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THE EFFECT OF MAGNETIC FIELD DEPENDENT VISCOSITY ON FERROMAGNETIC CONVECTION IN A ROTATING SPARSELY DISTRIBUTED POROUS MEDIUM - REVISITED

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The effect of magnetic field dependent (MFD) viscosity on the thermal convection in a ferrofluid layer saturating a sparsely distributed porous medium has been investigated by using the Darcy- Brinkman model in the simultaneous presence of a uniform vertical magnetic field and a uniform vertical rotation. A correction is applied to the study of Vaidyanathan *et al*. [11] which is very important in order to predict the correct behavior of MFD viscosity. A linear stability analysis has been carried out for stationary modes and oscillatory modes separately. The critical wave number and critical Rayleigh number for the onset of instability, for the case of free boundaries, are determined numerically for sufficiently large values of the magnetic parameter M_1. Numerical results are obtained and are illustrated graphically. It is shown that magnetic field dependent viscosity has a destabilizing effect on the system for the case of stationary mode and a stabilizing effect for the case of oscillatory mode, whereas magnetization has a destabilizing effect.

Key words: ferrofluid, convection, rotation, magnetic field dependent viscosity, porous medium.

1. Introduction

Synthetic magnetic fluids, also known as ferrofluids, are colloidal suspensions of solid single-domain ferromagnetic nano-particles, with typical dimensions of *10 nm*, dispersed in an organic carrier (e.g. kerosene or ester) or water. In recent years, the studies on ferrofluids have attracted attention due to their

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manifold applications in the areas such as acoustics, vacuum technology, lubrication, instrumentation, metals recovery, vibration damping, etc. The research studies have led to many commercial uses of ferrofluids which include medicine, chemical reactor, novel zero-leakage rotary shaft seals used in computer disk drives, contrast enhancement of magnetic resonance imaging (MRI), high speed silent printers, pressure seals of compressors and blowers, cooling of loud speakers (Rosensweig [1], Odenbach [2]).

Ferrohydrodynamics, the study of the magnetic properties of colloidal suspensions has drawn considerable attention since the 1930s (Elmore [3]), but the research on ferroconvection intensified noticeably, starting from the fundamental paper of Finlayson [4]. Currently, a significant body of literature exists devoted to ferroconvection. For a broad overview of the subject one may be referred to Shliomis [5], Odenbach [6], Suslov [7], Rahman and Suslov [8] and Sekar and Murugan [9].

The most specific property of ferrofluids is the possibility to exert a significant influence on their flow and physical properties by means of moderate magnetic fields (Odenbach [2]). The effect on the fluid's viscous behavior due to the presence of an external magnetic field seems to be most prominent and is one of the most challenging topics of magnetic fluid research. Several research papers have been published by eminent researchers. The effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments has been investigated by Shliomis [10]. Vaidyanathan *et al.* [11] studied the influence of MFD viscosity on ferrofluid-inducing convection in a sparsely distributed porous medium heated from below for stationary and oscillatory modes using linear stability analysis. Ramanathan and Suresh [12] investigated the effect of magnetic field on the viscosity of ferroconvection in an anisotropic porous medium using Darcy model. Prakash and Gupta [13] derived upper bounds for the complex growth rate of oscillatory motions in ferromagnetic convection with MFD viscosity in a rotating fluid layer. Prakash [14] also derived a sufficient condition for the validity of the principle of the exchange of stabilities for ferromagnetic convection with MFD viscosity in a rotating porous medium.

It is worth mentioning here that in the above cited papers on MFD viscosity, these researchers have carried out their analysis by describing MFD viscosity in the form $\eta = \eta_1(1 + \delta.\boldsymbol{B})$, where η_1 is the fluid viscosity in the absence of magnetic field $\boldsymbol{B}$ and δ is the variation coefficient of viscosity. They resolved η into components η_x, η_y and η_z, which is technically incorrect, since η, being a scalar quantity, cannot be decomposed in such a manner. Undoubtedly, they studied a very important problem of ferrohydrodynamics, but their results cannot be relied upon due to the wrong assumption. Recently, Prakash and Bala [15] and Prakash *et al.* [16]-[18] have rectified the above problem for some ferromagnetic convection configurations with MFD viscosity. In the present communication, the emphasis has, particularly, been put on the above cited paper by Vaidyanathan *et al.* [11] on ferromagnetic convection in a rotating sparsely distributed porous medium with MFD viscosity. Keeping in view the above fact, the basic equations have been reformulated accordingly and then mathematical and numerical analysis has been carried out to remedy the weaknesses in the existing results and to give a correct interpretation of the problem. It is also important to point out here that the role of viscosity for stationary convection is observed to destabilize the system which is in agreement with the result obtained by Chandrasekhar [19] for the case of ordinary fluid.

2. Mathematical formulation

Consider a ferromagnetic Boussinesq fluid layer of infinite horizontal extension and finite vertical thickness d saturating a rotating (with uniform angular velocity Ω about the vertical) sparsely distributed porous medium heated from below which is kept under the action of a uniform vertical magnetic field $\boldsymbol{H}$. The flow in the porous medium is described by the Darcy- Brinkman's law.

The fluid is assumed to be incompressible having a variable viscosity, given by $\eta = \eta_1\left(1 + \delta.\boldsymbol{B}\right)$, where η_1 is the viscosity of the fluid when there is no magnetic field applied, η is the magnetic field dependent viscosity and $\boldsymbol{B}$ is the magnetic induction. The variation coefficient of viscosity δ has been taken to be isotropic, i.e. $\delta_1 = \delta_2 = \delta_3 = \delta$. The effect of shear dependence on viscosity is not considered, since it has a negligible effect on a mono dispersive system of large rotation and high field. As a first approximation for small field variation, linear variation of magneto viscosity has been used (Vaidyanathan *et al*. [11]).

The basic governing equations for the above model are given by Vaidyanathan *et al*. [11]

$$\nabla.\boldsymbol{q} = 0, \tag{2.1}$$

$$\rho_0\left[\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{q}.\nabla\boldsymbol{q}\right] = -\nabla\overline{p} + \rho\boldsymbol{g} - \frac{\eta}{k_0}\boldsymbol{q} + \eta\nabla^2\boldsymbol{q} + \nabla.(\boldsymbol{HB}) + 2\rho_0(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\rho_0}{2}\nabla\left(|\boldsymbol{\Omega}\times\boldsymbol{r}|^2\right), \tag{2.2}$$

$$\left[\rho_0 C_{V,H} - \mu_0\boldsymbol{H}.\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + \mu_0 T\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}.\frac{d\boldsymbol{H}}{dt} = K_1\nabla^2 T + \Phi \tag{2.3}$$

where $\boldsymbol{q}$, $p = \overline{p} - \frac{\rho_0}{2}\nabla\left(|\boldsymbol{\Omega}\times\boldsymbol{r}|^2\right)$, $\boldsymbol{H}, \eta, \boldsymbol{g} = (0,0,-g)$, $\boldsymbol{\Omega} = (0,0,\Omega)$ and k_0 denote, respectively, the velocity, pressure, magnetic field, variable viscosity, acceleration due to gravity, the angular velocity and permeability of the porous medium. $C_{V,H}$ is the heat capacity at constant volume and magnetic field, μ_0 is the magnetic permeability, T is the temperature, $\boldsymbol{M}$ is the magnetization, K_1 is the thermal conductivity and Φ is the viscous dissipation containing second order terms in velocity.

The density equation of state is

$$\rho = \rho_0\left[1 + \alpha(T_0 - T)\right] \tag{2.4}$$

where α is a coefficient of volume expansion and ρ_0 is the density at some properly chosen mean temperature T_0.

For a non-conducting fluid with no displacement current, the Maxwell equations are given by

$$\nabla.\boldsymbol{B} = 0, \qquad \nabla\times\boldsymbol{H} = 0, \tag{2.5a,b}$$

where the magnetic induction is given by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}). \tag{2.6}$$

Combining Eqs (2.5a) and (2.6), we get

$$\nabla.(\boldsymbol{H} + \boldsymbol{M}) = 0. \tag{2.7}$$

We assume that the magnetization is aligned with the magnetic field, but allows a dependence on the magnitude of the magnetic field as well as the temperature as

$$\boldsymbol{M} = \left(\frac{\boldsymbol{H}}{H}\right)M(H,T). \tag{2.8}$$

The linearized magnetic equation of state is

$$M = M_0 + \chi(H - H_0) - K_2(T - T_0), \tag{2.9}$$

where M_0 is the magnetization when the magnetic field is H_0 and temperature T_0, $\chi = \left(\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{H}}\right)_{H_0,T_0}$ is a magnetic susceptibility, $K_2 = -\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{H_0,T_0}$ is the pyromagnetic coefficient.

The basic state is assumed to be a quiescent state and is given by

$$\begin{gathered} \boldsymbol{q} = \boldsymbol{q}_b = 0, \qquad \rho = \rho_b(z), \qquad p = p_b(z), \qquad T = T_b(z) = -\beta z + T_0, \\ \beta = \frac{T_1 - T_0}{d}, \qquad \boldsymbol{H}_b = \left(H_0 - \frac{K_2 \beta z}{1+\chi}\right)\hat{k}, \qquad \boldsymbol{M}_b = \left(M_0 + \frac{K_2 \beta z}{1+\chi}\right)\hat{k}, \qquad H_0 + M_0 = H_0^{ext}. \end{gathered} \tag{2.10}$$

Only the spatially varying parts of H_0 and M_0 contribute to the analysis, so that the direction of the external magnetic field is unimportant and the convection is the same whether the external magnetic field is parallel or antiparallel to the gravitational force.

Now following Finlayson [4] and Prakash *et al.* [18], and using the linear stability theory we obtain the following linear non-dimensional governing equations

$$\left(D^2 - a^2\right)\left\{(1+\delta M_3)\left(D^2 - a^2 - \frac{1}{k_o}\right) - \omega\right\} w = aR^{1/2}\left\{(1+M_1)\theta - M_1 D\phi\right\} + \mathrm{Ta}^{1/2} D\zeta, \tag{2.11}$$

$$\left(D^2 - a^2 - \Pr\omega\right)\theta + \Pr M_2 \omega D\phi = -(1 - M_2) aR^{1/2} w, \tag{2.12}$$

$$\left\{(1+\delta M_3)\left(D^2 - a^2 - \frac{1}{k_o}\right) - \omega\right\}\zeta = -\mathrm{Ta}^{1/2} Dw, \tag{2.13}$$

$$\left(D^2 - a^2 M_3\right)\phi = D\theta. \tag{2.14}$$

Since M_2 is of very small order (Finlayson [4]), it is neglected in the subsequent analysis and thus Eq.(2.12) takes the form

$$\left(D^2 - a^2 - \Pr\omega\right)\theta = -aR^{1/2} w. \tag{2.15}$$

The constant temperature boundaries are considered to be free. Hence the boundary conditions are

$$w = 0 = \theta = D^2 w = D\zeta = D\phi \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1, \ \text{(Both the boundaries are free)} \tag{2.16}$$

where z is the real independent variable such that $0 \le z \le 1$, D is differentiation with respect to z, a^2 is square of the wave number, $\Pr > 0$ is the Prandtl number, ω is the complex growth rate, $R > 0$ is the Rayleigh number, $\mathrm{Ta} > 0$ is the Taylor number, $M_1 > 0$ is the magnetic number which defines the ratio of magnetic forces due to temperature fluctuation to buoyant forces, $M_2 > 0$ is a non-dimensional parameter which defines the ratio of thermal flux due to magnetization to magnetic flux, $M_3 > 0$ is the measure of the nonlinearity of magnetization, $\omega = \omega_r + i\omega_i$ is a complex constant, in general, such that ω_r and ω_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi(z) = \phi_r(z) + i\phi_i(z)$ and $\zeta(z) = \zeta_r(z) + i\zeta_i(z)$ are complex valued functions of the real variable z such that $w_r(z)$, $w_i(z)$, $\theta_r(z)$,$\theta_i(z)$, $\phi_r(z)$, $\phi_i(z)$, $\zeta_r(z)$ and $\zeta_i(z)$ are real valued functions of the real variable z.

It may further be noted that Eqs (2.11) and (2.13) -(2.16) describe an eigenvalue problem for ω and govern ferromagnetic convection, with MFD viscosity, in a rotating sparsely distributed porous medium heated from below.

3. Mathematical analysis

Following the analysis of Finlayson [4], the exact solutions satisfying the boundary conditions (2.16) are given by

$$w = A\sin\pi z, \quad \theta = B\sin\pi z, \quad \phi = -\frac{C}{\pi}\cos\pi z, \quad D\phi = C\sin\pi z, \quad \zeta = -\frac{D}{\pi}\cos\pi z, \quad D\zeta = D\sin\pi z,$$

where A, B, C and D are constants. Substitution of the above solutions in Eqs (2.11) and (2.13) - (2.15) yields a system of four linear homogeneous algebraic equations in the unknowns A, B, C and D. For the existence of non-trivial solutions of this system, the determinant of the coefficients of A, B, C and D must vanish. This determinant on simplification yields

$$U\omega^3 + V\omega^2 + W\omega + X = 0 \tag{3.1}$$

where

$$U = \left(\pi^2 + a^2 M_3\right)\Pr k^2, \tag{3.2}$$

$$V = \left(\pi^2 + a^2 M_3\right)\left[k^4 + 2k^2\left(k^2 + \frac{1}{k_0}\right)(1 + \delta M_3)\Pr\right], \tag{3.3}$$

$$\begin{aligned} W = {} & \left(\pi^2 + a^2 M_3\right)\left[2k^4\left(k^2 + \frac{1}{k_0}\right)(1 + \delta M_3) + k^2\left(k^2 + \frac{1}{k_0}\right)^2 (1 + \delta M_3)^2 \Pr + \pi^2 \mathrm{Ta}\Pr\right] + \\ & - Ra^2\left[\pi^2 + a^2 M_3 (1 + M_1)\right], \end{aligned} \tag{3.4}$$

$$\begin{aligned} X = {} & \left(\pi^2 + a^2 M_3\right)\left[k^4\left(k^2 + \frac{1}{k_0}\right)^2 (1 + \delta M_3)^2 + \pi^2 \mathrm{Ta}\, k^2\right] + \\ & - Ra^2\left[(1 + \delta M_3)\left(k^2 + \frac{1}{k_0}\right)\right]\left[\pi^2 + a^2 M_3 (1 + M_1)\right], \end{aligned} \tag{3.5}$$

and $\quad k^2 = \left(\pi^2 + a^2\right)$.

By substituting $\omega = i\omega_i$ in Eq.(3.1), we obtain marginal state of convection. Further, when $\omega_i = 0$, the condition for stationary convection is determined which in turn yields the Rayleigh number for stationary convection as

$$R = \frac{\left(\pi^2 + a^2 M_3\right)\left[k^4\left(k^2 + \frac{1}{k_0}\right)^2 \left(1 + \delta M_3\right)^2 + \pi^2 \mathrm{Ta}\, k^2\right]}{a^2\left(1 + \delta M_3\right)\left(k^2 + \frac{1}{k_0}\right)\left[\pi^2 + a^2 M_3\left(1 + M_1\right)\right]} \tag{3.6}$$

In the expression (3.6), if we put $\delta = 0, \mathrm{Ta} = 0, k_0 \to \infty$, we obtain the Rayleigh number for classical ferroconvection (Finlayson [4]). If we put $\delta = 0, \mathrm{Ta} = 0$, we obtain the Rayleigh number for ferroconvection in a sparsely distributed porous medium with constant viscosity (Vaidyanathan *et al.* [20]). If we put $\mathrm{Ta} = 0$, we obtain the Rayleigh number for ferroconvection in a sparsely distributed porous medium with MFD viscosity (Prakash *et al.* [18]). If we put $\delta = 0 = M_3, \mathrm{Ta} \neq 0,$ we obtain the Rayleigh number for classical rotatory hydrodynamic convection (Chandrasekhar [19]) and if we put $\delta = 0 = M_3, \mathrm{Ta} = 0,$ we obtain the Rayleigh number for convection in an ordinary fluid heated from below (Chandrasekhar [19]). If we put $\delta = 0, \mathrm{Ta} \neq 0, M_3 \neq 0,$ we obtain the Rayleigh number for ferroconvection in a rotating ferrofluid saturated porous layer (Shivakumara *et al.* [21])and if we put $\delta = 0, \mathrm{Ta} \neq 0, M_3 \neq 0, k_0 \to \infty,$ we obtain the Rayleigh number for ferroconvection in a rotating ferrofluid layer (Venkatasubramanian and Kaloni [22]).

When M_1 is very large, the magnetic thermal Rayleigh number $N = RM_1$ for stationary mode can be obtained from Eq.(3.6) as

$$N = \frac{\left(\pi^2 + a^2 M_3\right)\left\{k^4\left[\left(k^2 + \frac{1}{k_0}\right)\left(1 + \delta M_3\right)\right]^2 + k^2\pi^2 \mathrm{Ta}\right\}}{a^4 M_3\left(1 + \delta M_3\right)\left(k^2 + \frac{1}{k_0}\right)}. \tag{3.7}$$

To find the minimum value of N (the critical magnetic Rayleigh number) with respect to the wave number a, Eq.(3.7) is differentiated with respect to a^2 and equated to zero and the following polynomial in a is obtained.

$$\begin{aligned}
& a^2\left(1 + \delta M_3\right)\left(\pi^2 + a^2 + \frac{1}{k_0}\right)\left[M_3\left\{\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2 + \frac{1}{k_0}\right)^2\left(1 + \delta M_3\right)^2 + \right.\right. \\
& \left. + \left(\pi^2 + a^2\right)\pi^2 \mathrm{Ta}\right\} + \left(\pi^2 + a^2 M_3\right)\left\{\pi^2 \mathrm{Ta} + 2\left(\pi^2 + a^2\right)\left(\pi^2 + a^2 + \frac{1}{k_0}\right)^2\left(1 + \delta M_3\right)^2 + \right. \\
& \left.\left. + 2\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2 + \frac{1}{k_0}\right)\left(1 + \delta M_3\right)^2\right\}\right] + \\
& - \left[\left(\pi^2 + a^2 M_3\right)\left\{\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2 + \frac{1}{k_0}\right)^2\left(1 + \delta M_3\right)^2 + \left(\pi^2 + a^2\right)\pi^2 \mathrm{Ta}\right\}\right] \\
& \left[2\left\{\left(1 + \delta M_3\right)\left(\pi^2 + a^2 + \frac{1}{k_0}\right)\right\} + a^2\left(1 + \delta M_3\right)\right] = 0.
\end{aligned}$$

The above equation is solved numerically for various values of M_3 (see Tabs 1 and 2) using the software scientific workplace and the minimum value of a is obtained each time, hence the critical wave number is obtained. Using this in Eq.(3.7), we obtain the critical magnetic Rayleigh number N_c, above which the instability sets in as stationary convection.

Table 1. Marginal stability of magnetic field dependent viscosity of a ferrofluid saturating a rotating sparsely distributed porous medium heated from below for stationary mode having $M_1 = 1000$, $k_0 = 0.10$, Ta $= 10^5$ and 10^7.

Taylor no. Ta	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	Critical magnetic Rayleigh no. $N_c = (RM_1)_c$
10^5	0.01	1	8.4988	23574
		3	8.2366	21553
		5	8.1294	21058
		7	8.0498	20791
	0.05	1	8.3830	23428
		3	7.9062	21055
		5	7.6116	20262
		7	7.3675	19741
	0.09	1	8.2730	23292
		3	7.6190	20640
		5	7.1939	19658
		7	6.8518	19010
10^7	0.01	1	18.881	4.2130×10^5
		3	18.666	4.1116×10^5
		5	18.524	4.0714×10^5
		7	18.396	4.0405×10^5
	0.05	1	18.633	4.1640×10^5
		3	17.972	3.9722×10^5
		5	17.443	3.8538×10^5
		7	16.975	3.7548×10^5
	0.09	1	18.397	4.1174×10^5
		3	17.367	3.8513×10^5
		5	16.568	3.6791×10^5
		7	15.898	3.5400×10^5

Table 2. Marginal stability of magnetic field dependent viscosity in a ferrofluid saturating a rotatory sparsely distributed porous medium heated from below for stationary mode having $M_1 = 1000$, $k_0 = 0.01$, $\mathrm{Ta} = 10^5$ and 10^7.

Taylor no. Ta	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	Critical magnetic Rayleigh no. $N_c = (RM_1)_c$
10^5	0.01	1	6.1271	21831
		3	5.5647	18567
		5	5.3692	17803
		7	5.2515	17439
	0.05	1	6.0368	21866
		3	5.3155	18483
		5	4.9874	17657
		7	4.7594	17287
	0.09	1	5.9525	21915
		3	5.1105	18509
		5	4.7059	17768
		7	4.4304	17561
10^7	0.01	1	16.964	3.9077×10^5
		3	16.677	3.7934×10^5
		5	16.510	3.7490×10^5
		7	16.363	3.7152×10^5
	0.05	1	16.693	3.8556×10^5
		3	15.911	3.6439×10^5
		5	15.310	3.5148×10^5
		7	14.782	3.4069×10^5
	0.09	1	16.436	3.8062×10^5
		3	15.241	3.5138×10^5
		5	14.333	3.3258×10^5
		7	13.569	3.1735×10^5

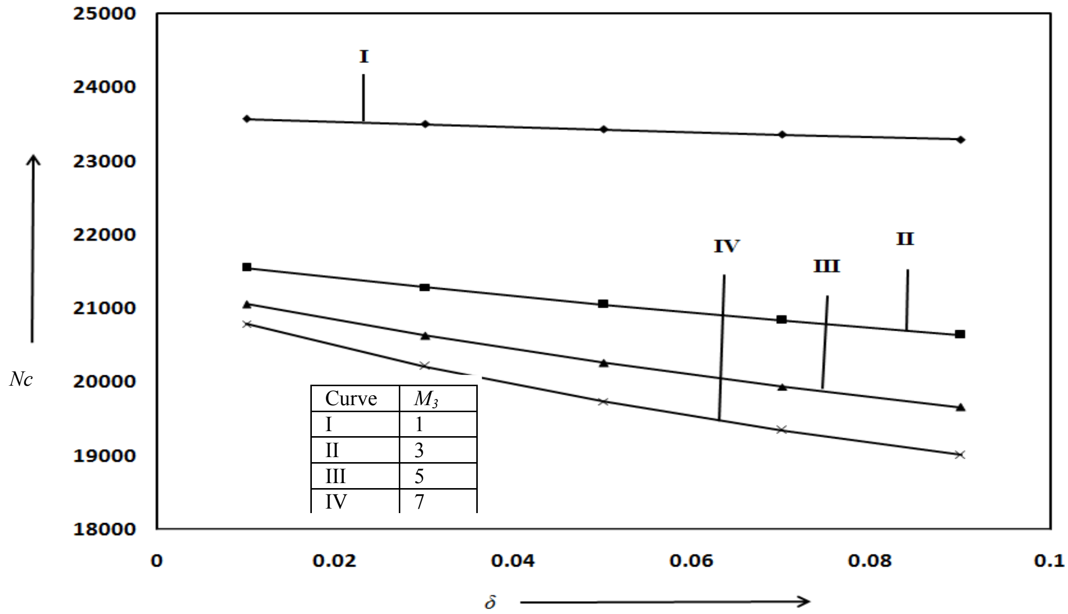

Fig.1. Variation of the magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for stationary convection for a medium of permeability $k_0 = 0.10$ and Taylor number $\mathrm{Ta} = 10^5$.

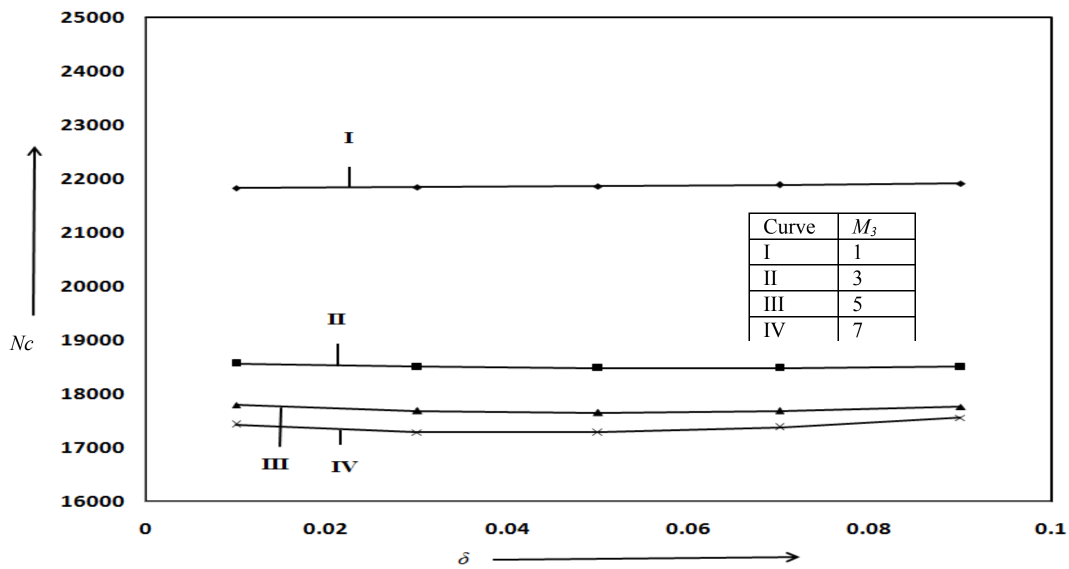

Fig.2. Variation of the magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for stationary convection for a medium of permeability $k_0 = 0.01$ and Taylor number $\mathrm{Ta} = 10^5$.

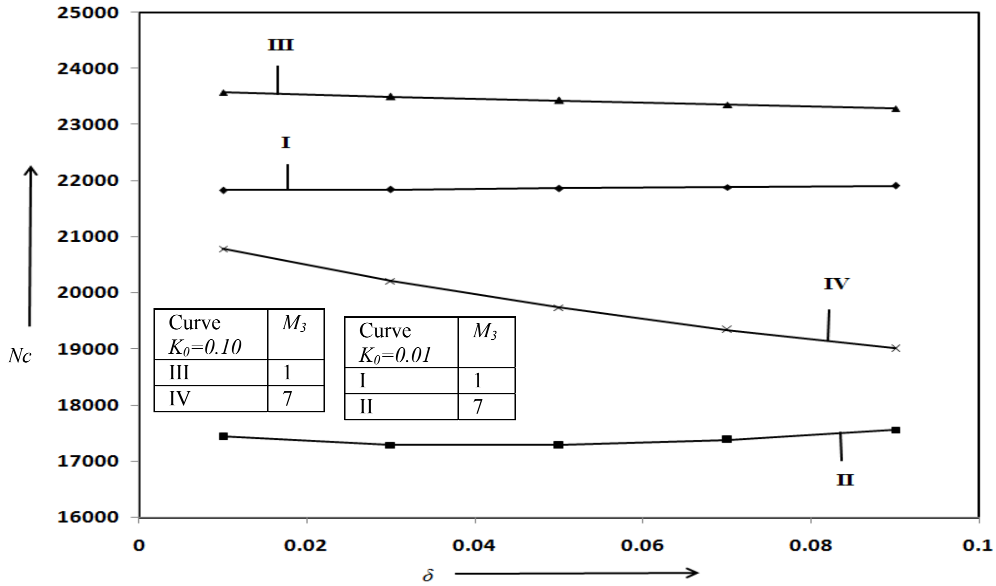

Fig.3. Variation of the thermal magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for stationary convection for a medium of permeability $k_0 = 0.01$, $k_0 = 0.10$ and Taylor number $\mathrm{Ta} = 10^5$.

When $\omega_i \neq 0$, we have a case of oscillatory convection. From Eq.(3.1), the Rayleigh number for oscillatory convection can be easily written as

$$R = \frac{\left(\pi^2 + a^2 M_3\right)\left[\begin{array}{l}\left\{2k^4\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3) + k^2\left(k^2+\dfrac{1}{k_0}\right)^2 (1+\delta M_3)^2 \,\mathrm{Pr} + \pi^2 \mathrm{Ta}\,\mathrm{Pr}\right\} \\ \left\{k^2 + 2\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3)\mathrm{Pr}\right\} - k^2\,\mathrm{Pr}\left\{k^2\left(\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3)\right)^2 + \pi^2\mathrm{Ta}\right\}\end{array}\right]}{a^2\left[k^2\left(1+\mathrm{Pr}(1+\delta M_3)\right) + \dfrac{1}{k_0}(1+\delta M_3)\mathrm{Pr}\right]\left[\pi^2 + a^2 M_3 (1+M_1)\right]}. \tag{3.8}$$

When M_1 is very large, the magnetic thermal Rayleigh number $N = RM_1$ for oscillatory mode can be obtained from Eq.(3.8) as

$$N = \frac{\left(\pi^2 + a^2 M_3\right)\left[\begin{array}{l}\left\{2k^4\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3) + k^2\left(k^2+\dfrac{1}{k_0}\right)^2 (1+\delta M_3)^2 \,\mathrm{Pr} + \pi^2 \mathrm{Ta}\,\mathrm{Pr}\right\} \\ \left\{k^2 + 2\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3)\mathrm{Pr}\right\} - k^2\,\mathrm{Pr}\left\{k^2\left(\left(k^2+\dfrac{1}{k_0}\right)(1+\delta M_3)\right)^2 + \pi^2\mathrm{Ta}\right\}\end{array}\right]}{a^4 M_3\left[k^2\left(1+\mathrm{Pr}(1+\delta M_3)\right) + \dfrac{1}{k_0}(1+\delta M_3)\mathrm{Pr}\right]}. \tag{3.9}$$

To find the minimum value of N (the critical magnetic Rayleigh number) with respect to wave number a, Eq.(3.9) is differentiated with respect to a^2 and equated to zero and the following polynomial in a is obtained.

$$\left\{a^2\left(1+P_r\left(1+\delta M_3\right)\right)\left(\pi^2+a^2\right)+\frac{1}{k_0}\left(1+\delta M_3\right)P_r\right\}\left[M_3\left\{\left\{2\left(\pi^2+a^2\right)^2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)+\right.\right.\right.$$

$$\left.+\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2P_r+\pi^2T_aP_r\right\}\left\{\left(\pi^2+a^2\right)+2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)P_r\right\}+$$

$$\left.\left.-\left(\pi^2+a^2\right)P_r\left\{\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2+\pi^2T_a\right\}\right\}+\left(\pi^2+a^2M_3\right)$$

$$\left\{\left\{4\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)+2\left(\pi^2+a^2\right)^2+\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2P_r+\right.\right.$$

$$\left.+2\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)^2P_r\right\}\left\{\left(\pi^2+a^2\right)+2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)P_r\right\}+$$

$$+\left\{2\left(\pi^2+a^2\right)^2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)+\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2P_r+\pi^2T_aP_r\right\}$$

$$\left\{1+2\left(1+\delta M_3\right)P_r\right\}-P_r\left\{2\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2+\right.$$

$$\left.\left.2\left(\pi^2+a^2\right)^2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)^2+\pi^2T_a\right\}\right\}-\left[\left(\pi^2+a^2M_3\right)\right.$$

$$\left[\left\{2\left(\pi^2+a^2\right)^2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)+\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2P_r+\pi^2T_aP_r\right\}\right.$$

$$\left.\left.\left\{\left(\pi^2+a^2\right)+2\left(\pi^2+a^2+\frac{1}{k_0}\right)\left(1+\delta M_3\right)P_r\right\}-\left(\pi^2+a^2\right)P_r\left\{\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{k_0}\right)^2\left(1+\delta M_3\right)^2+\pi^2T_a\right\}\right]\right]$$

$$\left[2\left\{\left(\pi^2+a^2\right)\left(1+P_r\left(1+\delta M_3\right)\right)+\frac{1}{k_0}\left(1+\delta M_3\right)P_r\right\}+a^2\left(1+P_r\left(1+\delta M_3\right)\right)\right]=0.$$

The above equation is solved numerically for various values of M_3 (see Tabs 3 and 4) using the software scientific workplace and the minimum value of a is obtained each time, hence the critical wave number is obtained. Using this in Eq.(3.9), we obtain the critical magnetic Rayleigh number, above which the instability sets in as oscillatory convection.

Table 3. Marginal stability of magnetic field dependent viscosity in a ferrofluid saturating a rotatory sparsely distributed porous medium heated from below for oscillatory mode having $M_1 = 1000$, $k_0 = 0.01$, $\mathrm{Ta} = 10^5$ and 10^7.

Taylor no. Ta	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	Critical magnetic Rayleigh no. $N_c = (RM_1)_c$
10^5	0.01	1	4.2881	24180
		3	3.7994	19354
		5	3.6379	18589
		7	3.5510	18469
	0.05	1	4.2880	25315
		3	3.7787	22111
		5	3.5903	23026
		7	3.4736	24669
	0.09	1	4.2872	26465
		3	3.7552	24989
		5	3.5379	27186
		7	3.3906	31567
10^7	0.01	1	8.0489	66091
		3	7.8621	61566
		5	7.8175	61556
		7	7.7965	62211
	0.05	1	8.0501	68778
		3	7.8546	68912
		5	7.7961	73583
		7	7.7574	78944
	0.09	1	8.0503	71473
		3	7.8397	76310
		5	7.7581	85741
		7	7.6913	95925

Table 4. Marginal stability of magnetic field dependent viscosity in a ferrofluid saturating a rotatory sparsely distributed porous medium heated from below for oscillatory mode having $M_1 = 1000$, $k_0 = 0.10$, Ta $= 10^5$ and 10^7.

Taylor no. Ta	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	Critical magnetic Rayleigh no. $N_c = (RM_1)_c$
10^5	0.01	1	3.9462	6239.7
		3	3.6118	4647.8
		5	3.5078	4355.0
		7	3.4573	4265.5
	0.05	1	3.9653	6500.0
		3	3.6521	5219.2
		5	3.5643	5234.2
		7	3.5265	5454.5
	0.09	1	3.9832	6762.1
		3	3.6855	5800.5
		5	3.6053	6135.9
		7	3.5702	6682.6
10^7	0.01	1	7.5480	31161
		3	7.3734	28589
		5	7.3379	28471
		7	7.3263	28716
	0.05	1	7.5675	32394
		3	7.4217	31911
		5	7.4087	33885
		7	7.4142	36226
	0.09	1	7.5855	33630
		3	7.4602	35247
		5	7.4566	39328
		7	7.4642	43780

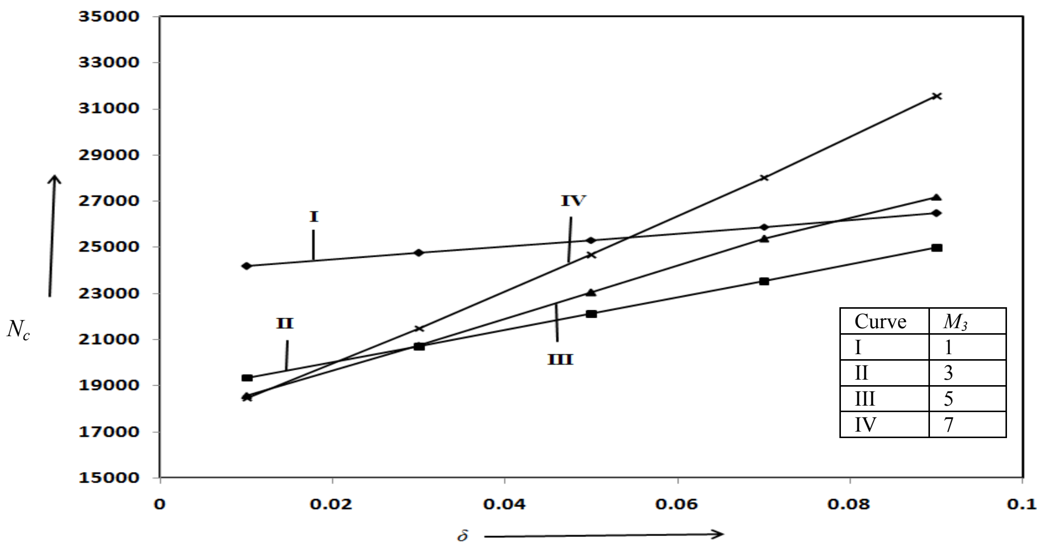

Fig.4. Variation of the thermal magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for oscillatory convection for a medium of permeability $k_0 = 0.01$ and Taylor number $\mathrm{Ta} = 10^5$.

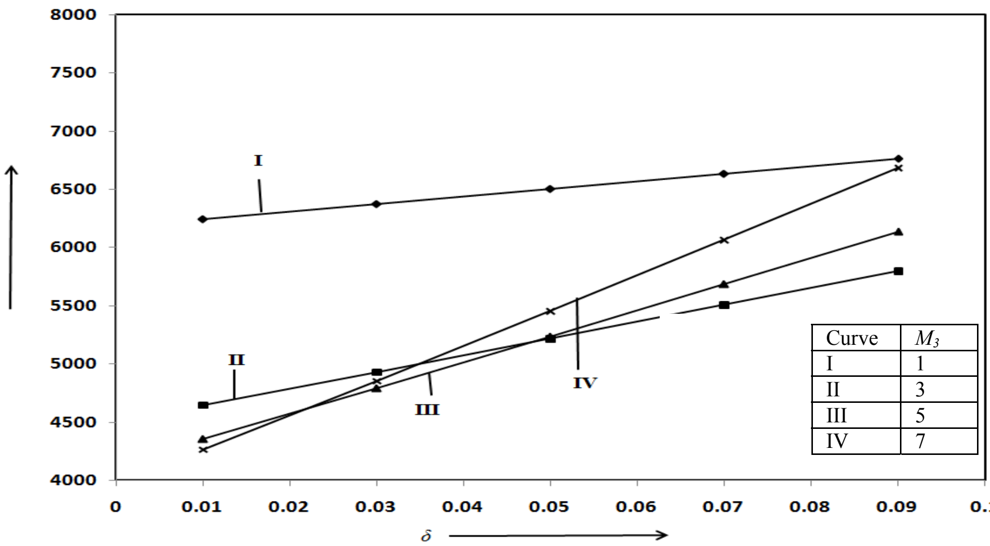

Fig.5. Variation of the thermal magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for oscillatory convection for a medium of permeability $k_0 = 0.10$ and Taylor number $\mathrm{Ta} = 10^5$.

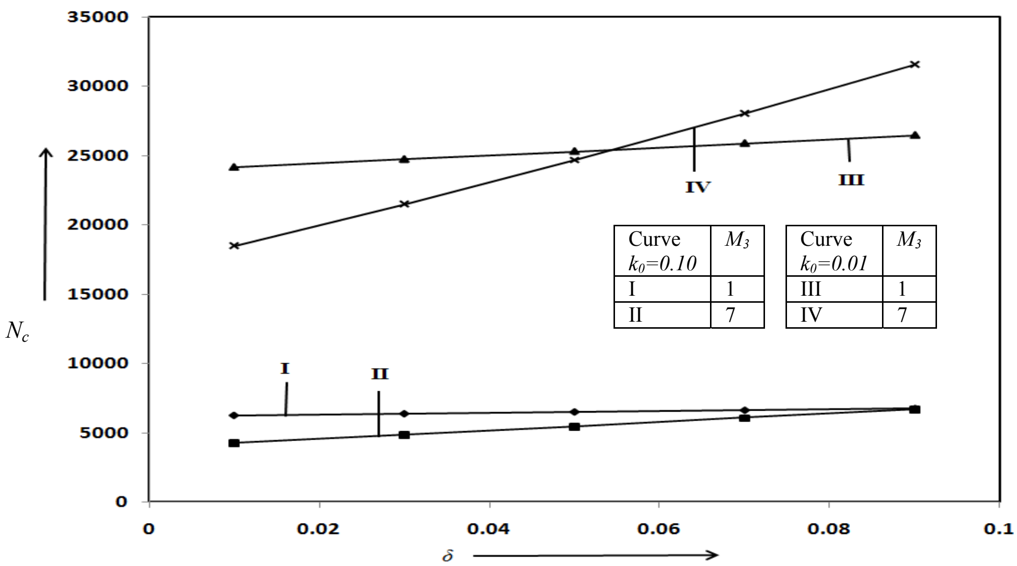

Fig.6. Variation of the thermal magnetic Rayleigh number (N_c) versus the variation coefficient of viscosity (δ) for oscillatory convection for a medium of permeability $k_0 = 0.10$, $k_0 = 0.01$ and Taylor number $\mathrm{Ta} = 10^5$.

4. Discussion and conclusion

In the present paper the influence of magnetic field dependent viscosity on the thermal convection in a rotating ferrofluid layer heated from below saturating a porous medium in the presence of a uniform vertical magnetic field has been investigated using the Darcy Brinkman model. The permeability values are used as proposed by Walker and Homsy [23]. The magnetization parameter M_1 is considered to be 1000 (Vaidyanathan *et al.* [24]). The value of M_2, being negligible (Finlayson [4]), has been taken as zero. The values of the parameter M_3 are varied from 1 to 7. The values of the coefficient of magnetic field dependent viscosity δ, has been varied from *0.01* to *0.09*.

Emphasize has been given to a paper published by Vaidyanathan *et al.* [11]. These researchers have performed their analysis by considering MFD viscosity as $\eta = \eta_1(1+\delta.\boldsymbol{B})$. But they further resolved η into components η_x, η_y and η_z along the coordinate axes which is technically wrong. This is because η, being a scalar quantity, cannot be resolved into components. Thus a correction to their analysis is very much sought after in order to give a correct interpretation of the problem. Keeping in view the above facts, the basic equations have been reformulated and then mathematical and numerical analysis has been performed. The results obtained herein have significant variations from the existing results which were otherwise obtained by using wrong assumptions.

From Tabs 1 and 2 and from Figs 1-3, it is evident that as the magnetization parameter M_3 increases, the critical value of the magnetic Rayleigh number, $N_c = (RM_1)_c$ decreases. Hence the magnetization has a destabilizing effect on the system. The physical interpretation of this may be given as follows: As the value of M_3 increases the departure of linearity in the magnetic equation of state increases resulting in an increase in the velocity of the ferrofluid in the vertical direction favoring the manifestation of instability. Thus the magnetization parameter destabilizes the system. This increase in magnetization releases extra energy, which adds up to thermal energy to destabilize the flow more quickly. A similar result is also obtained by Vaidyanathan *et al.* [11], but the difference in the values of N_c is quite significant and increases with the increase in the value of δ. It is also evident from Figs 1-3 that for stationary convection, the value of the magnetic Rayleigh number decreases as the MFD viscosity parameter increases, predicting the destabilizing behavior of the viscosity parameter δ. This unexpected result that 'the role of viscosity is inverted in the presence of rotation', has also been predicted by Chandrasekhar [19] for the case of ordinary fluid.

It is also found from Tabs 1 and 2 and Figs 1-3, that the magnetic Rayleigh number increases with an increase in the values of permeability k_0 of the porous medium. Thus permeability has a stabilizing effect on the system. Again, the difference in the existing values (Vaidyanathan *et al*. [11]) and the values obtained herein is significant.

It is interesting to note from Figs 4 and 5 that for the case of oscillatory motions the value of the magnetic Rayleigh number increases as the MFD viscosity parameter δ increases, thus resulting in the postponement of instability. Thus, MFD viscosity has a stabilizing effect on the system for the case of oscillatory convection, which is a result also obtained by Vaidyanathan *et al*. [11].

Further, we may note from Figs 4 and 5 that for the case of oscillatory convection also, M_3 prepones the onset of convection. Thus magnetization M_3 has a destabilizing effect on the system. Figure 6 predicts the destabilizing behavior of the permeability k_0 for oscillatory convection.

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Nomenclature

a^2 –.square of wave number
$\boldsymbol{B}$ – magnetic induction
D – differentiation w .r. t. z
$\boldsymbol{g}$ – acceleration due to gravity
$\boldsymbol{H}$ – magnetic field
K_1 – thermal conductivity
K_2 – pyromagnetic coefficient
k_0 – permeability of the porous medium
$\boldsymbol{M}$ – magnetization
M_1 – buoyancy magnetization
M_3 – magnetic parameter
Pr – Prandtl number
p – pressure
R – thermal Rayleigh number
T – temperature
Ta – Taylor number
T_0 – temperature at the lower boundary
T_1 – temperature at the upper boundary
$\boldsymbol{q}$ – velocity
z – vertical co-ordinate
α – coefficient of volume expansion
η – variable viscosity
μ_0 – magnetic permeability
ϕ' – perturbed magnetic potential
χ – magnetic susceptibility
ρ – density
ω – complex growth rate

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