refer to Boivin et al. (2008) for further information on the 111 macroeconomic variables. The source of data for the five additional variables is the FRED database of, the Federal Reserve Bank of St. Louis. Real defense spending is measured by the national defense consumption expenditures and gross investment deflated by the implicit GNP deflator, real non-defense government spending is equal to the real Government Consumption Expenditures & Gross Investment less real defense, and real public debt is the total outstanding national debt deflated by the implicit GNP deflator.

technique based on Kilian (1998) is implemented. This is necessary in constructing the 90 percent confidence intervals of the impulse responses.

4.   Empirical Results

Figure 1 displays the impulse response function of the growth rate of real GNP resulting from an increase in the growth rate of defense spending by one standard deviation, based on the FAVAR model described above. The real defense spending increases to approximately 0.25% and stays significant on for a short period. Following the increase in the defense spending, the initial impact on the growth rate is positive for around 10 quarters, but becomes insignificant after about two quarters. The effect, however, persists for quite a while, with signs of reversion back to the initial steady-state only being seen around the 17th quarter.
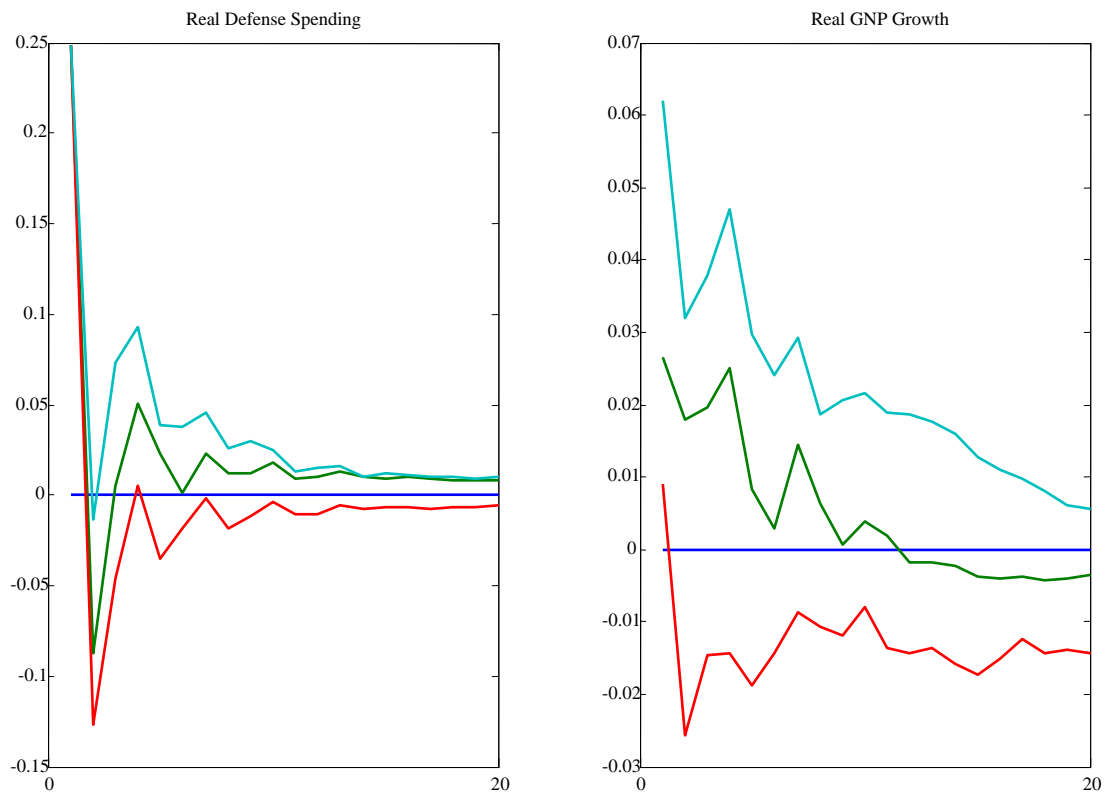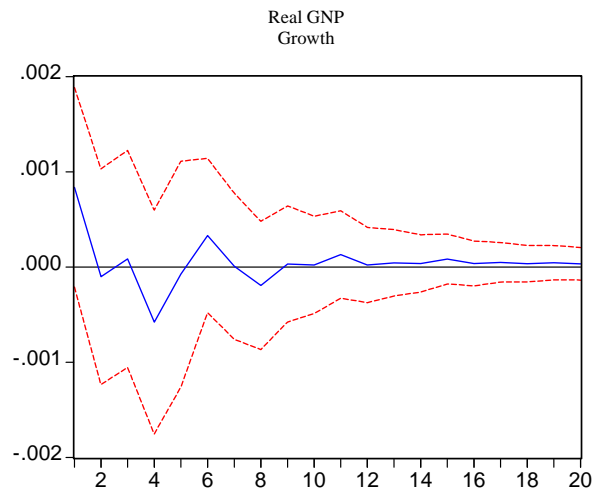

Figure 1: *IRF of Growth Rate of Real GNP to Growth Rate of Real Defense Spending (FAVAR)*

Recall that, in the introduction, we indicated that the mixed empirical evidence on the effect of defense spending on output could be a result of small information sets -- we now put this presumption of ours to test. In Figure 2, we present the impulse response function of the growth rate of real GNP resulting from an increase in the growth rate of defense spending by one standard deviation, based on a small-scale VAR model along the lines of Ireland and Otrok (1992) and Atesoglu (2009) for the period of 1976:01 till 2005:02. The

VAR[5] comprises of seven variables, namely, the growth rate of real outstanding public debt, the growth rate of real non-defense government spending, the Federal funds rate, the growth rate of real M2, the growth rate of the GNP deflator, besides the growth rate of real GNP and the growth rate of the real defense spending. As can be seen, following the increase in the defense spending, the initial impact on the growth rate is negative for around 5 quarters, but is generally insignificant. Beyond the 5th quarter, the effect stays positive but insignificant and dies down by the 16th quarter. Overall, the size of the effect is quite small as well.
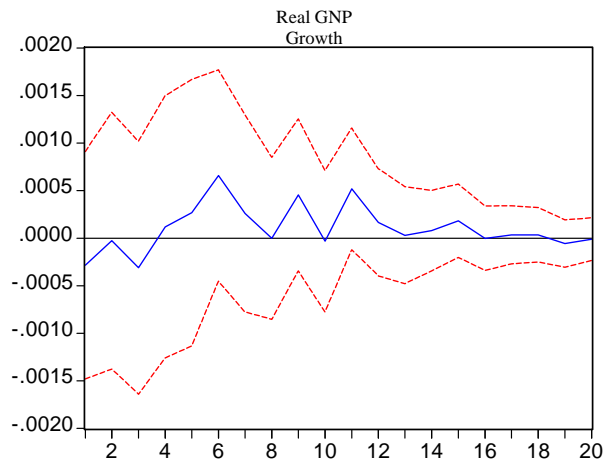
Figure 2: *IRF of Growth Rate of Real GNP to Growth Rate of Real Defense Spending (VAR)*: 1976:01-2005:03

Figure 3: *IRF of Growth Rate of Real GNP to Growth Rate of Real Defense Spending (VAR)*: 1959:01-2008:03

[5] Following Atesoglu (2009) the VAR is estimated with 4 lags and is found to be stable with no roots lying outside the unit circle. Following Ireland and Otrok (1992), the variables are ordered as follows: growth rate of real defense spending, growth rate of real outstanding debt, growth rate of real non-defense spending, the federal funds rate, growth rate of real M2, growth rate of the implicit GNP deflator and growth rate of real GNP. Alternative ordering schemes, however, does not affect the nature of the impact of the growth rate of real defense expenditure on the growth rate of output.

As seen from Figure 2 our claim that mixed empirical evidence on the effect of defense spending on output could be a result of small models incorporating lesser detailed information sets is not misplaced. To validate our point further, we re-estimated the small scale VAR[6] over a longer sample period of 1959:01 to 2008:03. As can be seen from Figure 3, following the increase in the defense spending, the initial impact on the growth rate is small but positive for around 3 quarters, but is generally insignificant, and dies down relatively faster by the 9th quarter. Clearly then, as indicated from Figures 2 and 3, reliability of small scale models for structural analysis, in this case, the impact of defense spending on output, is highly sensitive to the sample choice. Hence, the mixed evidence on the relationship between these two critical variables is far from being surprising.

The pertinent question now is: Is there an objective way to choose the FAVAR model over the VAR for the period of 1976:01 to 2005:02? Following Banbura et al. (2008), we look at the predictive capability of the FAVAR relative to the VAR, to choose a better model over this period. In this regard, we evaluate the two models in terms of their ability to forecast the real GNP growth rate. Table 1 reports the one- to four-quarters-ahead root mean squared errors[7] (RMSEs) generated from the VAR and the FAVAR[8] models in forecasting the growth rate of real GNP over an out-of-sample horizon of 2001:01-2005:02.[9] As can be seen from Table 1, the FAVAR outperforms the VAR at each quarters-ahead forecasts and, hence, in terms of the average. Again the result is not surprising given the span of information content of the FAVAR. Beck et al. (2000, 2004) points out that, forecasting is at the root of inference and prediction in time series analysis. As argued by Clements and Hendry (1998), in time series models, estimation and inference essentially means minimizing of the one-step (or multi-step) forecast errors. Hence, establishing a model's superiority boils down to showing that it produces smaller forecast errors than its competitors. So based on the ability of the FAVAR to forecast the real GNP growth rate better than the VAR, we could conclude that it is, perhaps, also the more reliable of the two models to analyze the impact of defense spending on output.

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[6] As with Figure 2, the VAR is estimated with 4 lags on a stable VAR, which, in turn, is based on the same ordering of the variables. Again, alternative ordering schemes, does not affect the positive impact of the growth rate of real defense expenditure on the growth rate of output.

[7] Note that if A_{t+n} denotes the actual value of a specific variable in period $t + n$ and $_tF_{t+n}$ is the forecast made in period t for $t + n$, the RMSE statistic can be defined as: $\sqrt{\frac{1}{N}\Sigma(A_{t+n} - _tF_{t+n})^2}$. For $n = 1$, the summation runs from 2001:01 to 2005:02, and for $n = 2$, the same covers the period of 2001:02 to 2005:02, and so on.

[8] As is standard in forecasting with the FAVAR models, it included the 5 factors and the 7 variables of the VAR. Note that both the FAVAR and the VAR were estimated based on 4 lags over an in-sample period of 1976:01 to 2000:04 and then recursively over the out-of-sample horizon of 2001:01 to 2005:02.

[9] The choice of the out-of-sample horizon is based on the fact that the level of real defense spending increased continuously, while its growth rate became more volatile, beyond the year 2000. See Figures A and B in the Appendix of the paper.

Table 1: *RMSEs for Growth Rate of Real GNP (2001:01-2005:02)*

QA	VAR	FAVAR
1	0.6799	0.6303
2	0.7554	0.7112
3	0.7079	0.5855
4	0.6099	0.5342
Average	0.6883	**0.6153**
Note: QA: Quarters Ahead.		

5.   Conclusions

Empirical evidence on the effect of defense spending on US output, measured in level or growth rate, is at best mixed. Against this backdrop, this paper assesses the impact of a positive defense spending shock on the growth rate of real GNP using a FAVAR model estimated with 116 variables spanning the period of 1976:01 to 2005:02. Overall, the results show that a positive shock to the growth rate of the real defense spending translates to a positive and long lasting effect on the growth rate of real GNP, but the effect on the growth rate of the output is significant only for a short-period of around two quarters. In addition, we indicate that the mixed empirical evidence could be a result of small information sets, by showing the sensitivity of the results to sample size using a small-scale VAR typically used in the literature to analyze the effect of defense spending on output. Finally, given that the FAVAR model was found outperform the VAR in forecasting the growth rate of real GNP, we concluded that the FAVAR framework is superior and should be relied upon more for the analysis of the impact of defense spending on output.

As part of future research, we intend to check for the robustness of our analysis by extending the data set both historically and to a more recent period. Recall that, the FAVAR approach requires us to make the data stationary, and, hence, we had to convert the two key variables of interest into their growth counterparts. This can be considered to be a drawback of the current analysis, especially if we are interested in studying the dynamics of output in level following a defense shock, as in the small-scale VECMs. In this regard, it would be interesting to repeat our analysis based on a large-scale Bayesian VAR (BVAR), developed recently by Banbura et al. (2008), since just like the FAVAR, the BVAR, given its estimation methodology, can also handle a data set of any size. But more importantly, unlike the FAVAR, the large-scale BVAR, via appropriate design of the priors can handle variables in levels without having to worry about the issues of non-stationarity. On could also use the Factor Augmented Error Correction Model (FAECM) developed by Banerjee and Marcellino (2008).

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Appendix
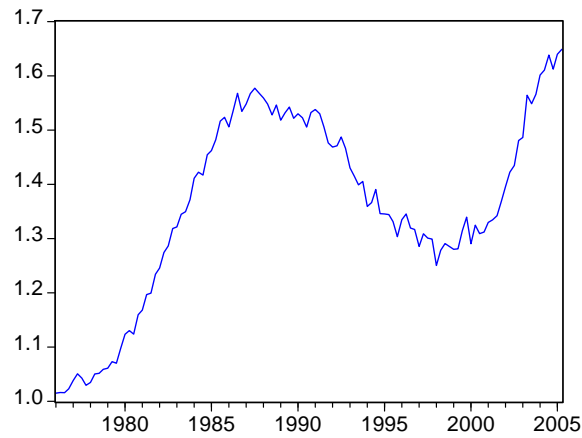
Figure A: *Log of Real Defense Spending*: 1976:01-2005:02
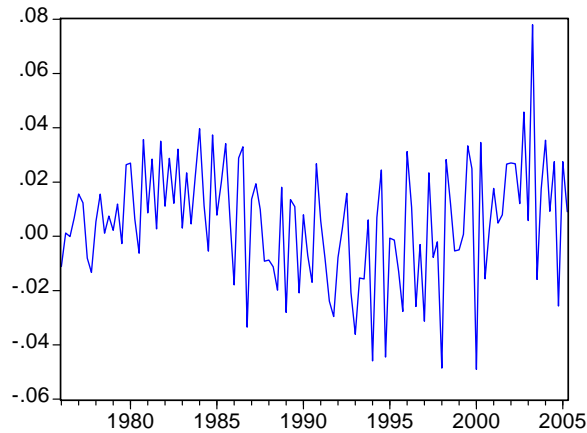
Figure B: *Growth Rate of Real Defense Spending*: 1976:01-2005:02.