Geophys. J. Int. (2000) **143,** 302–310

The effect of fluid pressure on wave speeds in a cracked solid

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Accepted 2000 May 31. Received 2000 May 30; in original form 1999 December 29

SUMMARY

In a porous or cracked elastic solid, the effective stress (defined in terms of the loads applied to the solid part of the outer boundary) and effective strain (defined in terms of the displacements at the solid part of the outer boundary) occurring in small-amplitude deformations are connected by a linear relation along with the pressure within the fluid occupying the pores and cracks. We derive here a formula of this kind for a static system in which enough time is allowed for pressure to be equalized throughout the fluid (on the assumption that all pockets of fluid are connected in some way). The formula depends on the overall stiffnesses relating stress to strain for the same material with the fluid removed (dry or empty cracks and pores). For undrained conditions where no fluid is allowed to enter or leave the body, the pressure is directly related to the effective stress and strain, and the Gassmann relations are obtained relating the stiffnesses for an isotropic material in dry and undrained conditions. For an anisotropic material, the Brown–Korringa relations are recovered. Externally imposed stresses and fluid pressure distort the material structure and influence the wave speeds of elastic waves. The main way in which this occurs is in changing the aspect ratios of flat cracks, the most compliant part of the microstructural geometry. This effect on the wave speeds is studied here both in terms of crack closure, with corresponding changes in crack number density, and in variations in crack aspect ratios. The principal way in which the latter influences the wave speeds is through the fluid incompressibility factor in the formula for the properties of materials with connected cracks. An increase in aspect ratio of the cracks is equivalent to a reduction in the bulk modulus of the fluid. This effect is apparent in the limits of both high frequencies, when the material behaves as if the cracks were isolated, and low frequencies, when undrained conditions apply.

Key words: cracks, effective medium theory, fluid flow.

1 INTRODUCTION

For many years now it has been clear that information about the microstructure of solid elastic materials is contained in their static or long-wavelength response (Mal & Knopoff 1967; Garbin & Knopoff 1973). As a corollary of this, it follows that changes of stress will be reflected in changes to the seismic wave speeds, particularly if the material contains cracks (Nur 1971). The first to confirm this in the laboratory appears to have been Gupta (1973), who measured *P*- and *S*-wave speeds in three orthogonal directions through a specimen subject to stress with a considerable deviatoric component. He interpreted his results for the ratios of the wave speeds in terms of dilatancy arising from the formation of cracks. This idea has been used to interpret the anisotropy of crustal rocks in terms of preferentially oriented cracks (Crampin *et al.* 1984). At the same time, the orientation of cracks has been taken to indicate the stress conditions of the rock mass (Crampin *et al.* 1986). Crustal rocks are subject not only to tectonic stresses but also to varying fluid pressure in the pore structure on account of the movement of fluids. For small deformations, there will clearly be a linear relation between the effective or overall stress on the solid skeleton, the effective strain and the fluid pressure. This relation must reduce to the Brown–Korringa (1975) equations when no fluid is allowed to enter or leave the material body (undrained conditions).

Flat cracks are the most significant form of microstructure in their effect on the speeds of seismic waves since they are the most compliant (Mavko & Nur 1979). As a result, they are the part of the structure that responds most dramatically to changes in tectonic stress and fluid pressure. The principal effect is in changes to the aspect ratios of the cracks and, ultimately, to the closure of some cracks (Gangi 1978; Horii & Nemat-Nasser 1983). It is to be expected, therefore, that such changes in stress and pressure may most easily be detected by seismic waves in the response of the wave speeds to variations in the aspect ratio and number density distribution of cracks.

Expressions exist for the overall properties of cracked materials, where the cracks are either dry or fluid-filled and isolated, calculated by a number of different methods; the most used are approximations accurate to first order (Garbin & Knopoff 1973, 1975a,b) and second order (Hudson 1980) in the number density of cracks, the self-consistent approximation (O'Connell & Budiansky 1974) and the differential effective medium method (Nishizawa 1982). All of these appear to agree at low crack densities but diverge at higher values (Sayers & Kachanov 1991). From the values of the overall parameters for dry conditions, it is possible to calculate the parameters for undrained conditions (Gassmann 1951; Brown & Korringa 1975). Undrained conditions correspond to the situation where a sample block of material is sealed so that no fluid can enter or leave the sample, although fluid does flow between cracks and pores within the sample. Under drained conditions, fluid is allowed to enter or leave the sample so that the fluid pressure remains fixed during deformation. This is effectively the same as having no fluid infill, i.e. dry conditions. The cracks and pores are said to be isolated if no fluid flows between them. However, the greatest effect of the presence of fluid on the wave speeds—and on the wave attenuation—occurs when the period of the wave is approximately the same as the relaxation time for fluid flow between cracks or into the porous matrix (Pointer *et al.* 2000). This corresponds to neither dry nor isolated nor undrained conditions but somewhere between the last two. The theory for this, accurate to first order in the crack number density, is provided by Hudson *et al.* (1996).

We begin by establishing the relationship between effective stress and strain and fluid pressure under static deformation in terms of the overall parameters for dry conditions. (It is assumed that, under static conditions, the pressure is uniform throughout the fluid.) This relationship is shown to be correct for both dry (zero pressure) and undrained conditions. Next we examine the effect of the imposed stress and pressure on the aspect ratios of flat cracks. In particular, we study the implications of crack closure. Finally, we consider the behaviour of the elastic wave speeds in cracked materials as the aspect ratios of the cracks vary. This effect is related in a clear way to changes in imposed stress and pressure.

2 STATIC DEFORMATION OF A FLUID-SATURATED POROUS SOLID

We consider a finite body of material occupying a region V; the material is solid with connected pores or cracks filled with fluid. A diagrammatic example of one such material is shown in Fig. 1. The mean strain and stress in the solid matrix material are denoted by $\mathbf{e}^{\mathrm{m}}$ and $\boldsymbol{\sigma}^{\mathrm{m}}$ respectively. The deformation regime is maintained by static loads or displacements on the solid part S_{s} of the exterior boundary of V and, following Hill (1963), we may define an effective strain by

$$\bar{e}_{ij}=\frac{1}{2V}\int_{S_{\mathrm{s}}}(u_i n_j+u_j n_i)\,dS\,, \tag{1}$$

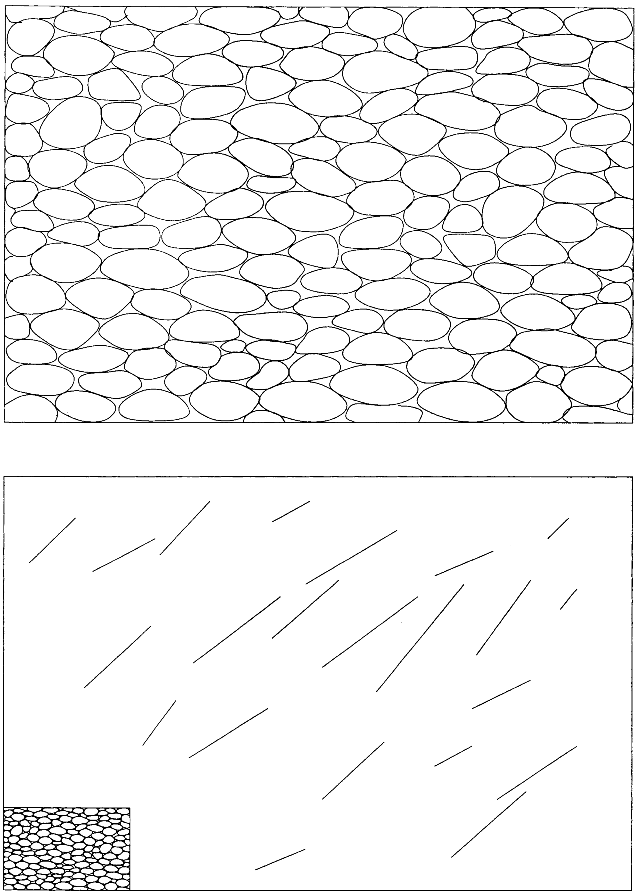

Figure 1. Illustration of the structure of one example of a material containing cracks and pores. The upper diagram shows the porous structure arising from a granular aggregate. The lower diagram shows the superimposed cracks, where, for clarity, the aggregate structure is shown only in the bottom left-hand corner. The scale of the lower figure is reduced by a factor of five from the upper figure. (Reproduced with permission of S. Tod.)

where $\mathbf{n}$ is the unit outward normal to S_{s}, and an effective stress by

$$\bar{\sigma}_{ij}=\frac{1}{V}\int_{S_{\mathrm{s}}} t_i x_j\,dS\,, \tag{2}$$

where $\mathbf{t}$ is the traction imposed on the surface S_{s} and the effect of body forces is ignored. Use of the divergence theorem gives

$$\bar{e}_{ij}=\left(1-\frac{V_{\mathrm{c}}}{V}\right)e^{\mathrm{m}}_{ij}+\frac{1}{2V}\int_{S_{\mathrm{p}}}(u_i n_j+u_j n_i)\,dS\,, \tag{3}$$

where S_{p} is the total surface of the cracks and pores inside V, V_{c} is the volume occupied by fluid and $\mathbf{n}$ is the outward normal to S_{p}; also,

$$\bar{\sigma}_{ij}=\left(1-\frac{V_{\mathrm{c}}}{V}\right)\sigma^{\mathrm{m}}_{ij}-\frac{V_{\mathrm{c}}}{V}\,p\delta_{ij}\,, \tag{4}$$

where p is the pressure in the fluid, assumed to be uniform. We assume that the solid matrix is homogeneous with known stress–strain relation given by

$$\boldsymbol{\sigma}^{\mathrm{m}}=\mathbf{c}^{\mathrm{o}}\mathbf{e}^{\mathrm{m}}\,, \tag{5}$$

where $\mathbf{c}^{\mathrm{o}}$ is a fourth-rank tensor of stiffnesses with inverse $\mathbf{s}^{\mathrm{o}}$ (the compliances).

These linear relations hold only when the magnitudes of the stress, strain and fluid pressure are small. While the ambient stress and pressure may be large, these linear relations will hold for small changes in these quantities. However, the structural parameters such as V_c/V will depend on the ambient stress and pressure.

Our aim in this section is to find the relationship between the effective stress $\bar{\boldsymbol{\sigma}}$, the effective strain $\bar{\mathbf{e}}$ and the fluid pressure p. When the pressure is zero, as in a drained or dry system, there will be a relation between $\bar{\boldsymbol{\sigma}}$ and $\bar{\mathbf{e}}$ of the form

$$\bar{\boldsymbol{\sigma}} = \bar{\mathbf{c}}^{\mathrm{d}}\bar{\mathbf{e}}\,, \quad \bar{\mathbf{e}} = \bar{\mathbf{s}}^{\mathrm{d}}\bar{\boldsymbol{\sigma}}\,, \tag{6}$$

where $\bar{\mathbf{c}}^{\mathrm{d}}$, $\bar{\mathbf{s}}^{\mathrm{d}}$ are the overall stiffnesses and compliances for dry (drained) pores. Once again, the linear relation holds for small $\|\bar{\boldsymbol{\sigma}}\|$ and $\|\bar{\mathbf{e}}\|$, although $\bar{\mathbf{c}}^{\mathrm{d}}$ and $\bar{\mathbf{s}}^{\mathrm{d}}$ depend on structural parameters that will change with sufficiently large changes in stress and pressure loads. In general, however, we have (eq. 4)

$$\begin{aligned}\bar{\boldsymbol{\sigma}} &= \left(1 - \frac{V_c}{V}\right)\boldsymbol{\sigma}^{\mathrm{m}} - \frac{V_c}{V}\,p\boldsymbol{I} \\ &= \left(1 - \frac{V_c}{V}\right)\mathbf{c}^{\mathrm{o}}\mathbf{e}^{\mathrm{m}} - \frac{V_c}{V}\,p\boldsymbol{I} \\ &= \mathbf{c}^{\mathrm{o}}(\bar{\mathbf{e}} - \boldsymbol{E}) - \frac{V_c}{V}\,p\boldsymbol{I}\end{aligned} \tag{7}$$

using eqs (5) and (3), where $\boldsymbol{I}$ is the second-order identity tensor and the elements of the tensor $\boldsymbol{E}$ are

$$E_{ij} = \frac{1}{2V}\int_{S_p}(u_i n_j + u_j n_i)\,dS\,. \tag{8}$$

$\boldsymbol{E}$ is determined by the displacements $\mathbf{u}$ on S_p, the interior part of the boundary of the solid material, corresponding to the stress field $\bar{\boldsymbol{\sigma}}$ imposed on the external part S_s of the boundary and pressure p on S_p.

As the effective stress and pressure change, the shapes of the cracks and pores change and the response of the material to small disturbances is also changed. Consider a static state of the material in which $\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{\sigma}}^{\mathrm{o}}$, $\bar{\mathbf{e}} = \bar{\mathbf{e}}^{\mathrm{o}}$, $p = p_o$, $\Sigma = \Sigma_o$, where Σ represents a set of shape parameters for the internal structure. We analyse the effects of changes in mean stress and pressure by considering the solid alone. The pressure is simply the normal traction on the interior surface of the solid. We assume that we can change the mean stress and the pressure independently. If we now change the effective stress to $\bar{\boldsymbol{\sigma}}^{\mathrm{o}} + \delta\bar{\boldsymbol{\sigma}}$ but keep the pressure in the cracks and pores fixed, the change $\delta\bar{\mathbf{e}}^1$ in the effective strain in the solid is given by

$$\delta\bar{\boldsymbol{\sigma}} = \bar{\mathbf{c}}^{\mathrm{d}}(\Sigma_o)\delta\bar{\mathbf{e}}^1\,, \tag{9}$$

where $\bar{\mathbf{c}}^{\mathrm{d}}(\Sigma)$ are the overall stiffnesses for dry or drained conditions and the shape parameter set Σ. (Here we have neglected any resistance of the crack faces to shear. The effect of the fluid viscosity on the crack faces is negligible at sufficiently long times.) If we now change the pressure to $p_o + \delta p$, keeping the effective stress fixed, there is again a corresponding change $\delta\bar{\mathbf{e}}^2$ in the effective strain. With the imposition of an additional uniform stress field $\delta p\boldsymbol{I}$ throughout the solid material, we see that the net effect on the solid is of an increment in effective stress of $\delta p\boldsymbol{I}$, and an effective strain of $\delta\bar{\mathbf{e}}^2 + \delta p\mathbf{s}^{\mathrm{o}}\boldsymbol{I}$. The net pressure change is zero, so we have the relationship

$$\delta p\boldsymbol{I} = \bar{\mathbf{c}}^{\mathrm{d}}(\Sigma_o)\{\delta\bar{\mathbf{e}}^2 + \delta p\mathbf{s}^{\mathrm{o}}\boldsymbol{I}\}\,. \tag{10}$$

The effect on the effective strain of a combined change in effective stress and fluid pressure is, therefore,

$$\delta\bar{\mathbf{e}} = \delta\bar{\mathbf{e}}^1 + \delta\bar{\mathbf{e}}^2 = \bar{\mathbf{s}}^{\mathrm{d}}(\Sigma_o)\delta\bar{\boldsymbol{\sigma}} + (\bar{\mathbf{s}}^{\mathrm{d}} - \mathbf{s}^{\mathrm{o}})\boldsymbol{I}\delta p\,, \tag{11}$$

where $\bar{\mathbf{s}}^{\mathrm{d}}(\Sigma_o)$ is the tensor of compliances, the inverse of $\bar{\mathbf{c}}^{\mathrm{d}}(\Sigma_o)$.

In general, the changes in effective stress and pressure will result in changes to the shapes of the cracks and pores, in particular to the aspect ratio r of cracks. However, the expressions for $\bar{\mathbf{c}}^{\mathrm{d}}$ and $\bar{\mathbf{s}}^{\mathrm{d}}$, which are the elastic constants for dry conditions, are independent of r when r is small (Hudson 1980). The relative change in the aspect ratio of cavities and the radii of pores is, in all other circumstances, small for small imposed stress. So, we assume that $\bar{\mathbf{c}}^{\mathrm{d}}$ and $\mathbf{s}^{\mathrm{d}}$ are, in general, independent of Σ and eq. (11) can be written

$$\bar{\mathbf{e}} = \bar{\mathbf{s}}^{\mathrm{d}}\bar{\boldsymbol{\sigma}} + (\bar{\mathbf{s}}^{\mathrm{d}} - \mathbf{s}^{\mathrm{o}})\boldsymbol{I}p \tag{12}$$

to give the total effective strain due to imposed effective stress $\bar{\boldsymbol{\sigma}}$ and fluid pressure p. Clearly, if the pressure change p is zero, the stress–strain relation is that for dry (fully drained) conditions as expected. The same relation is obtained if $\bar{\mathbf{s}}^{\mathrm{d}} = \mathbf{s}^{\mathrm{o}}$, which implies that the cracks and pores are non-compliant so pressure changes have no effect on the stress–strain relation.

If the matrix material is isotropic and the cracks and pores are randomly oriented, both $\mathbf{s}^{\mathrm{o}}$ and $\bar{\mathbf{s}}^{\mathrm{d}}$ are isotropic tensors and eq. (12) may be rewritten as

$$\bar{\mathbf{e}} = \bar{\mathbf{s}}^{\mathrm{d}}\bar{\boldsymbol{\sigma}} + \left(\frac{1}{\bar{\kappa}_d} - \frac{1}{\kappa}\right)\left(\frac{p}{3}\right), \tag{13}$$

where κ, $\bar{\kappa}_d$ are the overall bulk moduli of, respectively, the matrix and the dry (drained) material.

In general, we may define an 'apparent' stress $\boldsymbol{\sigma}^{\mathrm{a}}$ by

$$\boldsymbol{\sigma}^{\mathrm{a}} = \bar{\boldsymbol{\sigma}} + (\boldsymbol{I} - \bar{\mathbf{c}}^{\mathrm{d}}\mathbf{s}^{\mathrm{o}}\boldsymbol{I})p\,, \tag{14}$$

so that eq. (12) becomes

$$\bar{\mathbf{e}} = \bar{\mathbf{s}}^{\mathrm{d}}\boldsymbol{\sigma}^{\mathrm{a}}\,. \tag{15}$$

In this way, the deformation may be regarded as governed by the overall stiffnesses for the dry (drained) state.

If the matrix material and the interior structure are both isotropic, eq. (14) for the apparent stress becomes

$$\boldsymbol{\sigma}^{\mathrm{a}} = \bar{\boldsymbol{\sigma}} + \left(1 - \frac{\bar{\kappa}_d}{\kappa}\right)\boldsymbol{I}p\,, \tag{16}$$

a relationship established by Nur & Byerlee (1971) and proposed earlier by Geertsma (1957) and Skempton (1960) (see also Gangi & Carlson 1996).

3 FLUID BALANCE, UNDRAINED CONDITIONS

The increase in volume of the cracks and pores as a result of deformation of the matrix material is

$$\delta V_c = VE_{kk}\,, \tag{17}$$

where E_{ij} is given by eq. (8). Use of eq. (7) changes this to

$$\delta V_c = V\left\{\bar{e}_{kk} - s^0_{kkjj}\,\frac{V_c}{V}\,p - s^{\mathrm{o}}_{kkij}\bar{\sigma}_{ij}\right\}. \tag{18}$$

The increase in volume of the fluid within the cracks and pores due to the change in pressure is $(-V_c p/\kappa_f)$, so the volume of fluid squeezed out of the material by the deformation is

$$\delta V_f = pV_c(s^o_{kkjj} - 1/\kappa_f) + V(s^o_{kkij}\bar{\sigma}_{ij} - \bar{e}_{kk})\,. \tag{19}$$

With the use of eq. (12), this becomes

$$\delta V_f = pV_c(s^o_{kkjj} - 1/\kappa_f) + V(s^o_{kkij} - \bar{s}^d_{kkij})(\bar{\sigma}_{ij} + p\delta_{ij})\,. \tag{20}$$

If the matrix material is isotropic, $s^o_{kkij} = (1/3\kappa)\delta_{ij}$ with a similar eq. for $\bar{s}^d_{kkij}$ and

$$\delta V_f = pV_c(1/\kappa - 1/\kappa_f) + V(1/\kappa - 1/\bar{\kappa}_d)(\bar{\sigma}_{kk}/3 + p)\,. \tag{21}$$

If no fluid enters or leaves the material during deformation, the process is said to be undrained and the pressure is determined by the imposed stress field. We have $\delta V_f = 0$, so

$$p = -(s^o_{kkij} - \bar{s}^d_{kkij})\bar{\sigma}_{ij}/\{(s^o_{kkjj} - \bar{s}^d_{kkjj}) + \phi(s^o_{kkjj} - 1/\kappa_f)\}\,, \tag{22}$$

where $\phi = V_c/V$. Substituting this into eq. (12) we obtain the stress–strain relation for the undrained, fluid-saturated material in terms of the compliances for the dry material:

$$\bar{e}_{ij} = \left\{\bar{s}^d_{ijkl} + \frac{(\bar{s}^d_{ijpp} - s^o_{ijpp})(\bar{s}^d_{qqkl} - s^o_{qqkl})}{\phi(1/\kappa - 1/\kappa_f) - (1/\bar{\kappa}_d - 1/\kappa)}\right\}\bar{\sigma}_{kl}\,, \tag{23}$$

where

$$1/\kappa = s^o_{kkjj}\,,$$

$$1/\bar{\kappa}_d = \bar{s}^d_{kkjj}\,,$$

ϕ is the porosity of the material in the unstrained state, $1/\kappa$ is the ratio of the dilatation (e_{kk}) to the magnitude of an imposed hydrostatic stress field ($\sigma_{ij} = S\delta_{ij}$) in the solid matrix material and $1/\bar{\kappa}_d$ is the corresponding ratio for the dry cracked/porous material. Eq. (23) was obtained by Brown & Korringa (1975).

If the matrix material and the dry cracked material are both isotropic, their properties are given by shear and bulk moduli (μ, κ) and ($\bar{\mu}_d$, $\bar{\kappa}_d$) respectively. The undrained, fluid-saturated material is then also isotropic with moduli ($\bar{\mu}_u$, $\bar{\kappa}_u$), where

$$\bar{\mu}_u = \bar{\mu}_d,$$

$$\bar{\kappa}_u = \left\{\frac{1}{\bar{\kappa}_d} + \frac{\left(\dfrac{1}{\bar{\kappa}_d} - \dfrac{1}{\kappa}\right)^2}{\phi\left(\dfrac{1}{\kappa} - \dfrac{1}{\kappa_f}\right) - \left(\dfrac{1}{\bar{\kappa}_d} - \dfrac{1}{\kappa}\right)}\right\}^{-1}\,. \tag{24}$$

The first of these relations was given by Gassmann (1951) and also proposed by O'Connell & Budiansky (1977) on the grounds that shear deformation does not involve volume change and therefore there is no pressure change in the fluid within the pores of undrained material. They also proposed that the bulk modulus of the fluid-saturated material would be the same whether the pores and cracks were connected for fluid flow or not. This is because, under hydrostatic stress, the pores would be compressed equally, and it is certainly true for dilute concentrations of cracks (Pointer *et al.* 2000).

The second of the relations (24) is Gassmann's (1951) formula giving the bulk modulus of porous undrained material in terms of the bulk moduli of the dry and matrix materials.

4 THE EFFECT OF STRESS AND FLUID PRESSURE ON FLAT CRACKS

We now return to the general case where the mean stress on the solid and the fluid pressure can be applied independently to a representative sample of porous material. The effect of changes in mean stress and in pressure on the rock structure will be to distort it and hence to change its overall mechanical properties. The part of the structure that changes most easily, the most compliant part of the porous system, is that part consisting of cracks of small aspect ratio. We therefore consider the effects of a change $\bar{\boldsymbol{\sigma}}$ in mean stress and p in fluid pressure on the 'mean' crack, which we take to be flat and circular with small aspect ratio.

If we follow the procedure described in the previous section and consider the imposition of a mean stress field $(\delta\bar{\boldsymbol{\sigma}} + \delta p\boldsymbol{I})$ on the porous material with no change of pressure in the fluid, the change of volume of an individual crack of small aspect ratio is

$$\int_{S_c} [u_n]\,dS = (\delta\bar{\sigma}_{nn} + \delta p)\left(\frac{a^3}{\mu}\right)\bar{U}^d_{33} \tag{25}$$

approximately for a thin crack, where $[u_n]$ is the discontinuity in normal displacement across the face S_c of the crack, a is the radius of the crack and $\delta\bar{\sigma}_{nn}$ is the normal component of the traction on a surface oriented with the crack and corresponding to the stress field $\delta\bar{\boldsymbol{\sigma}}$; the quantity $\bar{U}^d_{33}$ is given (for a dry crack), neglecting interactions with other cracks and pores, by Hudson (1981) as

$$\bar{U}^d_{33} = \frac{4}{3}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right), \tag{26}$$

where λ, μ are the Lamé parameters of the matrix material.

We now subtract a uniform stress field $\delta p\boldsymbol{I}$ throughout the solid to obtain the conditions equivalent to an imposed stress $\delta\bar{\boldsymbol{\sigma}}$ and fluid pressure δp. The net increase in volume of the crack is

$$\delta V = \left(\frac{4a^3}{3\mu}\right)\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)(\delta\bar{\sigma}_{nn} + \delta p) - \frac{\delta p}{\kappa}V_o\,, \tag{27}$$

where V_o is the original volume of the crack.

For an ellipsoidal crack of diameter $2a$ and central thickness $2c$,

$$V_o = \frac{4\pi a^3}{3}r\,, \tag{28}$$

where $r = c/a$ is the aspect ratio of the crack. Changes in the diameter of the crack are negligible, so eq. (27) gives, for the change δr in aspect ratio,

$$\delta r = \left(\frac{1}{\pi\mu}\right)\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)(\delta\bar{\sigma}_{nn} + \delta p) - \frac{r}{\kappa}\delta p\,. \tag{29}$$

The last term on the right is of order $r = c/a$ compared with the others and hence may be neglected, so that

$$r = r_o + \frac{1}{\pi\mu}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)(\bar{\sigma}_{nn} + p)\,, \tag{30}$$

where r_o is the value of r before the stress field and pressure are applied. Thus the imposed stress and the pressure together form the crack opening stress

$$\begin{aligned}\sigma^{c(\mathbf{n})} &= \bar{\sigma}_{nn} + p \\ &= \bar{\sigma}_{ij} n_i n_j + p\end{aligned} \tag{31}$$

for the deformation of a crack with unit normal **n** (Zatsepin & Crampin 1997).

If the imposed effective stress $\bar{\boldsymbol{\sigma}}$ is isotropic, $\bar{\sigma}_{ij} = S\delta_{ij}$, then $\sigma^c = S + p$ is the same for all cracks, and their aspect ratios are all changed by the same amount, which is given by eq. (30). However, this will not be true if σ^c is negative and sufficiently large in magnitude that some cracks will close completely. The main effect of this is that the number density of cracks will decrease.

One of the principal parameters governing first- and second-order expressions for the overall properties of cracked materials is the crack density $\varepsilon = \nu a^3$, where ν is the number of cracks per volume and a^3 the mean cubed crack radius (Hudson 1981). If the number of cracks in some reference state of the material in unit volume and with aspect ratio in the range $(r, r+dr)$ is $\nu_o^c(r)dr$, then the same quantity measured after the imposition of a hydrostatic stress S and pressure change p is $\nu^c(r)dr$, where

$$\nu^c(r) = \nu_o^c(r - r') \quad \text{and} \quad r' = \frac{1}{\pi\mu}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)\sigma^c . \tag{32}$$

The total crack density in the reference state is

$$\varepsilon_o = a^3 \int_0^{r_{\max}} \nu_o^c(r)\, dr , \tag{33}$$

and, after the imposition of stress and pressure, it is

$$\begin{aligned}\varepsilon &= a^3 \int_{\max\{0, r'\}}^{r_{\max}+r'} \nu^c(r)\, dr \\ &= a^3 \int_{\max\{0, -r'\}}^{r_{\max}} \nu_o^c(r)\, dr\end{aligned} . \tag{34}$$

If r' is negative, the change in the crack density is

$$\varepsilon - \varepsilon_o = a^3 \int_0^{-r'} \nu_o^c(r)\, dr . \tag{35}$$

If now σ^c is changed by a small negative amount $\triangle\sigma^c$, the change in the crack density is

$$\begin{aligned}\triangle\varepsilon &= a^3 \int_0^{-\triangle\sigma^c} \nu^c(r)\, dr \\ &\simeq a^3(-\triangle\sigma^c)\nu^c(0) \\ &= a^3 \nu_o^c(-r')(-\triangle\sigma^c) .\end{aligned} \tag{36}$$

The measurement of $\triangle\varepsilon$ from elastic wave speeds in the material under negative imposed stress σ^c of increasing magnitude can therefore provide an estimate of $\nu_o^c(r)$, the number of cracks per volume with aspect ratio r, over a range of values of r. This conclusion holds whatever the distribution over orientation of the cracks, although ν_o^c and ν^c may also vary with the polar angles θ, ϕ.

These results may not be reproducible in reverse when σ^c is increased and $\triangle\sigma^c$ is positive since there is a possibility (and eventually the certainty) of the formation of new cracks.

If the imposed stress $\bar{\boldsymbol{\sigma}}$ is not hydrostatic then the crack opening stress $\sigma^c(\mathbf{n})$ varies with the orientation **n** of the crack. However, for aligned cracks, it is the same for every crack. The change in aspect ratio is given for all cracks by eq. (30) and the change in number density by eq. (32); also, the increment in the parameter ε is given in terms of the increment in σ^c by eq. (36).

For a general distribution of crack orientations, we need to define an extended number density function ν_o^c (r, θ, ϕ) such that the number of cracks in the reference state with aspect ratio in the range $(r, r+dr)$ and orientations in the range of polar angles $(\theta, \theta+d\theta)$, $(\phi, \phi+d\phi)$ is ν_o^c $(r, \theta, \phi)dr \sin\theta d\theta d\phi$. Defining $\nu^c(r, \theta, \phi)$ in the same way for a later state after the imposition of the external stress $\bar{\boldsymbol{\sigma}}$, we have an equation corresponding to eq. (32):

$$\nu^c(r, \theta, \phi) = \nu_o^c(r - r', \theta, \phi) ,$$

where (37)

$$r' = \frac{1}{\pi\mu}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)\bar{\sigma}_{ij} n_i n_j ,$$

thus making r' a function of θ and ϕ.

We also define a total crack density as a function of θ and ϕ:

$$\varepsilon_o(\theta, \phi) = a^3 \int_o^{r_{\max}(\theta,\phi)} \nu_o^c(r, \theta, \phi)\, dr \tag{38}$$

and $\varepsilon(\theta, \phi)$ similarly. Then, eq. (35) follows for each value of θ and ϕ, and eq. (36) for the increment in ε for a (negative) increment $\triangle\bar{\sigma}^c(\theta, \phi)$ in σ^c becomes

$$\triangle\varepsilon(\theta, \phi) = -a^3 \nu_o^c(-r', \theta, \phi)\triangle\sigma^c(\theta, \phi) . \tag{39}$$

First- and second-order changes in the elastic wave speeds are directly dependent on the parameter $\varepsilon(\theta, \phi)$ (Hudson 1986), so measurement of these (anisotropic) speeds as a function of θ and ϕ can provide information about the current number density ν^c as well as the number density ν_o^c in some earlier reference state. Although ν_o^c may not depend on orientation, ν^c will if the stress is not hydrostatic. So, a material that, in some reference state, has cracks randomly oriented and thus is isotropic to waves of long wavelength will become anisotropic due to the imposition of non-hydrostatic stress (Horii & Nemat-Nasser 1983). Zatsepin & Crampin (1997) give details of the effect on the wave speeds of the imposition of uniaxial stress on material that initially has randomly oriented cracks, all with the same aspect ratio.

Once again this analysis ignores the possibility of additional microfracturing of the material, which may occur even if the principal stresses imposed externally are negative (compressive); a sufficiently large difference between the maximum and minimum principal stresses will, in general, result in shear fracture (Li & Nordlund 1993).

5 ELASTIC WAVES

In dealing with the propagation of elastic waves we envisage unbounded material in which mean stress and fluid pressure are not controlled independently but vary according to the properties of the wave. Boundary and interface effects must be considered separately. Once again we concentrate on flat cracks as the most compliant part of the microstructure and, therefore, the part that generates the greatest variation in fluid pressure.

In contrast to the static systems considered in the previous sections, the propagation of waves implies dynamic conditions that, in the limit of low frequencies, might be expected to conform to the static picture of a completely drained system for a material with connected cracks and pores. At higher frequencies we expect locally drained conditions but the material will be undrained on a larger scale; that is, we will get pressure-relaxing flow between neighbouring cracks and pores that are distorted in different ways by the stresses in the wave, but this will be confined to a region smaller than a wavelength (Thomsen 1995) since the pressure gradients that drive the flow must change sign within a wavelength. At even higher frequencies there will be no flow between neighbouring cracks and pores and the system will behave as a distribution of isolated cracks and inclusions. This line of reasoning gives three separate models for cracked material in the three frequency ranges: dry cracks, undrained fluid-filled cracked material and a material with isolated fluid-filled cracks. It will be shown, however, that the limit of dry cracks is not, in general, achieved.

A number of different theories provide results, with good agreement at low crack densities, for both dry and isolated fluid-filled cracks (O'Connell & Budiansky 1974; Garbin & Knopoff 1975a,b; Hudson 1981; Nishizawa 1982). In order to obtain expressions for the overall material properties at frequencies where locally undrained conditions apply, it is possible to use the Brown–Korringa relations (eq. 23) or, for isotropic materials, the Gassmann relations (eqs 24) to derive these from the formulæ for dry cracks (Mukerji & Mavko 1994; LeRavalec & Guégen 1996). However, Hudson *et al.* (1996) have provided expressions for the overall properties of cracked fluid-saturated materials in which the cracks are connected by a network of small-scale pores; these expressions are valid across a broad frequency range.

Pointer *et al.* (2000) simplified and evaluated these formulæ and showed that, for randomly oriented cracks, the overall bulk and shear moduli are given to first order in the crack number density by

$$\bar{\kappa}=\kappa+\varepsilon\kappa_1\,,\qquad \bar{\mu}=\mu+\varepsilon\mu_1\,, \tag{40}$$

where

$$\frac{\kappa_1}{\kappa}=-\frac{\kappa}{\mu}\,U_{33}\left\{1-\left(\frac{q}{1+q}\right)(1+i\omega\tau)^{-1}\right.$$
$$\left.\times(1-i\omega\tau'[1+i\omega\tau(1+q)])^{-1}\right\},$$

$$\frac{\mu_1}{\mu}=-\frac{2}{15}\,(2U_{33}+3U_{11})\,,$$

$$U_{11}=\frac{16}{3}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right)(1+i\omega\tau'')^{-1}\,,$$

$$U_{33}=\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\frac{(1+i\omega\tau)}{[1+i\omega\tau(1+q)]}\,;$$

τ, τ', τ'' are relaxation times relating to three different processes. The first corresponds to diffusion between neighbouring cracks and is estimated to be

$$\tau=\frac{\phi_{\rm m}\eta_{\rm f}l^2}{\kappa_{\rm f}K_{\rm m}}\,, \tag{41}$$

where $\phi_{\rm m}$ and K_m are the porosity and permeability, respectively, of the uncracked rock, $\eta_{\rm f}$ and $\kappa_{\rm f}$ are the viscosity and bulk modulus, respectively, of the fluid infill and l is a measure of the intercrack spacing. τ' corresponds to diffusion on a wavelength scale:

$$\tau'=\frac{\kappa_{\rm f}K_{\rm r}}{4\pi\varepsilon\eta_{\rm f}rv^2}\,, \tag{42}$$

where $K_{\rm r}$ is the permeability of the cracked rock, v is the wave speed and ε and r were defined earlier as the crack density and aspect ratio respectively. Finally, τ'' corresponds to the fluid resistance to shearing of the crack:

$$\tau''=\frac{4\eta_{\rm f}}{\pi r\mu}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right). \tag{43}$$

The parameter q relates to the incompressibility of the fluid:

$$q=\frac{\kappa_{\rm f}}{\pi r\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right). \tag{44}$$

These expressions are for cracks with fixed aspect ratio but can easily be generalized to apply to cracks with a range of aspect ratios. The behaviour of the above formulæ has been illustrated for a range of parameter values by Pointer *et al.* (2000).

At high frequencies such that $\omega\tau$ is large, U_{33} becomes approximately

$$U_{33}=\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+q)^{-1}\,, \tag{45}$$

and the expression for (μ_1/μ) (eqs 40) is exactly the same as for fluid-filled isolated cracks (Hudson 1981). Similarly, the expression for κ_1/κ is

$$\frac{\kappa_1}{\kappa}=-\frac{\kappa}{\mu}\,\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+q)^{-1}\,, \tag{46}$$

which is also the form for isolated cracks as expected.

In the limit of low frequencies, we obtain

$$U_{11}=\frac{16}{3}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right),\qquad U_{33}=\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right). \tag{47}$$

Then (μ_1/μ) takes the form for dry cracks (Hudson 1981) while (κ_1/κ) is given once more by eq. (46) for fluid-filled isolated cracks. This is precisely the result for undrained conditions as proposed by O'Connell & Budiansky (1977) and referred to in Section 3.

We may now construct $\bar{\kappa}$ and $\bar{\mu}$ in the low-frequency limit from eqs (40) and (47):

$$\bar{\mu}=\bar{\mu}_{\rm d}\,,\qquad \bar{\kappa}=\kappa\left\{1-\frac{\varepsilon\kappa}{\mu}\,\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+q)^{-1}\right\}; \tag{48}$$

these equations are accurate to first order in ε.

For dry cracks, we have (Hudson 1981)

$$\bar{\mu}_{\rm d}=\mu\left\{1-\varepsilon\,\frac{16}{45}\,\frac{(\lambda+2\mu)(9\lambda+10\mu)}{(3\lambda+4\mu)(\lambda+\mu)}\right\},\qquad \bar{\kappa}_{\rm d}=\kappa\left\{1-\frac{\varepsilon\kappa}{\mu}\,\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}. \tag{49}$$

In the first place, it is clear that, in the limit of low frequencies, the behaviour of material with connected cracks is not as if the cracks were empty or dry as might have been expected. Using the expression above, we obtain the relation

$$\frac{1}{\bar{\kappa}} = \frac{1}{\bar{\kappa}_d} - \frac{\left(\frac{1}{\kappa_d} - \frac{1}{\kappa}\right)^2}{\phi\left(\frac{1}{\kappa_f}\right) + \left(\frac{1}{\bar{\kappa}_d} - \frac{1}{\kappa}\right)}, \qquad (50)$$

where

$$\phi = 4\pi a^2 cv/3 = (4\pi/3)\varepsilon r \qquad (51)$$

is the crack porosity. Eq. (50) is exactly Gassmann's (1951) equation relating the modulus for undrained to that for dry conditions with the neglect of a term in (κ_f/κ). This term was neglected in Hudson's (1981) equations since, if it is not small, U_{33} is effectively zero; that is, the bulk modulus of the fluid is large enough for the crack opening displacement under tension to be negligible.

Thus, the limiting values of the moduli of the cracked material at low frequencies are those for undrained, rather than dry, conditions. This result is physically plausible in that, to obtain dry conditions, the excess pressure within a crack needs to be released by allowing the fluid to diffuse into some kind of sink. Apart from neighbouring cracks with a different orientation and therefore a different pressure, the only sink available is provided by the oscillations of pressure in the wave itself, and highs and lows are separated by a wavelength $(2\pi v/\omega)$ distance. However, the diffusion length in matrix rock of permeability K_m and porosity ϕ_m is (Hudson *et al.* 1996) $(\kappa_f K_m/\omega\eta_f\phi_m)^{1/2}$, where κ_f and η_f are the bulk modulus and viscosity of the fluid, respectively. So, as ω decreases, the distance to the pressure sink increases faster than the diffusion length, so this mechanism of pressure release is ineffective. It follows that, at low frequencies, the conditions at any part of the wave are locally static in that there is ample time for pressure equalization between cracks and pores (unless the permeability is exceptionally low), but there is not enough time for fluid to diffuse to an area where the wave is at a different phase, so the conditions are equivalent to being undrained. The diffusion time for pressure relaxation over a wavelength increases as ω^{-2} as $\omega \to 0$, while the period of the wave increases, less quickly, as ω^{-1}. [See also the discussion by Dvorkin & Nur (1993), in connection with the BISQ model of local fluid flow.]

The situation is different in the model of equant porosity, where the matrix is assumed to be porous (with non-compliant pores as before) and, under increased pressure, fluid simply diffuses out of the crack and into the matrix (an infinite sink). First-order expressions for the overall bulk and shear moduli of such material with randomly oriented cracks are given by eqs (40) with

$$\frac{\kappa_1}{\kappa} = -\frac{\kappa}{\mu} U'_{33}, \qquad \frac{\mu_1}{\mu} = -\frac{2}{15}(2U'_{33} + 3U_{11}), \qquad (52)$$

where U_{11} is given by eqs (40),

$$U'_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+q')$$

and (53)

$$q' = \frac{\kappa_f}{\pi r\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left\{1 + \frac{3(1-i)}{2c}\left(\frac{\phi_m \kappa_f K_m}{2\omega\eta_f}\right)^{1/2}\right\}^{-1}$$

(Pointer *et al.* 2000).

In the limit of high frequencies, these expressions become those for isolated cracks as before, but at low frequencies they are equivalent to the formulæ for dry cracks. This is because the model treats the porous matrix as an unbounded sink for excess pressure and ignores crack–crack interactions. It would seem, therefore, that the model fails at sufficiently long wavelengths, that is, when the diffusion length is of the order of, or greater than, the intercrack spacing. On the physical grounds stated earlier, it is expected that the material properties will once more become those of the undrained system at low enough frequencies.

6 THE EFFECT OF STATIC PRESSURE AND STRESS ON ELASTIC WAVES

It is clear that, by changing the shape and size of cracks and pores, externally imposed stress and fluid pressure will have an effect on the speeds of elastic waves. The relative changes of pores of approximately cylindrical or spherical shape are much less than that of a crack of small aspect ratio, which can be closed completely, as we have seen. To analyse the effect on the wave speeds, we therefore concentrate on flat cracks on the grounds that other cavities, inclusions and pores are less compliant (Mavko & Nur 1979).

The effect of crack closure was studied in Section 4; the number density of cracks of given aspect ratio and orientation is changed in a specific way by imposed stress $\bar{\boldsymbol{\sigma}}$ and pressure p. In addition, the aspect ratios of specific sets of cracks are changed, according to eq. (29), by

$$\triangle r = \frac{1}{\pi\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(\bar{\sigma}_{nn} + p), \qquad (54)$$

where

$$\bar{\sigma}_{nn} = \bar{\sigma}_{ij} n_i n_j \qquad (55)$$

and $\mathbf{n}$ is the unit normal to the crack. The dependence of $\triangle r$ on $\bar{\sigma}_{nn}$ shows that the level of anisotropy of cracked material will change under the imposition of non-hydrostatic stress.

Expressions for the overall properties of materials with connected, fluid-filled cracks whose distributions are not spherically symmetric are fairly complicated (Hudson *et al.* 1996), but the effects of the fluid infill are controlled by the parameters τ, τ', τ'' and q defined in Section 5 for overall isotropic material.

The relaxation time τ for diffusion between cracks is given by eq. (41) and is independent of crack aspect ratio since it depends only on the porous structure connecting the cracks. O'Connell & Budiansky (1977) give quite a different expression for τ but this too is independent of aspect ratio. However, estimates of τ for high crack densities where cracks are connected directly through some kind of constriction or throat depend critically on aspect ratio (O'Connell & Budiansky 1977; Mavko & Nur 1975).

The relaxation time τ' for long-range diffusion (on a wavelength scale) is inversely proportional to r (eq. 42). On the other hand, it is directly proportional to K_r, the permeability of the cracked material, which may be expected to increase with r. The net effect, therefore, is likely to be small.

Fluid resistance to shearing of the crack has a relaxation time τ'' that, according to eq. (43), is inversely proportional to r and is, therefore, critically affected by imposed stress and pressure. However, this process has a vanishingly small effect at seismic frequencies since τ'' is so small (Pointer *et al.* 2000) and so its variation due to changing aspect ratio is unlikely to be detected. It looks as if, by taking r small enough, τ'' could be made to take any value, but the faces of a crack are not, of course, optically smooth and when they are close enough together they make contact and the crack is effectively closed (see Gangi 1978). There is the possibility of frictional movement of the contacting rock faces (Winkler & Nur 1982) and of closed cracks opening under wave rarefaction but remaining closed under pressure (Smyshlyaev & Willis 1994); the latter is, of course, a non-linear process that changes the waveform in a complex way. We do not consider either of these processes here.

Finally there is the factor q (eq. 44), which is controlled by the fluid compressibility. This too is inversely proportional to r. O'Connell & Budiansky (1977) assumed that q is very large but this need not be the case (Pointer *et al.* 2000). Overall properties of materials with isolated cracks—the limit at high frequencies for connected cracks—is dependent on q (see, for instance, eq. 46) and, at the other end of the frequency spectrum, the properties for undrained connected cracks are also governed by q; Gassmann's equations (eqs 23) contain q in the form $(\kappa_f/\kappa\phi)$. The only system where q is not a critical parameter is one with dry cracks, which, of course, is not affected by the aspect ratio at all. However, it appears that material with connected cracks does not behave as if the cracks are dry at any frequency.

When the process of diffusion out of the cracks into the porous matrix dominates (equant porosity), the system is governed by the parameter q' (eqs 53). The dependence of q' on the aspect ratio is given by the fact that q' is directly proportional to q; q' is inversely proportional to r.

It appears, therefore, that the main effect on the wave speeds of externally imposed stress and fluid pressure is through the changes in aspect ratios of flat cracks, and, in turn, these changes are most likely to be observed in the factor $q\ [=(\kappa_f/\pi r\mu)(\lambda+2\mu)/(\lambda+\mu)]$. It follows that changes in aspect ratio are effectively equivalent to changes in fluid compressibility.

7 CONCLUSIONS

The effective stress and strain and the fluid pressure are related by a linear relationship for an elastic material with connected cracks and pores. Such a relationship must be regarded as applying to a static system since enough time must be allowed so that the pressure is equalized throughout the fluid. By linking the stress and pressure together into an apparent stress, we have obtained a simple stress–strain relation governed by the stiffnesses of the material under dry conditions. The isotropic form of this relation is well known (Geertsma 1957; Skempton 1960). Under undrained conditions, the pressure is given in terms of the effective stress and strain, and, once again, a straightforward linear relation between stress and strain is obtained. The equations relating the stiffnesses in undrained and dry conditions are those of Brown & Korringa (1975); they revert to the Gassmann (1951) relations when the material is isotropic.

The most compliant structure in the region occupied by fluid is flat cracks and the main effect of externally imposed stress and pressure on porous media is to change the aspect ratios of cracks. If the imposed stress is not isotropic, the change in aspect ratio depends on the orientation of the crack, and the net effect is to change the level or type of anisotropy of the material at long wavelengths, as noted by Horii & Nemat-Nasser (1983). In particular, part of the crack population may close altogether, thus changing the crack density distribution.

The nature and magnitude of the static stress field and fluid pressure can be monitored by elastic waves whose wavelengths are long compared to the length scale of the microstructure. In the first place changes in crack number density, brought about by crack closure, directly influence the overall properties of the cracked material. Second, changes in crack aspect ratios lead to changes in the parameters involved in expressions for the overall properties of materials with connected cracks. It appears that these influences will be seen most clearly through the fluid incompressibility factor q. This term contains the ratio (κ_f/r), where κ_f is the bulk modulus of the fluid and r is the crack aspect ratio. A rise in the aspect ratio is, therefore, equivalent to an increase in the compressibility of the fluid as far as this term is concerned. The factor q appears in the formula for the stress–strain relations both in the high-frequency limit, where the cracks behave as if they were isolated, and in the low-frequency limit, where undrained conditions apply. There appears to be no reason to suggest that materials with fluid-saturated cracks will respond as if the cracks were dry or empty at low frequencies.

O'Connell & Budiansky (1977) proposed that, for an isotropic material under undrained conditions, the shear modulus is the same as for dry conditions and the bulk modulus the same as for isolated (unconnected) cracks. This is confirmed here, and, furthermore, with first-order expressions for the bulk modulus for both dry and isolated cracks, we show that the Gassmann conditions are satisfied.

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