Int. J. of Applied Mechanics and Engineering, 2020, vol.25, No.4, pp.96-115
DOI: 10.2478/ijame-2020-0052

THE EFFECT OF MODULATION ON HEAT TRANSPORT BY A WEAKLY NONLINEAR THERMAL INSTABILITY IN THE PRESENCE OF APPLIED MAGNETIC FIELD AND INTERNAL HEATING

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The present paper deals with a weakly nonlinear stability problem under an imposed time-periodic thermal modulation. The temperature has two parts: a constant part and an externally imposed time-dependent part. We focus on stationary convection using the slow time scale and quantify convective amplitude through the real Ginzburg-Landau equation (GLE). We have used the classical fourth order Runge-Kutta method to solve the real Ginzburg-Landau equation. The effect of various parameters on heat transport is discussed through GLE. It is found that heat transport analysis is controlled by suitably adjusting the frequency and amplitude of modulation. The applied magnetic field (effect of Ha) is to diminish the heat transfer in the system. Three different types of modulations thermal, gravity, and magnetic field have been compared. It is concluded that thermal modulation is more effective than gravity and magnetic modulation. The magnetic modulation stabilizes more and gravity modulation stabilizes partially than thermal modulation.

Key words: Ginzburg-Landau equation, temperature modulation, applied magnetic field, internal heating.

1. Introduction

In this paper, we study the impact of time-periodic oscillations on Rayleigh-Benard convection in the presence of an applied magnetic field by weakly nonlinear analysis. We derive the Ginzburg-Landau equation focusing on stationary finite amplitude convection. We study heat transfer through GLE and discuss the impact of thermal modulation on heat transport. An excellent review of the studies related to magneto convection is presented by Yu *et al*. [1], Thomson [2] and Chandrasekhar [3]. The effect of thermal modulation on linear instability of Rayleigh Benard convection is reported by Venezian [4]. The shift in the critical Rayleigh number has been calculated as a function of frequency modulation and wavenumber. It has

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been reported that frequency of modulation has a significant effect on instability of the layer with its proper tuning.

Among the early studies on thermal modulation, Venezian [4] and Greshuni *et al.* [5] using small amplitude approximation studied the effect of thermal modulation in a fluid layer. They showed that the system could be stabilized by three different types of modulation with periodically varying temperature of the plane. They also investigated unsteady equilibrium nature of a layer. Double diffusive convection under an applied magnetic field is reported by Rudraiah *et al*. [6]. They showed that the magnetic field acts like third diffusing component to suppress onset convection. In general, the effect of thermal modulation is of three forms:

1. In-phase modulation ($\theta=0$)
2. Out of phase modulation ($\theta=\pi$)
3. Only lower boundary modulation ($\theta=-I\infty$)

where θ is the phase angle. Most of the published studies considered only these three different types of thermal modulation on convective flows. The effect of thermal modulation on different models related to linear or nonlinear problems was well documented and reported by Bhadauria [7-10] and Bhadauria *et al.* [11-19]. In their studies, the effect of thermal modulation was investigated on different fluid models either for linear or nonlinear theory.

The study of gravity modulation on Rayleigh Benard convection was made by Gresho and Sani [20]. A linearized stability analysis was performed to show stability limits of the system under gravity modulation. The effect of gravity modulation on RBC with rigid, isothermal boundaries was investigated by Clever *et al*. [21]. The effect of resonance ranging from *100* to *3000* and Pr from *0.71* to *50* on thermal instability was presented. It was concluded that both synchronous and subharmonic modes of convection are identified. The effect of gravity modulation for oscillatory mode of convection for fluid and porous media was investigated by Bhadauria and Kiran [22, 23]. It was concluded that oscillatory modes enhance heat transfer more than stationary modes. A number of studies have been devoted to gravity modulation on different models, e.g., on chaotic convection [24,25], on throughflow [26], on rotating nanofluid convection [27], rotating oscillatory convection [28], on throughflow and double diffusive oscillatory convection [29]. The effect of gravity modulation was extensively investigated on different fluid or porous convection.

Other models of magnetic field modulation were investigated by Aniss *et al*. [30, 31]. These authors proposed theoretical and experimental investigations of RBC confined in a horizontal annular Hele–Shaw cell and subjected to radial temperature and magnetic field modulation. With their geometrical configuration, the possibility of magneto convection and its control by an external magnetic field gradient in the absence of gravity was shown. Their studies are restricted to only linear models. The effect of magnetic field modulation on a weakly nonlinear thermal instability was investigated by Bhadauria and Kiran [32] for stationary mode convection. The comparison of thermal, gravity, and magnetic field modulation was investigated. They concluded that magnetic modulation reduces heat transfer and stabilizes the system. The same problem has been extended to oscillatory mode of thermal convection by Kiran and Bhadauria [33]. It was concluded that oscillatory flows produce better heat transfer results.

In situations like radioactive decay or relative weak exothermic reactions the fluid layer offers its own internal heat generation (IHG). Due to internal heat generation a thermal gradient is formed between interior and exterior layers of the earth's crust with multi component liquids. Other important and relevant applications can be seen in geophysics, reactor safety analyses, fire and combustion studies. However, there are few studies on internal heating of the convective flow, some of them have been published by Tveitereid *et al*. [34, 35], Tasaka *et al*. [36], Takashima [37], Bhadauria *et al*. [38-40], Kiran *et al*. [41, 42, 61]. No data have been reported on thermal convection in the presence of an applied magnetic field and internal heat generation.

An unsteady flow of an incompressible fluid an infinite vertical channel in the presence of an applied magnetic field was investigated by Rao *et al*. [43]. They considered viscous dissipate heat along with the free convection currents. It is reported that variations of velocity field, temperature field and skin friction are influenced by the applied magnetic field. The study of heat transfer in the presence of magneto convection is reported by Bhadauria *et al.* [16]. It is reported that under the effect of magnetic field modulation heat transfer can be suppressed more than that of thermal and gravity modulation. Recently Keshri *et al.* [44] studied the effect of solutal and gravity modulation on thermal instability in a fluid layer under applied magnetic field. They concluded that the effect of the applied magnetic field is to suppress the mass transfer irrespective of the modulation. The effect of concentration modulation on weakly nonlinear thermal instability in a rotating porous media has been investigated by Kiran [45]. The investigation on stability analysis of RBC under an applied magnetic field and internal heat source has not been carried out yet.

To the best of the authors' knowledge, there is no nonlinear study available in the literature in which the effect of thermal modulation has been considered in a magnetic fluid layer with internal heating. This motivated us to make a nonlinear stability analysis and study the combined effect of internal heating and thermal modulation. Further, three types of different modulations, thermal, gravity and magnetic field modulations are investigated and the results compared.

2 Governing equations

We consider two infinite horizontal and parallel planes at $z = 0$, $z = d$ and between these two planes there is an electrically conducting liquid of depth 'd'. We have taken Cartesian coordinates with the z axis vertically upwards and the origin at the bottom of the layer. The layer is heated and salted from below to maintain a variable temperature across the layer.

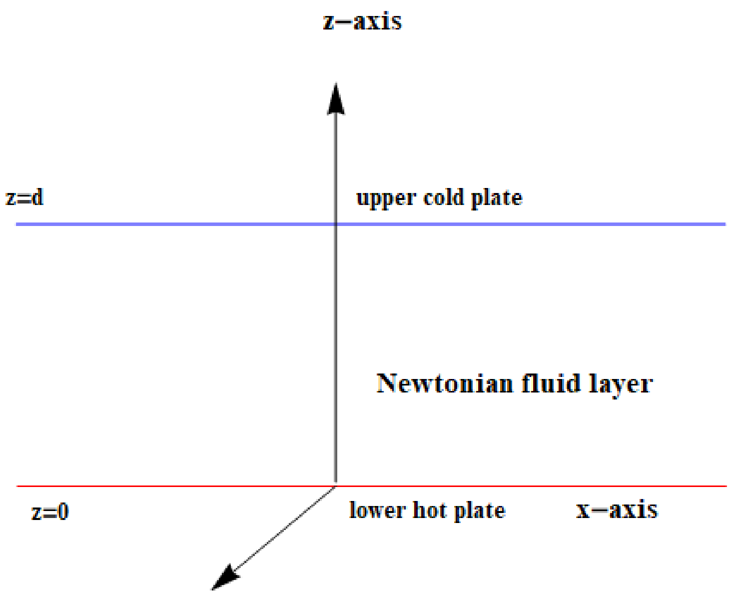

Fig.1. Physical configuration of the problem.

The surfaces are maintained at a constant gradient $\frac{\Delta T}{d}$ and a constant magnetic field $H_b\boldsymbol{K}$ is applied across the liquid region (as shown in Fig.1). Under the Boussinesq approximation, the dimensional governing equations for the study of applied magneto-convection in a fluid layer are

$$\nabla.\boldsymbol{q} = 0, \tag{2.1}$$

$$\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}.\nabla)\boldsymbol{q} = \frac{1}{\rho_0}\nabla_p + \frac{\rho}{\rho_0}\boldsymbol{g} - \frac{\mu}{\rho_0}\nabla^2\boldsymbol{q} - \sigma\mu_e^2 B_0^2\boldsymbol{q}, \tag{2.2}$$

$$\frac{\partial \boldsymbol{q}}{\partial t}+(\boldsymbol{q}.\nabla)\boldsymbol{q}=\frac{1}{\rho_0}\nabla_p+\frac{\rho}{\rho_0}\boldsymbol{g}-\frac{\mu}{\rho_0}\nabla^2\boldsymbol{q}-\sigma\mu_e^2 B_0^2\boldsymbol{q}, \tag{2.3}$$

$$\rho=\rho_0\left[1-\beta_T(T-T_0)\right] \tag{2.4}$$

where $\boldsymbol{q}$ is velocity (u, v, w), μ is the viscosity, K_T is the thermal diffusivity tensor, T is temperature, β_T is the thermal expansion coefficient, γ is the ration of heat capacity. For simplicity γ is taken to be unity in this paper, ρ is density, $\boldsymbol{g} = (0, 0, -g)$ is the gravitational acceleration, while ρ_0 is the reference density, μ_e is the magnetic permeability, B_0 is the strength of the applied magnetic field. The externally imposed thermal boundaries considered in this paper are given by Venezian [4] and Kiran *et al*. [10, 12, 18, 41, 56, 61].

$$\begin{aligned} &T=T_0+\frac{\Delta T}{2}(1+\in^2\delta_1\cos(\omega t)) \qquad \text{at} \qquad z=0, \\ &T=T_0-\frac{\Delta T}{2}(1-\in^2\delta_1\cos(\omega t+\theta)) \qquad \text{at} \qquad z=d \end{aligned} \tag{2.5}$$

where δ_1 is the small amplitude of temperature modulation, ΔT is the temperature difference across the fluid layer, ω is modulation frequency and θ is the phase difference. The basic state is assumed to be quiescent and the quantities in the state are given by

$$\vec{q}_b=0,\ \ \rho=\rho_b(z,t), \qquad T=T_b(z,t), \tag{2.6}$$

$$\frac{\partial \rho_b}{\partial z}=-\rho_b\boldsymbol{g}, \tag{2.7}$$

$$\frac{\partial T_b}{\partial t}=kT\frac{\partial^2 T_b}{\partial z^2}+Q(T_b-T_0), \tag{2.8}$$

$$\rho_b=\rho_0[1-\beta_T(T_b-T_0)]. \tag{2.9}$$

The solution of Eq.(2.8), subjected to the boundary conditions Eq.(2.5), is given by

$$T_b(z,\ t)\ =\ T_s(z)\ +\in^2\delta_1\,\mathrm{Re}\left[T_1(z,\ t)\right] \tag{2.10}$$

where $T_s(z)$ is the study temperature field and $T_1(z,t)$ is the oscillating part while Re stands for the real part. We assume finite amplitude perturbations on the basic state in the form.

$$q=q_b+\boldsymbol{q}',\ \rho=\rho_b+\rho',\ p=p_b+p',\ T=T_b+T' \tag{2.11}$$

where primes denote the quantities at the perturbations. Substituting Eq.(2.11) in Eqs (2.1)-(2.4) and using the basic state results, we obtain

$$\nabla.\boldsymbol{q}'=0, \tag{2.12}$$

$$\frac{\partial \boldsymbol{q}'}{\partial t}+(\boldsymbol{q}'.\nabla)\boldsymbol{q}=\frac{1}{\rho_0}\nabla p'-\frac{\rho'}{\rho_0}\boldsymbol{gk}+\nu\nabla^2\boldsymbol{q}'+-\sigma\mu_e^2 B_0^2\boldsymbol{q}'\frac{\partial \boldsymbol{H}'}{\partial z}, \tag{2.13}$$

$$\frac{\partial T'}{\partial t}+(\boldsymbol{q}'.\nabla)T'+\boldsymbol{w}'\frac{\partial T_b}{\partial z}k_T\nabla^2 T'+QT', \tag{2.14}$$

$$\rho'=\rho_0\beta_T T'. \tag{2.15}$$

Further, we consider only two dimensional disturbances in our study and hence the stream functions ψ are introduced as $(u,w)=\left(\frac{\partial\Psi}{\partial z},-\frac{\partial\Psi}{\partial x}\right)$. We eliminate density and pressure terms from Eqs (2.12)-(2.15), and the resulting systems can be dimensionless through the following transformations: $(x',y',z')=d(x^*,y^*,z^*)$, $\psi=k_T\psi^*$, $t=\frac{d^2}{k_T}t^*$, $\boldsymbol{q}'=\frac{k_T}{d}\boldsymbol{q}^*$, $T'=\Delta TT^*$, and $\omega=\frac{k_T}{d^2}\omega^*$. For simplicity we drop the asterisk. Then the non-dimensionalized governing system is

$$-\nabla^4\psi+\mathrm{Ha}^2\nabla^2\psi+R_{aT}\frac{\partial T}{\partial x}=-\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2\psi+\frac{1}{\Pr}\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(x,z)}, \tag{2.16}$$

$$-\frac{\partial\Psi}{\partial x}\frac{\partial T_b}{\partial}-\left(\nabla^2+R_i\right)T=-\frac{\partial T}{\partial t}+\frac{\partial(\Psi,T)}{\partial(x,z)}. \tag{2.17}$$

The non-dimensional parameters in the above equations are given in the nomenclature. Equation (2.17) shows that the basic state solution influences the stability problem through the factor $\frac{\partial T_b}{\partial z}$ which is given by

$$\frac{\partial T_b}{\partial z}=f_1(z)+\epsilon^2\,\delta_1\left[f_2(z,t)\right] \tag{2.18}$$

where

$$f_1(z)=\frac{\sqrt{R_i}}{2\sin\sqrt{R_i}}\left(\cos\sqrt{R_i}\,(1-z)+\cos\sqrt{R_i}\,(z)\right), \tag{2.19}$$

$$f_2(z)=R_e\left[f(z)e^{-i\omega t}\right], \tag{2.20}$$

$$f(z)=\left[A(m)e^{mz}+A(-m)e^{-mz}\right],\quad A(m)=\frac{m}{2}\frac{\left(e^{-i\theta}-e^{-m}\right)}{\left(e^{m}-e^{-m}\right)},\quad m=\sqrt{\lambda^2-R_i}\ \text{ and } \lambda^2=-i\omega.$$

We assume small variations of time and re-scale it as $\tau=\epsilon^2 t$ to study the stationary convection of the system Eqs (2.16)-(2.17). We use the following boundary conditions to solve the above system. The stress free and isothermal boundary conditions are given by Kiran *et al*. [10, 16, 22, 28], Bhadauria and Kiran [32], Manjula *et al*. [47], Bhadauria *et al*. [22, 32]

$$\psi=\frac{\partial^2\psi}{\partial z^2}=T\qquad\text{at}\qquad z=0,\qquad z=1. \tag{2.21}$$

3. Finite amplitude equation and heat transport for stationary instability

We now introduce the following asymptotic expansions (Malkus and Veronis [46], Manjula *et al.* [47, 58], Kiran *et al.* [48, 49, 57]) in the system Eqs (2.16)-(2.17)

$$Ra_T = R_{0c} + \in^2 R_2 + \in^4 R_4 + \ ...,$$
$$\psi = \in \psi_1 + \in^2 \psi_2 + \in^3 \psi_3 + ...,\tag{3.1}$$
$$T = \in T_1 + \in^2 T_2 + \in^3 T_3 + ...$$

where R_{0c} is the critical value of the Rayleigh number at which the onset of convection takesplace in the absence of temperature modulation. Now we solve the system for different orders of $\in$.

3.1. Lowest order system

The lowest order system case is similar to the problem of linear system. At this order we get the following relation

$$\begin{bmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\dfrac{\partial}{\partial x} \\ -\dfrac{dT_b}{dz}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{3.2}$$

The solutions of the lowest order system subjected to the boundary conditions Eq.(2.21) are

$$\psi_1 = B(\tau)\sin(k_c x)\sin(\pi z),$$
$$T_1 = \frac{4\pi^2 k_c}{\delta_R^2 (4\pi^2 - R_i)} B(\tau)\cos(k_c x)\sin(\pi z), \tag{3.3}$$

where $\delta^2 = k_c^{\ 2} + \pi^2, \quad \delta_R^2 = \delta^2 - R_i.$

The critical value of the Rayleigh number for the onset of magneto-convection in the absence of temperature modulation is

$$R_{0c} = \frac{\delta_R^2 \left(\delta^4 + Ha^2 \delta^2\right)\left(4\pi^2 - R_i\right)}{4\pi^2 k_c^{\ 2}} \tag{3.4}$$

when $R_i = 0$, $Ha = 0$ the classical results of Chandrasekhar [3] are obtained.

3.2. Second order system

The second order system is obtained based on the first order system. Because the nonlinear Jacobian term in Eq.(17) is clearly dependent on the previous solutions, thus we have.

$$\begin{bmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\dfrac{\partial}{\partial x} \\ -\dfrac{dT_b}{dz}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i) \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}, \tag{3.5}$$

$$R_{21} = 0, \tag{3.6}$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_1}{\partial x}. \tag{3.7}$$

The second order solutions subjected to the boundary conditions Eq.(2.21) is obtained as follows

$$\psi_2 = 0, \tag{3.8}$$

$$T_2 = \frac{2\pi^3 k_c^2}{\delta_R^2 \left(4\pi^2 - R_i\right)^2} B^2(\tau)\sin(2\pi z). \tag{3.9}$$

3.3. Estimation of heat transport in terms of the Nusselt number

The horizontally averaged Nusselt number $\mathrm{Nu}(\tau)$ for the stationary mode of convection is given by (Bhaduria and Kiran [39, 40], Kiran [41, 42, 45], Keshri *et al.* [44], Manjula *et al.* [47])

$$\mathrm{Nu}(\tau) = 1 + \frac{\left[\dfrac{k_c}{2\pi}\displaystyle\int_0^{\frac{2\pi}{k_c}} \left(\dfrac{\partial T_2}{\partial z}\right) dx\right]_{z=0}}{\left[\dfrac{k_c}{2\pi}\displaystyle\int_0^{\frac{2\pi}{k_c}} \left(\dfrac{\partial T_b}{\partial z}\right) dx\right]_{z=0}}, \tag{3.10}$$

$$\mathrm{Nu}(\tau) = 1 + \frac{8\pi^4 k_c^2 \sin\sqrt{R_i}}{\delta_R^2\left(4\pi^2 - R_i\right)^2 \sqrt{R_i}(\cos\sqrt{R_i} + 1} B^2(\tau). \tag{3.11}$$

Here one can notice that $f_2(z, \tau)$ is effective at second order and affects the above Nusselt number Eq.(3.11), through factor $B(\tau)$ because this amplitude is obtained from GLE.

3.4. Third order system

In this order we get the following system, where the modulation effect will take place. We restrict ourselves up to 3^{rd} order system and find the finite amplitude. Thus the third order system is given by

$$\begin{bmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\dfrac{\partial}{\partial x} \\ -\dfrac{dT_b}{dz}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \end{bmatrix}. \tag{3.12}$$

The terms in the RHS of Eq.(33), i.e. R_{31} and R_{32}, are given by

$$R_{31} = -\frac{1}{\Pr}\frac{\partial}{\partial \tau}(\nabla^2 \psi_1) - R_{0c}\frac{\partial T_2}{\partial x} R_2 \frac{\partial T_1}{\partial x}, \tag{3.13}$$

$$R_{32} = \frac{\partial T_1}{\partial \tau} + \delta_1 f_2(z,\tau)\frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_1}{\partial x}\frac{\partial T_2}{\partial z} - \frac{\partial \psi_2}{\partial x}\frac{\partial T_1}{\partial z} \tag{3.14}$$

where the second term in Eq.(3.14) represents the modulation term. Substituting ψ_1, T_1 and T_2 into Eqs (3.13)-(3.14), we can obtain expressions for R_{31}, R_{32}easily. Now by applying the solvability condition for the existence of third order solution, we get the Ginzburg-Landau equation (Bhaduria and Kiran [39, 40, 54], Kiran [41, 42, 45, 61], Keshri *et al.*[44], Manjula *et al.*[47],) for stationary convection with time-periodic coefficients in the form

$$\left(\frac{\delta^2}{\Pr} + \frac{4\pi^2 k_c^2 R_{0c} k_c^2}{\delta_R^4\left(4\pi^2 - R_i\right)}\right)\frac{dB(\tau)}{d\tau} + \left(\frac{4\pi^2 k_c^2 R_{0c} k_c^2}{\delta_R^4\left(4\pi^2 - R_i\right)} - 2\frac{R_{0c} k_c^2}{\delta_R^2}\delta_1 I_1\right)B(\tau) + \left(\frac{2\pi^4 k_c^4 R_{0c}}{\delta_R^4\left(4\pi^2 - R_i\right)^2}\right)B(\tau)^3 = 0 \tag{3.15}$$

where $I_1 = \int_0^1 f_2(z,t)\sin^2(\pi z)dz.$

The Ginzburg Landau equation given in Eq.(3.15) is a Bernoulli equation and obtaining its analytical solution is difficult, due to its non-autonomous nature. So it is solved numerically using the in-built function NDSolve of Mathematica, subjected to the initial condition $B(0) = b_0$; where b_0 is the chosen initial amplitude of convection. In our calculations we may use $R_2 = R_{0c}$; to keep the parameters to the minimum. We assume that $R_2 = R_{0c}$ which shows that the nonlinear influence considered in this paper are in the neighborhood of critical state of convection onset.

4. GLE in the presence of non-uniform gravity field

The effect of gravity modulation is discussed in the studies of (Gresho and Sani [20], Bhadauria and Kiran [22-24], Kiran *et al.*[28], Manjula *et al.* [29]). The momentum equation takes the form

$$\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}.\nabla)\boldsymbol{q} = \frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0} g_0\left(1 + \epsilon^2 \delta_g \cos(\omega_g t)\right) - \frac{\mu}{\rho_0}\nabla^2 \boldsymbol{q} - \sigma\mu_e^2 B_0^2 \boldsymbol{q} \tag{4.1}$$

where δ_2, δ_m are the amplitude and frequency of the applied magnetic field.

Similarly, the finite amplitude (GLE) equation is given by

$$\begin{aligned}&\left(\frac{\delta^2}{\Pr} + \frac{\left(\delta^4 + H\delta^2 a^2\right)k_c^2}{\delta_R^2}\right)\frac{dB(\tau)}{d\tau} + \\ &+ \frac{R_{0c}k_c^2}{\delta_R^2}\left(1 + \frac{4\pi^2}{\left(4\pi^2 - R_i\right)}\delta_g \cos(\omega_g \tau)\right)B(\tau) + \left(\frac{\pi^2 k_c^4\left(\delta^4 + H\delta^2 a^2\right)}{2\delta_R^2\left(4\pi^2 - R_i\right)}\right)B(\tau)^3 = 0.\end{aligned} \tag{4.2}$$

There are many studies on gravity modulation well documented in [50]-[55].

5. GLE in the presence of non-uniform applied magnetic field

According to the studies of Bhadauria and Kiran [32], Kiran and Bhadauria [33], under the effect of magnetic modulation the momentum equation takes the form

$$\frac{\partial \boldsymbol{q}}{\partial t}+(\boldsymbol{q}.\nabla)\boldsymbol{q}=\frac{1}{\rho_0}\nabla p+\frac{\rho}{\rho_0}\boldsymbol{g}-\frac{\mu}{\rho_0}\nabla^2\boldsymbol{q}-\sigma\mu_e^2 B_0^2\left(1+\in^2\delta_2\cos(\omega_m t)\right)\boldsymbol{q} \tag{5.1}$$

where δ_2 and δ_m are the amplitude and frequency of the applied magnetic field.

Similarly, the finite amplitude (GLE) equation is given by

$$\begin{gathered}\left(\frac{\delta^2}{\Pr}+\frac{\left(\delta^4+H\delta^2 a^2\right)k_c^2}{\delta_R^2}\right)\frac{dB(t)}{dt}+\\+\left(\frac{4\pi^2 k_c^4 R_{0c}}{\delta_R^4\left(4\pi^2-R_i\right)}+Ha^2\delta_2\cos(\omega_m t)\right)B(t)+\left(\frac{\pi^2 k_c^4\left(\delta^4+H\delta^2 a^2\right)}{2\delta_R^2\left(4\pi^2-R_i\right)}\right)B(t)^3=0.\end{gathered} \tag{5.2}$$

6. Results and discussions

In this paper, we discuss the effect of thermal modulation and internal heating on RBC in the presence of an applied magnetic field. The magnetic field and thermal modulation are applied externally to the system. Using the method of GLM the finite amplitude of convection is quantified regarding the Nusselt number. The systems of nonlinear partial differential equations are simplified using perturbation analysis. The GLE is derived under the solvability condition. Three types of temperature modulations (i) out of phase modulation (OPM) (ii) in phase modulation (IPM) (iii) and lower boundary modulations are considered.

We have also discussed three different modulations; (i) thermal modulation (ii) gravity modulation (iii) applied magnetic field modulation. These three different modulations have been compared and presented in the results. The effect of various system parameter values on heat transport has been presented. The values of parameters are considered within the range of the solutions. The Nusselt number Eq.(3.11) is obtained at second order.

Variations of Nu with slow time for various parameters are presented in Figs 2-7. Here the Nusselt number oscillates with slow time τ. The solution of the Ginzburg-Landau equation gives the amplitude of convection which helps to quantify heat transfer through the Nusselt number. Before interpreting the results we assume $R_2=R_{0c}$ which means that the disturbances are near to critical state of convection onset.

Because we solve the nonlinear system at every order, every order depends on the previous solution. Thus, our analysis is not a direct solution to the nonlinear model problem. Since our study is related to slow convective flow we consider the slow time as $t=\chi 2\,\tau$. We present our results in the case of OPM only for convenience and later we compare three different types of modulation.

The effect of internal heat source and sink is presented in Figs 2a and 2b. From the figures we observe that the effect of internal heating on thermal instability is destabilizing, as heat transport increases on increasing R_i. The heat transport is greater at higher positive values of R_i Fig.2a. This confirms the results obtained most recently by Kiran *et al.* [15, 16, 52, 61]. The effect of heat sink, i.e. negative values of R_i, is to diminish heat transport and shows a stabilizing effect. Thus, one needs to understand that any composite mixture of material stabilizes or destabilizes the system. The stability criteria are very important in many chemical experiments or reactions.

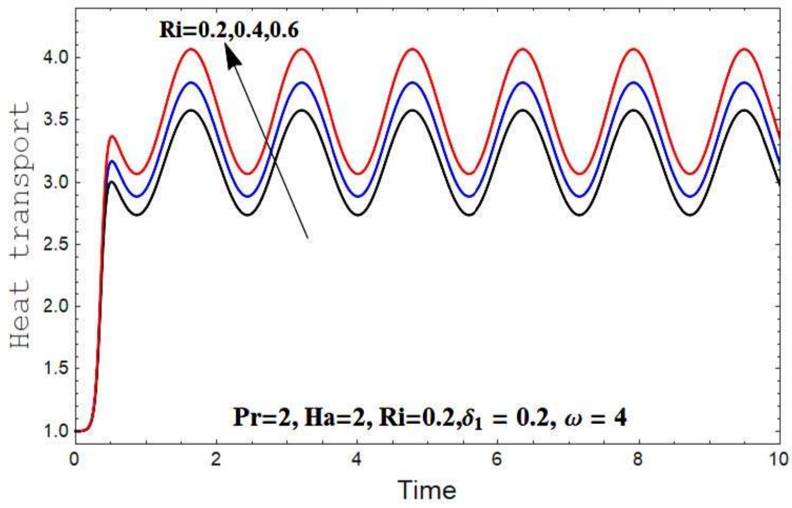

Fig.2a. The effect of internal heat source on heat transport.

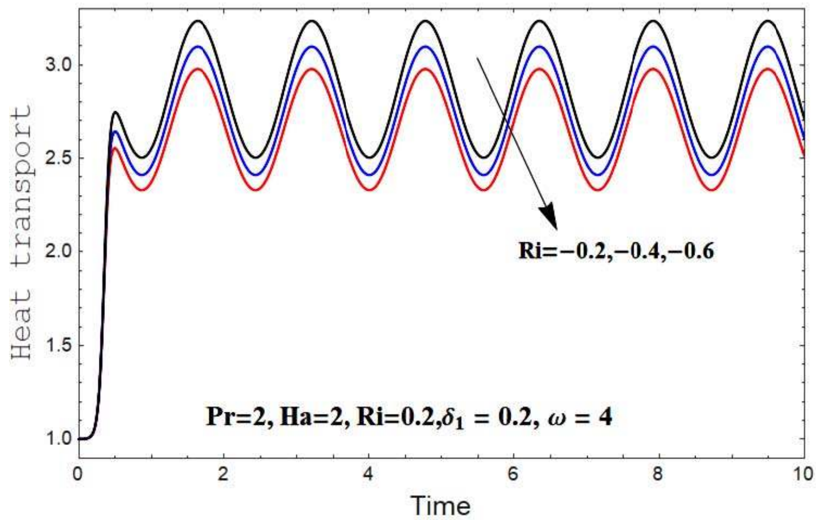

Fig.2b. The effect of internal heat sink on heat transport.

The strength of the fluid flow in the presence of a magnetic field is presented in Fig.3a. The Hartman number is the ratio of an electromagnetic force to the viscous force. It is clear from the figure that upon increasing the value of Ha heat transfer enhances in the layer. To see the effect of the magnetic field on the Nusselt number the value of $\boldsymbol{R_i}$ is chosen near *0.2* which does not affect the magnetic field. In Fig.3b we find Nu increases on increasing the value of the Prandtl number Pr for fixed values of other parameters. This may happen due to the dominating role of thermal diffusivity κ_T over kinematic viscosity υ .

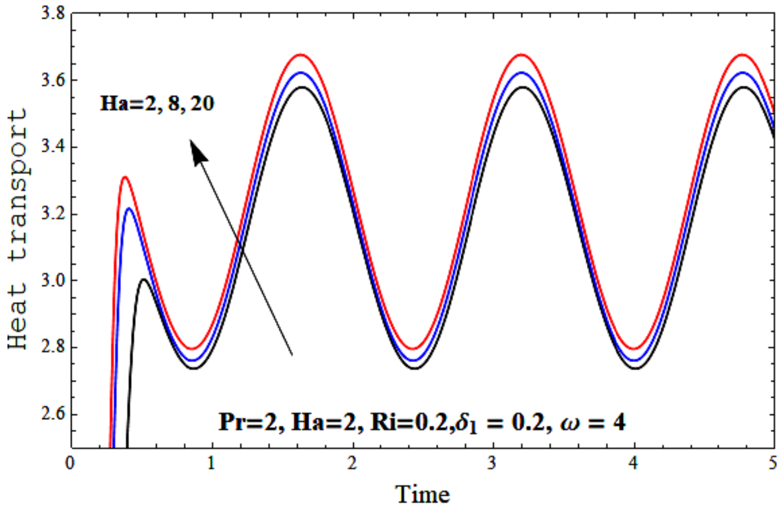

Fig.3a. The effect of Hartmann number on heat transport.

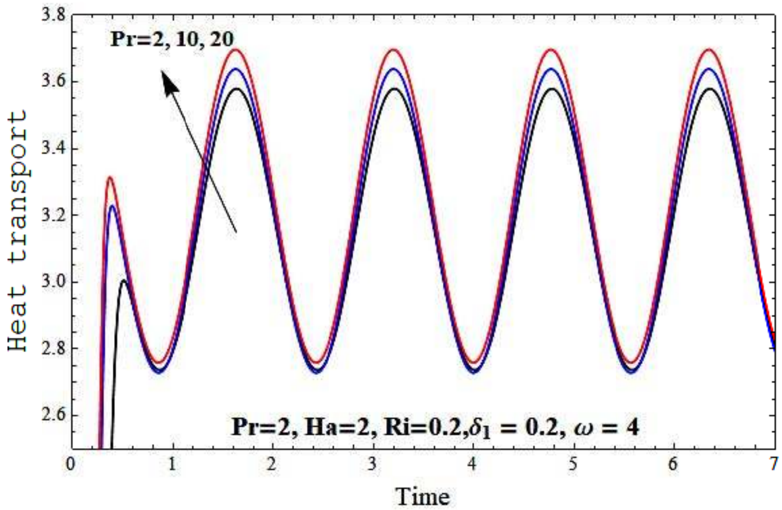

Fig.3b. The effect of internal heat sink on heat transport.

As the Prandtl number Pr increases, then for no change in kinematic viscosity probably there is a large decrement in thermal diffusivity, and this causes a sudden increase in the temperature gradient. So convection takes place early, and there is an enhancement in heat transfer. Thus, the effect of an increment in the Prandtl number Pr is to advance convection. A similar nature of Pr is observed in the studies of Bhadauria and Kiran [15, 18, 24, 32, 54], Kiran *et al*. [10, 28, 33], Manjula *et al*. [47, 59]. We have the following mathematical expression

$$Nu_{Pr=2} < Nu_{Pr=10} < Nu_{Pr=20}$$

In Fig.4a, we depict the effect of amplitude of modulation for moderate values of Ri and for the fixed values of other parameters. Upon increasing the value of δ_1, the value of Nu increases, hence advancing heat transport. This means that an increasing amplitude of modulation increases heat transfer. In the case of un-modulated system, δ_1 shows no influence on heat transport for larger values of time τ. The above results are compared with the studies of Manjula *et al*. [13, 59], Bhadauria and Kiran [14, 15], Kiran *et al*. [16,19, 41] and are found in good agreement. We have the following mathematical expression

$$Nu_{\delta 1=0.2} < Nu_{\delta 1=0.4} < Nu_{\delta 1=0.6}.$$

From Fig.4b, we see the effect of frequency of modulation. For small values of ω heat transport is greater. An increment in the value of ω decreases the magnitude of $Nu(\tau)$, and shortens the wavelength of oscillations. As the frequency increases from 4 to 40, the magnitude of $Nu(\tau)$ decreases, and the effect of modulation on heat transport diminishes. On further increment of ω the effect of modulation on thermal instability disappears altogether. Hence the effect of ω is to stabilize the system. These results agree with many other studies on thermal instability by Bhadauria and Kiran [15, 18, 24, 32, 38], Kiran *et al*. [10, 28, 33, 41, 42], Manjula *et al*. [47, 58], and Kiran and Manjula [48, 57]. We have the following mathematical expression

$$Nu_{\omega=40} < Nu_{\omega=20} < Nu_{\omega=10} < Nu_{\omega=4}.$$

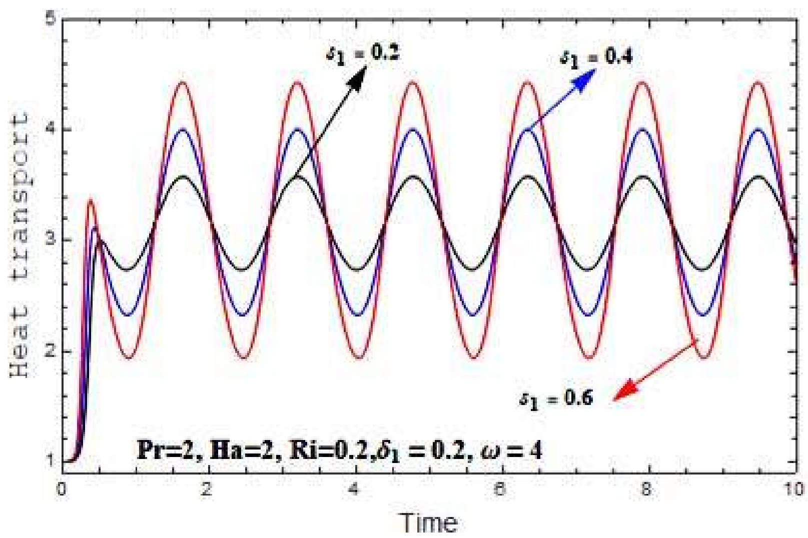

Fig.4a. The effect of amplitude of modulation on heat transport.

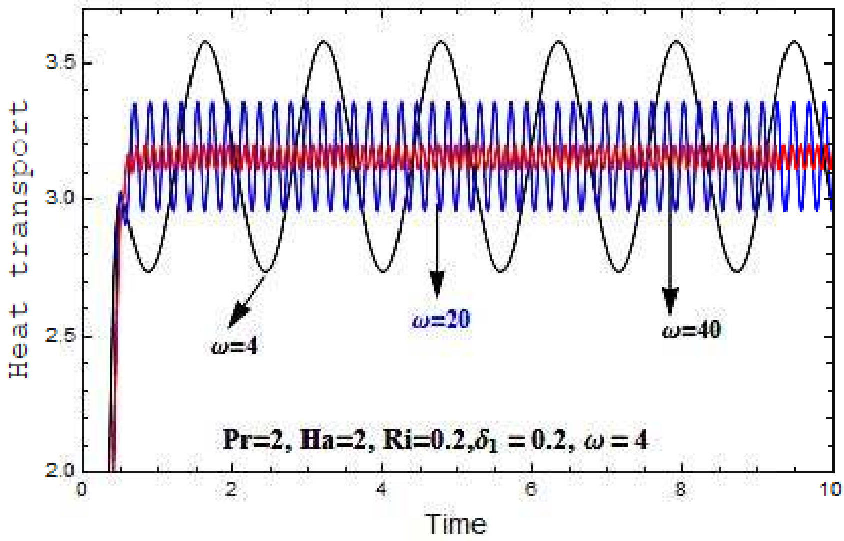

Fig.4b. The effect of frequency of modulation on heat transport.

In Fig.5, we have depicted the comparison between with or without internal heating. It is clearly evident that internal heating of the system enhances heat transport in the media.

$$\mathrm{Nu}_{Ri\neq 0}<\mathrm{Nu}_{Ri=0}.$$

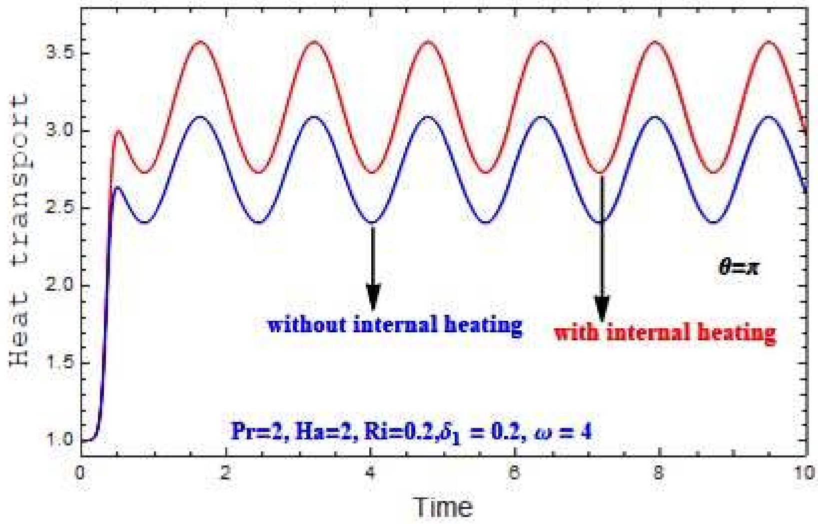

Fig.5. With and without internal heating.

Figure 6 shows the stability curves.

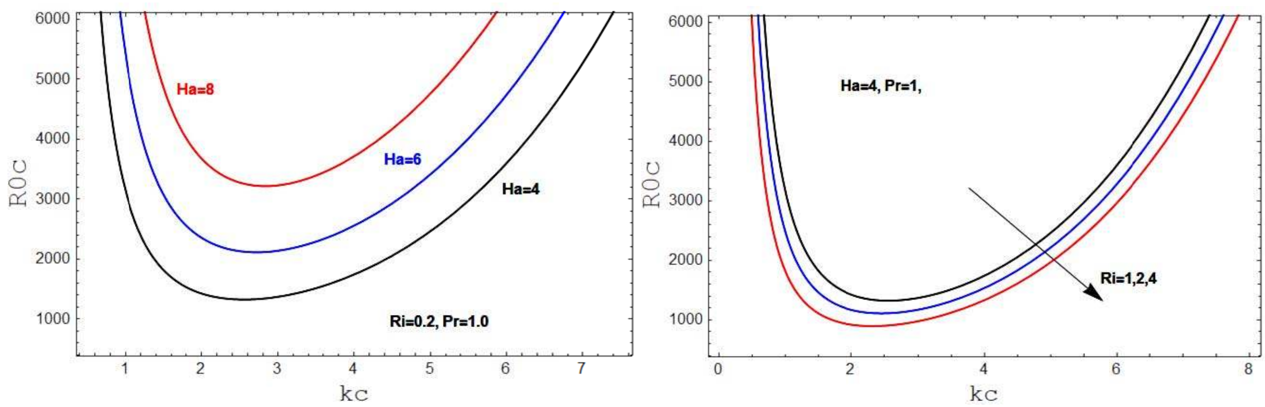

Fig.6. Stability curves R0c versus kc for different values of Ha and Ri.

Here we plot R_{0c} versus wavenumber k_c. The effects of Ha (in Fig.6a) and R_i (in Fig.6b) show that Ha stabilizes the system i.e. as Ha increases R_{0c} increases. This means that as the thermal Rayleigh number increases buoyancy enhances and more viscous force is required to destabilize the system.

In Fig.7 the effect of R_i and different modulations have been compared and presented. Figure 7a presents the results of R_i for large values of time, and shows the destabilizing effect on heat transport. The corresponding studies of Ri have been compared with Tveitereid *et al*. [34], Bhadauria *et al*. [38], Kiran *et al*. [41, 42, 52, 57] and found similar. The results have also been compared with Kiran and Manjula [48] and Manjula *et al.* [58, 59] and Kiran *et al*. [60] for internally soluted media. The internal solutal Rayleigh number S_i has a reverse nature of *Ri*.

In Fig.7b the effects of thermal (solution of Eq.(3.15)), gravity (solution of Eq.(4.2)) and magnetic field modulations (solution of Eq.(5.2)) are compared. It is clear that thermal modulation advances stability and enhances heat transfer more than the other two modulations. It is concluded that magnetic field modulation stabilizes the system more than the other two modulations. These results have been compared with the studies of Bhadauria and Kiran [32] and Kiran and Bhadauria [33] and found in good agreement. The following relation is observed clearly.

$$Nu_{\text{thermal modulation}} > Nu_{\text{gravity modulation}} > Nu_{\text{magnetic field modulation}}.$$

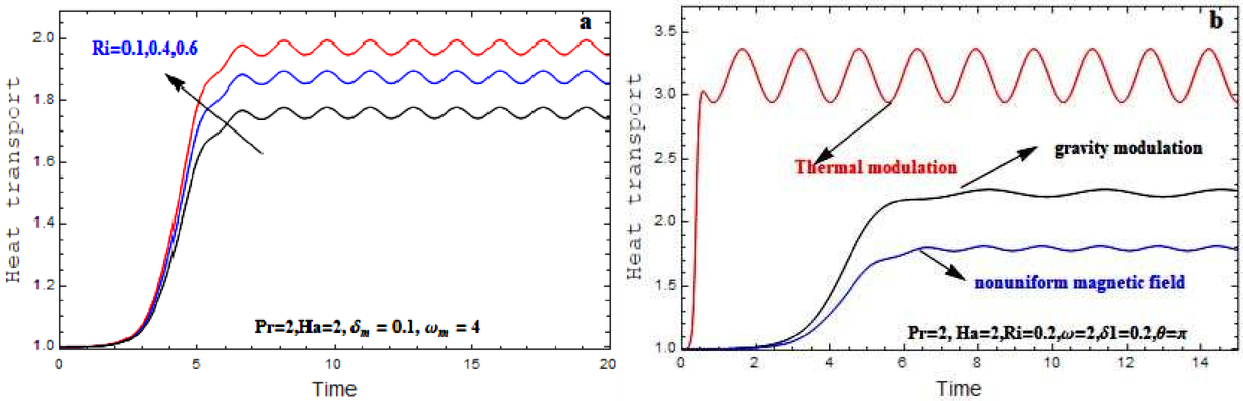

Fig.7. (a) The effect of Ri on Nu for magnetic modulation (b) Different modulation profiles.

In Figs 8 and 9, the streamlines and the corresponding isotherms are depicted for rotation speed modulation, respectively at $\tau = 0.0$, 0.10, 0.14, 0.16, 0.2 and 0.4. From the figures, we found that initially when time is small, the magnitude of streamlines is also small (Figs 8a, b), and isotherms are straight showing the system in conduction state, Figs 9a, b. However, as the time increases the magnitude of streamlines increases and the isotherms lose their evenness. This shows that convection is in progress in the system. The layer is more vibrant, i.e. convection becomes faster on further increasing the value of time τ.

However, the system achieves its steady state beyond $\tau = 0.6$ as there is no change in the streamline, and isotherms Figs 8c, d - 9c, d. The results of streamlines and isotherms have been compared with the studies of Bhadauria and Kiran [22-29] and Kiran [51, 55] for gravity modulation. The readers may find similar results for gravity modulation (porous convection) in the studies of Kiran *et al*. [51, 52, 55], Bhadauria *et al*. [53, 54] and Kiran *et al*. [60].

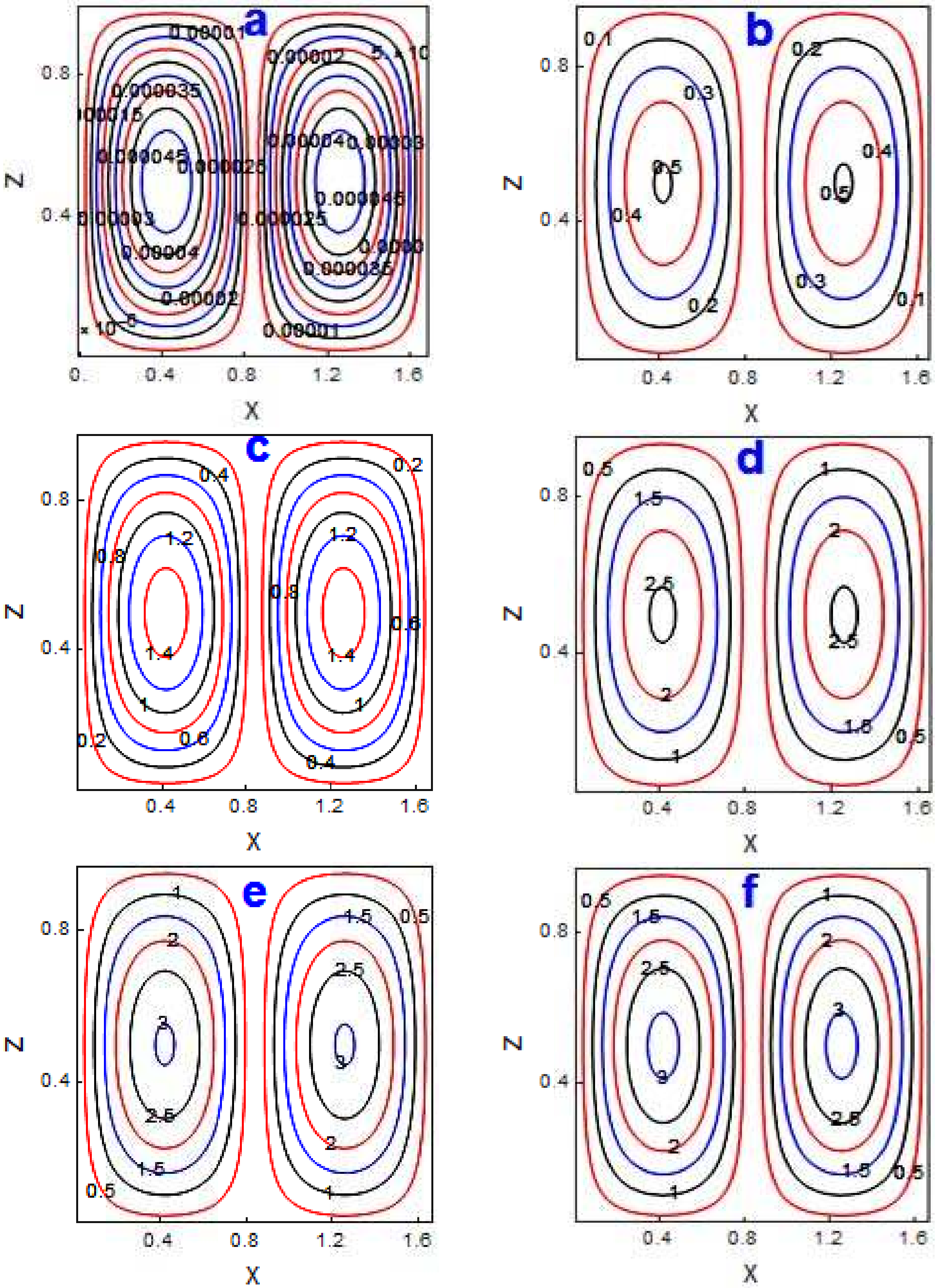

Fig.8. Streamlines for various values of time τ (a) $\tau = 0.0$ (b) $= 0.10$ (c) $\tau = 0.14$ (d) $\tau = 0.16$ (e) $\tau = 0.2$ (f) $\tau = 0.4$.

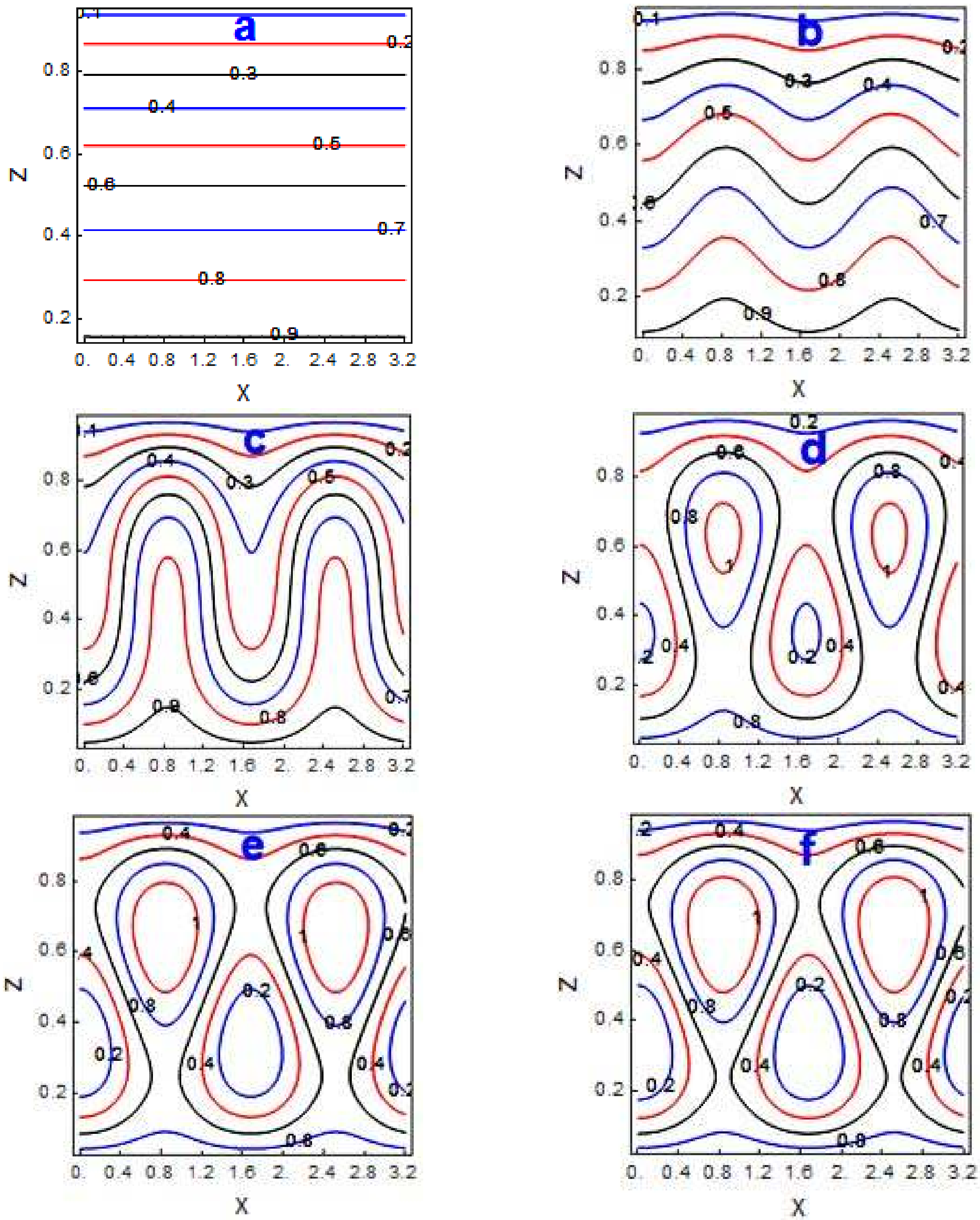

Fig.9. Isotherms for various values of time τ (a) τ *=0.0*(b) τ *=0.10* (c) τ *=0.14*(d) τ *=0.16* (e) τ *=0.2*(f) τ *=0.4.*

7. Conclusions

The following conclusions are drawn from the analysis:

1. The Prandtl number Pr, is to increase heat transfer.
2. The modulation loses its effect at sufficiently large values of frequency ω .

3. The effect of the magnetic field (Ha), frequency of modulation (ω), heat sink ($R_i<0$) is to suppress heat transport.
4. The effect of an increase in the values of Ha decreases the value of the Nusselt number. Thus, the amount of heat transfer decreases and hence the system is more stable.
5. The effect of amplitude of modulation (δ_1), heat source ($R_i>0$) is to enhance heat transport.
6. Upon increasing the value of R_i, Nu increases.
7. The magnitude of streamlines increases as time τ passes and isotherms lose their evenness, showing that convection takes place. At $\tau=1.0$ the system achieves equilibrium state.
8. Thermal modulations enhance heat transfer.
9. Magnetic modulation diminishes heat transfer.

Acknowledgement

The author SHM is grateful to the Division of Mathematics, Department of Science and Humanities, VFSTR for the support of research. The authors are thankful to the referees for their comments to refine the paper into its current form. Further, the authors PK and GNR gratefully acknowledge the support of CBIT to promote the research. This work is done during the promotion of Dr. Palle Kiran as a Research coordinator of the Department of Mathematics, CBIT.

Nomenclature

B − amplitude of convection
b − basic state
c − critical
d − depth of the fluid layer
g − acceleration due to gravity
Ha − Hartman number
k − vertical unit vector
k − wavenumber
p − reduced pressure
q − fluid velocity
Ra_T − thermal Rayleigh number
R_i − internal Rayleigh number
δ_1 − amplitude of thermal modulation
δg − amplitude of gravity modulation
δm − amplitude of magnetic modulation
Pr − Prandtl number
$R0c$ − critical Rayleigh-number
t − temperature
t − time
β_T − coefficient of thermal expansion
τ − slow time (dimensionless)
$\in$ − perturbation parameter
θ − phase angle
κ_t − effective thermal diffusivity
ω − thermal modulation frequency
ω_g − gravity modulation frequency
ω_m − magnetic modulation frequency
μ − dynamic viscosity of the fluid
μ_e − magnetic permeability
v − kinematic viscosity
ρ − fluid density
ψ − stream function
/ − perturbed quantity
* − dimensionless quantity
0 − reference value

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Received: April 21, 2020

Revised: August 9, 2020