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The Effect of Promotional Cost Sharing on the Decisions of Two-level Supply Chain with Uncertain Demand

Abstract

In much of today's competitive marketplace, consumers have the opportunity to choose where they spend their money based on their examination of a company's corporate social responsibility (CSR). A company's social reputation and its promotional efforts may influence consumer choice. Sustainable development is highly regarded by governments, experts, decision-makers and managers in organizations and two-level supply chains. This supply chain includes a manufacturer (acting as leader) and a retailer (acting as follower), both of whom face demand uncertainty. In this article, the cost of advertising is considered in two ways for the manufacturer and the retailer. In the first model, the retailer determines the optimal retail price and order quantity, and the manufacturer determines the optimal wholesale price and the promotional efforts value so that their profits are maximized. In the second model, the retailer determines the optimal retail price, order quantity, and promotional efforts value, and the manufacturer determines the optimal wholesale price. We will also study the impact of promotional cost sharing on the coordination of the supply chain and the issue will be further explained with numerical examples. Therefore, in this research, the simultaneous effect of increased advertising and value consumer surplus is studied We examine cases when the retailer is a provider of promotional cost and profit, when consumer surplus is directed toward the retailer, and when both retailer and manufacturer achieve higher profits than when the manufacturer is responsible for promotional cost and profit.

Keywords: Promotional efforts; Social responsibility; Two-level supply chain; Consumer surplus; Stackelberg; Cost sharing policy.

1. Introduction

The topic of the supply chain has shifted from its traditional emphasis on maximizing profits. Instead, a good deal of research has studied social, non-economic objectives such as customer welfare or, more comprehensively, social issues in the supply chain. In addition to the social improvement of the system, such shifts in focus can also have a positive impact on the system profit.

In recent years, corporate social responsibility (CSR) has been much discussed. The reasons for this are rooted in social concerns of companies, governmental pressures and regulations, and of course, the positive impact CSR has on attracting customers. A simple definition identifies CSR as a doctrine that promotes the expansion of social stewardship by businesses and organizations, (Modak et al., 2014), responsibility of an organization for the impacts of its decisions and activities on society and the environment through transparent and ethical behavior (Panda et al., 2016). In fact, CSR is part of both sustainable development and social welfare, and the impact of CSR on customer attraction is undeniable. The main CSR theories and related approaches are classified into four groups: (1) instrumental theories, (2) political theories, (3) integrative theories, and (4) ethical theories (Krishnan et al., 2004). In this regard, many articles have been published and each of them seeks to improve the profitability of the system. In this article, we will try to examine the profits of the system in two scenarios and compare the results.

2. Literature review

This section reviews pertinent related articles. Because of the importance of CSR, a large number of review articles have been published in this field during the past years; see Garriga and Melé (2004); Montiel (2008), Moir (2001), Jenkins and Yakovleva (2006). In Cruz and Wakolbinger (2008), a company's CSR level is studied in a multi-period supply chain in which the chain members have determined the level of CSR activity, production level, and transaction quantities. This paper has three objective functions of risk minimization, net return maximization, and emissions minimization. Palanivel and Uthayakumar (2015) created a bi-objective mathematical programming model to design the supply chain network with the aim of maximizing social responsibility of the supply chain and minimizing total costs in uncertainty conditions. In this study, CSR is expressed in terms of minimizing the total produced wastes and number of hazardous products and maximizing job opportunities. In most studies, CSR is expressed in terms of consumer surplus. For example, Goering (2008) has studied social welfare and consumer surplus in a two-period supply chain. In the articles by Panda et al. (2016) and Panda (2014), consumer surplus profit is separately considered, once for the retailer and once for the manufacturer. In the article by Nematollahi et al. (2016), the amount of CSR investment in the two-level supply chain including a supplier and a retailer has been investigated in centralized, decentralized, and collaborative models. The demand function was probable and used the newsvendor approach. The order quantity is the decision variable of the retailer and the amount of CSR investment is the decision variable of the supplier.

In the investigated models in the field of supply chain, the system management was not always integrated and interactive. In fact, it is observed that, in some cases, system members are only looking to optimize their profits. In such models, coordination plays an important role in creating the integrity of the system members. They will lead to improve the system profits. In the models presented in this article, our results will examine the agreement of sharing the advertising costs to establish coordination in the system.

Moreover, each of the articles has respectively used quantity discount contracts and revenue sharing for system coordination. Cárdenas-Barrón and Sana (2015) has studied consumer surplus and consumer rebates in a decentralized supply chain which includes a manufacturer and retailer, and each product is uniformly distributed to the customer. Yao and Wu (1999) has studied consumer surplus and producer surplus in a model of linear fuzzy supply and linear fuzzy demand. A two-level social responsibility supply chain is studied in the articles by Modak et al. (2015) and Modak et al. (2014), which respectively have a manufacturer and two competitive retailers. In both the papers, CSR is the responsibility of the manufacturer. Panda et al. (2015) has studied channel coordination in a three-level social responsibility supply chain. CSR can also be defined in other terms such as in Wu (2015), where it has been defined as the variable affecting demand function in a two-level supply chain, or in Ni et al. (2012), in which CSR costs have been considered for the manufacturer and retailer. These costs have an impact on the demand function and channel profit function. In Hsueh (2014), the author has addressed CSR costs as the responsibility of the manufacturer.

Chintapalli et al. (2017) have provided the two-level model including a supplier and a manufacturer in which the manufacturer faces the uncertain demand from the retailer. In the following, in this decentralized model, the discount contract has been used. Finally, the multi-channel model including two manufacturers has been reviewed. The article stated that when a combination of discount contract and minimum pre-order amounts in the model is implemented, the best coordination occurs. CSR has been investigated in other fields of science. For instance, Lamata et al. (2016)'s work proposes an MCDM model for the analysis of social criteria. In Garcia-Melón et al. (2016), an AHP-TOPSIS approach is used and socially responsible investment is investigated based on social criteria; AHP and multi-stakeholder approaches are employed. In today's competitive world of famous brands, advertising efforts play an important role in increasing sales and attracting customers. Advertising costs are provided by the manufacturer or retailer. Therefore, in this paper two types of scenarios are expressed. In the first scenario, the manufacturer is responsible for paying the advertising costs, and in the second scenario, responsibility rests with the retailer.

Advertising and marketing can be used for the reputation of a product, as in Chopra et al. (2007). Hence, we investigated a supply chain including CSR and promotional efforts. Many promotional efforts examine and evaluate costs, such as in Tsao and Sheen (2012), De and Sana (2015), Maihami and Karimi (2014), and so on. Moreover, the impact of promotional efforts on the replenishment policy and product sales price has been studied in Maihami and Karimi (2014). Tsao and Sheen (2012) has considered promotional efforts costs for the retailer and shared advertising costs to coordinate the supply chain. In papers by Cárdenas-Barrón and Sana (2015), De and Sana (2015), Palanivel and Uthayakumar (2015), and Pattnaik (2012), an order quantity inventory model in a supply chain is presented where demand is dependent on promotional efforts. In a two-level supply chain with demand uncertainty, Roy et al. (2015) has defined promotional efforts as a parameter affecting retailer's demand. Further, Wu et al. (2016) has assumed that demand is influenced by promotional efforts and consumer return policies and that promotional efforts costs are the responsibility of the retailer. Tsao (2015) has investigated a decentralized supply chain with demand uncertainty; the manufacturer's profit function includes promotional efforts costs. Jenkins and Yakovleva (2006) and Khouja and Zhou (2010) both offered a decentralized supply chain with a non-linear uncertain demand function where promotional efforts costs are the responsibility of the retailer. In order to coordinate the supply chain, Jenkins and Yakovleva (2006) has used Buy-Back contract. In addition, Khouja and Zhou (2010) has applied revenue sharing contract.

In the article by Karray (2011), the author introduces horizontal joint promotions (HJP) which means the collaborative effort of competitive retailers for joint products advertising, leading to reducing costs and increasing sales. HJP has been defined as a positive parameter in demand function. In Sheu (2011), the effect of revenue-sharing contracts is studied on a channel with supplier and retailer members. In this channel, three types of promotional demand have been defined. In Giri et al. (2013), a two-level supply chain with demand function is modeled which is sensitive to retail price and advertising. The author examined different contracts including wholesale price-only, revenue sharing, two-part tariff, and continuous wholesale price discount to obtain coordination in the system and to maximize profit. In the article by Amrouche and Yan (2016), a multi-channel dual-level model was presented. This system includes a manufacturer who sells products through both a retail channel and a direct channel. The scenario examined in the wholesale price model was the same in the two sales channels. Finally, the profit sharing agreement coordinated the system. Chen et al. (2017) compared the system profit in the two-level model when it includes the retail channel, the direct channel, and both channels. The price and quality levels have been investigated under these three scenarios. In a two-level newsvendor model, Jadidi et al. (2016) modeled pricing in two scenarios in which the retailer has two opportunities to purchase a product. These two modes of the model were examined under four contract methods. Each one is superior to the other.

Priyan et al. (2015) offered a two-level production-inventory model that includes a manufacturer and a retailer. Demand depends on advertising and the rate of production is uncertain. Ramanathan and Muyldermans (2010) studied promotional efforts and a set of factors affecting demand in soft drinks. Studying two-level supply chain models is common. Pan et al. (2009), for instance, studied pricing and ordering model in a two-period system characterized by uncertainty. Taleizadeh et al. (2015) pursued pricing and vendor-managed inventory (VMI) models in a two-level supply chain with a vendor and some retailers. A model considering the pricing and inventory regarding perishable products was presented by Herbon and Khmelnitsky (2017). In this model, the optimal program and time to replenish products, and the optimal product price are decision variables realized to maximize the retail profit. The demand function is compared in both linear and non-linear modes depending on the price of the product. Jaggi et al. (2017) proposed a solution algorithm for two inventory models. In this model, demand depends on price. The goal was to determine the retailer's optimal policy of refilling so that he achieves the average of profit maximum in each period.

Giri and Sarker (2016) also defined a two-level supply chain with a manufacturer and two retailers where the manufacturer was the leader and wholesale discount was employed for coordination in the system. Li et al. (2013) studied two two-level supply chains: 1) one supplier and one buyer with uncertain demand, and 2) multiple suppliers and one buyer with uncertain supply. They studied the coordination in a model characterized by uncertain demand, which was also studied by Heydari and Norouzinasab (2015). Chen (2014) offered a model with a Nash game and cooperation games in a manufacturer-retailer channel. Zhang et al. (2016) studied the retailer`s ordering policy and supplier`s pricing policy in a two-level supply chain characterized by demand and supply uncertainties. Li et al. (2016) investigated the coordination in a supply chain with demand uncertainty. Here, three contracts (risk-return, revenue sharing, and wholesale price) were employed. The study also offered a vendor-retailer channel where the vendor is the leader and the demand is uncertain.

A brief literature summary is presented in Table 1.

Table 1. The literature review summary

	Reference	Pricing model	Coordination	Inventory model	Corporate social responsibility	Promotional efforts	Demand uncertainty	Review article
1	Amrouche and Yan (2016)	X	X					
2	Cárdenas-Barrón and Sana (2015)			X		X		
3	Chen (2014)	X	X					
4	Chen et al. (2017)	X	X					
5	Chopra et al. (2007)							X
6	Chintapalli et al. (2017)	X	X				X	
7	Cruz and Wakolbinger (2008)	X			X			
8	De and Sana (2015)			X		X		
9	Garriga and Melé (2004)				X			X
10	García-Melón et al. (2016)				X			
11	Giri et al. (2013)	X	X			X		
12	Giri and Sarker (2016)	X	X					
13	Goering (2008)	X			X			
14	Gunasekaran and Spalanzani (2012)				X			
15	Herbon and Khmelnitsky (2017)	X	X					
16	Heydari and Norouzinasab (2015)		X	X			X	
17	Hsueh (2014)	X	X		X			
18	Jadidi et al. (2016)	X						
19	Jaggi et al. (2017)							
20	Jenkins and Yakovleva (2006)				X			X
21	Karray (2011)	X				X		
22	Khouja and Zhou (2010)	X	X		X			
23	Krishnan et al. (2004)	X	X			X	X	
24	Lamata et al. (2016)				X			
25	Li et al. (2016)	X	X				X	
26	Li et al. (2013)	X	X				X	
27	Maihami and Karimi (2014)	X	X	X		X	X	
28	Modak et al. (2015)	X	X		X			
29	Modak et al. (2014)	X	X		X			
30	Moir (2001)				X			X

31	Montiel (2008)				X			X
32	Nematollahi et al. (2017)	X			X			
33	Ni et al. (2012)	X	X		X			
34	Pal et al. (2015)	X	X			X		
35	Palanivel and Uthayakumar (2015)			X		X		
36	Pan et al. (2009)	X		X			X	
37	Panda (2014)	X	X		X			
38	Panda et al. (2015)	X	X		X			
39	Panda et al. (2016)	X	X		X			
40	Pattnaik (2012)			X		X		
41	Priyan et al. (2015)			X		X		
42	Ramanathan and Muyldermans (2010)					X		
43	Roy et al. (2015)	X				X	X	
44	Sheu (2011)	X	X			X		
45	Taleizadeh et al. (2015)	X		X			X	
46	Tsao (2015)	X				X	X	
47	Tsao and Sheen (2012)	X	X			X	X	
48	Wu (2015)	X			X		X	
49	Wu et al. (2016)		X	X		X	X	
50	Yao and Wu (1999)				X		X	
51	Zhang et al. (2016)	X		X			X	

Since few studies have been conducted concerning the concurrent effect of promotional efforts and consumer surplus on an uncertain supply chain, this article aims to study the two-level supply chain characterized by uncertainty and to investigate the effect of promotional efforts and consumer surplus on this supply chain. The impact of advertising costs on this supply chain will be compared in two cases: when the manufacturer is responsible for paying the costs, and when the retailer is responsible. The research questions outlined here are as follows: 1. What is the effect on retailer`s profit when consumer surplus profit is taken by the manufacturer?, 2. Do promotional efforts cost sharing alone reduce retailer's profit?, 3. Are cost sharing and consumer surplus alone able to increase the order quantity?, 4. In the two examined scenarios (when the manufacturer is responsible for paying the costs and when the retailer is responsible), which one will have a better effect on the system profit?, 5. Will the contract have the same effect on both scenarios? Most above-mentioned studies consider promotional efforts cost and consumer surplus profit in retailer`s profit function; however, we consider the cost and profit not only for the manufacturer but also for the chain.

3. Problem description

Consider a supply chain that consists of one or more raw material suppliers, manufacturers, retailers, distributors, and final consumers. Each can play the role of a buyer or seller. In fact, each member of the supply chain can have both roles, seller and buyer. For example, a manufacturer facing the supplier has the role of a buyer, but has the role of a seller when he faces the retailer. A supply chain, however, should at least have a buyer and seller. Here, we considered a single-product and single-period supply chain with one manufacturer and one retailer (Figure 1). Promotional efforts costs were taken into account in order to increase order quantity for the manufacturer, and CSR profit was also considered for the manufacturer. In the first scenario, advertising cost was considered for the manufacturer, and in the second scenario, they were considered for the retailer. In this system, the manufacturer is the leader and the retailer is the follower. In the first scenario, the retailer determines retail price and order quantity and the manufacturer determines wholesale price and promotional efforts. These decisions are enacted in order to calculate the profit of system members. In the second scenario, the retailer determines the optimal retail price, the order quantity, and the promotional efforts value. The manufacturer determines the optimal wholesale price. Here, the retailer faces demand uncertainty. This article aims to study the effect of consumer surplus and promotional efforts cost sharing on the supply chain.

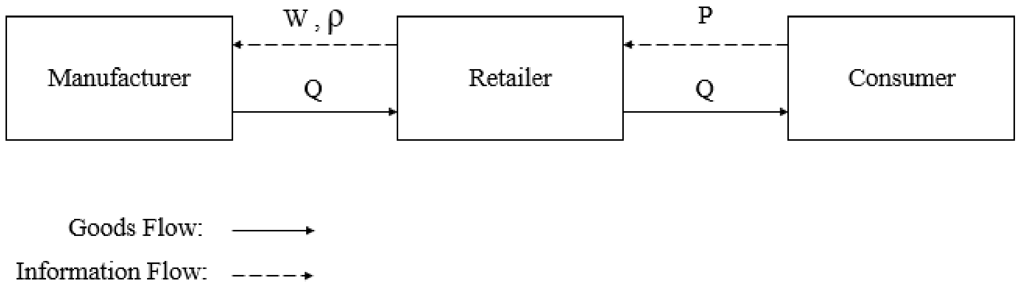


Figure 1. The two-level supply chain

In Fig. 1, a supply chain scheme is displayed. Based on the variables and parameters defined in the figure, the first scenario considers a traditional supply chain with the goal of optimizing supply chain profit. However, in this paper, in addition to optimizing the system profit, answering the social needs, and accessing sustainable development, the issue of Corporate Social Responsibility (CSR) is also considered.

4. The models

This section reviews the article's purposes and seeks to answer the study questions we presented in the proposed model. As mentioned in the literature review, a different method is implemented for examining CSR. However, in this article we follow the work of Modak et al. (2015), and we consider CSR as a consumer surplus problem and examine its effect on the profit of supply chain members. In the following, it is explained that consumer surplus is calculated as the difference between the greatest payable amount for the product (by client) and product market price. In this supply chain, we implemented consumer surplus and promotional efforts. We intend to study the effect of promotional efforts cost sharing on the channel member`s profit, retail price, wholesale price, order quantity, and promotional efforts. The manufacturer and retailer decide about the selling price and promotional efforts in two scenarios and two conditions: No promotion cost sharing (N.P.C.Sh) and Promotion cost sharing (P.C.Sh).

Notations:

P_α	Retail price under N.P.C.Sh
P_θ	Retail price under P.C.Sh
W_α	Wholesale price under N.P.C.Sh
W_θ	Wholesale price under P.C.Sh
ρ_α	The promotional effort of manufacturer under N.P.C.Sh
ρ_θ	Manufacturer promotional effort under P.C.Sh
ζ	Basic demand
D	Demand function
$P_{\max}$	Consumer's maximum willingness to pay per product
P_{mkt}	Market price
π_R^α	Retailer's profit under N.P.C.Sh
π_R^θ	Retailer's profit under P.C.Sh
π_M^α	Manufacturer's profit under N.P.C.Sh

π_M^{θ}	Manufacturer's profit under P.C.Sh
V_M^{α}	The total profit function of the manufacturer under N.P.C.Sh
V_M^{θ}	The total profit function of the manufacturer under P.C.Sh
CS	Consumer surplus
τ	Socially responsible manufacturer's concern, $0 \le \tau \le 1$
θ	Retailer’s fraction of promotional cost

The demand function is as follows: $D = f(\rho) + U$, where $f(\rho) = \rho\zeta$ and $\zeta = A - bP$. U is a stochastic variable following a continuous uniform distribution function within $[-\rho\zeta, \rho\zeta]$ and $A > 0, b > 0$. Following Tsao (2015), promotional effort cost is defined as $k(\rho - 1)^2$ where $k > 0, \rho \ge 1$. In this study, we used advertising cost sharing for system coordination and investigated its effect on each member’s profit. Two scenarios and two conditions are taken into account:

1. In the first scenario, the manufacturer is responsible for paying the advertising cost. In the second scenario, the retailer is responsible for paying the advertising cost. Each of these two scenarios will be checked in two ways.
2. The results from No promotion cost sharing (N.P.C.Sh) and Promotion cost sharing (P.C.Sh) are then compared. Section 4.1 presents the first scenario and section 4.2 models the second scenario.

4.1. First scenario

In the first scenario, the manufacturer is responsible for paying the advertising cost and the profit from consumer surplus is directed to the manufacturer. This scenario will be examined in two conditions: N.P.C.Sh (4.1.1) and P.C.Sh (4.1.2).

4.1.1. No promotion cost sharing (N.P.C.Sh)

In the N.P.C.Sh model, the whole promotional effort cost is paid by the manufacturer. Indeed, total promotional effort cost is the manufacturer`s responsibility. In this mode, the retailer`s profit is revenue resulting from sales deducted by the cost of purchasing the product from the manufacturer. The retailer's profit is as follows:

$$\pi_R^{\alpha} = P_\alpha \int_0^{q_\alpha} \frac{x}{2\rho_\alpha (A - bP_\alpha)} dx + P_\alpha \int_{q_\alpha}^{2\rho_\alpha (A - bP_\alpha)} \frac{q_\alpha}{2\rho_\alpha (A - bP_\alpha)} dx - W_\alpha q_\alpha \tag{1}$$

$$\pi_R^{\alpha} = q_\alpha (P_\alpha - W_\alpha) - \frac{P_\alpha q_\alpha^2}{4\rho_\alpha (A - bP_\alpha)} \tag{2}$$

The manufacturer`s profit is the revenue resulting from product sales deducted by promotional effort costs:

$$\pi_M^{\alpha} = (W_\alpha - C) q_\alpha - k(\rho_\alpha - 1)^2 \tag{3}$$

The difference between the consumers’ maximum willingness to pay and the market price (the actual price paid for the product) is called consumer surplus. Since the demand function follows uniform distribution function, order quantity will be either of the following:

$$Q = \begin{cases} \int_0^{q} \frac{x}{2\rho(A - bP)} dx \, ; if \ (x < q) \\ \int_q^{2\rho(A - bP)} \frac{q}{2\rho(A - bP)} dx \, ; if \ (x > q) \end{cases} \tag{4}$$

Therefore, consumer surplus is as follows in this model:

$$CS_{Total} = \underbrace{\int_{P(\mathrm{I})_{mkt}}^{P(\mathrm{I})_{\max}} \int_0^q \frac{x}{2\rho(A-bP)} dx(dP)}_{CS_{\mathrm{I}}} + \underbrace{\int_{P(\mathrm{II})_{mkt}}^{P(\mathrm{II})_{\max}} \int_q^{2\rho(A-bP)} \frac{q}{2\rho(A-bP)} dx(dP)}_{CS_{\mathrm{II}}} \tag{5}$$

$$= \frac{(A-bP_\alpha)}{b}[q_\alpha^2 - 2\rho_\alpha(A-bP_\alpha)] + \frac{(A-bP_\alpha)}{b}[q_\alpha - q_\alpha^2 - \rho_\alpha(A-bP_\alpha)]$$

$$= \frac{(A-bP_\alpha)}{b}[q_\alpha - \rho_\alpha(A-bP_\alpha)]$$

Total manufacturer`s profit equals the profit in Eq. 3 plus consumer surplus:

$$V_M^\alpha = (W_\alpha - C)q_\alpha - k(\rho_\alpha - 1)^2 + \tau.CS_{Total} \tag{6}$$

$$\mathrm{V}_M^\alpha = (W_\alpha - c)q_\alpha - k(\rho_\alpha - 1)^2 + (\frac{\tau.(A-bP_\alpha)}{b})[q_\alpha - \rho_\alpha(A-bP_\alpha)] \tag{7}$$

In order to find the retailer’s maximum profit, the retailer`s decision variables need to be determined. By differentiating π_R^α with respect to q_α and equating it to zero, the first optimum value of the order quantity is determined:

$$\frac{d\pi_R^\alpha}{dq_\alpha} = 0 \rightarrow q_\alpha(W_\alpha, P_\alpha) = \frac{2\rho_\alpha(P_\alpha - W_\alpha)(A-bP_\alpha)}{P_\alpha} \tag{8}$$

Since $\frac{d^2\pi_R^\alpha}{dq_\alpha{}^2} = -\frac{P_\alpha}{2\rho_\alpha(A-bP_\alpha)} < 0$, then π_R^α concave with respect to q_α. By substituting Eq. 8 into Eq. 2 and differentiating π_R^α with respect to P_α and equating it to zero, the optimal retail price will be achieved:

$$\frac{d\pi_R^\alpha}{P_\alpha} = 0 \rightarrow P_\alpha = \frac{A + \sqrt{A(A+8bW_\alpha)}}{4b} \tag{9}$$

Since the equations are based on a manufacturer-Stackelberg scenario in this supply chain, the manufacturer is the leader and the retailer is the follower. Eqs. 8 and 9 give the retailer`s optimal profit function, optimal values of the manufacturer`s decision variables, and optimal value of manufacturer`s profit function. By substituting the initial optimal order quantity and retail price into the manufacturer`s overall profit function, and then by differentiating with respect to the wholesale price, a 5-degree equation is obtained without analytical response. Therefore, we obtained the optimal wholesale price by coding the relationship of this supply chain into MATLAB software.

$$W_\alpha^* = \frac{5A(1-2\tau) + 32bc + 9A\tau^2 + 3(1+\tau)\sqrt{A(9A\tau^2 - 38A\tau + 17A + 64bc)}}{64b} \tag{10}$$

Finally, optimal promotional effort is obtained by substituting the optimal order quantity value, wholesale price, and retailing price into the producer`s overall profit function and then differentiating with respect to the promotional effort:

$$\frac{dV_M^\alpha}{d\rho_\alpha} = 0 \rightarrow \rho_\alpha^* = \frac{2(A-bP_\alpha^*)(P_\alpha^* - W_\alpha^*)(b(W_\alpha^* - c) + \tau(A-bP_\alpha^*)) - \tau P_\alpha^*(A-bP_\alpha^*)^2}{2bkP_\alpha^*} + 1 \tag{11}$$

4.1.2. Promotion cost sharing (P.C.Sh)

In the (P.C.Sh) model, the retailer tends to share promotional effort costs. As a result, the retailer`s profit function equals Eq. 1 deducted by a fraction of promotional effort costs:

$$\pi_R^\theta = P_\theta \int_0^{q_\theta} \frac{x}{2\rho_\theta (A - bP_\theta)} dx + P_\theta \int_{q_\theta}^{2\rho_\theta (A - bP_\theta)} \frac{q_\theta}{2\rho_\theta (A - bP_\theta)} dx - W_\theta q_\theta - \theta k (\rho_\theta - 1)^2 \quad (12)$$

$$\pi_R^\theta = q_\theta (P_\theta - W_\theta) - \frac{P_\theta q_\theta^2}{4\rho_\theta (A - bP_\theta)} - \theta k (\rho_\theta - 1)^2 \quad (13)$$

The manufacturer`s profit is as follows:

$$\pi_M^\theta = (W_\theta - C) q_\theta - (1-\theta) k (\rho_\theta - 1)^2 \quad (14)$$

Like Eq. 7, the manufacturer`s overall profit equals Eq. 18 plus the profit resulting from consumer surplus:

$$V_M^\theta = (W_\theta - c) q_\theta - (1-\theta) k (\rho_\theta - 1)^2 + (\frac{\tau.(A - bP_\theta)}{b})[q_\theta - \rho_\theta (A - bP_\theta)] \quad (15)$$

To find the maximum profit for the retailer, the optimal order quantity needs to be determined. By differentiating π_R^θ with respect to q_θ and equating it to zero, the first optimal order quantity is:

$$\frac{d\pi_R^\theta}{dq_\theta} = 0 \rightarrow q_\theta (W_\theta, P_\theta) = \frac{2\rho_\theta (P_\theta - W_\theta)(A - bP_\theta)}{P_\theta} \quad (16)$$

Since $\frac{d^2 \pi_R^\theta}{dq_\theta^2} = -\frac{P_\theta}{2\rho_\theta (A - bP_\theta)} < 0$, π_R^θ is concave compared to q_θ. By substituting Eq. 16 into Eq. 13 and by differentiating π_R^θ with respect to P_θ and equating it to zero, we can obtain the optimal retail price:

$$\frac{d\pi_R^\theta}{P_\theta} = 0 \rightarrow P_\theta = \frac{A + \sqrt{A(A + 8bW_\theta)}}{4b} \quad (17)$$

As in the N.P.C.Sh case, MATLAB is employed to obtain the optimal value of wholesale price:

$$W_\theta^* = \frac{5A(1-2\tau) + 32bc + 9A\tau^2 + 3(1+\tau)\sqrt{A(9A\tau^2 - 38A\tau + 17A + 64bc)}}{64b} \quad (18)$$

Optimal promotional effort equals:

$$\frac{dV_M^\theta}{d\rho_\theta} = 0 \rightarrow \quad \rho_\theta^* = \frac{2(A - bP_\theta^*)(P_\theta^* - W_\theta^*)(b(W_\theta^* - c) + \tau(A - bP_\theta^*)) - \tau P_\theta^* (A - bP_\theta^*)^2}{2(1-\theta) bkP_\theta^*} + 1 \quad (19)$$

4.2. Second scenario

In the second scenario, the retailer is responsible for paying the advertising cost and consumer surplus profit. This scenario will be examined in the same two conditions: N.P.C.Sh (4.2.1) and P.C.Sh (4.2.2).

4.2.1. Promotion cost sharing (P.C.Sh)

According to Eq. 1 and the explanation given in this section, the retailer's objective function is as follows:

$$\pi_R^\alpha = q_\alpha (P_\alpha - W_\alpha) - \frac{P_\alpha q_\alpha^2}{4\rho_\alpha (A - bP_\alpha)} - k(\rho_\alpha - 1)^2 \quad (20)$$

The manufacturer's profit function is as follows in this scenario:

$$\pi_M^\alpha = (W_\alpha - C) q_\alpha \quad (21)$$

$$V_M^{\alpha} = (W_\alpha - c)q_\alpha + (\frac{\tau.(A - bP_\alpha)}{b})[q_\alpha - \rho_\alpha(A - bP_\alpha)] \quad (22)$$

Based on Stackelberg approach, the retailer determines the optimal amount of decision variables relevant to himself. The optimal amount of order quantity in accordance with the Eq. 8 is equal to:

$$\frac{d\pi_R^{\alpha}}{dq_\alpha} = 0 \rightarrow q_\alpha(W_\alpha, P_\alpha) = \frac{2\rho_\alpha(P_\alpha - W_\alpha)(A - bP_\alpha)}{P_\alpha} \quad (23)$$

$$\frac{d\pi_R^{\alpha}}{d\rho_\alpha} = 0 \rightarrow \rho_\alpha^* = \frac{2k\,p_\alpha^* + (p_\alpha^* - w_\alpha^*)^2(\mathrm{A} - \mathrm{b}\,p_\alpha^*)}{2k\,p_\alpha^*} \quad (24)$$

$$\frac{d\pi_R^{\alpha}}{dP_\alpha} = 0 \rightarrow P_\alpha = \frac{A + \sqrt{A(A + 8bW_\alpha)}}{4b} \quad (25)$$

In order to find the manufacturer's decision variables optimal solution, the retailer decision variables optimal amount substitutes for the manufacturer's objective function. In the following, the optimal value of the manufacturer's decision variables will be obtained.

4.2.2. Promotion cost sharing (P.C.Sh)

According to section 4.1.2, a member of the system who is responsible for advertising cost is willing to share this cost with another member so that this will increase the overall system profits and provide synchronization. So, in this section, the objective functions are defined as follows:

$$\pi_R^{\theta} = q_\theta(P_\theta - W_\theta) - \frac{P_\theta q_\theta^2}{4\rho_\theta(A - bP_\theta)} - (1-\theta)k(\rho_\theta - 1)^2 \quad (26)$$

$$\pi_M^{\theta} = (W_\theta - C)q_\theta - \theta k(\rho_\theta - 1)^2 \quad (27)$$

$$V_M^{\theta} = (W_\theta - c)q_\theta - \theta k(\rho_\theta - 1)^2 + (\frac{\tau.(A - bP_\theta)}{b})[q_\theta - \rho_\theta(A - bP_\theta)] \quad (28)$$

As previous sections, the optimal decision variables will be obtained as follows:

$$\frac{d\pi_R^{\theta}}{dq_\theta} = 0 \rightarrow q_\theta(W_\theta, P_\theta) = \frac{2\rho_\theta(P_\theta - W_\theta)(A - bP_\theta)}{P_\theta} \quad (29)$$

$$\frac{d\pi_R^{\theta}}{P_\theta} = 0 \rightarrow P_\theta = \frac{A + \sqrt{A(A + 8bW_\theta)}}{4b} \quad (30)$$

$$\frac{d\pi_R^{\theta}}{d\rho_\theta} = 0 \rightarrow \rho_\theta^* = \frac{2k\,p_\theta^*(\theta - 1) + (p_\theta^* - w_\theta^*)^2(\mathrm{A} - \mathrm{b}\,p_\theta^*)}{2k\,p_\theta^*(\theta - 1)} \quad (31)$$

The optimal decision variables values are presented in Table 2.

Proposition 1.

1.a. Considering Eqs. 10 and 18, we realize that $W_\theta^* = W_\alpha^*$ and consequently $P_\theta^* = P_\alpha^*$

1.b. Considering Eqs. 11 and 19 and $(1-\theta) \le 1$, we see that $\rho_\theta^* \ge \rho_\alpha^*$.

1.c. According to Eqs. 24 and 30 in the second scenario, it is observed that although the retailer is responsible for the cost of advertising, in general, the retail price is independent of advertising efforts as in the first scenario.

Table 2. The optimal values of decision variables in both scenarios

		First Scenario	Second Scenario
No promotion sharing	W	$\frac{5A(1-2\tau) + 32bc + 9A\tau^2 + 3(1+\tau)\sqrt{A(9A\tau^2 - 38A\tau + 17A + 64bc)}}{64b}$	W^*
	P	$\frac{A + \sqrt{A(A + 8bW_\alpha^*)}}{4b}$	$\frac{A + \sqrt{A(A + 8bW_\alpha)}}{4b}$

	ρ	$\frac{2(A-bP_\alpha^*)(P_\alpha^*-W_\alpha^*)(b(W_\alpha^*-c)+\tau(A-bP_\alpha^*))-\tau P_\alpha^*(A-bP_\alpha^*)^2}{2bkP_\alpha^*}+1$	$\frac{2k\,p_\alpha^*+(p_\alpha^*-w_\alpha^*)^2(\mathrm{A}-\mathrm{b}\,p_\alpha^*)}{2k\,p_\alpha^*}$
	q	$\frac{2\rho_\alpha^*(P_\alpha^*-W_\alpha^*)(A-bP_\alpha^*)}{P_\alpha^*}$	$\frac{2\rho_\alpha(P_\alpha-W_\alpha)(A-bP_\alpha)}{P_\alpha}$
	π_R^*	$q_\alpha^*(P_\alpha^*-W_\alpha^*)-\frac{P_\alpha^* q_\alpha^{*2}}{4\rho_\alpha^*(A-bP_\alpha^*)}$	π_R^*
	V_M^*	$(W_\alpha^*-c)q_\alpha^*-k(\rho_\alpha^*-1)^2+(\frac{\tau.(A-bP_\alpha^*)}{b})[q_\alpha^*-\rho_\alpha^*(A-bP_\alpha^*)]$	V_M^*
Promotion sharing	W	$\frac{5A(1-2\tau)+32bc+9A\tau^2+3(1+\tau)\sqrt{A(9A\tau^2-38A\tau+17A+64bc)}}{64b}$	W^*
	P	$\frac{A+\sqrt{A(A+8bW_\theta^*)}}{4b}$	$\frac{A+\sqrt{A(A+8bW_\theta)}}{4b}$
	ρ	$\frac{2(A-bP_\theta^*)(P_\theta^*-W_\theta^*)(b(W_\theta^*-c)+\tau(A-bP_\theta^*))-\tau P_\theta^*(A-bP_\theta^*)^2}{2(1-\theta)bkP_\theta^*}+1$	$\frac{2k\,p_\theta^*+(p_\theta^*-w_\theta^*)^2(\mathrm{A}-\mathrm{b}\,p_\theta^*)}{2k\,p_\theta^*}$
	q	$\frac{2\rho_\theta^*(P_\theta^*-W_\theta^*)(A-bP_\theta^*)}{P_\theta^*}$	$\frac{2\rho_\theta(P_\theta-W_\theta)(A-bP_\theta)}{P_\theta}$
	π_R^*	$q_\theta^*(P_\theta^*-W_\theta^*)-\frac{P_\theta^* q_\theta^{*2}}{4\rho_\theta^*(A-bP_\theta^*)}-\theta k(\rho_\theta^*-1)^2$	π_R^*
	V_M^*	$(W_\theta^*-c)q_\theta^*-(1-\theta)k(\rho_\theta^*-1)^2+(\frac{\tau.(A-bP_\theta^*)}{b})[q_\theta^*-\rho_\theta^*(A-bP_\theta^*)]$	V_M^*

Proof.

1.a. According to Eq. 10, the optimal wholesale prices in the no incentive and incentive policies are $W_\alpha^* = \left(5A(1-2\tau)+32bc+9A\tau^2+3(1+\tau)\sqrt{A(9A\tau^2-38A\tau+17A+64bc)}\right)/64b$ and $W_\theta^* = \left(5A(1-2\tau)+32bc+9A\tau^2+3(1+\tau)\sqrt{A(9A\tau^2-38A\tau+17A+64bc)}\right)/64b$, respectively. As a result, these optimal wholesale prices are equal to ($W_\theta^*=W_\alpha^*$). Sinceand $P_\alpha^* = \frac{A+\sqrt{A(A+8bW_\alpha^*)}}{4b}$). $P_\theta^*=P_\alpha^*$, then the retail prices will be equal to ($P_\theta^* = \frac{A+\sqrt{A(A+8bW_\theta^*)}}{4b}$

1.b. The optimal promotional efforts in the no incentive and incentive policies, respectively, are $\rho_\alpha^* = \frac{2(A-bP_\alpha^*)(P_\alpha^*-W_\alpha^*)(b(W_\alpha^*-c)+\tau(A-bP_\alpha^*))-\tau P_\alpha^*(A-bP_\alpha^*)^2}{2bkP_\alpha^*}+1$ and $\rho_\theta^* = \frac{2(A-bP_\theta^*)(P_\theta^*-W_\theta^*)(b(W_\theta^*-c)+\tau(A-bP_\theta^*))-\tau P_\theta^*(A-bP_\theta^*)^2}{2(1-\theta)bkP_\theta^*}+1$, according to Proposition 1.a and knowing that $(1-\theta)\le 1$, so $\rho_\theta^* \ge \rho_\alpha^*$.

5. Numerical Studies and Computations

In this section, the designed models are investigated by numerical examples. The two models will be also compared in two conditions of N.P.C.Sh and P.C.Sh.

In this section, we are going to prove that in the first scenario, $V_M^\theta \ge V_M^\alpha$ and $\pi_R^\theta \ge \pi_R^\alpha$ through numerical examples. We first assign a number to the problem parameters and obtain the optimal value of decision variables in each model and in both N.P.C.Sh and P.C.Sh states. Both the models are evaluated with different numerical examples. The problem model is solved using Lingo 11.0. Here, two numerical examples are given to assess the impact of sharing costs on the profit of channel members. A sensitivity analysis is also done on the examples. Parameters in these examples are set based on model assumptions (*e.g.,* $A>0, b>0, k>0$, $0\le\tau\le 1$ and $A-bP\ge 0$) as well as on positivity of the consumer surplus and feasibility of the problem. In the first numerical example, the value of parameters is considered as such: $A = 100, b = 2, c = 5, k = 100$. The results of the first scenario and the N.P.C.Sh state are shown in Table 3.

Table 3. The optimal values of decision variables in the first scenario and N.P.C.Sh case under the influence of parameter τ.

τ	W	Q	P	Π_R	V_M	ρ
0	17.74380	73.78190	36.99173	710.0746	642.6463	2.725154
0.1	17.13533	77.63029	36.67919	758.5977	647.6532	2.734325
0.2	16.43111	82.40228	36.31234	819.1295	655.6527	2.748914
0.3	15.60040	88.47952	35.87221	896.8198	667.6233	2.770602
0.4	14.59399	96.51261	35.32761	1000.528	685.0994	2.801962
0.5	13.32376	107.7454	34.62112	1147.346	710.6974	2.847275
0.6	11.59847	125.0656	33.62372	1377.301	749.5232	2.914658
0.7	8.765625	158.9741	31.87500	1836.896	814.7464	3.024478

Results of the first numerical example in N.P.C.Sh case in the first scenario are shown in Table 3. By changing the parameter τ, we have calculated the profit amounts of channel members, wholesale prices, retail prices, promotional effort, and order quantity. According to Table 3, we can say that when τ increases, the profit amounts of channel members, promotional effort, and order quantities also increase. Actually, an increase in the value of τ parameter in this supply chain, including promotional effort, has a positive impact on the increase of order quantity which is followed by increased profits of channel members. These results are shown in Figures 2 and 3. When τ increases, the wholesale and retail prices decrease (Figure 4).

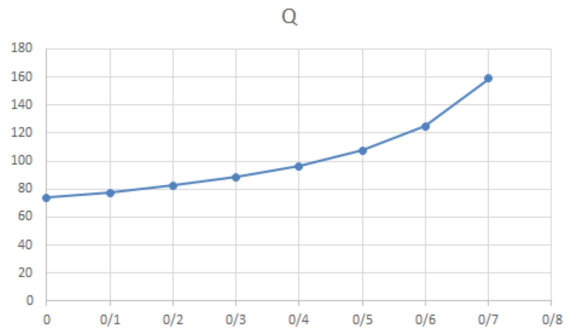


Figure 2. The effect of τ increase on order quantity in (N.P.C.Sh) case

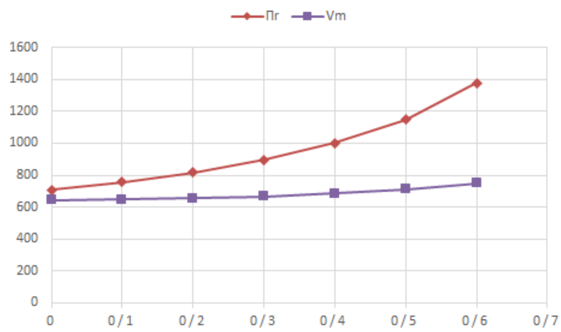


Figure 3. The effect of τ increase on profits of channel members in (N.P.C.Sh) case

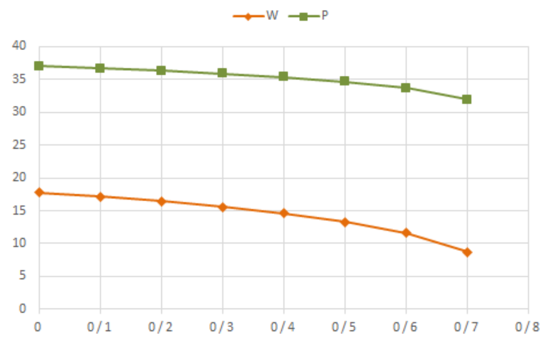


Figure 4. The effect of τ increase on wholesale and retail prices in N.P.C.Sh case

With respect to the question posed at the beginning of this article and the impact of consumer surplus on the retailer's profits, we recognize, according to Figure 3, that an increase in τ profits leads to the rise in retailer sales. But the important point in Figure 3 is the higher increase of the retailer's profit compared to the manufacturer's profit. The manufacturer's profit occurs from the increase in the order quantity and the further decline in wholesale prices compared to retail prices (Figure 4). For sensitivity analysis, the values of each parameter have been changed and the results are reviewed. First, we increased the value of parameter *A* whose results are given in Table 4; then, we have respectively reduced the values of parameters *b, c*, and *k*.

Table 4. The optimal values of the decision variables of the first scenario and N.P.C.Sh case under the effects of the parameter τ and *A*=200, *b*=2, *c*=5, and *k*=100

τ	W	Q	P	Π_R	V_M	ρ
0	31.38019	594.2521	71.84026	12021.74	8674.992	9.367493
0.1	29.82906	637.8674	71.00493	13132.37	8894.642	9.484009
0.2	28.00000	694.0800	70.00000	14575.68	9192.960	9.640000
0.3	25.78125	769.5389	68.75000	16533.06	9602.442	9.850098
0.4	22.96626	877.4003	67.11072	19366.18	10180.12	10.13909
0.5	19.07559	1050.273	64.73386	23976.83	11042.77	10.55593
0.6	12.00000	1438.720	60.00000	34529.28	12533.76	11.24000
0.7	5.000000	2111.599	54.58040	52346.95	0.0001824	12.79484

According to Table 4 and Figure 5, the increase in parameter *A* leads to a predictable increase in profits of channel members. An increase in the retailer's profit compared to the manufacturer's profit is completely obvious in this Figure. In sensitivity analysis, we see that for $\tau = 0.7$, retail prices have equaled production costs, and that causes the problem to become infeasible afterwards. The results of the sensitivity analysis for increasing parameter *b* are given in Table 5. In the first numerical example, the problem becomes infeasible for a reduction in parameter *b* which was predictable.

Table 5. The optimal values of the decision variables of the first scenario and N.P.C.Sh case under the effects of the parameter τ and *A*=100, *b*=1, *c*=5, and *k*=100

τ	W	Q	P	Π_R	V_M	ρ
0	31.38019	164.4225	71.84026	3326.272	2587.123	5.183746
0.1	29.82906	176.2811	71.00493	3629.264	2647.861	5.242004
0.2	28.00000	191.5200	70.00000	4021.920	2730.240	5.320000
0.3	25.78125	211.9160	68.75000	4552.882	2843.115	5.425049
0.4	22.96626	240.9842	67.11072	5319.058	3001.985	5.569546
0.5	19.07559	287.4424	64.73386	6562.060	3238.488	5.777965
0.6	12.00000	391.6800	60.00000	9400.320	3645.440	6.120000
0.7	5.000000	569.1584	54.58040	14109.55	4657.438	6.897418

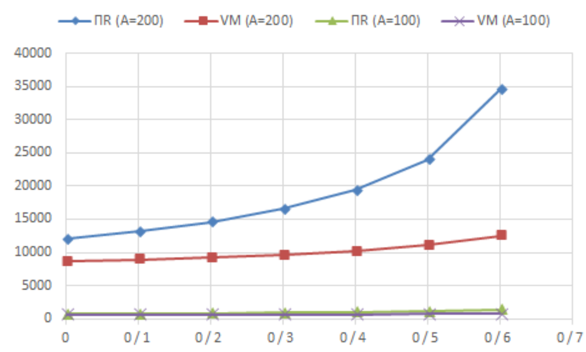


Figure 5. The effect of the increase of parameter *A* on profits of channel members in the first scenario and N.P.C.Sh case

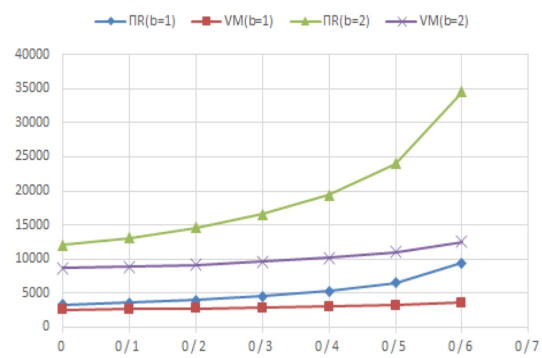


Figure 6. The effect of the decrease of parameter *b* on profits of channel members in the first scenario and N.P.C.Sh case

Separately, a reduction has occurred in the value of parameters *c* and *K* whose results are respectively given in Table 6, Figure 7, Table 7, and Figure 8.

Table 6. The optimal values of the decision variables of the first scenario and N.P.C.Sh case under the effects of the parameter τ and A=100, b=2, c=2, and k=100

τ	W	Q	P	Π_R	V_M	ρ
0	15.27188	103.8575	35.69584	1060.590	906.4355	3.172437
0.1	14.45936	111.5619	35.25377	1159.932	928.1765	3.206519
0.2	13.49683	121.4925	34.71871	1289.149	957.3495	3.251691
0.3	12.32069	134.8756	34.04686	1465.165	997.0894	3.312234
0.4	10.80855	154.2358	33.15100	1723.003	1053.011	3.395602
0.5	8.651074	186.3927	31.80095	2157.484	1137.153	3.517318
0.6	2.750155	302.4303	27.50013	3742.571	1294.089	3.733750
0.7	2.000000	358.9005	26.86141	4461.385	1655.336	4.189673

Table 7. The optimal values of the decision variables of the first scenario and N.P.C.Sh case under the effects of the parameter τ and A=100, b=2, c=2, and k=100

τ	W	Q	P	Π_R	V_M	ρ
0	17.74380	120.4894	36.99173	1159.586	940.2618	4.450307
0.1	17.13533	126.8695	36.67919	1239.760	948.4414	4.468649
0.2	16.43111	134.8283	36.31234	1340.276	961.5226	4.497828
0.3	15.60040	145.0239	35.87221	1469.948	981.1263	4.541203
0.4	14.59399	158.5806	35.32761	1643.975	1009.806	4.603925
0.5	13.32376	177.6492	34.62112	1891.729	1051.940	4.694550
0.6	11.59847	207.2220	33.62372	2282.059	1116.115	4.829316
0.7	8.765625	265.3857	31.87500	3066.449	1224.597	5.048955

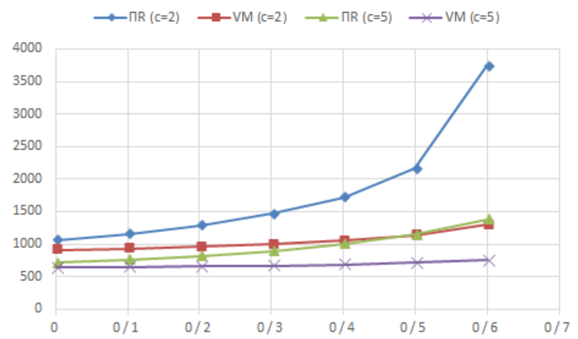


Figure 7. The effect of the decrease of parameter c on profits of channel members in the first scenario and N.P.C.Sh case

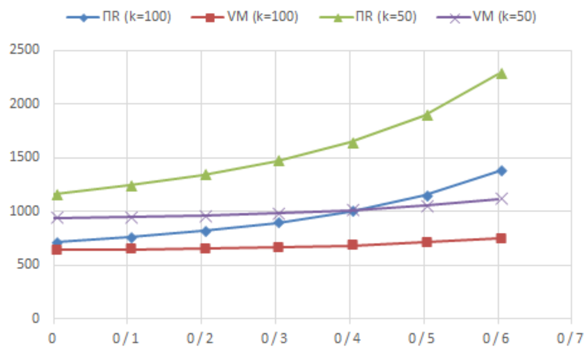


Figure 8. The effect of the decrease of parameter k on profits of channel members in the first scenario and N.P.C.Sh case

Finally, to sum up, we can say that by increasing τ, members' profits also increase. Moreover, in the sensitivity analysis, with constant parameters of k, c, b, and increasing A, members' profits increase more sharply, as compared to when A does not increase. Similarly, in cases when each of the parameters of c, b, and k reduce, the same results can be achieved. But τ can be increased to the extent that the problem is feasible. In this instance, it was observed that when τ is greater than 0.6, amount of wholesale prices becomes less than production costs and thus, the problem becomes infeasible for τ of more than 0.6. Now, we study the first numerical example in (P.C.Sh) state. For the parameters $A=100, b=2, c=5, k=100$, the effect of changes in parameters τ and θ on the values of decision variables is obtained (See Table 8).

Table 8. The optimal values of decision variables in the first scenario and P.C.Sh case affected by changes in the parameters θ and τ

	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	17.74380	73.78190	36.99173	710.0746	642.6463	2.725154
	0.1	17.74380	78.97163	36.99173	723.2776	675.7147	2.916837
	0.2	17.74380	85.45878	36.99173	729.4476	717.0502	3.156442

	0.3	17.74380	93.79940	36.99173	720.5087	770.1958	3.464505
	0.4	17.74380	104.9202	36.99173	679.0649	841.0566	3.875256
	0.5	17.74380	120.4894	36.99173	564.3550	940.2618	4.450307
$\tau = 0.2$	0	16.43111	82.40228	36.31234	819.1295	655.6527	2.748914
	0.1	16.43111	88.22739	36.31234	839.2730	689.6383	2.943238
	0.2	16.43111	95.50877	36.31234	853.8318	732.1202	3.186142
	0.3	16.43111	104.8706	36.31234	855.2107	786.7398	3.498448
	0.4	16.43111	117.3529	36.31234	826.7051	859.5660	3.914856
	0.5	16.43111	134.8283	36.31234	728.5362	961.5226	4.497828
$\tau = 0.4$	0	14.59399	96.51261	35.32761	1000.528	685.0994	2.801962
	0.1	14.59399	103.4091	35.32761	1031.935	721.1779	3.002181
	0.2	14.59399	112.0296	35.32761	1059.919	766.2761	3.252453
	0.3	14.59399	123.1132	35.32761	1077.491	824.2595	3.574232
	0.4	14.59399	137.8913	35.32761	1068.707	901.5706	4.003271
	0.5	14.59399	158.5806	35.32761	994.5609	1009.806	4.603925
$\tau = 0.6$	0	11.59847	125.0656	33.62372	1377.301	749.5232	2.914658
	0.1	11.59847	134.1941	33.62372	1432.571	790.2556	3.127398
	0.2	11.59847	145.6047	33.62372	1488.931	841.1711	3.393323
	0.3	11.59847	160.2755	33.62372	1540.611	906.6339	3.735226
	0.4	11.59847	179.8366	33.62372	1573.149	993.9176	4.191097
	0.5	11.59847	207.2220	33.62372	1548.876	1116.115	4.829316

In Table 8, optimal values of channel members' profits, wholesale price, retail price, promotional effort, and order quantity are calculated for different values of τ. With regard to Proposition 1, with constant τ and increased θ, wholesale and retail prices do not change, but other decision variables increase. The effect of consumer surplus and increased value of τ on the retailer's profits, manufacturer's profits, order quantity, and promotional effort are respectively shown in Figures 9, 10, 11, and 12. When sharing costs, with constant τ and increased θ, we see that the profit increases for both of the channel members, which is due to the increase in the amount of orders. When τ increases, the wholesale and retail prices decrease. With respect to Figure 9, by increasing τ and θ, the retail profits begin to decrease later despite the increased percent of sharing costs. If, as in the case of sensitivity analysis in N.P.C.Sh case, we increase parameter A, the obtained results are shown in Table 9.

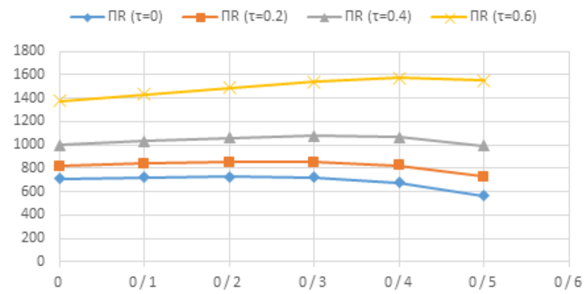


Figure 9. Impact of τ increase on the retailer's profit in the first scenario and P.C.Sh case

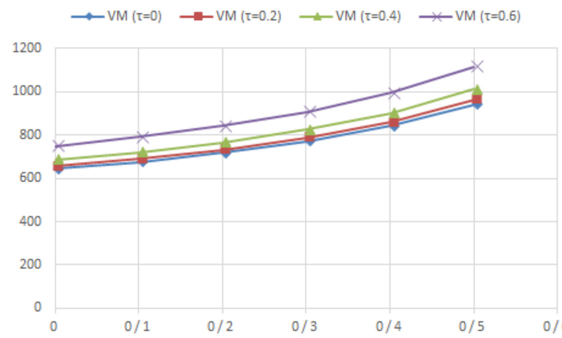


Figure 10. Impact of τ increase on the manufacturer's profit in the first scenario and P.C.Sh case

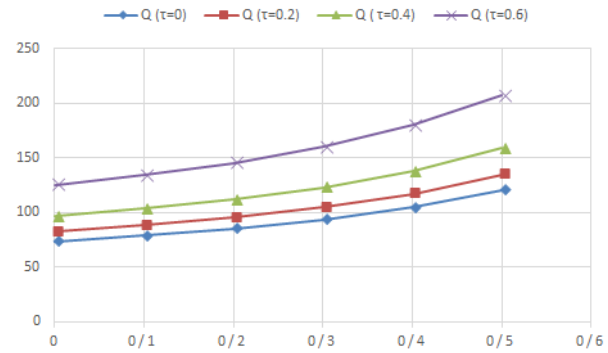


Figure 11. Impact of τ increase on order quantity in the first scenario and P.C.Sh case

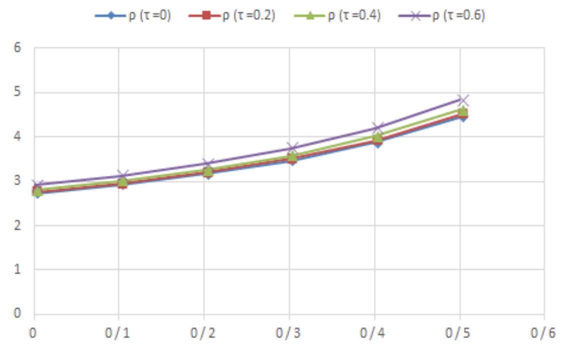


Figure 12. . Impact of τ increase on promotional efforts in the first scenario and P.C.Sh case

Table 9. The optimal values of decision variables in the first scenario and P.C.Sh case affected by changes in the two parameters of τ and θ with *A*=200, *b*=2, *c*=5, and *k*=100

	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	31.38019	594.2521	71.84026	12021.74	8674.992	9.367493
	0.1	31.38019	653.2315	71.84026	12350.51	9452.936	10.29721
	0.2	31.38019	726.9557	71.84026	12518.37	10425.37	11.45937
	0.3	31.38019	821.7440	71.84026	12337.28	11675.63	12.95356
	0.4	31.38019	948.1284	71.84026	11401.23	13342.65	14.94582
	0.5	31.38019	1125.067	71.84026	8757.146	15676.49	17.73499
$\tau = 0.2$	0	28.00000	694.0800	70.00000	14575.68	9192.960	9.640000
	0.1				*infeasible*		
	0.2	28.00000	849.6000	70.00000	15508.80	11059.20	11.80000
	0.3	28.00000	960.6857	70.00000	15604.02	12392.23	13.34286
	0.4	28.00000	1108.800	70.00000	14990.40	14169.60	15.40000
	0.5	28.00000	1316.160	70.00000	12709.44	16657.92	18.28000
$\tau = 0.4$	0	22.96626	877.4003	67.11072	19366.18	10180.12	10.13909
	0.1	22.96626	965.2740	67.11072	20274.60	11108.15	11.15455
	0.2	22.96626	1075.116	67.11072	21120.12	12268.20	12.42387
	0.3	22.96626	1216.342	67.11072	21733.73	13759.68	14.05585
	0.4	22.96626	1404.643	67.11072	21723.27	15748.32	16.23182
	0.5	22.96626	1668.264	67.11072	20117.71	18532.42	19.27819
$\tau = 0.6$	0	12.00000	1438.720	60.00000	34529.28	12533.76	11.24000
	0.1	12.00000	1584.356	60.00000	36730.00	13698.84	12.37778
	0.2	12.00000	1766.400	60.00000	39116.80	15155.20	13.80000
	0.3	12.00000	2000.457	60.00000	41591.12	17027.66	15.62857
	0.4	12.00000	2312.533	60.00000	43849.96	19524.27	18.06667
	0.5	12.00000	2749.440	60.00000	45015.04	23019.52	21.48000

In Table 9, with an increase in τ, we observe an increase in channel members' profits and a reduction in the amounts of W and P as shown in Table 8. With an increase in θ, the manufacturer's and retailer's profits, the amount of order quantity, and promotional effort also increase. But when θ is more than 0.4, the retailer's profits decrease. By comparing Tables 8 and 9, we come to the conclusion that an increase in parameter A leads to increase in decision variables of the problem. For example, in constant state of $\tau = 0.4$, the difference between Tables 8 and 9 is shown in Figures 13, 14, and 15. By reducing the value of parameters *c, b,* and *k*, we observe some increase in problem decision variables compared to Table 8. For reliability of the obtained results, we have defined another numerical example with $A = 320$, $b = 5$, $c = 8$, $k = 180$. We have reviewed it first in the N.P.C.Sh state and then in the P.C.Sh case. Results of the second numerical example are shown in Table 10 N.P.C.Sh case.

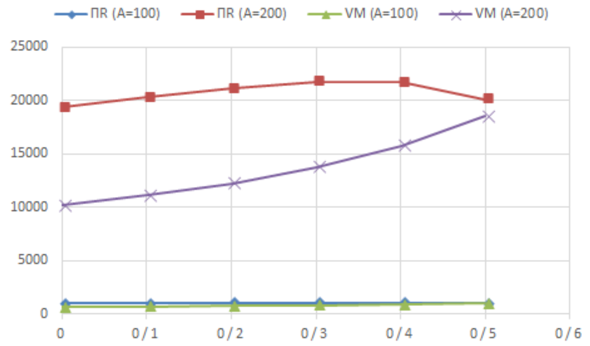


Figure 13. Impact of increase in the parameter *A* on profits of channel members in the first scenario and P.C.Sh case

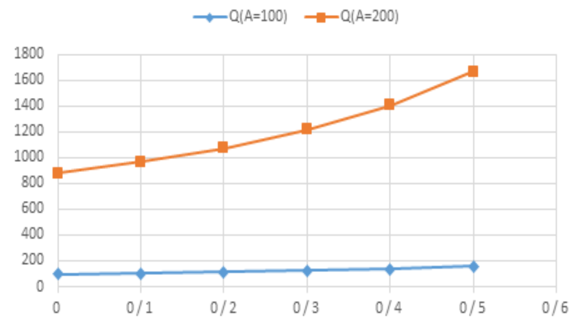


Figure 14. Impact of increase in the parameter *A* on order quantity in the first scenario and P.C.Sh case

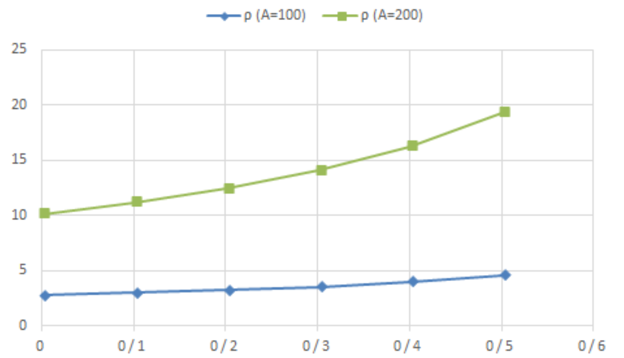
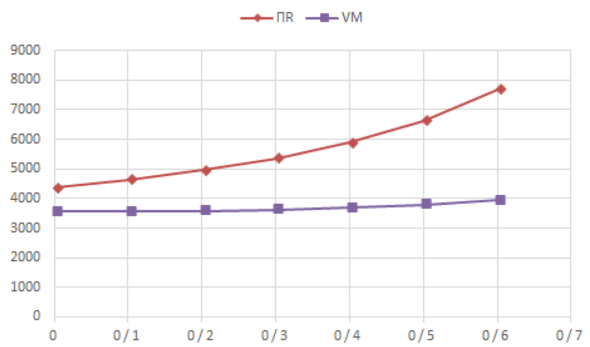


Figure 15. Impact of increase in the parameter *A* on promotional effort in the first scenario and P.C.Sh case

Table 10. The optimal values of decision variables in the second numerical example and in the first scenario and N.P.C.Sh case under the influence of the parameter τ

τ	W	Q	P	Π_R	V_M	ρ
0	24.00000	364.4444	48.00000	4373.333	3555.556	4.555556
0.1	23.31656	380.5837	47.65644	4631.680	3561.844	4.559388
0.2	22.53134	400.5340	47.25705	4951.743	3582.902	4.572200
0.3	21.61460	425.7338	46.78420	5357.775	3623.120	4.596569
0.4	20.52097	458.5137	46.21044	5889.489	3689.159	4.636306
0.5	19.17527	502.9930	45.48913	6617.844	3791.812	4.697406
0.6	17.43543	567.4932	44.52952	7687.858	3950.356	4.790242
0.7	14.95916	673.4372	43.10522	9477.303	4205.464	4.935959

According to Table 10 and Figures 16, 17 and 18, channel members' profits and the amount of orders increase and the wholesale and retail prices decrease. These results were also obtained in the first instance. Next, we conducted a sensitivity analysis. First, we discuss an increase in the value of parameter *A* (Table 11) and then, we separately address reductions in the values of parameters *c*, *b*, and *k*.



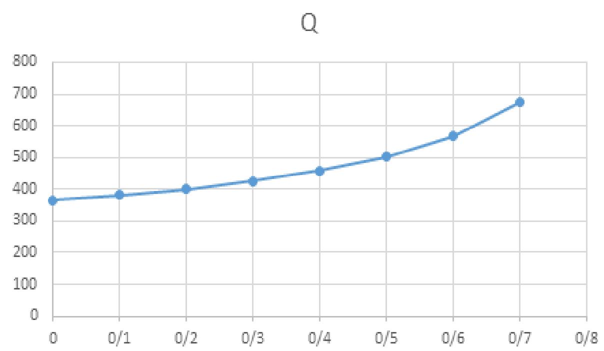


Figure 16. The effect of τ increase on the profit of channel members in the second numerical example in the first scenario and N.P.C.Sh case

Figure 17. The effect of τ increase on the order quantity in the second numerical example in the first scenario and N.P.C.Sh case

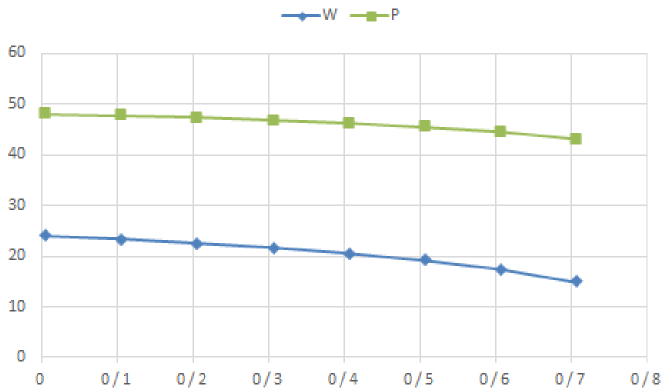


Figure 18. The effect of τ increase on the wholesale and retail price in the second numerical example in the first scenario and N.P.C.Sh case

Table 11. The optimal values of decision variables in the first scenario and N.P.C.Sh case influenced by changes in the parameter τ and with A = 400, b = 5, c = 8, and k = 180

τ	W	Q	P	Π_R	V_M	ρ
0	28.39008	772.5821	59.18677	11896.49	8980.604	7.133880
0.1	27.41653	813.8558	58.68671	12724.71	9064.539	7.166488
0.2	26.28978	865.5191	58.09975	13766.07	9198.850	7.218360
0.3	24.96064	931.9274	57.39553	15113.48	9400.305	7.295472
0.4	23.35038	1020.523	56.52417	16927.31	9695.397	7.406978
0.5	21.31802	1145.554	55.39380	19517.81	10129.67	7.568088
0.6	18.55755	1340.084	53.79796	23612.55	10792.76	7.807673
0.7	14.02500	1723.659	51.00000	31866.15	11917.72	8.198142

Table 12. The optimal values of decision variables in the first scenario and N.P.C.Sh case influenced by changes in the parameter τ and with A=320, b=3, c=8, and k=180

τ	W	Q	P	Π_R	V_M	ρ
0	35.67857	770.0478	77.79365	16215.31	11954.84	8.210711
0.1	34.20945	818.6159	77.02156	17523.34	12160.02	8.279834
0.2	32.49454	880.2055	76.10505	19193.10	12450.78	8.376812
0.3	30.44620	960.9067	74.98756	21400.04	12858.81	8.511043
0.4	27.91509	1071.819	73.56993	24466.86	13436.62	8.697581
0.5	24.60415	1236.563	71.64814	29086.43	14281.34	8.963302
0.6	19.70664	1523.440	68.64442	37276.89	15613.09	9.366931
0.7	8.000000	2451.753	60.39763	64233.02	18473.44	10.17989

Table 13. The optimal values of decision variables in the first scenario and N.P.C.Sh case influenced by changes in the parameter τ and with A=320, b=5, c=5, and k=180

τ	W	Q	P	Π_R	V_M	ρ
0	21.57125	489.8111	46.76166	6169.273	4827.815	5.274580
0.1	20.70305	519.7899	46.30672	6654.266	4901.538	5.313263
0.2	19.69075	557.6460	45.76750	7270.798	5006.861	5.368044
0.3	18.48369	606.9986	45.11148	8081.515	5155.263	5.444295
0.4	16.99627	674.3824	44.28216	9200.563	5365.588	5.550569
0.5	15.06080	773.4931	43.16515	10869.26	5672.265	5.701981
0.6	12.23784	942.6031	41.44820	13766.89	6151.481	5.930842
0.7	5.000000	1517.450	36.39608	23821.00	7130.017	6.372692

Table 14. The optimal values of decision variables in the first scenario and N.P.C.Sh case influenced by changes in the parameter τ and with A=320, b=5, c=8, and k=100

τ	W	Q	P	Π_R	V_M	ρ
0	24.00000	592.0000	48.00000	7104.000	5376.000	7.400000
0.1	23.31656	618.2726	47.65644	7524.340	5386.215	7.406899
0.2	22.53134	650.8796	47.25705	8046.728	5420.430	7.429960
0.3	21.61460	692.2249	46.78420	8711.513	5485.805	7.473824
0.4	20.52097	746.2076	46.21044	9584.841	5593.231	7.545350
0.5	19.17527	819.7243	45.48913	10785.06	5760.408	7.655330
0.6	17.43543	926.7130	44.52952	12554.22	6019.051	7.822436
0.7	14.95916	1103.039	43.10522	15523.10	6436.280	8.084726

Looking at Tables 11, 12, 13 and 14, we see that by changes made on the parameters of the problem, members' profit has increased. Results of the second numerical example when sharing the costs of advertising are seen in Table 15.

Table 15. The optimal values of decision variables in the first scenario and P.C.Sh case affected by variations in the two parameters of τ and θ and with A = 320, b = 5, c = 8, and k = 180

	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	24.00000	364.4444	48.00000	4373.333	3555.556	4.555556
	0.1	24.00000	396.0494	48.00000	4471.660	3808.395	4.950617
	0.2	24.00000	435.5556	48.00000	4515.556	4124.444	5.444444
	0.3	24.00000	486.3492	48.00000	4442.993	4530.794	6.079365
	0.4	24.00000	554.0741	48.00000	4120.494	5072.593	6.925926
	0.5	24.00000	648.8889	48.00000	3235.556	5831.111	8.111111
$\tau = 0.2$	0	22.53134	400.5340	47.25705	4951.743	3582.902	4.572200
	0.1	22.53134	435.3042	47.25705	5098.033	3838.114	4.969111
	0.2	22.53134	478.7670	47.25705	5201.141	4157.129	5.465250
	0.3	22.53134	534.6477	47.25705	5203.499	4567.292	6.103143
	0.4	22.53134	609.1553	47.25705	4978.775	5114.175	6.953666
	0.5	22.53134	713.4660	47.25705	4226.654	5879.812	8.144400
$\tau = 0.4$	0	20.52097	458.5137	46.21044	5889.489	3689.159	4.636306
	0.1	20.52097	498.4712	46.21044	6108.894	3953.614	5.040340
	0.2	20.52097	548.4180	46.21044	6300.509	4284.182	5.545382
	0.3	20.52097	612.6354	46.21044	6411.944	4709.198	6.194722
	0.4	20.52097	698.2586	46.21044	6324.406	5275.886	7.060509
	0.5	20.52097	818.1310	46.21044	5748.501	6069.249	8.272611
$\tau = 0.6$	0	17.43543	567.4932	44.52952	7687.858	3950.356	4.790242
	0.1	17.43543	617.3849	44.52952	8044.499	4237.675	5.211380
	0.2	17.43543	679.7494	44.52952	8400.514	4596.823	5.737803
	0.3	17.43543	759.9324	44.52952	8711.655	5058.585	6.414632
	0.4	17.43543	866.8430	44.52952	8869.976	5674.268	7.317071
	0.5	17.43543	1016.518	44.52952	8599.079	6536.225	8.580485

Looking at Table 16 and comparing it with Table 15, we see that as the first numerical example, an increase in the parameter A improves the profit amount of the channel members.

Finally, we proved through these numerical examples that consumer surplus alone increases the order quantity as well as retailer's and manufacturer's profits. When sharing advertising costs, members' profits increase more sharply compared to when the costs are not shared. Consumer surplus has a positive effect on willingness of the manufacturer towards advertising (Fig. 12). The primary point about this model is that, although consumer surplus profit is for the manufacturer, it has increased the retailer's profit more than the manufacturer's profit in both the N.P.C.Sh and P.C.Sh cases. Hence, for better distribution of profits between these two members, some changes should be made in the systems or new contracts must be defined.

Table 16. The optimal values of decision variables in the first scenario and P.C.Sh case affected by variations in the two parameters of τ and θ and with A=400, b=5, c=8, and k=180

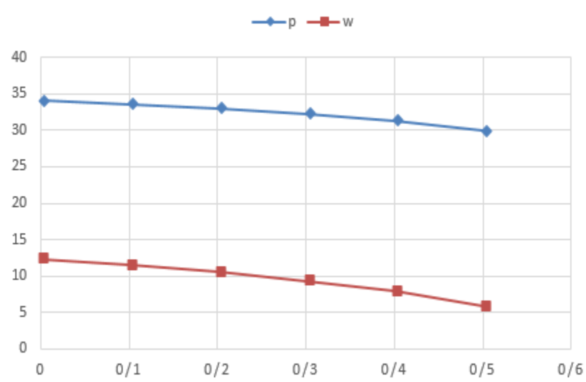
	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	28.39008	772.5821	59.18677	11896.49	8980.604	7.133880
	0.1	28.39008	846.3915	59.18677	12196.93	9733.093	7.815422
	0.2	28.39008	938.6532	59.18677	12337.33	10673.71	8.667350
	0.3	28.39008	1057.275	59.18677	12133.92	11883.06	9.762686
	0.4	28.39008	1215.438	59.18677	11190.85	13495.54	11.22313
	0.5	28.39008	1436.867	59.18677	8580.556	15753.01	13.26776
$\tau = 0.2$	0	26.28978	865.5191	58.09975	13766.07	9198.850	7.218360
	0.1	26.28978	948.3651	58.09975	14224.45	9972.210	7.909289
	0.2	26.28978	1051.923	58.09975	14555.74	10938.91	8.772950
	0.3	26.28978	1185.068	58.09975	14587.12	12181.81	9.883372
	0.4	26.28978	1362.595	58.09975	13938.46	13839.01	11.36393
	0.5	26.28978	1611.133	58.09975	11704.57	16159.09	13.43672
$\tau = 0.4$	0	23.35038	1020.523	56.52417	16927.31	9695.397	7.406978
	0.1	23.35038	1118.606	56.52417	17641.99	10516.38	8.118864
	0.2	23.35038	1241.209	56.52417	18278.78	11542.62	9.008722
	0.3	23.35038	1398.842	56.52417	18678.64	12862.06	10.15283
	0.4	23.35038	1609.019	56.52417	18478.76	14621.32	11.67830
	0.5	23.35038	1903.267	56.52417	16791.53	17084.28	13.81396
$\tau = 0.6$	0	18.55755	1340.084	53.79796	23612.55	10792.76	7.807673
	0.1	18.55755	1469.911	53.79796	24870.26	11719.65	8.564082
	0.2	18.55755	1632.195	53.79796	26152.74	12878.26	9.509592
	0.3	18.55755	1840.847	53.79796	27328.75	14367.90	10.72525
	0.4	18.55755	2119.048	53.79796	28069.18	16354.09	12.34612
	0.5	18.55755	2508.530	53.79796	27516.83	19134.75	14.61535

We will now analyze the second model. In the second model, it was assumed that the retailer is responsible for paying the advertising costs. The results of the first example are presented in Table 17.

Table 17. The optimal values of decision variables in the second scenario and N.P.C.Sh case under the influence of parameter τ

τ	W	Q	P	Π_R	V_M	ρ
0	12.34620	130.3303	34.06166	928.2011	957.4326	3.206557
0.1	11.52126	145.8153	33.57798	1041.985	1008.088	3.379326
0.2	10.55198	166.3709	32.99511	1193.062	1073.585	3.595918
0.3	9.380719	195.1247	32.26785	1404.317	1161.063	3.878552
0.4	7.899738	238.7825	31.30807	1724.519	1283.817	4.271439
0.5	5.850626	316.1926	29.89298	2289.309	1471.604	4.888056

The effect of consumer surplus and increased value of τ on the retail price and wholesale price, retailer's profits, manufacturer's profits, and order quantity are respectively shown in Figures 19, 20, and 21.



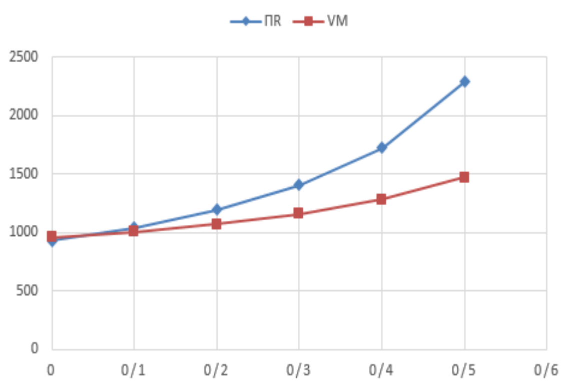


Figure 19. The effect of τ increase on wholesale and retail prices in the second scenario and N.P.C.Sh case

Figure 20. The effect of τ increase on profits of channel members in the second scenario and N.P.C.Sh case

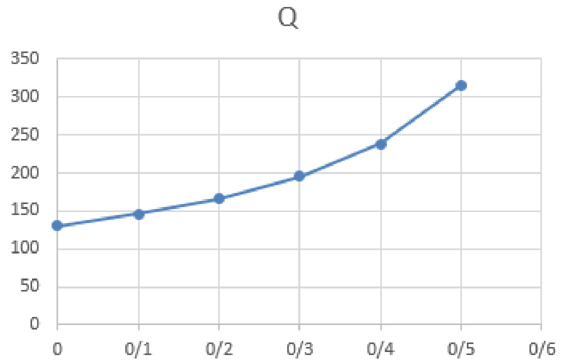


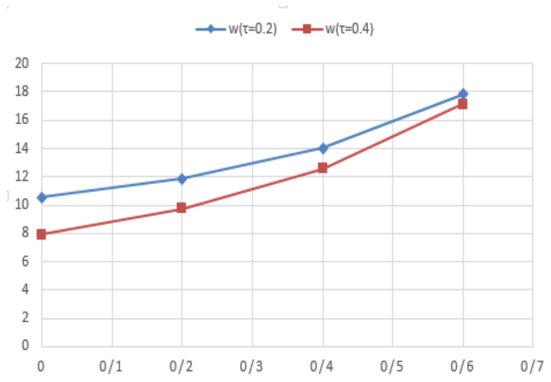
Figure 21. The effect of τ increase on order quantity in the second scenario and N.P.C.Sh case

In the comparison of the first and second models in the mode of N.P.C.Sh, it is observed that in the second scenario in which the retailer is responsible for advertising costs, the wholesale and retail prices have fallen compared to the first scenario. As a result, the order quantity rate increases. Each member gains higher profits. According to Table 3, by increasing the parameter τ, wholesale and retail prices will be reduced to a greater extent. We will see an increase in the profits of the system members. In Table 18, the optimal value of the decision variables of the second model in the mode of P.C.Sh is given.

Table 18. The optimal values of the decision variables of the second scenario and P.C.Sh case under the effects of the parameter τ

	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	12.34620	130.3303	34.06166	928.2011	957.4326	3.206557
	0.2	13.37583	132.1744	34.65052	903.4906	981.4461	3.506228
	0.4	15.13215	126.5207	35.62042	817.5038	962.8652	3.824273
	0.6	18.39265	103.9948	37.32068	613.8231	837.1882	4.042966
$\tau = 0.2$	0	10.55198	166.3709	32.99511	1193.062	1073.585	3.595918
	0.2	11.87675	163.1484	33.78776	1124.048	1077.228	3.879510
	0.4	14.03046	148.7353	35.01692	968.8090	1022.410	4.140871
	0.6	17.83118	113.6762	37.03629	674.8936	844.1096	4.227569
$\tau = 0.4$	0	7.899738	238.7825	31.30807	1724.519	1283.817	4.271439
	0.2	9.760267	219.3811	32.50642	1525.955	1240.241	4.480451
	0.4	12.56172	184.2619	34.18624	1212.454	1117.688	4.605167
	0.6	17.13583	126.8394	36.67945	758.4355	859.4110	4.467761

As can be seen, in the cost sharing mode, unlike the first model and Table 8, by increasing the sharing percentage due to rising the prices and reducing the product order quantity, the system members' profit decreased remarkably.



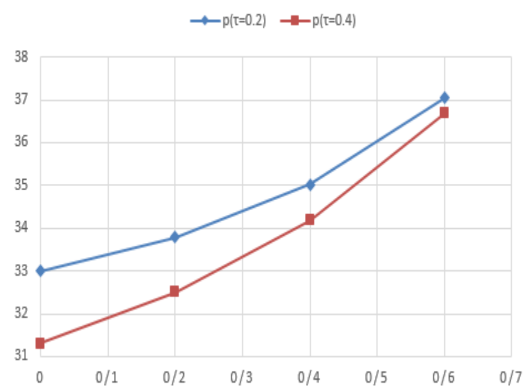
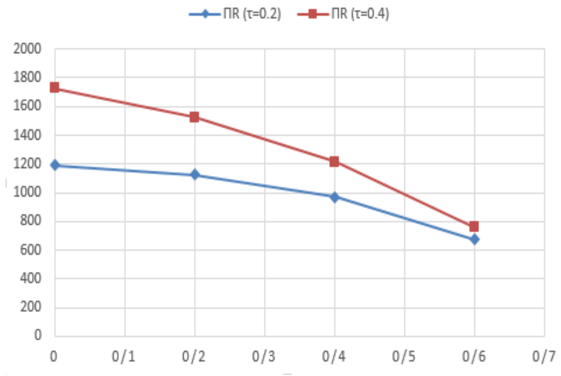


Figure 22. Impact of τ increase on wholesale price in the second scenario and P.C.Sh case

Figure 23. Impact of τ increase on retail price in the second scenario and P.C.Sh case



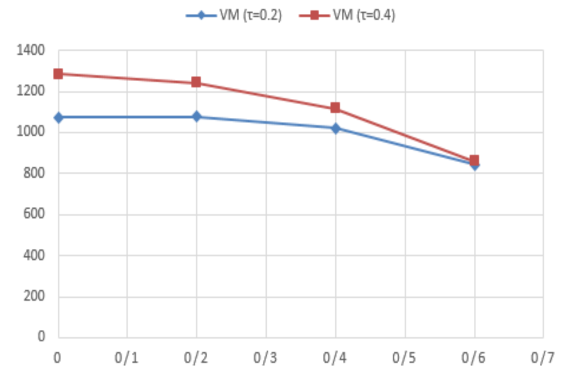


Figure 24. Impact of τ increase on retailer's profit in the second scenario and P.C.Sh case

Figure 25. Impact of τ increase on manufacturer's profit in the second scenario and P.C.Sh case

In order to ensure the results of the model, the second scenario will be investigated with numerical example of A = 320, b = 5, c = 8, k = 18 like the first scenario, and optimal values of the second model in the cases of N.P.C.Sh and P.C.Sh are presented in Tables 19 and 20.

Table 19. The optimal values of decision variables in the second scenario and (P.C.Sh) case influenced by changes in the parameter τ and with A=320, b=5, c=8, and k=100

τ	W	Q	P	Π_R	V_M	ρ
0	16.52575	719.6121	44.01471	5803.114	6135.229	5.765373
0.1	15.61770	800.2767	43.49121	6492.558	6457.097	6.088495
0.2	14.55992	905.1504	42.86852	7393.457	6866.662	6.486506
0.3	13.29918	1047.421	42.10697	8622.195	7402.215	6.992931
0.4	11.74425	1252.808	41.13595	10405.93	8130.560	7.668814
0.5	9.710625	1581.476	39.80630	13275.15	9184.380	8.645857

As expected, according to the results of the first example, in the case of N.P.C.Sh, as in the first scenario, by increasing the τ value, the amount of demand and profits increases. However, in the case of P.C.Sh, as stated in the previous example, sharing the advertising costs has no positive effect on the coordination of system members. There is no improvement on the system members' profit. According to Table 20, product selling prices increase. Consequently, the demand of product decreases, resulting in a reduction in profits compared to the case of N.P.C.Sh.

Table 20. The optimal values of decision variables in the second scenario and P.C.Sh case affected by variations in the two parameters of τ and θ and with A = 320, b = 5, c = 8, and k = 180

	θ	W	Q	P	Π_R	V_M	ρ
$\tau = 0$	0	16.52575	719.6121	44.01471	5803.114	6135.229	5.765373
	0.2	18.02916	724.9050	44.86058	5630.434	6246.515	6.332475
	0.4	20.46072	685.6935	46.17852	5046.764	6030.572	6.908640
	0.6	24.81137	549.4060	48.40315	3688.723	5048.258	7.227189
$\tau = 0.2$	0	14.55992	905.1504	42.86852	7393.457	6866.662	6.486506
	0.2	16.40355	881.0734	43.94483	6931.893	6818.460	7.009861
	0.4	19.29552	793.5072	45.55430	5907.131	6343.793	7.462952
	0.6	24.27728	590.7665	48.13834	3993.084	5017.112	7.513954
$\tau = 0.4$	0	11.74425	1252.808	41.13595	10405.93	8130.560	7.668814
	0.2	14.16271	1149.120	42.63094	9193.950	7758.889	8.052756
	0.4	17.76636	958.9177	44.71452	7243.300	6840.513	8.250309
	0.6	23.62062	645.4772	47.80974	4398.739	5019.836	7.879949

6. Conclusion

As mentioned at the beginning of this article, promotional efforts and consumer surplus have a significant impact on consumer choice and growth of product sales. In this article, we examined the impact of consumer surplus profit and advertising costs on the retailer's and manufacturer's profits, the amount of order quantity, wholesale and retail prices, and the promotional effort value. Due to uncertainties in the real world, customer demand is considered as non-conclusive. Two models were presented in this paper, and two conditions of N.P.C.Sh and P.C.Sh were considered. In this two-level supply chain, the retailer follows the manufacturer and sets the retail price and order quantity. Further, the manufacturer determines the wholesale price.

In the first mode, the retailer determines the advertising efforts. In the latter case, this variable will be determined by the manufacturer's profit function.

Using two numerical examples, we proved that an increase in the socially responsible manufacturer's concern (τ) leads to increase in profits of channel members in both the N.P.C.Sh and P.C.Sh cases. Moreover, sharing advertising costs leads to an increase in profits. In addition, as a sensitivity analysis, we separately increased the value of parameter A and reduced the value of parameters c, b, and k which led to an increase in the manufacturer's and retailer's profits.

In the first scenario according to Proposition 1 and the numerical examples, with constant value of τ and an increase of θ, the W and P values remain unchanged and the increase in τ leads to increased profits. But as one might predict, by increasing the cost sharing fraction, the retailer's profits decrease after a point. As a result, the retailer accepts that the increase in the sharing fraction reduces his profits. According to Figure 9, when there is no consumer surplus profit, the value of $\theta = 0.3$ is acceptable by the retailer. However, considering the consumer surplus profit, the acceptable value of cost sharing fraction by the retailer becomes $\theta = 0.4$. This θ value is obtained for $\tau = 0.6$. The τ value cannot be greater than 0.6 because the problem becomes infeasible due to a lower wholesale price compared to production costs. However, in the second scenario, given that the retailer is responsible for the advertising costs, in the mode of N.P.C.Sh. The results are similar to the first scenario. In fact, by increasing the τ value, the amount of ordering and profits of the members of the system increases. However, in the case of P.C.Sh, given that the purpose of this mode is to create more coordination between the two members of the system, the profit will increase for the members. In the second scenario, the opposite results have been achieved. By sharing the advertising costs and increasing the share percentage, wholesale prices increase, leading to a corresponding increase in retail price. According to these cases, members' profits will also decrease. According to the obtained results, it can be concluded that the agreement of sharing the advertising cost has no positive effect on the system. As a result, in order to improve the profitability of the system members, other coordination mechanisms must be examined.

For future research, we can either study the CSR effect on this chain using another method or use other contracts. We should also consider inventory control and inventory costs in the supply chain. It is necessary to implement other coordination mechanisms or a combination of mechanisms on the models of this article. To reach new and important topics, such as the sustainable supply chain, the impact on the environment and social factors can be implemented in the model.

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Appendix A.

Calculate the consumer surplus:

$$M = \int_0^{q_\alpha} \frac{x}{2\rho_\alpha (A - bP_\alpha)} dx$$

$$M = \frac{q_\alpha^2}{4\rho_\alpha (A - bP_\alpha)} \rightarrow P(\mathrm{I})_{mkt} = \frac{A}{b} - \frac{q_\alpha^2}{4\rho_\alpha bM} \rightarrow P(\mathrm{I})_{\max} = \frac{A}{b}$$

$$CS_{\mathrm{I}} = (\frac{(A - bP_\alpha)}{b})[q_\alpha^2 - 2\rho_\alpha (A - bP_\alpha)]$$

$$\mathrm{N} = \int_{q_\alpha}^{2\rho_\alpha (A - bP_\alpha)} \frac{q_\alpha}{2\rho_\alpha (A - bP_\alpha)} dx$$

$$N = q_\alpha - \frac{q_\alpha^2}{2\rho_\alpha (A - bP_\alpha)} \rightarrow P(\mathrm{II})_{mkt} = \frac{A}{b} - \frac{q_\alpha^2}{2\rho_\alpha b (q_\alpha - N)} \rightarrow P(\mathrm{II})_{\max} = \frac{A}{b}$$

$$CS_{\mathrm{II}} = (\frac{(A - bP_\alpha)}{b})[q_\alpha - q_\alpha^2 - \rho_\alpha (A - bP_\alpha)]$$

$$\mathrm{CS}_{Total} = CS_{\mathrm{I}} + CS_{\mathrm{II}}$$

$$\mathrm{CS}_{Total} = (\frac{(A - bP_\alpha)}{b})[q_\alpha - \rho_\alpha (A - bP_\alpha)]$$