THE EFFECT OF SERVICE LEVEL CONSTRAINT ON EPQ MODEL WITH RANDOM DEFECTIVE RATE

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We study the effect of service level constraint on the economic production quantity (EPQ) model with random defective rate. We first prove that the expected overall cost for imperfect quality EPQ model with backlogging permitted is less than or equal to that of the same model without backlogging. Secondly, the relationship between "imputed backorder cost" and maximal shortage level is derived for decision-making on whether the required service level is achievable. Then an equation is proposed for calculating the intangible backorder cost for the situation when the required service level is not attainable. By including this intangible backorder cost in the mathematical analysis, one can derive a new optimal lot-size policy that minimizes expected total costs as well as satisfies the service level constraint. Numerical example is provided to demonstrate its practical usage.

1. Introduction

The primary operation strategies and goals of most manufacturing firms are to seek a high satisfaction to customer's demands and to become a low-cost producer. To achieve these goals, the company must be able to effectively utilize resources and minimize costs. The economic order quantity (EOQ) model was the first mathematical model introduced several decades ago to assist corporations in minimizing total inventory costs. It balances inventory holding and setup costs, and derives the optimal order quantity. Regardless of its simplicity, the EOQ model is still applied industry-wide today [1, 13].

In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier, the economic production quantity (EPQ) model is often employed to determine the optimal production lot size that minimizes overall production/inventory costs. It is also known as the finite production model, because of its assumption that the production rate must be much larger than the demand rate. The classic EPQ model assumes that manufacturing facility functions perfectly during a production run. However, due to process deterioration or other factors, the generation of

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imperfect quality items is inevitable. A considerable amount of research has been carried out [4, 7, 8, 12] to address the imperfect quality EPQ problem and some additional examples are surveyed as follows. Chung [8] investigated bounds for production lot sizing with machine breakdown conditions. Rosenblatt and Lee [16] proposed an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items are produced. Hayek and Salameh [10] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items. They assumed that all defective items are repairable and allowed backorders. Chiu [5] examined an EPQ model with scrap items and the reworking of repairable items.

This paper studies the effect of service level constraint on EPQ model with random defective rate. In the realistic inventory control and management, due to certain internal orders of parts/materials and other operating considerations, the planned backlogging is the strategy to effectively minimize overall inventory costs. While allowing backlogging, abusive shortage in an inventory model, however, may cause an unacceptable service level and turn into possible loss of future sales (because of the loss of customer goodwill). Therefore, the maximal allowable shortage level per cycle is always set as an operating constraint of the business in order to achieve minimal service level while deriving the optimal lot-size decision. A noticeable amount of research has been conducted to address the service level constraint issue [2, 11, 14, 17] and other examples are surveyed below. de Kok [9] considered a lost-sales production/inventory control model with two adjustable production rates to meet demand. He obtained the practical approximations for optimal switch-over levels to such a model under the service level constraints. Chen and Krass [3] investigated inventory models with minimal service constraints. They showed that the minimal service level constraint (SLC) model to be qualitatively different from their shortage cost counterparts and the transformation from SLC model to a shortage cost model may not be always possible.

For the reason that little attention was paid to the area of investigating the effect of service level constraint on the EPQ model with random defective rate, this paper intends to serve this purpose.

2. Mathematical modeling and analysis

2.1. Assumptions and notations. The imperfect production process, due to process deterioration or other factors, may randomly generate x percent of defective items at a constant rate d. The inspection cost per item is involved when all items are screening. In this paper, all defective items are assumed to be scrap items. The production rate P is much larger than demand rate λ, as the basic assumption of the finite production model. It follows that the production rate of the scrap items d can be expressed as $d = Px$. The following notations are used in our analysis:

(1) Q is the production lot size in the EPQ model with shortage not permitted,
(2) Q_b is the production lot size per cycle in the EPQ model with shortage allowed and backordered,
(3) B is the allowable backorder level in the EPQ model with backlogging permitted,
(4) K is the fixed setup cost for each production run,

(5) H_1 is the maximum level of on-hand inventory in units, when the regular production process stops,
(6) c is the production cost per item (\$/item, inspection cost per item is included),
(7) h is the holding cost per item per unit time (\$/item/unit time),
(8) b is the backordering cost per item per unit time,
(9) c_s is the disposal cost for each scrap item (\$/scrap item),
(10) T is the cycle length,
(11) t_1 is the production uptime,
(12) t_2 is the production downtime,
(13) t_3 is the time shortage permitted,
(14) t_4 is the time needed to satisfy all the backorders by the next production,
(15) $TC(Q)$ is the total inventory costs per cycle in the EPQ model with shortage not permitted,
(16) $TCU(Q)$ is the total inventory costs per unit time in the EPQ model with shortage not permitted,
(17) $TC(Q_b,B)$ is the total inventory costs per cycle in the EPQ model with backlogging permitted,
(18) $TCU(Q_b,B)$ is the total inventory costs per unit time in the EPQ model with backlogging permitted.

2.2. Formulation of the EPQ model with backlogging. The EPQ model assumes that the production rate P must always be greater than or equal to the demand rate λ. The production rate of perfect quality items must always be greater than or equal to the sum of the demand rate and the production rate of defective items. Therefore, we must have the following: $(P-d-\lambda)\geq 0$ or $(1-x-\lambda/P)\geq 0$. Figure 2.1 depicts the on-hand inventory level and allowable backorder level for the EPQ model with backlogging permitted.

For the following derivation, we employ the solution procedures used by Hayek and Salameh [10]. From Figure 2.1, one can obtain the cycle length T, production uptime t_1, the maximum level of on-hand inventory H_1, production downtime t_2, shortage permitted time t_3, and t_4 as follows:

$$T=\sum_{i=1}^{4}t_i=\frac{Q_b(1-x)}{\lambda}, \tag{2.1}$$

$$t_1=\frac{H_1}{P-d-\lambda}, \tag{2.2}$$

$$H_1=(P-d-\lambda)\frac{Q_b}{P}-B, \tag{2.3}$$

$$t_2=\frac{H_1}{\lambda}, \tag{2.4}$$

$$t_3=\frac{B}{\lambda}, \tag{2.5}$$

$$t_4=\frac{B}{P-d-\lambda}, \tag{2.6}$$

$$(t_1+t_4)=\frac{Q_b}{P}. \tag{2.7}$$

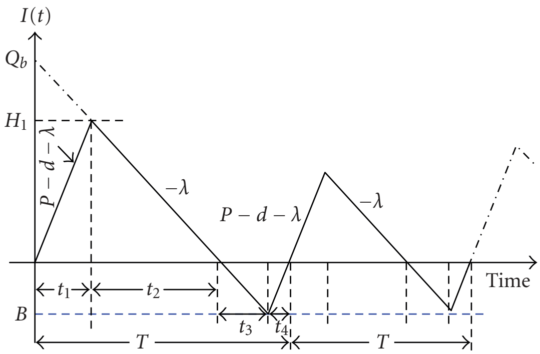

Figure 2.1. On-hand inventory of the EPQ model with random defective rate and backlogging permitted.

The scrap items built up randomly during production uptime "$t_1 + t_4$" are

$$d \cdot (t_1 + t_4) = x \cdot Q. \tag{2.8}$$

The inventory cost per cycle, $TC(Q_b, B)$, is

$$\begin{aligned} TC(Q_b,B) &= c \cdot Q_b + c_s \cdot x \cdot Q_b + K + h\left[\frac{H_1}{2}(t_1+t_2)\right] \\ &\quad + b\left[\frac{B}{2}(t_3+t_4)\right] + h\left[\frac{d(t_1+t_4)}{2}(t_1+t_4)\right]. \end{aligned} \tag{2.9}$$

Since scrap items are produced randomly during a regular production run, the cycle length T is a variable (see Figure 2.1). One may employ the renewal reward theorem [18] to cope with the variable cycle length. By substituting variables form (2.1) to (2.8) in (2.9), the expected cost $E[TCU(Q_b,B)] = E[TC(Q_b,B)]/E[T]$ can be obtained as follows [6]:

$$\begin{aligned} E[TCU(Q_b,B)] &= \lambda\left[\frac{c}{1-E[x]} + \frac{c_S E[x]}{1-E[x]}\right] + \frac{K\lambda}{Q_b}\frac{1}{1-E[x]} \\ &\quad + \frac{h}{2}\left[\left(1-\frac{\lambda}{P}\right)Q_b - 2B\right]\frac{1}{1-E[x]} + \frac{B^2}{2Q_b}\frac{(b+h)}{1-E[x]}E\left(\frac{1-x}{1-x-\lambda/P}\right) \\ &\quad + h\left[B - \left(1-\frac{\lambda}{P}\right)Q_b\right]\frac{E[x]}{1-E[x]} + \frac{hQ_b}{2}\frac{E[x^2]}{1-E[x]}. \end{aligned} \tag{2.10}$$

For the proof of convexity of $E[TCU(Q_b,B)]$, one can utilize the Hessian matrix equation [15]:

$$\begin{bmatrix} Q_b & B \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\partial^2 E[TCU(Q_b,B)]}{\partial Q_b^2} & \dfrac{\partial^2 E[TCU(Q_b,B)]}{\partial Q_b \partial B} \\ \dfrac{\partial^2 E[TCU(Q_b,B)]}{\partial B \partial Q_b} & \dfrac{\partial^2 E[TCU(Q_b,B)]}{\partial B^2} \end{bmatrix} \cdot \begin{bmatrix} Q_b \\ B \end{bmatrix} = \frac{2K\lambda}{Q_b} \frac{1}{1-E[x]} > 0. \tag{2.11}$$

Equation (2.11) is resulting positive, because all parameters are positives. Hence, the expected inventory cost function $E[TCU(Q_b,B)]$ is a strictly convex function for all Q_b and B different from zero.

Hence, it follows that for the optimal production lot size Q_b and the maximal level of backorder B, one can differentiate $E[TCU(Q_b,B)]$ with respect to Q_b and with respect to B and solve the linear system of (2.12) by letting these partial derivatives equal to zero:

$$\begin{aligned} \frac{\partial E[TCU(Q_b,B)]}{\partial Q_b} &= \frac{-K\lambda}{Q_b^2}\frac{1}{1-E[x]} + \frac{h}{2}\left(1-\frac{\lambda}{P}\right)\frac{1}{1-E[x]} \\ &\quad - \frac{B^2}{2Q_b^2}(b+h)E\left(\frac{1-x}{1-x-\lambda/P}\right)\frac{1}{1-E[x]} \\ &\quad - h\cdot\left(1-\frac{\lambda}{P}\right)\frac{E[x]}{1-E[x]} + \left(\frac{h}{2}\right)\frac{E[x^2]}{1-E[x]} = 0 \\ \frac{\partial E[TCU(Q_b,B)]}{\partial B} &= -h\frac{1}{1-E[x]} + \frac{B}{Q_b}(b+h)E\left(\frac{1-x}{1-x-\lambda/P}\right)\frac{1}{1-E[x]} \\ &\quad + h\frac{E[x]}{1-E[x]} = 0. \end{aligned} \tag{2.12}$$

Hence, one derives the optimal production policy, Q_b^* and B^* as shown below:

$$Q_b^* = \sqrt{\frac{2K\lambda}{h(1-\lambda/p) - h^2\cdot\{1-E[x]\}^2/(b+h)\cdot E((1-x)/(1-x-\lambda/P))} - 2h(1-\lambda/p)E[x] + h\cdot E[x^2]}, \tag{2.13}$$

$$B^* = \left(\frac{h}{b+h}\right)\frac{1-E[x]}{E((1-x)/(1-x-\lambda/P))}Q_b^*. \tag{2.14}$$

2.3. Formulation of the EPQ model with shortage not permitted. For the EPQ model with random defective rate and shortage not permitted, the cycle length $T = t_1 + t_2$ (see Figure 2.1). The expected annual cost $E[TCU(Q)] = E[TC(Q)]/E[T]$ can be obtained as follows [7]:

$$\begin{aligned} E[TCU(Q)] &= \lambda\left[\frac{c}{1-E[x]} + c_S\frac{E[x]}{1-E[x]}\right] + \frac{K\lambda}{Q}\frac{1}{1-E[x]} \\ &\quad + \frac{hQ}{2}\left(1-\frac{\lambda}{P}\right)\frac{1}{1-E[x]} - hQ\left(1-\frac{\lambda}{P}\right)\frac{E[x]}{1-E[x]} + \frac{hQ}{2}\frac{E[x^2]}{1-E[x]}. \end{aligned} \tag{2.15}$$

Differentiating $E[TCU(Q)]$ with respect to Q twice, we find that $E[TCU(Q)]$ is convex and by minimizing the expected annual cost $E[TCU(Q)]$, one can derive the optimal production quantity Q^* as shown in (2.16).

$$Q^* = \sqrt{\frac{2K\lambda}{h(1-\lambda/p) - 2h(1-\lambda/p)E[x] + h\cdot E[x^2]}}. \tag{2.16}$$

2.4. Effects of backlogging and service level constraint on the EPQ model

Property 2.1. *The expected annual cost per unit time for the EPQ model with random defective rate and backlogging not permitted is always greater than or equal to that of the EPQ model with random defective rate and backlogging allowed. That is,* $E[TCU(Q)] \geqq E[TCU(Q_b,B)]$, *for any given* $Q = Q_b$.

Proof. Assume that for any given $Q = Q_b$, employing (2.10) and (2.15), one obtains

$$\begin{aligned} &E[TCU(Q)] - E[TCU(Q,B)] \\ &\quad = hB\frac{1}{1-E[x]} - \frac{B^2}{2Q}\frac{(b+h)}{1-E[x]}E\left(\frac{1-x}{1-x-\lambda/P}\right) - hB\frac{E[x]}{1-E[x]}. \end{aligned} \tag{2.17}$$

Substituting B from (2.14), one has

$$\therefore E[TCU(Q)] - E[TCU(Q,B)] = \frac{h^2}{2(b+h)}\frac{1-E[x]}{E((1-x)/(1-x-\lambda/P))}Q \geq 0. \tag{2.18}$$

Since cost-related parameters h and b are nonnegative numbers, the random defective rate x and $(1-x-\lambda/P) \geq 0$, and the production lot-size $Q \geq 0$, hence (2.18) ≥ 0. □

Property 2.1 confirms that it is better (in terms of total inventory costs) to permit shortage and have them backordered for the EPQ model with random defective rate. While allowing backlogging, abusive shortage in an inventory model, however, may cause an unacceptable service level and turn into possible loss of future sales. Hence, the maximal allowable shortage level per cycle is always set as an operating constraint for the business in order to attain the minimal service level.

Suppose that we set α to be the maximum proportion of shortage permitted time per cycle (i.e., the service level $= (1-\alpha)\%$), then

$$\alpha = \frac{t_3+t_4}{T}, \tag{2.19}$$

$$\therefore \frac{\alpha}{1-\alpha} = \frac{t_3+t_4}{t_1+t_2}. \tag{2.20}$$

Substituting t_1, t_2, t_3, and t_4 in (2.20), one obtains

$$\therefore \frac{\alpha}{1-\alpha} = \frac{B}{(1-x-\lambda/P)Q_b - B}. \tag{2.21}$$

Substituting B in (2.21), one has the following:

$$\frac{1}{\alpha}=\left(\frac{b+h}{h}\right)E\left(\frac{1-x}{1-x-\lambda/P}\right)\frac{1}{1-E(x)}\left(1-x-\frac{\lambda}{P}\right), \tag{2.22}$$

$$\therefore b=h\cdot\left\{\frac{1}{\alpha}\cdot\left[\frac{[1-E(x)]}{(1-x-\lambda/P)}\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)^{-1}\right]-1\right\}. \tag{2.23}$$

Assume that

$$f(\alpha,x)=h\cdot\left\{\frac{1}{\alpha}\cdot\left[\frac{[1-E(x)]}{(1-x-\lambda/P)}\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)^{-1}\right]-1\right\}. \tag{2.24}$$

Equation (2.24) represents the relationship between the imputed backorder cost $f(\alpha,x)$ and the maximum proportion of shortage permitted time α. In other words, when the service level $(1-\alpha)\%$ of the EPQ model is set, the corresponding imputed backorder cost $f(\alpha,x)$ can be obtained. Hence, one can utilize this information to determine whether or not the service level is achievable. For the computation of $E[(1-x)/(1-x-\lambda/P)]$, one can refer to the appendix.

Let b_t be the tangible backorder cost per item. If $b_t > f(\alpha,x)$ then the service level $(1-\alpha)\%$ is achievable. Otherwise, we need to increase the tangible backorder cost to $f(\alpha,x)$ and then use it to derive the new optimal operating policy (in terms of Q_b^* and B^*), so that the overall inventory costs can be minimized and the service level constraint will be attained.

Let b_i be the adjustable intangible backorder cost (per item per unit time), then b_i should satisfy the following condition in order to attain the $(1-\alpha)\%$ service level:

$$bi \geq [f(\alpha,x)-bt]. \tag{2.25}$$

Therefore, by using $b=f(\alpha,x)$ one can derive the new optimal production lot size Q_b^* and the optimal backorder level B^* that minimizes the expected annual inventory costs as well as achieves the minimal service level $(1-\alpha)\%$.

3. Numerical example and discussion

A company produces a product for several regional clients. It has experienced a relatively flat demand $\lambda = 4000$ units per year. This item is produced at a rate $P = 10000$ units per year. The percentage of imperfect quality items produced is uniformly distributed over the interval $[0,0.1]$. All of the defective items are scrap, they cannot be repaired and the disposal cost $c_s = \$0.3$ per scrap item. The accounting department has estimated that it costs \$450 to initiate a production run and each unit costs the company \$2 to manufacture. The cost of holding h is \$0.6 per item per year. The service level of this item, according to the firm's policy, is set at 70% or above (i.e., the maximum proportion of shortage permitted time per cycle α is 0.3). Thus, we have

(1) $K = \$450$ for each production run,

(2) x is the proportion of imperfect quality items produced and is uniformly distributed over the interval $[0,0.1]$,

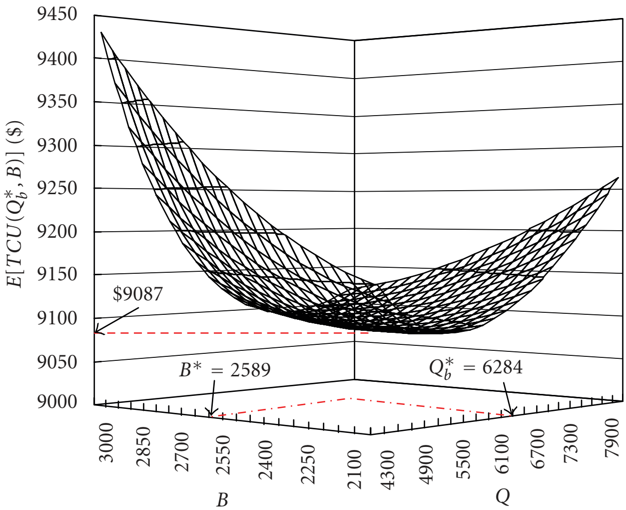

Figure 3.1. Convexity of the expected cost function $E[TCU(Q_b^*, B^*)]$.

(3) $c = \$2$ per item,

(4) $b_t = \$0.2$ per item backordered per unit time (the tangible backorder cost),

(5) $\alpha = 0.3$; the maximum proportion of shortage permitted time per cycle.

First let $b = b_t$. From (2.10), (2.13), and (2.14), one obtains the overall costs $E[TCU(Q_b^*, B^*)] = \$9087$, the optimal production quantity $Q_b^* = 6284$, and the optimal backorder level $B^* = 2589$. Convexity of the expected cost function for this example is displayed in Figure 3.1.

For EPQ model with backlogging not allowed, from (2.15) and (2.16), we obtain the total cost $E[TCU(Q^*)] = \$9625$ and the optimal production quantity $Q^* = 3,323$. One notices that the EPQ model with backlogging permitted has a lower overall cost than that of the EPQ model with no shortage allowed, as proved by Property 2.1.

3.1. Effect of service level constraint on EPQ model. In this example, suppose we ignore the 70% service level constraint for now, then from (2.22) the proportion of shortage time per cycle $\alpha = 0.824$. This represents a 17.6% service level only. In order to achieve the required 70% service level (i.e., $\alpha = 0.3$), one can use the proposed equations (2.23) and (2.24) and obtain the intangible backorder cost $b_i \geqq [f(\alpha,x) - b_t] = 1.4$. Variation of x effects on $f(\alpha,x)$ is depicted in Figure 3.2. One notices that as the random defective rate x increases, $f(\alpha,x)$ increase too.

Variation of λ/P effects on $f(\alpha,x)$ is displayed in Figure 3.3. One notices that as λ/P increases, $f(\alpha,x)$ increases slightly too. Change of h effects on the $f(\alpha,x)$ is illustrated in Figure 3.4.

Then, using a minimum $b = (b_t + b_i) = 1.4$ and (2.10), (2.13), and (2.14), one can recalculate the optimal production quantity $Q_b^* = 3869$, the optimal backorder level $B^* = 580$, and the expected annual costs $E[TCU(Q_b^*, B^*)] = \$9464$. Table 3.1 presents the variation of α effects on the $f(\alpha,x)$, the optimal operating policy (Q_b^* and B^*), the

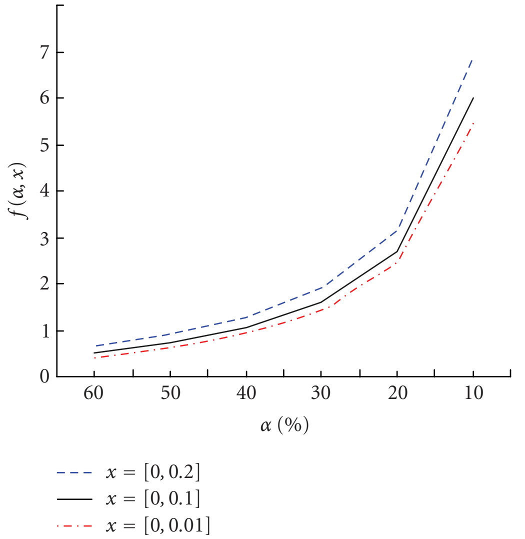

Figure 3.2. Variation of x effects on the $f(\alpha,x)$.

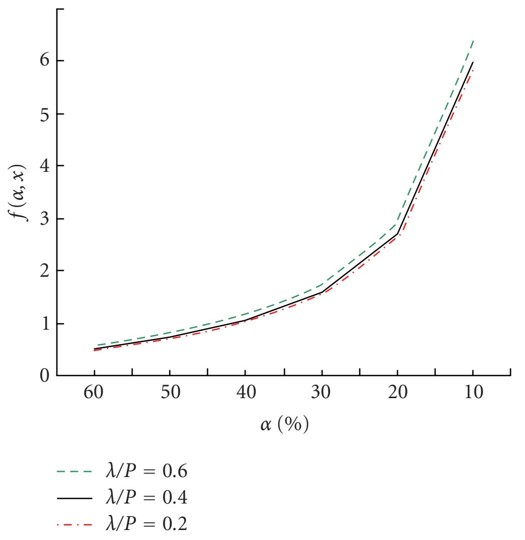

Figure 3.3. Change of λ/P effects on the $f(\alpha,x)$.

expected cost function $E[TCU(Q_b^*,B^*)]$, the $E[TCU(Q_b^*,B^*)]$ excluding an intangible backorder cost, and the price for raising the service level from 17.6%.

From Table 3.1, one notices that though $E[TCU(Q_b^*,B^*)]$ for 70% service level is \$9464, if we exclude the intangible backorder cost b_i (which merely helps us to achieve the 70% service level) from the computation of (2.10), we will obtain the actual cost \$9353.

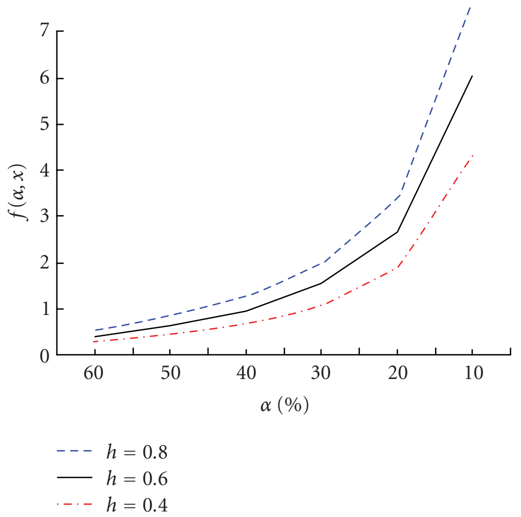

Figure 3.4. Variation of h effects on the $f(\alpha,x)$.

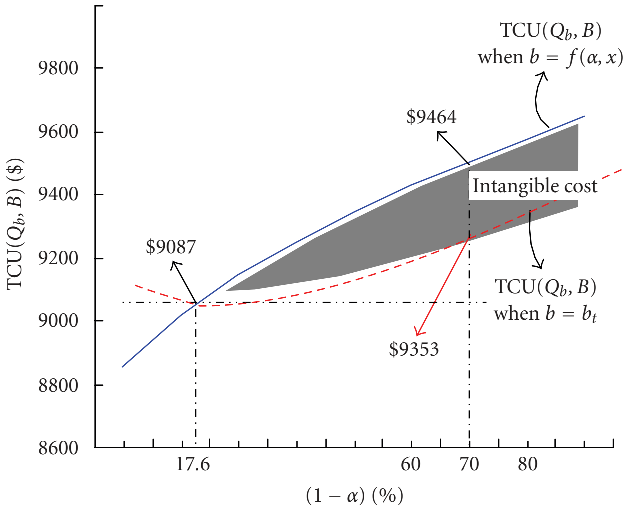

Figure 3.5. Variation of α effects on the optimal cost $E[TCU(Q_b,B)]$ and the $E[TCU(Q_b,B)]$ excluding the intangible backorder cost.

Comparing to \$9087, there is an increase of \$266 in cost. In other words, \$266 is the price that we actually pay for raising the service level from 17.6% to 70% (see both Table 3.1 and Figure 3.5 for details). One also notices that as the service level $(1-\alpha)$% increases, both $E[TCU(Q_b^*,B^*)]$ and the actual price for raising service level increase too.

Table 3.1. Variation of α effects on the optimal policy, $E[TCU(Q_b^*, B^*)]$ and $[TCU(Q_b^*, B^*)]$ excluding the intangible backorder cost, and price for raising the service level.

Service level $(1-\alpha)\%$	α	$f(\alpha,x)$	b_i	Q_b^*	B^*	$E[TCU(Q_b^*,B^*)]$	$TCU(Q_b^*,B^*)$ excluding intangible backorder cost	Price for raising service level
100.0%	0.00	∞	∞	3323	0	$9625	$9625	$537
95.0%	0.05	12.58	12.38	3398	85	$9599	$9575	$488
90.0%	0.10	5.99	5.79	3479	174	$9574	$9528	$440
85.0%	0.15	3.79	3.59	3565	267	$9547	$9482	$394
80.0%	0.20	2.70	2.50	3658	366	$9520	$9437	$350
75.0%	0.25	2.04	1.84	3759	470	$9492	$9394	$307
70.0%	0.30	1.60	1.40	3869	580	$9464	$9353	$266
65.0%	0.35	1.28	1.08	3989	698	$9434	$9314	$227
60.0%	.40	1.05	0.85	4121	824	$9404	$9277	$189
55.0%	0.45	0.86	0.66	4267	960	$9372	$9242	$154
50.0%	0.50	0.72	0.52	4429	1107	$9340	$9209	$122
45.0%	0.55	0.60	0.40	4612	1268	$9306	$9179	$92
40.0%	0.60	0.50	0.30	4819	1446	$9271	$9153	$65
35.0%	0.65	0.41	0.21	5057	1644	$9234	$9129	$42
30.0%	0.70	0.34	0.14	5335	1867	$9195	$9110	$23
25.0%	0.75	0.28	0.08	5663	2124	$9153	$9096	$9
20.0%	0.80	0.22	0.02	6061	2424	$9109	$9088	$1
17.6%	0.82	0.20	0.00	6284	2589	$9087	$9087	$0

4. Concluding remarks

In practical inventory control and management, due to process deterioration or other factors, the generation of defective items is inevitable. Also, owing to the existence of internal orders and other operating considerations, the planned backlogging is the strategy to effectively minimize overall inventory costs. While allowing backlogging, abusive shortage in an inventory model, however, may cause an unacceptable service level and turn into possible loss of future sales. This paper studies the effect of service level constraint on EPQ model with random defective rate. We first derive and prove that the expected overall inventory costs for EPQ model with backlogging permitted is less than or equal to that of the same model without backlogging permitted. Secondly, the relationship between imputed backorder cost and maximal shortage level is derived for decision-maker to judge on whether the required service level is achievable. Then we propose an equation for calculating the intangible backorder cost for the situation when the required service level is not attainable. By including this intangible backorder cost in the mathematical analysis, one can derive a new optimal lot-size policy that minimizes expected total costs as well as satisfies the service level constraint. Numerical example is provided to demonstrate its practical usage.

For future research, one interesting direction among others will be to investigate the effect of service level constraint on an imperfect quality EPQ model with rework process.

Appendix

Computation of $E((1-x)/(1-x-\lambda/P))$

In this paper, the proportion of imperfect quality x is assumed to be a random variable; for example, with a uniform distribution over the interval $[X_l, X_u]$, the probability density function $f(x)$ is

$$f(x) = \begin{cases} \dfrac{1}{Xu - Xl} & \text{for } Xl < x < Xu, \\ 0 & \text{otherwise.} \end{cases} \tag{A.1}$$

The expectation value of $E((1-x)/(1-x-\lambda/P))$ can be calculated using the following integration equation:

$$\begin{aligned} E\left(\frac{1-x}{1-x-\lambda/P}\right) &= \int_{Xl}^{Xu} \left(\frac{1-x}{1-x-\lambda/P}\right) \cdot f(x)dx \\ &= \left[-1 + x - \frac{\lambda}{P}\ln\left(1 - x - \frac{\lambda}{P}\right)\right] f(x)\Big|_{Xl}^{Xu}. \end{aligned} \tag{A.2}$$

If the proportion of imperfect quality items x follows some other probability distribution, one may derive the expectation value of $E((1-x)/(1-x-\lambda/P))$ accordingly; perhaps with extra efforts.

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