THE EFFECT OF FADING IN MOBILE ALOHA RADIO COMMUNICATIONS

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ABSTRACT

The application of unslotted Aloha to mobile communication systems is studied. In such systems, the terminal-to-base channel can undergo various types of fade such as shadowing (log-normal fading) or unshadowed multipath reflections with a direct component (Rician fading), or the near/far effect. It is shown that fading may give rise to a capture effect that improves the Aloha throughput.

I. INTRODUCTION

With the development of portable and mobile radio communications for local data transmission in buildings or in an urban area, the best known random access protocol Aloha has recently received considerable interest. In an idealized Aloha channel, a collision of two or more packets arriving at the receiver simultaneously destroys all packets. In an unslotted Aloha channel where packets are allowed to start at any point in time, the maximum channel throughput is 0.184. On the other hand, if time on the channel is organized into uniform slots with size equal to the packet length, and packets are restricted to arrive at the receiver only within slots (slotted Aloha), the maximum channel throughput is doubled to 0.368. The idealized channel takes no accounts of the propagation effect encountered in realistic mobile radio channels, namely, fading.

However recent work [1]–[4] have shown that the near/far effect and fading in local and mobile radio environments which cause signals from different mobile terminals to reach the base station at different powers enhance the performance of Aloha systems. This phenomenon is called power capture, and applies to all mobile terminals not just the ones with the strongest received signals. Even the most distant terminals maintain a finite probability of capturing the base station receiver. In the capture model of [2], the near/far effect was considered where the receiver captures the strongest arriving packet if the ratio of the geometrical distance between this user and the next to the strongest user exceeds a capture parameter. A more elaborate capture model [1] was studied in which the multipath fading effect of the channel was considered in the form of Rayleigh fading. Both models clearly indicated that spatially distributed terminals and Rayleigh fading reduce mutual packet interference and improve the channel throughput. Both models also considered slotted Aloha only. Most recently [5] unslotted Aloha systems is considered assuming the fading effect is Rayleigh distributed. It is the purpose of this paper to review the results in [2], [5] to provide a framework for mobile unslotted Aloha channels which encounter fading. In Section II we discuss the system dynamics of an unslotted Aloha channel by establishing the criterion for capture. The channel fading model is presented in Section III.

II. SYSTEM DYNAMICS

We consider a mobile Aloha network consisting of many mobile terminals accessing the base station via an unslotted Aloha channel, and a broadcast time division multiplexing (TDM) channel used by the base station. A mobile terminal transmitting a packet must wait for an acknowledgment from the base station to conclude that its packet is successful. Otherwise a timeout will expire and the terminal must retransmit a packet after a random timeout. Assume that packets arrive at the terminals are transmitted immediately. Then a successful transmission happens when a packet capture the base station receiver in the presence of interfering packets, and that no errors occur due to thermal noise, and the packet carrying the acknowledgment and requested data from the base station is received correctly by the corresponding terminal. This success probability of a packet is given by

$$p = Pr\{capture\}(1 - p_{\mathrm{A}})(1 - p_{\mathrm{TDM}}) \tag{1}$$

where p_{A} is the packet error probability due to thermal noise, and p_{TDM} is the packet error probability of the TDM channel.

As shown in [2], the capture effect in Aloha channels occurs naturally when fading reduces the joint power of interfering packets. Assume that all packets reach the receiver with uncorrelated fading and that packet power X is characterized by a density function $f_X(x)$. For a given number of interfering packets n during the transmission of a tagged packet there are 2^n

realizations r_n^k [6], each with a maximum number of interferers j where $\lceil n/2 \rceil \leq j \leq n$ ($\lceil a \rceil = integer \geq a$). Associated with 2^n realizations r_n^k is a set of parameters $C_j(n)$ that represent the number of realizations r_n^k in which the maximum number of interferers equals j. The parameters $C_j(n)$ can be calculated using the binary mapping technique in [6], and are shown in Table 1.

A closed form expression of $C_j(n)$ was also derived as follows:

$$C_j(n) = \begin{cases} \binom{n}{j} \frac{(2j-n+1)^2}{j+1}, j \geq n/2 & \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Table 1: LIST OF $C_j(n)$

n	$C_1(n)$	$C_2(n)$	$C_3(n)$	$C_4(n)$	$C_5(n)$	$C_6(n)$
1	2	0	0	0	0	0
2	1	3	0	0	0	0
3	0	4	4	0	0	0
4	0	2	9	5	0	0
5	0	0	10	16	6	0
6	0	0	5	27	25	70

Thus given $n \geq 1$, the conditional density function of the power Y of n independent interfering packets is

$$f_Y(y|n) = 2^{-n} \sum_{j=\lceil n/2 \rceil}^{n} C_j [f_X(x)]^{j\otimes} \tag{3}$$

where $j\otimes$ denotes the j-fold convolution. Knowing $f_X(x)$ and $f_Y(y)$ we can find the conditional density function of the signal-to-interference ratio

$$Z = \frac{X}{Y} \tag{4}$$

which is given by

$$f_Z(z|n) = \int_0^\infty y f_X(yz) f_Y(y|n) dy \tag{5}$$

A tagged packet can capture the base station receiver if its received signal-to-interference ratio Z exceeds a given threshold γ_o. Thus the conditional capture probability is given by

$$\begin{aligned} Pr\{capture|n\} &= 1 - Pr\{0 \leq Z \leq \gamma_o|n\} & (6) \\ &= 1 - \int_0^{\gamma_o} f_Z(z|n) dz & (7) \end{aligned}$$

hence the capture probability in a fading Aloha channel is

$$Pr\{capture\} = \sum_{n=0}^{\infty} Pr\{capture|n\} Pr\{n\} \tag{8}$$

$$Pr\{n\} = \frac{(2G)^n}{n!} e^{-2G} \tag{9}$$

where G is the channel attempted rate in packets/packet time. The system throughput is then given by

$$S = Gp \tag{10}$$

III. CHANNEL MODELING

There are many models that can be used to describe the fading effects in a mobile communication environment. Fading can be classified into two categories: the random signal power variations and the inverse α-power propagation law.

In the first category the total received signal at the base station is consider to be the sum of the direct and diffuse components. The direct component can be an unattenuated or unshadowed line-of-sight (LOS) signal or an LOS signal under vegetative attenuation (shadowing). The diffuse component is caused by reflections from nearby terrain such as mountains, hill, and buildings (multipath). In the unshadowed situation, normally there is an incomming signal which is stronger than the reflected signals. Thus, the received signal may consist of a constant direct component plus a diffuse component. The ensuing amplitude distribution of the total received signal is the Rician density function [7],

$$p_A(a) = \frac{2a}{\beta} e^{-(a^2+A^2)/\beta} I_0 \left(\frac{2Aa}{\beta} \right), a \geq 0 \tag{11}$$

where a is the amplitude, β is the average power of the diffuse component, A^2 is the power of the direct component, and $I_0(x)$ is the modified Bessel function of zero order. When $A = 0$, the Rician density function reduces to the Rayleigh density function

$$p_A(a) = \frac{2a}{\beta} e^{-a^2/\beta}, a \geq 0 \tag{12}$$

In the shadowed situation, experiments show that the total received signal amplitude a obeys a log-normal distribution superimposed on a Rayleigh distribution [8]. For large values of $a \gg \sqrt{\beta}$ where β is the average power of the diffuse component, the density function of a is essentially log-normal

$$p_A(a) = \frac{1}{\sqrt{2\pi\sigma^2 a}} e^{-\frac{(\ln a - \mu)^2}{2\sigma^2}}, a \gg \sqrt{\beta} \tag{13}$$

where μ and σ^2 are the mean and the variance of the natural logarithm of the amplitude of the shadowed direct component. For $a \ll \sqrt{\beta}$, the density function of a is essentially Rayleigh.

Figure 1 shows the throughput of the mobile Aloha channel with Rayleigh fading effect assuming $p_A = p_{TDM} = 0$ for threshold levels $\gamma_o = 3, 5$ and 10. The maximum throughput for $\gamma_o = 3(4.77dB)$ is about 0.26 as compared to 0.184 of nonfading Aloha.

In the second category of fading, the signal power varies with the distance r as $r^{-\alpha}$. If we assume that users are independent and uniformly distributed in a circle of radius R with the base station at its center, then the density function for the position of a mobile user at distance r from the base station is

$$f_R(r) = \frac{2r}{R^2}, 0 \leq r \leq R \qquad (14)$$

This means that the packet attempted rate per unit area is constant at any distance within the circle of radius R. Therefore the density function of packet power x at distance r from a mobile transmitter can be calculated as follows:

$$f_X(x) = \frac{2}{R^2\alpha} x^{-\frac{2}{\alpha}-1}, R^{-\alpha} \leq x \leq \infty \qquad (15)$$

Other spatial distribution can also be considered such as the following one [1]

$$f_X(x) = \frac{2}{R^2\alpha} x^{-\frac{2}{\alpha}-1} e^{-k/x}, 0 \leq x \leq \infty, k \geq 0 \qquad (16)$$

This type of spatial distribution models the attempted rate per unit area as near constant inside the circle of radius R and falls off rapidly outside.

IV. CONCLUSION

We have shown that fading improves the throughput of the Aloha channel. This is because every packet has a finite probability to capture the base station receiver and that certain type of fading that causes random variations of signal power tend to reduce the joint interference power.

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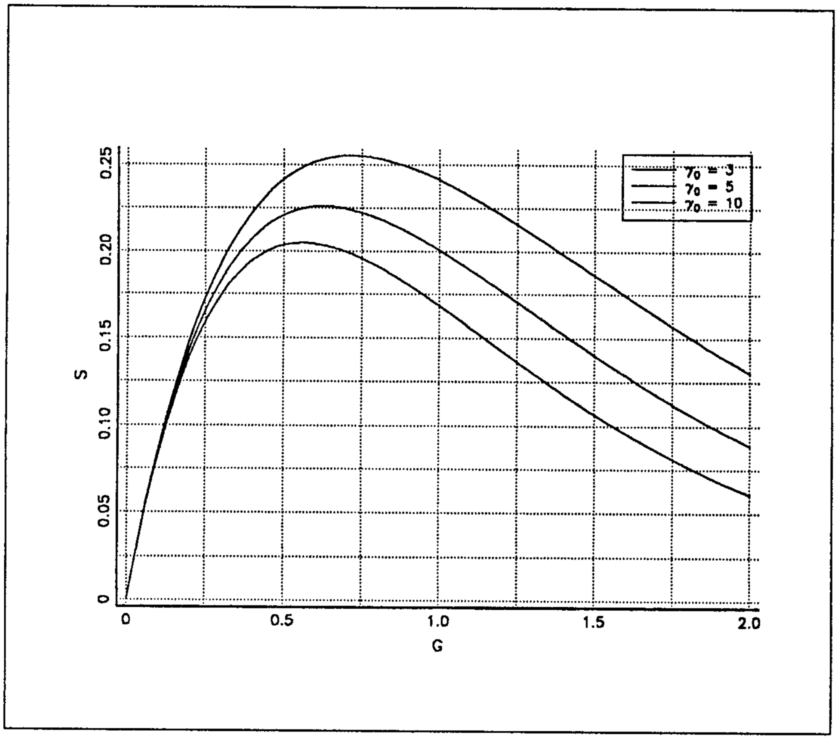

FIGURE 1. THROUGHPUT OF UNSLOTTED ALOHA IN RAYLEIGH FADING ENVIRONMENT.