this direction over FG. The north current through Yucatan Channel at the point ⊗, derived from observations extending from 1887 April 13–17, is given as 12·9 cm./sec., 245°, whereas the average current $-V$ over DE is found to be 3·8 cm./sec., 170°. It is, of course, doubtful how far the currents derived from observation correspond with the average currents U and $-V$. The magnitudes of the latter seem to be of the order that might be expected for average currents corresponding to isolated values 6·8 and 12·9 cm./sec. respectively, especially as there seems to be a tendency for current observations to refer to places of maximum current,* but apart from this there is little agreement.

From the values obtained for ζ a chart has been constructed (fig. 7) to exhibit the co-tidal and co-range lines for the K_1 tide of the Gulf of Mexico, as determined by theory. The inference derived from this chart has been given in § 1.

It may be noted from Table II that by far the largest discrepancy in amplitude between theory and observation occurs for Campeche. Reference to fig. 1 suggests, however, that this is partly due to the sharp edge assumed at C, whereas the actual coast-line is rounded off. An amplitude nearer to the observed value at Campeche is obtained slightly to the north.

Finally it should be remarked that the K_1 tidal motion of the Gulf of Mexico is dependent almost entirely on the external currents, the equilibrium tide for the basin forming only a small fraction of the actual tide.

THE EFFECT OF GEOLOGICAL STRUCTURE UPON MICROSEISMIC DISTURBANCE.

A. W. Lee, M.Sc., D.I.C.

(Communicated by Dr. F. J. W. Whipple)

(Received 1932 March 23)

1. *Introduction*.—The small oscillations of the ground, termed "microseisms" or "microseismic unrest" (*Bodenunruhe*), are reproduced in the records of sensitive seismographs in most parts of the world. The period of the oscillations is from 3 to 10 seconds, and the amplitudes range from 0 to 20 μ. The suggestion that these oscillations are Rayleigh waves transmitted through the outer layers of the earth's crust has been made to account for the occurrence of microseisms in the vertical component seismograms as well as in the horizontal components ; the possibility that the movements might consist of Rayleigh waves and Love waves superposed has hitherto been overlooked. The investigations recorded in this paper have been

* The point × is approximately on a line marked on the Admiralty Chart as the "Axis or strongest part of the Gulf Stream," also it is noted on the same chart that strong indraughts are sometimes experienced along the south coast of Cuba near Yucatan Channel.

developed on the assumption that the movements are Rayleigh waves, and tend to show that the Love waves may be neglected.

The amplitude and period of the microseisms at selected hours are published by a number of observatories. The measurements usually refer to the largest group of well-developed microseisms recorded during an interval centred at the selected hour. The tabulations give a satisfactory representation of the changes in microseismic activity from hour to hour, but are less satisfactory if we wish to compare the horizontal and vertical movements at a given observatory. In a number of comparisons which have been published the horizontal amplitudes have been computed from the amplitudes of the north and east components by the parallelogram law; since the north and east amplitudes are each maxima of a large number of oscillations the horizontal amplitude must be overestimated. It would be simpler, and probably more accurate, to take the larger of the north and east tabulated amplitudes as an approximation to the horizontal amplitude.

These published values and comparisons between the microseisms in different places show that although the major fluctuations in disturbance extend over wide areas, there are large differences between the amplitudes at the various observatories. For example, a comparison of the microseisms recorded during 1930 January at seven observatories in Great Britain shows that the mean amplitudes varied from 2·3 μ at Kew to 5·4 μ at Edinburgh, but at each observatory the amplitudes from the 11th to 12th were abnormally high, being in every case about three times the average.* The records of Galitzin, Wiechert and Milne-Shaw seismographs are most generally used for the study of microseisms, and comparisons between these types have shown that there is reasonable agreement between the amplitudes either would record at a given observatory. Thus the variations in amplitude from place to place cannot be due to instrumental inconsistencies, and geographical and geological causes have been suggested as the explanation. According to Professor Gutenberg and other writers, the amplitudes of the microseisms recorded depend upon distance from the region in which the microseisms are generated and upon the material on which the observatory is located. Gutenberg has compared the amplitudes for a number of stations and classified them according to the subsoil; † the results are not very satisfactory, as large variations in amplitude between neighbouring stations on similar material are not explained. Several authors have suggested that the microseisms at observatories on the more recent geological formations are larger than those on rock,‡ but have not been able to explain why this should happen. Others have suggested that the movements would be absorbed by loose sandy soil and propagated with larger amplitudes in rock.

More detailed investigations of the connexion between microseisms and

* A. W. Lee, *M.N.R.A.S., Geophys. Suppl.*, 3, 105, 1932.

† B. Gutenberg, *Die seismische Bodenunruhe*. Berlin, 1924.

‡ D. Kituchi, *Publications of Earthquake Investigation Commission*, Tokyo, 1904; F. Omori, *Bulletin of Earthquake Investigation Commission*, Tokyo, 1908; E. Rudolph, *Comptes-Rendus de l'Association Internationale de Sismologie*, Zermatt, 1909.

the geological formations have recently been carried out at Kew Observatory and are described in the following sections. Two possible effects are considered: (*a*) that on some formations the seismograph pillar might be tilted by the microseismic oscillations and behave like an inverted pendulum, and (*b*) that the motion at the surface depends upon the material in the upper layers of the earth's crust.

2. *Effect of Subsoil upon a Seismograph Pillar.*—The possibility that the motion recorded by a seismograph might be complicated by tilting of the pillar has been recognised for many years. G. W. Walker concluded * that a pier of concrete 1 metre square embedded in clay to a depth of 1 metre would not be affected in this way, but no analysis of the motion of piers actually in use appears to have been published. The behaviour of a seismograph pillar can be computed if we assume that the force due to earth resistance is similar to the resistance of a number of springs distributed below the base of the pillar.

The forces acting on a pillar which rests upon a level surface (fig. 1*a*) are :

(*a*) The reaction of the ground, F.
(*b*) The shearing force, S.
(*c*) The turning couple, G.

In equilibrium S and G are zero, and $F = -Mg$ (the weight of the pillar).

For pure horizontal movement of the ground, u, with the pillar sliding and tipping, the horizontal movement of the centre of gravity of the pillar is $u - \xi - \alpha h$, where α is the angular deflexion of the pillar, h the height of its centre of gravity, and ξ denotes the slip which depends upon the friction between pillar and ground. If η is the vertical motion of the centre of gravity of the pillar, the equations of motion are :

$$\begin{aligned} M(\ddot{u} - \ddot{\xi} - \ddot{\alpha}h) &= S, \\ M\ddot{\eta} &= F - Mg, \\ Mk^2\ddot{\alpha} &= Sh - G + F\alpha h, \end{aligned}$$

where k^2 is the radius of gyration of the pillar about P (fig. 1*b*).

Assuming that Hooke's law holds for the ground below the pillar, let $\mu\alpha$ be the moment of force due to earth resistance. The damping of the motion is proportional to $\dot{\alpha}$, but since we only require the maximum effect of tilt this term may be neglected.

For a first approximation to F, η may be neglected, and unless S is very large $\xi = 0$. If S and G be eliminated from the equations of motion (simplified by these assumptions) we get

$$M(\ddot{u} - \ddot{\alpha}h) = \frac{Mk^2}{h}\ddot{\alpha} + \frac{\mu}{h}\alpha - Mg\alpha$$

or

$$M(k^2 + h^2)\ddot{\alpha} + (\mu - Mgh)\alpha = Mh\ddot{u}.$$

The motion of the pendulum is due to the motion of the top of the pillar ($u - 2h\alpha$) and to the tilt of the pillar. If u' is the apparent motion,

* G. W. Walker, *Modern Seismology*, p. 31. London, 1913.

i.e. the motion which would be computed on the assumption that the motion of pier and ground are identical :

$$\begin{aligned} \ddot{u}' &= \ddot{u} - 2h\ddot{a} + ga \\ &= \ddot{u}\left[1 - \frac{2h\ddot{a}}{\ddot{u}} + \frac{ga}{\ddot{u}}\right] \\ &= \ddot{u}\left[1 + \frac{ga - 2h\ddot{a}}{\left(\frac{k^2+h^2}{h}\right)\ddot{a} + \left(\frac{\mu}{Mh} - g\right)a}\right]. \end{aligned}$$

When the ground movement is harmonic with period τ :

$$\frac{\ddot{u}'}{\ddot{u}} = \frac{\frac{\mu}{Mh}\tau^2 - 4\pi^2 \cdot \frac{k^2 - h^2}{h}}{\left(\frac{\mu}{Mh} - g\right)\tau^2 - 4\pi^2 \cdot \frac{k^2 + h^2}{h}}.$$

The values of M, h and k for a particular pillar are easily obtained. μ may be determined experimentally by placing a weight mg on one side of the pillar and observing the deflexion ; then if x is the distance of the weight from the axis of the pillar, $mgx = (\mu - Mgh)a$. Experiments have been made at Kew Observatory and at Durham University Observatory ; * the pillar at Kew is large, to accommodate the three pendulums of the Galitzin seismographs ; that at Durham is smaller, only carrying one Milne-Shaw instrument. The results obtained are—

	Kew	Durham
M (gm.)	10^7	2.10^6
h (cm.)	100	50
k^2 (cm.^{2})	2.10^4	5.10^3
μ (dyne-cm.)	10^{15}	10^{15}

On substitution of these values in the above equation we find that—

$$\left(\frac{\ddot{u}'}{\ddot{u}}\right)_{\text{Kew}} = \frac{\tau^2 - \cdot 004}{\tau^2 - \cdot 012}$$

and

$$\left(\frac{\ddot{u}'}{\ddot{u}}\right)_{\text{Durham}} = \frac{\tau^2 - \cdot 0002}{\tau^2 - \cdot 0006}.$$

These equations show that for movements with periods exceeding 1 second $\ddot{u}'$ cannot differ appreciably from $\ddot{u}$. Thus for these oscillations the effect of tilting of the pillar is negligible, and we conclude that pillars of the type usually employed are so stable that the motion cannot be affected by differences in the subsoil.

3. *The Effect of a Superficial Layer on the Amplitude of Rayleigh Waves.* —In considering whether the motion at the surface is related to geological structure, we shall assume that when no more recent formations are present

* I am indebted to Mr. F. Sargeant for the data relating to the pillar at Durham Observatory and for carrying out the experiments for the determination of μ.

Fig 1a.

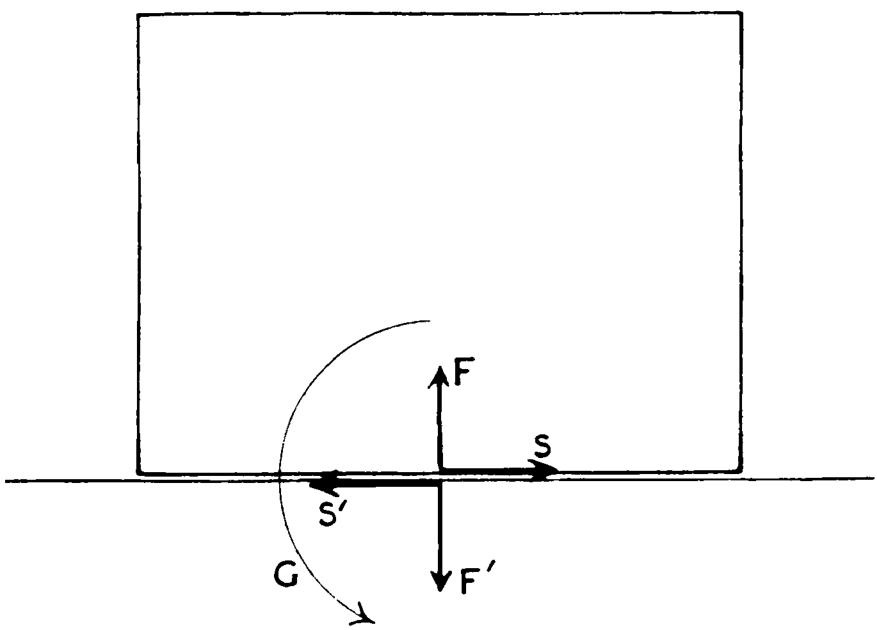
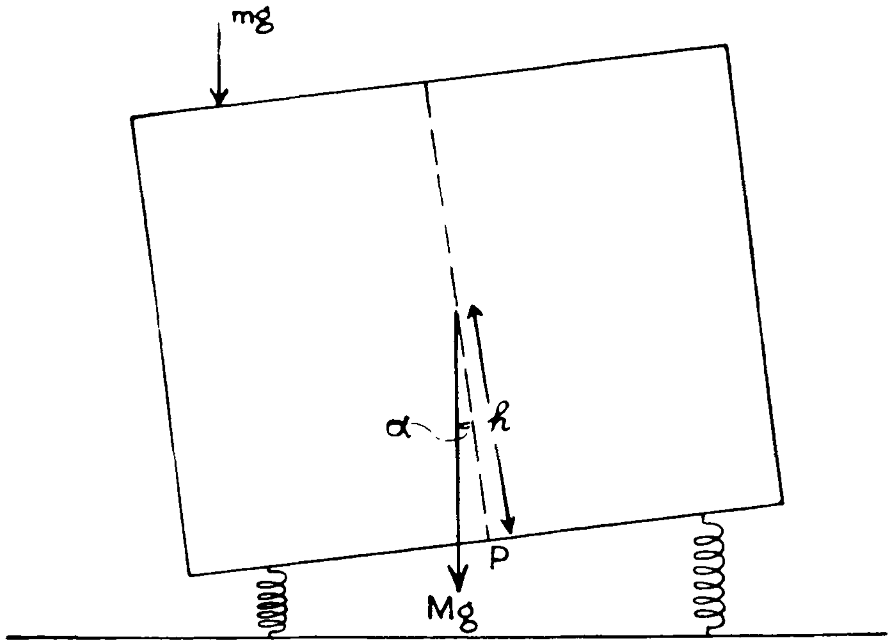


Fig. 1b.



the microseisms are Rayleigh waves in granite (granite is here taken as including the sub-continental granitic layer, and also the Archæan rocks for which, roughly speaking, the elastic constants of granite are appropriate). The thickness of the granite being comparable with the wave-length of Rayleigh waves in granite with the period of microseisms, no allowance will be made for the discontinuity between the granitic and basaltic layers. A. E. H. Love has developed the theory of propagation of waves analogous to Rayleigh waves when the medium is covered with a superficial layer.* He assumes that the materials in the layer and below are incompressible. R. Stoneley gives the equation of wave-velocity in the more general case,† but neither author has considered how the amplitudes are affected for oscillations with a period comparable with that of the microseisms. Here we follow Love's method, but take the compressibility into account.

Taking the origin at the bottom of the layer, let the axis of Z be vertically upwards and the axis of X in the direction of propagation of the waves. It is assumed that the subjacent rock extends to $Z = -\infty$.

Let u, w = components of displacement along OX and OZ.
ρ, λ, μ = density and elastic constants in superficial layer.
ρ', λ', μ' = " " " subjacent rock.
T = thickness of layer.
$2\pi/f$ = wave-length.
$2\pi/p$ = period of the waves.
κ^2 and $\kappa'^2 = p^2\rho/\mu$ and $p^2\rho'/\mu'$ respectively.
h^2 " $h'^2 = p^2\rho/(\lambda+2\mu)$ " $p^2\rho'/(\lambda'+2\mu')$ "
s^2 " $s'^2 = \kappa^2 - f^2$ " $f^2 - \kappa'^2$ "
r^2 " $r'^2 = h^2 - f^2$ " $f^2 - h'^2$ "

It will be noticed that the notation is not quite symmetrical. The desirability of avoiding imaginary quantities as far as possible has dictated the choice of sign.

The equations of motion are—

In the layer:

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2} &= (\lambda+\mu)\frac{\partial \Delta}{\partial x} + \mu\nabla^2 u \\ \rho\frac{\partial^2 w}{\partial t^2} &= (\lambda+\mu)\frac{\partial \Delta}{\partial z} + \mu\nabla^2 w \end{aligned}\right\}. \tag{1}$$

Below:

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2} &= (\lambda'+\mu')\frac{\partial \Delta}{\partial x} + \mu'\nabla^2 u \\ \rho\frac{\partial^2 w}{\partial t^2} &= (\lambda'+\mu')\frac{\partial \Delta}{\partial z} + \mu'\nabla^2 w \end{aligned}\right\}, \tag{2}$$

where

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}. \tag{3}$$

---

* A. E. H. Love, *Problems of Geodynamics*, p. 165. Cambridge, 1926.

† R. Stoneley, *Proc. Leeds Phil. Soc.* (Sci. Section), 1, 219, 1928.

When $f^2 < h^2 < \kappa^2$ and $f^2 > \kappa'^2 > h'^2$, the definitions give real values of s, r, s', r', and the motion in the layer is given by :

$$\left.\begin{aligned} -u &= [f(P\cos rz - Q\sin rz) + s(A\sin sz + B\cos sz)]\sin(pt+fx+\epsilon) \\ -w &= [r(P\sin rz + Q\cos rz) + f(A\cos sz - B\sin sz)]\cos(pt+fx+\epsilon) \end{aligned}\right\}; \tag{4}$$

and beneath the layer :

$$\left.\begin{aligned} u &= (-fP'e^{r'z} + s'A'e^{s'z})\sin(pt+fx+\epsilon) \\ w &= (r'P'e^{r'z} - fA'e^{s'z})\cos(pt+fx+\epsilon) \end{aligned}\right\}, \tag{5}$$

where P, Q, A, B, P', A' are constants.

By taking the boundary conditions that stress and displacement are continuous across the interface, and that the upper surface of the layer is free from traction, five relations are found between the constants, and the equation of wave-velocity is obtained as :

$$\xi\eta' - \xi'\eta = 0, \tag{6}$$

where

$$\left.\begin{aligned} \xi &= \left(2 - \frac{\kappa^2}{f^2}\right)\left[X\cos rT + \frac{r'}{r}Y\sin rT\right] + 2\frac{s}{f}\left[\frac{r'}{f}W\sin sT - \frac{f}{s}Z\cos sT\right] \\ \xi' &= \left(2 - \frac{\kappa^2}{f^2}\right)\left[\frac{s'}{f}W\cos rT + \frac{f}{r}Z\sin rT\right] + 2\frac{s}{f}\left[X\sin sT - \frac{s'}{s}Y\cos sT\right] \\ \eta &= \left(2 - \frac{\kappa^2}{f^2}\right)\left[\frac{r'}{f}W\cos sT + \frac{f}{s}Z\sin sT\right] + 2\frac{r}{f}\left[X\sin rT - \frac{r'}{r}Y\cos rT\right] \\ \eta' &= \left(2 - \frac{\kappa^2}{f^2}\right)\left[X\cos sT + \frac{s'}{s}Y\sin sT\right] + 2\frac{r}{f}\left[\frac{s'}{f}W\sin rT - \frac{f}{r}Z\cos rT\right] \end{aligned}\right\} \tag{7}$$

X, Y, Z, W having the values given by Love :

$$\left.\begin{aligned} X &= \frac{\mu'}{\mu}\cdot\frac{\kappa'^2}{f^2} - 2\left(\frac{\mu'}{\mu} - 1\right), & Y &= \frac{\kappa^2}{f^2} + 2\left(\frac{\mu'}{\mu} - 1\right) \\ Z &= \frac{\mu'}{\mu}\cdot\frac{\kappa'^2}{f^2} - \frac{\kappa^2}{f^2} - 2\left(\frac{\mu'}{\mu} - 1\right), & W &= 2\left(\frac{\mu'}{\mu} - 1\right) \end{aligned}\right\}. \tag{8}$$

When $T = 0$ equation (6) becomes

$$\frac{4s'r'}{f^2} = \left(2 - \frac{\kappa'^2}{f^2}\right)^2, \tag{9}$$

which is the equation for a Rayleigh wave in the subjacent material with no superficial layer ; if it is assumed that $\lambda' = \mu'$ (or that Poisson's ratio for the subjacent rock is 0·25) the solution is :

$$\left.\begin{aligned} &\frac{\kappa'^2}{f^2} = 0{\cdot}8453 \quad \text{or} \quad f = 1{\cdot}0877\kappa', \\ &\frac{s'}{f} = 0{\cdot}3933, \qquad \frac{r'}{f} = 0{\cdot}8475, \\ &u = -fP'(e^{r'z} - 0{\cdot}5773e^{s'z})\sin(pt+fx+\epsilon) \\ &w = fP'(0{\cdot}8475e^{r'z} - 1{\cdot}4679e^{s'z})\cos(pt+fx+\epsilon) \end{aligned}\right\}. \tag{10}$$

The solution shows that

(*a*) u, the horizontal displacement, is zero when $e^{(r'-s')z}=0{\cdot}5773$;

(*b*) w, the vertical displacement, is a maximum when $e^{(r'-s')z}=\frac{s'}{r'}\cdot\frac{1{\cdot}4679}{0{\cdot}8475}$;

(*c*) the ratio of horizontal to vertical displacements is always between $+0{\cdot}68$, the value at the surface, and $-0{\cdot}39$, the value at great depths.

On the assumption that $\lambda=\mu$, $\lambda'=\mu'$ and that T is small, the equations of (7) reduce to:

$$\left.\begin{aligned}\xi&=\frac{\kappa^2}{f^2}[r'T(2W+2-Y)+(2-X)]\\ \xi'&=\frac{\kappa^2}{f^2}\left[fT(2X-2-Z)-\frac{s'}{f}(2+W)\right]\\ \eta&=\frac{\kappa^2}{f^2}\left[fT(\tfrac{8}{3}X-2-Z)-\frac{r'}{f}(2+W)\right]\\ \eta'&=\frac{\kappa^2}{f^2}[s'\mathrm{T}(\tfrac{8}{3}W+2-Y)+(2-X)]\end{aligned}\right\}, \tag{11}$$

and equation (6) becomes

$$\left[\left(2-\frac{\kappa'^2}{f^2}\right)^2-\frac{4s'r'}{f^2}\right]+T\frac{\mu}{\mu'}\frac{\kappa'^2}{f^2}\left[(r'+s')\frac{\kappa^2}{f^2}-\tfrac{8}{3}s'\right]=0. \tag{12}$$

If the layer is composed of the same material as that below, $\mu=\mu'$, $\kappa=\kappa'$, etc., and equation (12) reduces to that for a Rayleigh wave (9).

For solution of equation (12) it is convenient to assume that the required value of f differs from that for a Rayleigh wave by δf, so that

$$4\frac{\delta f}{f}\left[\frac{2\kappa'^2}{f^2}-\frac{\kappa'^4}{f^4}-\frac{r'}{s'}-\frac{s'}{r'}+\frac{2r's'}{f^2}\right]+T\frac{\mu}{\mu'}\frac{\kappa'^2}{f^2}\left[(r'+s')\frac{\kappa^2}{f^2}-\tfrac{8}{3}s'\right]=0. \tag{13}$$

On substitution of the values given above for $\frac{\kappa'}{f}$, $\frac{r'}{f}$ and $\frac{s'}{f}$ this equation becomes

$$\delta f=\frac{T}{3{\cdot}70}\frac{\mu}{\mu'}[\kappa^2-\kappa'^2],$$

and the solution of (12) is

$$f=1{\cdot}0877\kappa'+\frac{T}{3{\cdot}70}\frac{\mu}{\mu'}[\kappa^2-\kappa'^2]. \tag{14}$$

As T increases this will not give an accurate solution of (6) and a closer approximation must be obtained. In numerical work it has been found that the solution can be obtained in reasonable time from successive approximations.

The energy of the wave being half kinetic and half potential, the total energy (E) per unit width per wave-length is given by

$$E=\frac{p^2\pi}{f}\int_{-\infty}^{T}\rho(\bar{u}^2+\bar{w}^2)dz, \tag{15}$$

where $\bar{u}$ and $\bar{w}$ are the maximum values of u and w for a given value of Z.

On substitution from equation (5) for $\bar{u}$ and $\bar{w}$ we obtain the energy below the interface (E_1):

$$E_1 = \tfrac{1}{2}p^2\pi\rho' P'^2\left[\frac{f}{r'}+\frac{r'}{f}+\left(\frac{f}{s'}+\frac{s'}{f}\right)\left(\frac{A'}{P'}\right)^2-4\left(\frac{A'}{P'}\right)\right]. \tag{16}$$

Similarly from equation (4) the energy in the layer (E_2) is given by

$$\left.\begin{aligned} E_2 = \frac{p^2\pi\rho}{f}\Bigg\{&\frac{T}{2}[(f^2+r^2)(P^2+Q^2)+(f^2+s^2)(A^2+B^2)] \\ &+\sin rT\frac{f^2-r^2}{2r}[(P^2-Q^2)\cos rT-2PQ\sin rT] \\ &+\sin sT\frac{f^2-s^2}{2s}[(A^2-B^2)\cos sT-2AB\sin sT] \\ &-2f[A\cos sT-B\sin sT][P\cos rT-Q\sin rT]+2fAP\Bigg\} \end{aligned}\right\}. \tag{17}$$

When T is small equation (17) reduces to

$$E_2 = \frac{p^2\pi\rho T}{f}[(sB+fP)^2+(rQ+fA)^2],$$

or

$$E_2 = \frac{p^2\pi\rho T}{f}[\bar{u}_0^2+\bar{w}_0^2]. \tag{18}$$

When $f^2 > \kappa^2 > h^2$ the values of r and s are imaginary, and it is convenient to change to a more symmetrical notation. We therefore take

$$S^2 = f^2-\kappa^2, \qquad R^2 = f^2-h^2,$$

and the motion in the layer is given by

$$\left.\begin{aligned} u &= [-f(P\cosh Rz+Q\sinh Rz)+S(A\sinh Sz+B\cosh Sz)]\sin(pt+fx+\epsilon) \\ w &= [R(P\sinh Rz+Q\cosh Rz)-f(A\cosh Sz+B\sinh Sz)]\cos(pt+fx+\epsilon) \end{aligned}\right\}, \tag{19}$$

and the equations of (7) become

$$\left.\begin{aligned} \xi &= \left(2-\frac{\kappa^2}{f^2}\right)\left(X\cosh RT+\frac{r'}{R}Y\sinh RT\right)-2\frac{S}{f}\left(\frac{r'}{f}W\sinh ST+\frac{f}{S}Z\cosh ST\right) \\ \xi' &= \left(2-\frac{\kappa^2}{f^2}\right)\left(\frac{s'}{f}W\cosh RT+\frac{f}{R}Z\sinh RT\right)-2\frac{S}{f}\left(X\sinh ST+\frac{s'}{S}Y\cosh ST\right) \\ \eta &= \left(2-\frac{\kappa^2}{f^2}\right)\left(\frac{r'}{f}W\cosh ST+\frac{f}{S}Z\sinh ST\right)-2\frac{R}{f}\left(X\sinh RT+\frac{r'}{R}Y\cosh RT\right) \\ \eta' &= \left(2-\frac{\kappa^2}{f^2}\right)\left(X\cosh ST+\frac{s'}{S}Y\sinh ST\right)-2\frac{R}{f}\left(\frac{s'}{f}W\sinh RT+\frac{f}{R}Z\cosh RT\right) \end{aligned}\right\}. \tag{20}$$

Corresponding equations may be written out for the intermediate case when $\kappa^2 > f^2 > h^2$, and (using our earlier notation) s is real with r imaginary. In each case the equations for the motion and wave-velocity are real.

4. *Numerical Examples—Rayleigh Waves.*—The cases which have been

selected as most appropriate for application of the formulæ to the study of microseisms are those of Rayleigh waves in granite and of the analogous waves when the granite is covered by a superficial layer. The materials chosen for the layers are limestone, sandstone and clay, but before considering the motion with a superposed layer the properties of true Rayleigh waves in these media will be compared with those of the Rayleigh waves in granite. Values of μ and κ^2/p^2 have been computed for each medium from the density and the velocity of longitudinal waves (V_P); assuming λ and μ equal, the formulæ are $\mu = \frac{1}{3}\rho V_P{}^2$ and $\frac{\kappa^2}{p^2} = \frac{\rho}{\mu}$. The data are given in Table I, the density and velocity of longitudinal waves in granite being taken from H. Jeffreys.* The properties of limestone, sandstone and clay are more variable, being influenced to a greater extent by conditions during deposition and the subsequent history of the materials. The densities in the table are those given by Jeffreys; the velocities are taken from data published by B. Gutenberg.†

Table I

Properties of Granite, Limestone, Sandstone and Clay

Material	Granite	Limestone	Sandstone	Clay
Velocity of longitudinal wave (km./sec.)	5·50	3·0	2·5	1·75
Density ρ (gm./c.c.)	2·65	2·5	2·2	1·9
$\mu \times 10^{-10}$	26·72	7·50	4·58	1·94
$(\kappa^2/p^2) \times 10^{12}$	9·9	33·3	48·1	98·0

The solutions of equation (9) show that for Rayleigh waves with period 2π seconds:‡

$f = 3{\cdot}42 \,.\, 10^{-6}$, $r' = 2{\cdot}90 \,.\, 10^{-6}$, $s' = 1{\cdot}35 \,.\, 10^{-6}$ in granite.
$f = 6{\cdot}28 \,.\, 10^{-6}$, $r' = 5{\cdot}32 \,.\, 10^{-6}$, $s' = 2{\cdot}47 \,.\, 10^{-6}$ in limestone.
$f = 7{\cdot}54 \,.\, 10^{-6}$, $r' = 6{\cdot}39 \,.\, 10^{-6}$, $s' = 2{\cdot}97 \,.\, 10^{-6}$ in sandstone.
$f = 10{\cdot}77 \,.\, 10^{-6}$, $r' = 9{\cdot}13 \,.\, 10^{-6}$, $s' = 4{\cdot}23 \,.\, 10^{-6}$ in clay.

The wave-lengths therefore are 18·4 km. in granite, 10·0 km. in limestone, 8·3 km. in sandstone and 5·8 km. in clay.

On substitution of the values of f, r' and s' in the equations of (10) we can obtain for each medium the ratio of the horizontal or vertical movement at any depth to the horizontal amplitude at the surface. The relations between amplitude and depth are shown in fig. 2; the data from which the curves for granite have been prepared will be found in Table II. The

* H. Jeffreys, *The Earth* (2nd ed.), chap. vi. Cambridge, 1929.

† B. Gutenberg, *Lehrbuch der Geophysik*, p. 607, Berlin, 1929; *Müller Pouillets Lehrbuch der Geophysik*, p. 781. Brunswick, 1928.

‡ The period of maximum frequency of microseisms at Kew.

RELATION BETWEEN DISPLACEMENT AND DEPTH FOR RAYLEIGH WAVES.

$T = 2\pi$ SECOND.

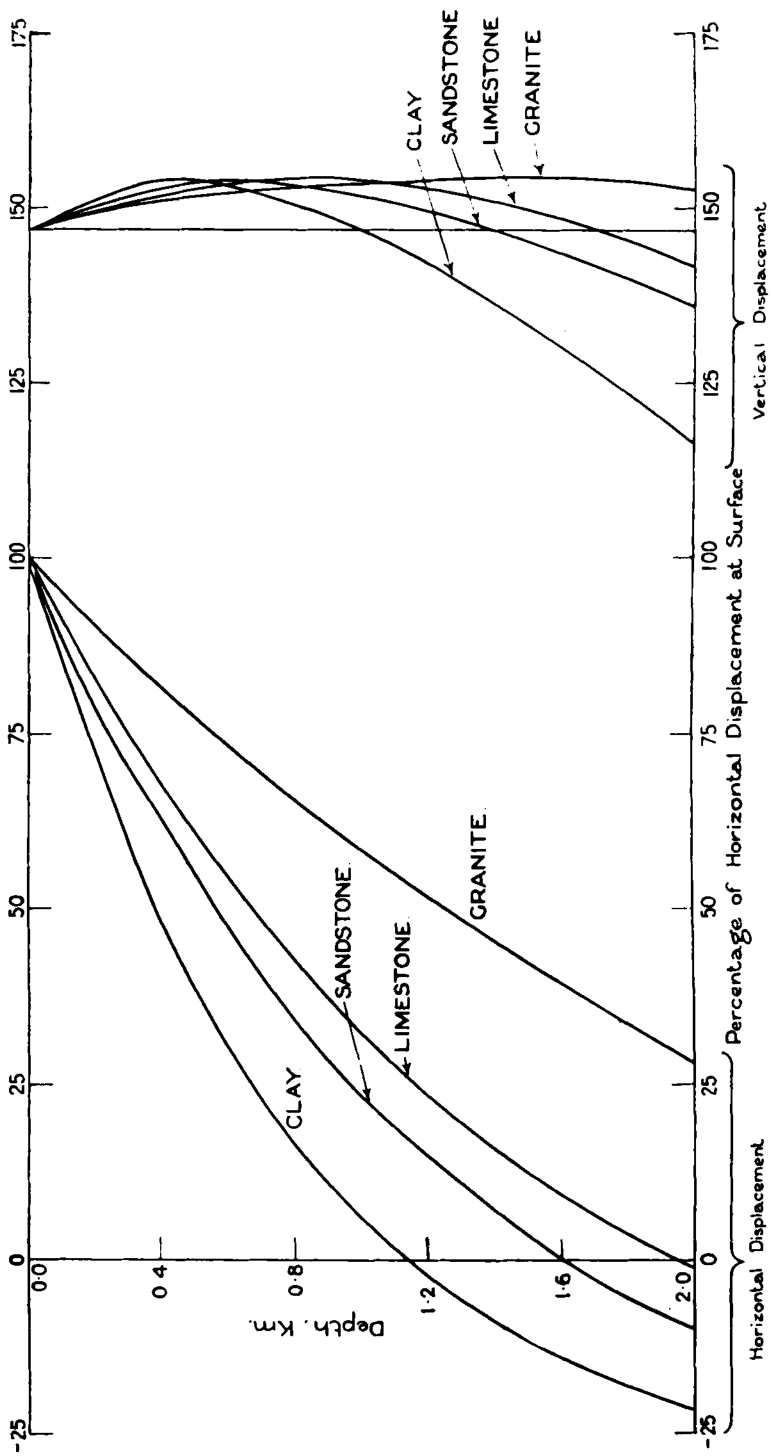


Fig. 2.

depths at which the tabulated ratios would be appropriate in the other media are less than the depths in granite in proportion to the wave-lengths.

TABLE II

Relation between Displacement and Depth for Rayleigh Waves in Granite

($\tau = 2\pi$ seconds)

Depth ($-z$) km.	0·0	0·2	0·4	0·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0
$\bar{u}(-z)$.	1·00	0·91	0·82	0·74	0·66	0·58	0·51	0·45	0·40	0·34	0·28
$\bar{w}(-z)$.	1·47	1·49	1·51	1·52	1·53	1·53	1·54	1·54	1·54	1·54	1·53

In granite for waves of period 2π seconds the horizontal displacement falls to zero at a depth of 3·5 km., and the maximum vertical displacement occurs at a depth of 1·4 km. For the other media :

Limestone	$\bar{u}=0$ at	1·9 km.	Maximum $\bar{w}$ at	0·7 km.
Sandstone	,,	1·6 km.	,, ,,	0·6 km.
Clay	,,	1·1 km.	,, ,,	0·4 km.

Since the relation between displacement and depth depends upon a function of the product pz, the tabulated values for $p=1$ may be applied to any other period by changing the vertical scale. Thus the numerical results for earthquake waves of period 6π seconds ($p=\frac{1}{3}$) at a depth $-z$ are the same as those tabulated for $p=1$ at three times this depth.

The curves of fig. 1 depict the amplitudes at different depths in relation to the horizontal amplitudes at the surface, but give no basis for comparing the waves in the various media owing to the large differences in energy. The energy per unit width per wave-length (equation (15)) is $1{\cdot}2377\pi\rho p^2 P'^2$, and P' is proportional to $\bar{u}/f$. Consequently with equal energy per wave-length the surface amplitudes for the media would be in the proportion

Granite : Limestone : Sandstone : Clay = 1·00 : 1·89 : 2·42 : 3·72.

5. *Numerical Examples—Wave-motion with a Superficial Layer.*—The motion with a superficial layer has been computed for waves with period 2π seconds propagated through granite covered by a given thickness of limestone, of sandstone or of clay. Again the results are appropriate for other periods subject to a change in the vertical scale, which now includes thickness of the layer as well as depth below the surface or interface. The thicknesses of the layers are taken for each fifth of a kilometre up to 1 km., and the motion of the granite, when needed, is taken at similar intervals to a depth of 1 km. below the interface. Computations of the motion for thicker layers or at greater depths might be of interest in connexion with the mathematical analysis, but could be of no practical use. The computations are carried out in three stages : (*a*) Investigation of the effect

of the layers upon the frequency and upon the ratio of horizontal to vertical displacement; (*b*) determination of the ratio of the energy in the layer to that in the granite; (*c*) the effect of the layer upon the movements at the surface, and the distribution of displacement with depth.

The effect of the layers upon frequency is obtained from solution of the equations represented by (6). With layers of sandstone or limestone, satisfactory solutions for f are obtained by the approximate methods of section 3. In the case of clay the solutions were found by successive approximations. The frequencies and corresponding wave-lengths are given in Table III; identical values were found for limestone and sandstone throughout the range of thicknesses covered in the computations.

TABLE III

Frequency and Wave-length with Superficial Layers

($\tau = 2\pi$ seconds)

Material in Layer		Thickness of Layer (km.)					
		0·0	0·2	0·4	0·6	0·8	1·0
Limestone or sandstone	$f \times 10^6$	3·42	3·46	3·49	3·53	3·56	3·60
	wave-length (km.)	18·4	18·2	18·0	17·8	17·7	17·5
Clay	$f \times 10^6$	3·42	3·47	3·52	3·57	3·62	3·68
	wave-length (km.)	18·4	18·1	17·9	17·6	17·4	17·1

Thus with layers up to 1 km. in thickness the wave-length is much nearer to that of Rayleigh waves in the granite than to that of Rayleigh waves in the material of the layer. For these layers the changes in frequency and wave-length with increasing thicknesses of superposed material are approximately linear. The variation with thickness is only slightly affected by large changes in the composition of the layer; for waves of period 2π seconds, replacement of the first kilometre of granite by limestone or sandstone reduces the wave-length from 18·4 km. to 17·5 km., and substitution of clay in the top kilometre yields a wave-length only 0·4 km. lower.

The relation between horizontal and vertical amplitudes $\left(\frac{\bar{u}}{\bar{w}}\right)$ with superficial layers is found from equations (4) and (5); numerical values may readily be obtained from Table VI in the latter part of this section. Fig. 3 shows how the ratio of horizontal to vertical amplitude at the surface depends upon the material and thickness of the layer. For layers of thickness 1 km. or less the ratio is positive and increases with the thickness. The distribution of the curves, granite-limestone-sandstone-clay, is that of diminishing density and elastic constants for the layer.

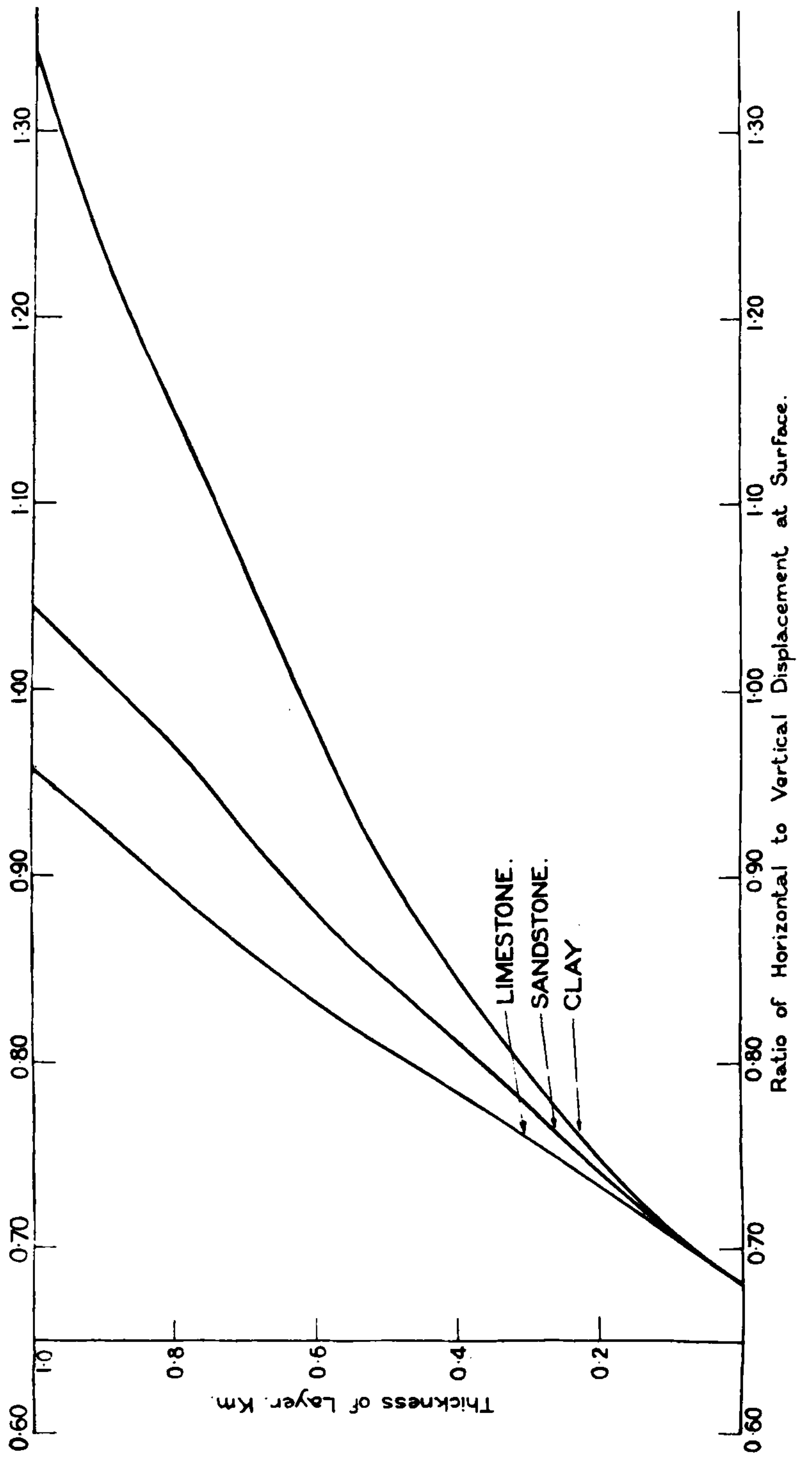
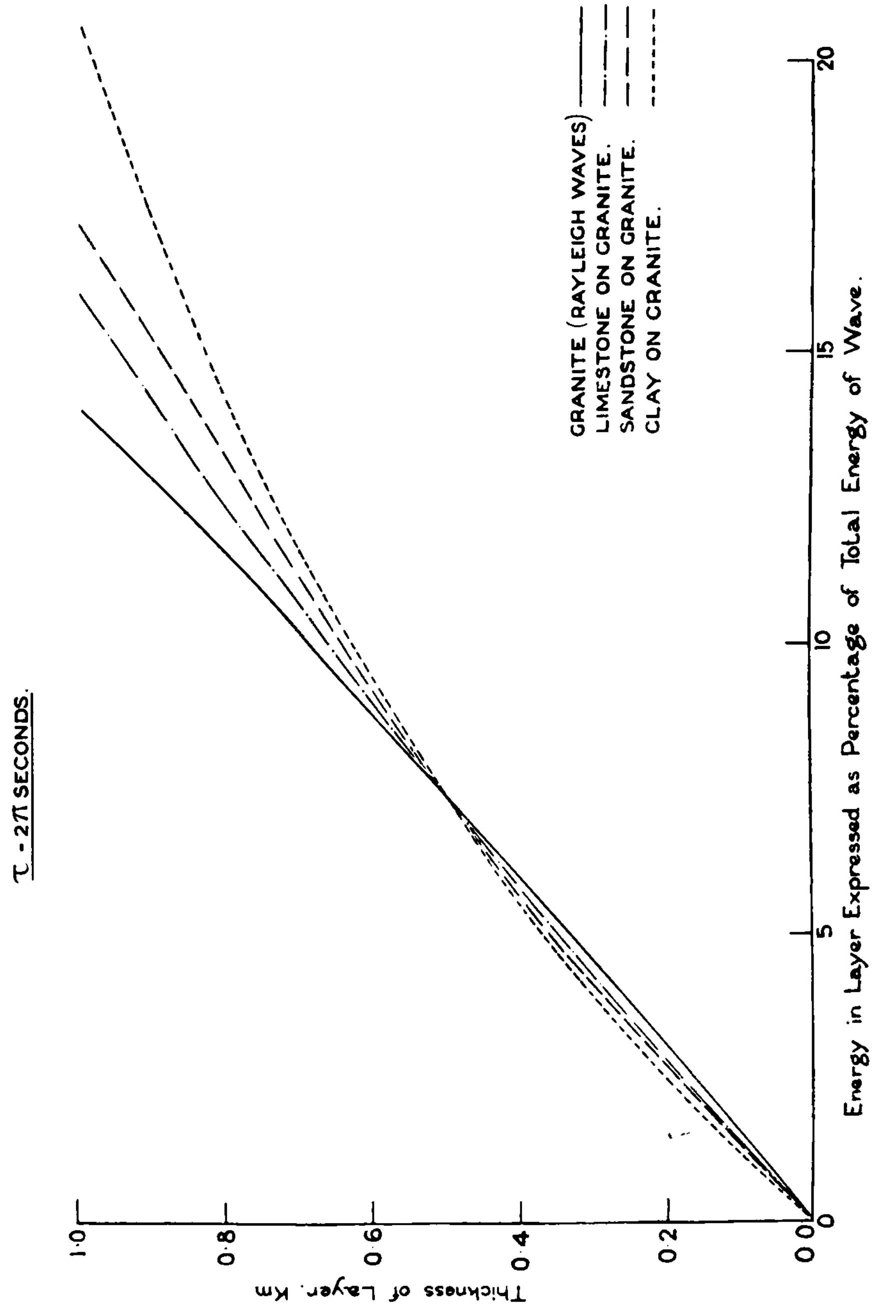


Fig. 3.

RATIO OF HORIZONTAL TO VERTICAL DISPLACEMENTS AT SURFACE WITH SUPERFICIAL LAYERS.

$T = 2\pi$ SECONDS.

DISTRIBUTION OF ENERGY WITH LAYERS OF DIFFERENT MATERIALS. Fig. 4



The amounts of energy in the subjacent granite and in the layer are determined from equations (16)–(18). Table IV gives the percentages of the total energy which are located in the respective layer, and the total energy expressed in terms of the P'^2 which is appropriate for the specified thickness and material. The distribution of energy in the top kilometre for Rayleigh waves (of period 2π seconds) in granite is included for comparison. The distribution of energy between the layers and the subjacent granite is shown graphically in fig. 4.

TABLE IV

Values of $(E_1+E_2)/P'^2$ *and of* $100E_2/E_1$

($\tau = 2\pi$ seconds)

	Material	Thickness of Layer (km.)					
		0·0	0·2	0·4	0·6	0·8	1·0
$(E_1+E_2)/P'^2$	Limestone on granite	10·30	9·21	8·55	7·89	7·45	7·01
	Sandstone on granite	10·30	9·10	8·35	7·72	7·18	6·69
	Clay on granite	10·30	8·87	7·59	6·77	5·99	5·39
$100E_2/E_1$	Granite on granite	0·0	3·0	6·0	8·7	11·4	14·0
	Limestone on granite	0·0	2·9	5·7	8·9	12·1	16·0
	Sandstone on granite	0·0	2·8	5·6	9·1	12·9	17·2
	Clay on granite	0·0	2·5	5·4	9·3	14·0	20·6

For the thinner layers of limestone, sandstone or clay a smaller fraction of the total energy is carried in the layer than is carried in the same thickness with Rayleigh waves in granite; for the greater thicknesses this effect is reversed. In each case the departure of the distribution with a superficial layer from that for Rayleigh waves in granite depends upon the difference between the properties of the layer and of the subjacent granite, and upon the thickness of the layer.

The amplitudes at the surface with a superficial layer can be compared with those for Rayleigh waves in granite, if we assume that the total energy per wave-length does not change as the waves pass from one material to another. This appears to be the only reasonable hypothesis open to us, but it neglects the losses of energy due to reflection when the medium changes. In comparing the amplitudes for the different layers the values of E/P'^2 (Table IV) are used for elimination of P' from the equations of (4), thereby expressing the displacements in terms of the total energy. The ratios of the amplitudes of the horizontal and vertical displacements at the surface with the layers to those for Rayleigh waves in granite appear in Table V and are plotted in fig. 5.

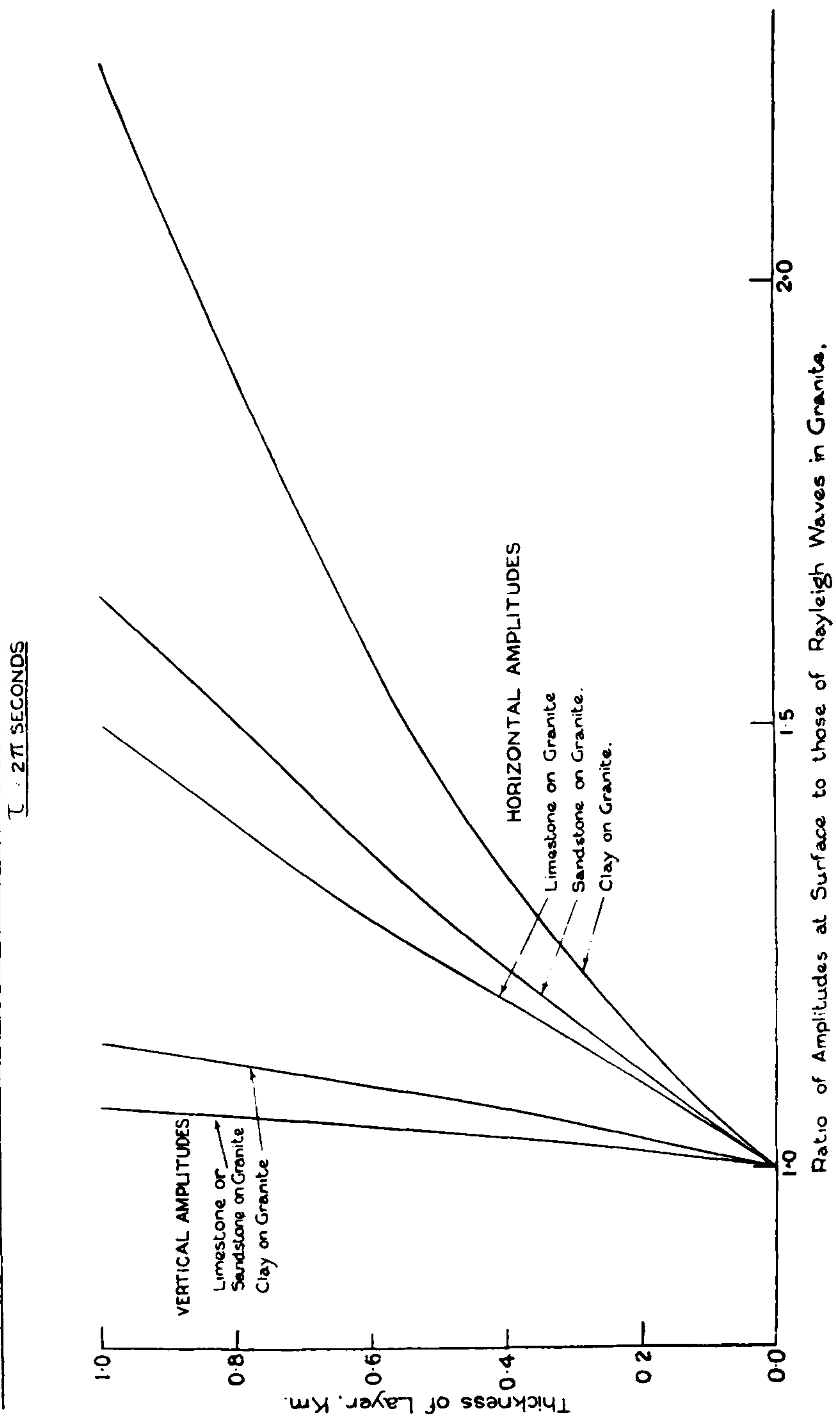


Fig 5.

RELATIVE SURFACE AMPLITUDES WITH SUPERFICIAL LAYERS FOR CONSTANT TOTAL ENERGY PER WAVELENGTH

TABLE V

Relative Surface Amplitudes for Constant Total Energy per Wave-length

	Material in Layer	Thickness of Layer (km.)				
		0·2	0·4	0·6	0·8	1·0
Ratio of horizontal amplitude at surface to that for Rayleigh waves in granite.	Limestone	1·10	1·19	1·28	1·39	1·50
	Sandstone	1·11	1·23	1·35	1·51	1·64
	Clay	1·14	1·33	1·57	1·90	2·25
Ratio of vertical amplitude at surface to that for Rayleigh waves in granite.	Limestone or sandstone	1·02	1·03	1·05	1·06	1·07
	Clay	1·03	1·07	1·09	1·12	1·14

Thus for the same total energy there would be larger movements at the surface with either of the layers than there are for Rayleigh waves in granite. The effect of the layer upon the horizontal amplitude is much greater than that upon the vertical amplitude. For the thicknesses examined the ratios for clay are both greater than those for sandstone or limestone. The horizontal ratios for sandstone are appreciably greater than those for limestone, but in the case of the vertical ratios the values for these materials are the same.

TABLE VI

Relative Amplitudes with Layers 1 km. in Thickness for Constant Total Energy per Wave-length

($\tau = 2\pi$ seconds)

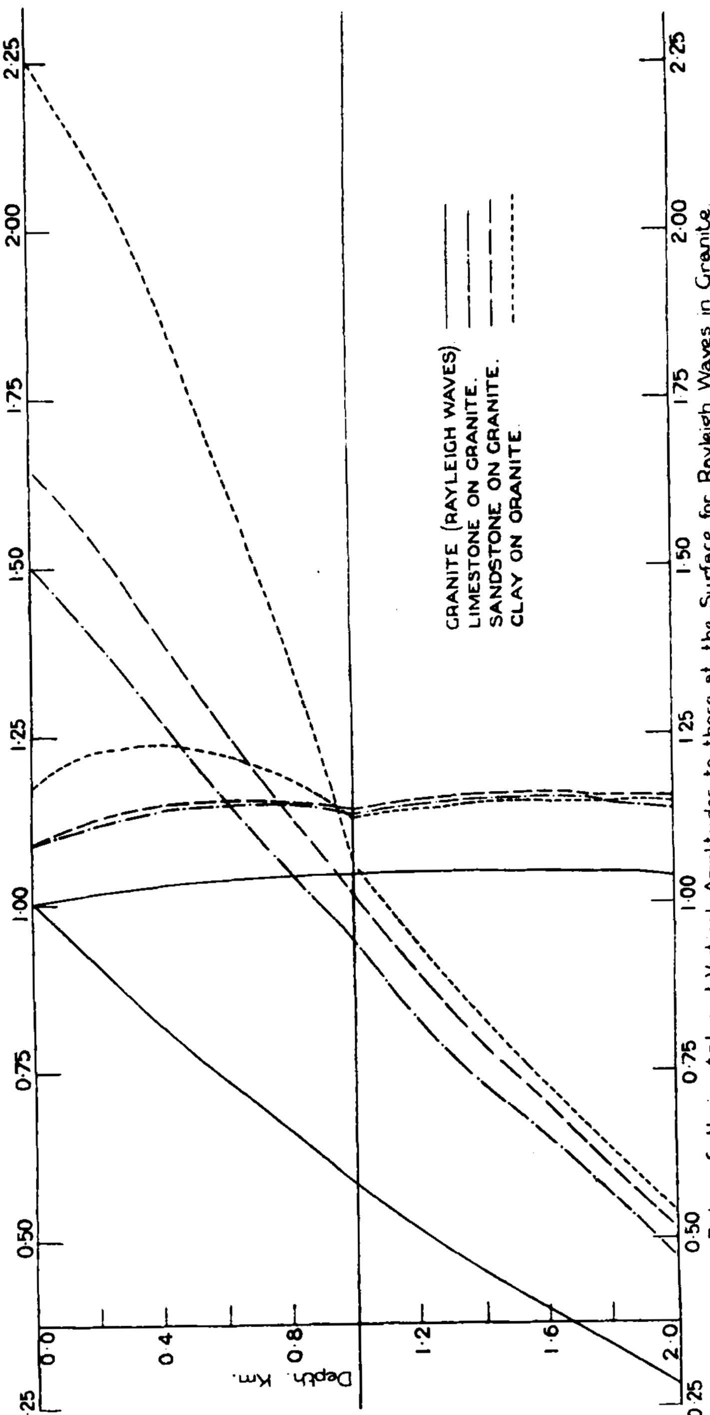
Depth km.	Ratio of Horizontal Amplitudes $\frac{\bar{u}(z)}{\bar{u}(0)\text{ Granite}}$			Ratio of Vertical Amplitudes $\frac{\bar{w}(z)}{\bar{w}(0)\text{ Granite}}$		
	Limestone	Sandstone	Clay	Limestone	Sandstone	Clay
0·0	1·50	1·64	2·25	1·07	1·07	1·14
0·2	1·39	1·54	2·10	1·09	1·10	1·18
0·4	1·27	1·40	1·89	1·11	1·12	1·19
0·6	1·16	1·26	1·63	1·11	1·12	1·17
0·8	1·05	1·14	1·37	1·12	1·12	1·15
1·0	0·94	1·01	1·06	1·11	1·11	1·10
1·2	0·82	0·89	0·94	1·11	1·12	1·11
1·4	0·72	0·79	0·83	1·12	1·13	1·12
1·6	0·63	0·69	0·73	1·13	1·13	1·12
1·8	0·55	0·60	0·63	1·12	1·13	1·12
2·0	0·47	0·52	0·54	1·11	1·13	1·12

Table VI and fig. 6 show the variations in amplitude with depth in and

Fig. 6.

VARIATION OF AMPLITUDE WITH DEPTH FOR LAYERS 1 Km IN THICKNESS CONSTANT TOTAL ENERGY PER WAVELENGTH.

$\tau = 2\pi$ SECONDS.



below layers 1 km. thick for waves of period 2π seconds, assuming that the total energy per wave-length is constant. The amplitudes throughout the layer and in the top kilometre of the granite are considerably greater than those of Rayleigh waves in granite. The sequence of the curves for horizontal movements is that of diminishing density and elastic constants in the layer. The sequence for vertical movements is less regular owing to the location in the layer of their maximum amplitudes; the depths at which these maxima occur are in agreement with the depths for Rayleigh waves in the material of the layer.

6. *Microseismic Disturbance and Geological Formation.*—The theoretical investigations of the previous sections have shown that—

(*a*) The recording of microseisms by a seismograph mounted upon a suitable pillar is not affected by the nature of the subsoil.

(*b*) The microseismic movements depend upon the thickness and nature of the formations on which the observatory is located.

Result (*b*) has been obtained on the assumptions that the layers are uniform over wide areas and that the energy of the oscillations remains constant. The result gives a qualitative explanation of the phenomena noted by Kituchi, Omori and Rudolph, but close agreement between the theoretical and observed distribution of microseisms cannot be expected owing to the numerous changes in the structure of the earth's crust and irregularities in the stratification. Ignorance of the composition and properties of the material at depths below about half a kilometre, and of the geographical variation in microseisms, are further difficulties.

Data for a number of European observatories, used for further test of the hypothesis, are given in Table VII. The amplitudes are mean values for the month of January, but do not all refer to the same year. Taking the horizontal amplitude as the amplitude of the larger horizontal component, the ratio A_H/A_Z is 0·9 at Kew, 1·2 at Göttingen, 2·7 at de Bilt and 4·1 at Hamburg. There has recently been an opportunity to examine at Kew the records of various other stations for 1931 February 19–20; the following ratios, A_H/A_Z, have been obtained: Sverdlovsk (Archæan) 0·6, Abisko (Lower Silurian) 0·7 and Strasbourg (Recent) 2·2.

At Sverdlovsk and Abisko the ratio A_H/A_Z is the theoretical value for Rayleigh waves. The data for these stations provide some justification for our neglect of Love waves, since in movements consisting of Rayleigh waves and Love waves the ratio would be larger. The values for Kew and Göttingen would correspond with layers of thickness about 1 km. superposed upon granite; those for Strasbourg, de Bilt and Hamburg would require much greater thicknesses.

The observations of Table VII are arranged in geological sequence. More detailed information is available concerning the structure below the British observatories:

Dyce.—Thin layer of gravel on granite.

Edinburgh.—Augite and hypersthene andesite of the Lower Old Red Sandstone. Carboniferous formations are very close.

Eskdalemuir.—Boulder clay on Silurian rock traversed by dykes.

TABLE VII

Mean Amplitudes of Microseisms for Month of January

Geological Formation	Observatory	Type of Seismograph	Years	Mean Amplitude		
				N–S	E–W	Z
				μ	μ	μ
Granite	Dyce (1)	Milne-Shaw	1930	2·9	...	...
	Pulkovo (2)	Galitzin	1912–14	1·2	...	...
	Upsala (3)	Wiechert	1928	0·8	...	...
Silurian	Eskdalemuir (4)	Galitzin	1911–24	2·5	...	...
Devonian	Edinburgh (1) *	Milne-Shaw	1930	...	5·5	...
	Graz (2)	Wiechert	1911, 12, 13, 15	0·6	...	...
Carboniferous	Durham (1)	Milne-Shaw	1930	3·4	...	...
	Stonyhurst (1)	,,	1930	...	3·7	...
Triassic	Bidston (1)	,,	1930	4·6	...	...
Jurassic	Göttingen (2)	Wiechert	1906–10	1·1	0·9	0·9
	Oxford (1)	Milne-Shaw	1930	...	2·9	...
Recent	de Bilt (5)	Galitzin	1927	5·2	6·5	2·4
	Hamburg (6)	Wiechert	1920–23	6·5	5·5	1·6
	Kew (1)	Galitzin	1930	2·3	2·5	2·7
	Strasbourg (7)	,,	1930	6·9	3·9	...

* Devonian lava ; carboniferous formations adjacent.

The figure following the name of each station indicates the source from which the data have been obtained, as follows :—

(1) A. W. Lee, *M.N.R.A.S., Geophys. Suppl.*, 3, 105, 1932.
(2) B. Gutenberg, *Die seismische Bodenunruhe.* Berlin, 1924.
(3) E. Lindberg, *Observations Séismographiques faites à l'Observatoire d'Upsala*, 1928–29.
(4) F. J. W. Whipple and F. J. Scrase, *M.N.R.A.S., Geophys. Suppl.*, 2, 2, 88, 1928.
(5) *Seismische Registrierungen in de Bilt*, 1927.
(6) H. Mendel, *Die seismische Bodenunruhe in Hamburg.* Hamburg, 1929.
(7) Communicated by M. J. Lacoste. Kew File 52m/31.

Durham.—Sand with bands of clay and gravel on sandstones.

Stonyhurst.—Millstone grit overlying limestones and shales.

Bidston.—Sandstone (Kuyper). Borings in the vicinity show marls and sandstones (to 220 m.) on Bunter beds, the latter being over 280 m. in thickness.*

Oxford.—Gravel on Oxford Clay. A boring at the City Brewery entered the Great Oolite Series at 80 m. and Lower Lias at 120 m.†

* "The Geology of Liverpool," *Mem. Geol. Surv. Eng.*, pp. 148–150. London, 1923.

† "The Geology of the Country around Oxford," *Mem. Geol. Surv. Eng.*, pp. 174–176. London, 1926.

Kew.—River gravel and London Clay on chalk. A boring in Richmond (about 1½ km. from the observatory) has been carried down to the Old Red Sandstone at a depth of 440 m.*

The horizontal displacements at the top of the granite at Dyce and Eskdalemuir, where the superficial strata are thin, would be equal to the amplitudes at the surface (2·9 μ at Dyce, 2·5 μ at Eskdalemuir). At Kew the ratio A_H/A_Z corresponds with our computed value for 1 km. of limestone on granite; actually the observatory is separated from the granite by 440 m. of clay and chalk and an unknown thickness of sedimentary rock. Thus the equivalent layer of 1 km. limestone on granite seems reasonable; with this layer a displacement of 1·8 μ at the top of the granite would give rise to the observed amplitude at the surface.

Around Durham, Stonyhurst, Bidston and Oxford the top strata are known, but these are separated from the granite by other material about which we have no information. The observatory at Stonyhurst is only about 60 km. north-east of that at Bidston, but the microseisms are considerably larger at Bidston than at Stonyhurst. Below the Triassic formations at Bidston, presumably, are the carboniferous series which outcrop towards Stonyhurst and also to the south. In the absence of fuller details of geological structure, and with no vertical seismograph at these observatories, the motion at the top of the granite cannot be obtained directly. The difference between the microseisms is about what would be expected if the amplitude at the top of the granite were 2·2 μ and the equivalent layers at Bidston about half a kilometre thicker than at Stonyhurst.

An explanation of the large amplitudes recorded at Edinburgh is probably to be found in the geological peculiarities of that district. The observatory is located on an intrusion of Devonian lava amid the large coalfields of Central Scotland, and the microseismic disturbance would therefore be expected to be closer to that appropriate to carboniferous formations than to that for Devonian.

As a working hypothesis in dealing with microseismic amplitudes in Britain during 1930 January we suggest that (*a*) the amplitudes at the top of the granite fall off from about 3 μ in the north of Scotland to 1·8 μ in the south of England; (*b*) the surface amplitude in a given locality depends upon the surrounding strata over the granite. Apart from the comparisons of microseisms during 1930 January there are very few data to test this hypothesis, since systematic measurements have been published only for Eskdalemuir and for Kew. At Eskdalemuir the mean amplitude from 1911 to 1924 January was 2·3 μ and the mean period 6·1 seconds; the corresponding values at Kew from 1926 to 1930 were 2·5 μ and 6·5 seconds. Thus the mean amplitude at Kew was slightly less than that at Eskdalemuir and the mean period was appreciably longer. Comparison of the mean periods suggests that in the aggregate the region in which the microseisms were generated was nearer to Eskdalemuir than to Kew, and that owing to dispersion the period lengthens as the waves travel to greater distances. The data would therefore be in agreement with

* *Journ. Geol. Soc.*, **40**, London, 1884; *ibid.*, **41**, 523, 1885.

our hypothesis that the disturbance in the granite diminishes from north to south.

7. *Summary.*—The suggestion that the amplitude of microseismic disturbance depends upon the geological structure leads to investigation of the effects of the underlying formations upon the stability of a seismograph pillar and upon the propagation of Rayleigh waves through the granitic continental layer.

Analysis of the motion of a seismograph pillar, and measurements of the earth resistance at Durham and Kew, show that the tilting of the pillars due to microseismic oscillations is negligible; consequently the accuracy with which these oscillations are recorded cannot depend upon the subsoil.

Love's theory of propagation of waves analogous to Rayleigh waves in a medium covered with a superficial layer is extended for compressible material, and applied for layers of limestone, sandstone and clay upon granite. The ratio of horizontal to vertical movements at the surface depends upon the material and thickness of the layer. Assuming the same total energy per wave-length, the surface movements with the layers are larger than those with Rayleigh waves in granite; the effect of layers up to 1 km. in thickness upon the horizontal amplitude is greater than that upon the vertical amplitude.

Taking the horizontal amplitude of the microseisms as the larger of the north and east components, the ratio of horizontal to vertical motion is obtained for a number of European observatories; at stations on the earlier geological formations the ratio agrees with the theoretical value for Rayleigh waves. The variations from place to place in microseismic amplitude are apparently due to geographical as well as geological causes.

MICROSEISMIC DISTURBANCE IN GREAT BRITAIN DURING 1930 JANUARY: A COMPARISON OF THE RECORDS OF SEVEN OBSERVATORIES.

A. W. Lee, M.Sc., D.I.C.

(Communicated by Dr. F. J. W. Whipple)

(Received 1932 March 23)

1. *Introduction.*—Values of the amplitude and period of the microseisms recorded on the north-south component Galitzin seismograph at Eskdalemuir from 1911 to 1925, and at Kew since 1926, have been included in the annual publications of these observatories.* In a recent discussion of this material it was noted that although the amplitudes of the north-south component microseisms at Eskdalemuir and at Kew were nearly

* *British Meteorological and Magnetic Year Book*, 1911–21; *Observatories Year Book*, 1922–30.