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CORRIGENDUM

The effect of parameterized ice microphysics on the simulation of vortex circulation with a mesoscale hydrostatic model

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I have noted several typographical errors, particularly in the mathematical formulae, in my article, "The effect of parameterized ice microphysics on the simulation of vortex circulation with a mesoscale hydrostatic model" which appeared in *Tellus 41A*, 132–147.

Eqs. (1)–(4) on p. 135 should read:

$$\frac{\partial p^* q_v}{\partial t} = -m^2 \left[\frac{\partial(up^* q_v/m)}{\partial x} + \frac{\partial(vp^* q_v/m)}{\partial y} \right] - \frac{\partial p^* q_v \dot{\sigma}}{\partial \sigma} + p^*(P_{ced} + P_{red} - P_{gci}) + p^*(F_{CON} + F_{PBL} + F_{HD} + F_{VD}) q_v, \tag{1}$$

$$\frac{\partial p^* q_{wi}}{\partial t} = -m^2 \left[\frac{\partial(up^* q_{wi}/m)}{\partial x} + \frac{\partial(vp^* q_{wi}/m)}{\partial y} \right] - \frac{\partial p^* q_{wi} \dot{\sigma}}{\partial \sigma} + p^*(P_{gci} - P_{ced} - P_{aut} - P_{acr}) + p^*(F_{PBL} + F_{HD} + F_{VD}) q_{wi}, \tag{2}$$

$$\frac{\partial p^* q_m}{\partial t} = -m^2 \left[\frac{\partial(up^* q_m/m)}{\partial x} + \frac{\partial(vp^* q_m/m)}{\partial y} \right] - \frac{\partial p^* q_m \dot{\sigma}}{\partial \sigma} + p^*(P_{aut} + P_{acr} - P_{red}) - g \frac{\partial(\rho q_m v_t)}{\partial \sigma} + p^* F_{HD} q_m, \tag{3}$$

$$\frac{\partial p^* T}{\partial t} = -m^2 \left[\frac{\partial(up^* T/m)}{\partial x} + \frac{\partial(vp^* T/m)}{\partial y} \right] - \frac{\partial p^* T\dot{\sigma}}{\partial \sigma} + \frac{RT_v \omega}{C_{pm}(\sigma + p_t/p^*)} + \frac{Lp^*(P_{gci} - P_{ced} - P_{red})}{C_{pm}} - \frac{\delta L_f p^*[\dot{\sigma}(q_{wi} + q_m) + \rho g q_m v_t]}{C_{pm} \Delta\sigma} + p^*(F_{CON} + F_{PBL} + F_{HD} + F_{VD}) T, \tag{4}$$

The corrected formulae and associated descriptions in the Appendix should read as follows:

A.1. Generations of cloud water and ice (P_{gci})

When $T > 0°\mathrm{C}$ and water vapor is supersaturated with respect to water, the condensation rate of water vapor into cloud water is diagnostically computed using

$$P_{gci} = \begin{cases} \dfrac{(q_v - q_{vs})/\Delta t}{1 + L_v^2 q_{vs}/C_{pm} R_v T^2} & q_v > q_{vs} \\ 0 & q_v \leqslant q_{vs} \end{cases} \tag{A1}$$

where q_{vs} is the saturation specific humidity with respect to water (or ice). Whenever $T \leqslant 0°\mathrm{C}$ and the air is supersaturated with respect to ice, the initiation rate of cloud ice from water vapor is given by

$$P_{gci} = \min \begin{cases} (M_0 n_c/\rho)/\Delta t \\ (q_v - q_{vs})/\Delta t \end{cases} \tag{A1'}$$

and it is always

$$P_{\mathrm{gci}} \geqslant 0,$$

where M_0 is the initial mass of cloud ice crystals, and n_c is the number concentration of cloud ice crystals which is assumed to be dependent on temperature (Fletcher, 1962):

$$n_c = n_0 \exp[\beta(T_0 - T)]. \tag{A1''}$$

A.2. Autoconversion of cloud water to rainwater and ice to snow (P_{aut})

The autoconversion process is obtained from

$$P_{\mathrm{aut}} = \begin{cases} k_1(q_{\mathrm{wi}} - q_{\mathrm{w0}}) & T > 0^\circ\mathrm{C} \\ (q_{\mathrm{wi}} - q_{\mathrm{i0}})/\Delta t & T \leqslant 0^\circ\mathrm{C}' \end{cases} \tag{A2}$$

and it is always

$$P_{\mathrm{aut}} \geqslant 0,$$

where q_{w0} and q_{i0} are critical values for the onset of autoconversion of cloud water to rainwater and cloud ice to snow, respectively, and equal to the respective values of 0.5 g kg^{-1} and $M_{\max} n_c/\rho$.

A.3. Evaporation of cloud water and depositional growth of cloud ice (P_{ced})

Below the level of 0°C, whenever the air is subsaturated and cloud water is available, the evaporation rate of cloud water is computed using

$$P_{\mathrm{ced}} = \min \begin{cases} \dfrac{(q_{\mathrm{vs}} - q_{\mathrm{v}})/\Delta t}{1 + L_{\mathrm{v}}^2 q_{\mathrm{vs}}/C_{\mathrm{pm}} R_{\mathrm{v}} T^2} \\ q_{\mathrm{wi}}/\Delta t \end{cases} \tag{A3}$$

Above the level of 0°C, the sublimation rate of cloud ice and vapor deposition rate onto ice crystals in respective sub- and super-saturated conditions (w.r.t. ice) are given by

$$P_{\mathrm{ced}} = \min \begin{cases} \dfrac{65.2(1 - \mathrm{RH})(\rho q_{\mathrm{wi}} n_c)^{1/2}}{\rho(L_s^2/K_a R_v T^2 + 1/\rho q_{\mathrm{vs}} D_f)} \\ (q_{\mathrm{vs}} - q_{\mathrm{v}})/\Delta t \end{cases} \tag{A3'}$$

where RH, K_a and D_f are the relative humidity, thermal conductivity of air and diffusivity of water vapor, respectively. Note that $P_{\mathrm{ced}} = 0$ when available water vapor (i.e., $q_v - q_{vs}$) has been used up by the ice generation process.

A.4. Evaporation of rainwater and depositional growth of snow (P_{red})

When the air is subsaturated with respect to water surface, the evaporation rate of raindrops below the 0°C isotherm is given by

$$P_{\mathrm{red}} = \min \begin{cases} \dfrac{2\pi(1 - \mathrm{RH}) n_{\mathrm{w}} \left[0.78\lambda_{\mathrm{w}}^{-2} + 0.32 S_c^{1/3} \Gamma\left(\dfrac{b_{\mathrm{w}} + 5}{2}\right)(a_{\mathrm{w}}/\nu)^{1/2} \lambda_{\mathrm{w}}^{-0.5(b_{\mathrm{w}}+5)}\right]}{\rho\left(\dfrac{L_v^2}{K_a R_v T^2} + \dfrac{1}{\rho q_{\mathrm{vs}} D_f}\right)} \\ q_{\mathrm{rn}}/\Delta t \end{cases} \tag{A4}$$

where λ_w is the slope of raindrop size distribution, S_c the Schmidt number (ν/D_f) and ν kinetic viscosity of air. When the air is supersaturated with respect to ice above the 0°C isotherm, snow crystals will grow at the expense of water vapor, but sublimate into water vapor when subsaturated with respect to ice. These processes are computed by

$$P_{\mathrm{red}} = \min \begin{cases} \dfrac{2\pi(1-\mathrm{RH})n_{\mathrm{s}}\left[0.78\lambda_{\mathrm{s}}^{-2} + 0.32S_{\mathrm{c}}^{1/3}\Gamma\left(\dfrac{b_{\mathrm{s}}+5}{2}\right)(a_{\mathrm{s}}/\nu)^{1/2}\lambda_{\mathrm{s}}^{-0.5(b_{\mathrm{s}}+5)}\right]}{\rho\left(\dfrac{L_{\mathrm{s}}^2}{K_{\mathrm{a}}R_{\mathrm{v}}T^2} + \dfrac{1}{\rho q_{\mathrm{vs}}D_{\mathrm{f}}}\right)} \\ q_{\mathrm{rn}}/\Delta t \end{cases} \tag{A4$'$}$$

where λ_{s} is the slope of snow size distribution. Note also that $P_{\mathrm{red}} = 0$ when available moisture has all been used up by the ice generation and depositional ice growth processes (i.e., $(q_{\mathrm{v}} - q_{\mathrm{vs}}) \leqslant (P_{\mathrm{gci}} + P_{\mathrm{ced}})\Delta t$). That is, the priority of transferring available moisture into frozen particles in the model is given to ice generation, deposition onto cloud ice and onto snow, in that order.

A.5. Accretion of cloud water by raindrops and ice crystals by snow (P_{acr})

Below the level of $T = 0\,^{\circ}\mathrm{C}$, the collection of cloud water by rainwater in the vertical is given by

$$P_{\mathrm{acr}} = \begin{cases} \dfrac{\pi E_{\mathrm{w}} n_{\mathrm{w}} a_{\mathrm{w}} \Gamma(3+b_{\mathrm{w}}) q_{\mathrm{wi}}}{4\lambda_{\mathrm{w}}^{3+b_{\mathrm{w}}}} & q_{\mathrm{rn}} > 0 \\ 0 & q_{\mathrm{rn}} \leqslant 0 \end{cases} \tag{A5}$$

where E_{w} is the collection efficiency for raindrops. The collection of cloud ice by snow is given by

$$P_{\mathrm{acr}} = \begin{cases} \dfrac{\pi E_{\mathrm{s}} n_{\mathrm{s}} a_{\mathrm{s}} \Gamma(3+b_{\mathrm{s}}) q_{\mathrm{wi}}}{4\lambda_{\mathrm{s}}^{3+b_{\mathrm{s}}}} & q_{\mathrm{rn}} > 0 \\ 0 & q_{\mathrm{rn}} \leqslant 0 \end{cases} \tag{A5$'$}$$

where E_{s} is the collection efficiency for snow. Note that the accretion of cloud water by snow is not considered.

A.6. Terminal velocity of raindrops and snow (v_t)

All precipitable particles are assumed to fall at their mass-weighted terminal velocity. For raindrops, v_{t} is computed according to

$$v_{\mathrm{t}} = \frac{a_{\mathrm{w}}\Gamma(4+b_{\mathrm{w}})}{6}\lambda_{\mathrm{w}}^{-b_{\mathrm{w}}}\left(\frac{p_0}{p}\right)^{0.4}. \tag{A6}$$

For snow, it is

$$v_{\mathrm{t}} = \frac{a_{\mathrm{s}}\Gamma(4+b_{\mathrm{s}})}{6}\lambda_{\mathrm{s}}^{-b_{\mathrm{s}}}\left(\frac{p_0}{p}\right)^{0.4}. \tag{A6$'$}$$