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THE EFFECT OF SPACE CHARGE IN BEAM TRANSPORT LINES*

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Introduction

Space-charge effects can be large in transport lines. For the 50 MeV proton lines at Brookhaven or CERN, an average current of 100 mA corresponds to a peak current of 2.5 A due to the short bunch length (∿ 2 cm). As a result, the space-charge defocusing force is an appreciable fraction of the average external focusing force (1/3 for the CERN line), which leads to significant departures from the beam envelope calculated for zero current, i.e. waists as moved, aperture requirements are increased, and the beam is mismatched for the following synchrotron. In addition, longitudinal space-charge forces cause an increase in energy spread, typically an increase of ± 200 or ± 300 keV for the CERN line. Finally, non-linear components of the space-charge force twist and filament the distribution leading to an increase in emittance.

Two computer programs, LINEAR and BEAM[1], have been used to examine these effects. Both trace the evolution of particle distributions in six-dimensional phase space. The first includes only the linear part of the space-charge force and has been incorporated into the program TRANSPORT[2]; this modified CERN version[3] can be used to trace and optimize beam envelopes with space charge for any combination of drift spaces, quadrupoles, bending magnets and solenoids. The second program is similar to the linac codes of Chasman[4] and Martini[5] and includes non-linear effects. It computes trajectories for 650 super-particles; at each step, the space-charge force on a given particle is the resultant of the Coulomb forces of the remaining N-1 particles. Both programs neglect image forces and the forces due to adjacent bunches. LINEAR requires about 5 seconds of CDC 6600 computer time for 100 meters of transfer line while BEAM requires 40 minutes.

LINEAR

The method follows that of TRANSPORT[2]. The beam is specified by its 6×6 correlation matrix σ which is related to the RMS envelope by $\tilde{\chi}\,\sigma^{-1}\,\chi = 1$, where the 6 components of χ are the particle coordinates from bunch center. Since only linear forces are included, σ_1 at distance s_1 along the line is related to its initial value σ_0 by $\sigma_1 = M\sigma_0\tilde{M}$ where M is the 6×6 transfer matrix from s_0 to s_1, $\chi_1 = M\,\chi_0$. Our procedure is to transform σ through the system in a series of small steps with the zero-current transfer matrices; after each step a space-charge correction is applied as an impulse. In effect, a series of thin lens are added to the line to include the defocusing action of space charge. The space-charge force is determined by the 3×3 correlation matrix in x,y,z space, i.e. the nine components σ_{xx}, σ_{xy}, ... of σ without velocities. If correlations are present ($\sigma_{xy} \neq 0,\ldots$), this matrix is rotated into normal form.[7] Then assuming that the charge density ρ is uniform (uniform ellipsoïd model), the beam boundary is $X = \sqrt{5\sigma_{xx}}$,.. and the electric field is $E = k_x . x$

$$k_x = 2\pi\rho\, XYZ \int_0^{\infty} \frac{dt}{(X^2+t)^{3/2}(Y^2+t)^{1/2}(Z^2+t)^{1/2}} \qquad (1)$$

with similar expressions for E_y and E_z[6]. The integral (1) is evaluated by 20 point gaussian quadrature to within 0.1% accuracy. The electric fields are then rotated back to the original direction, the impulse matrix constructed, and the σ matrix updated. The resulting RMS envelope is insensitive to the type of distribution assumed; for example, the same envelope is obtained for a gaussian distribution provided the beam retains the ellipsoïdal form and the emittance growth is negligible (see Ref. 7).

One application which emphasizes the importance of space charge is shown in Fig. 1, namely a preliminary version of the linac-booster transfer line. The zero-current envelope is matched at the booster and within the design energy-spread of ± 150 keV. The 100 and 200 mA envelopes differ considerably : apertures are exceeded, large mismatches occur, and the energy-spread is more than twice the design limit. Because of space charge, the final design (Fig. 2) includes three RF cavities (debunchers) to control energy spread (two at the fundamental 200 MHz and one at 400 MHz) and additional quadrupoles.

The program is also used to optimize the existing 50 MeV linac-CPS transfer line. A comparison between calculated and measured envelopes[8] for 100 mA is shown in Fig. 3, and indicates good agreement. As a consequence, the time consuming, trial-and-error adjustments from the calculated zero-current envelope to the desired envelope is largely avoided. In practice, this ease of adjustment results in a more reliable performance of the line.

BEAM

This program is used to follow the evolution of an initial six-dimensional gaussian distribution consisting of 650 super-particles. At suitable intervals, statistical information is calculated including the six standard deviations in x,x',y,y',z,z' and the 15 correlations, i.e. the σ matrix. To test for non-linear effects, the two-dimensional projections in the (x,x'), (y,y') and (z,z') planes are examined. For example, the standard deviations and correlation in the x,x' plane specify a family of ellipses. If the space-charge forces were linear, the projected distribution would remain gaussian with 86% of the particles within the ellipse for two standard deviations. In fact, the projected distributions do remain gaussian (within the statistical limits imposed by the small sample size) but the phase area containing 86% of the particles increases. For 100 mA the transverse emittance increases about 5% while for 200 mA it increases about 10%. Because these emittance growths are small, the RMS envelope (Fig. 1) is very close to that obtained with LINEAR.

References

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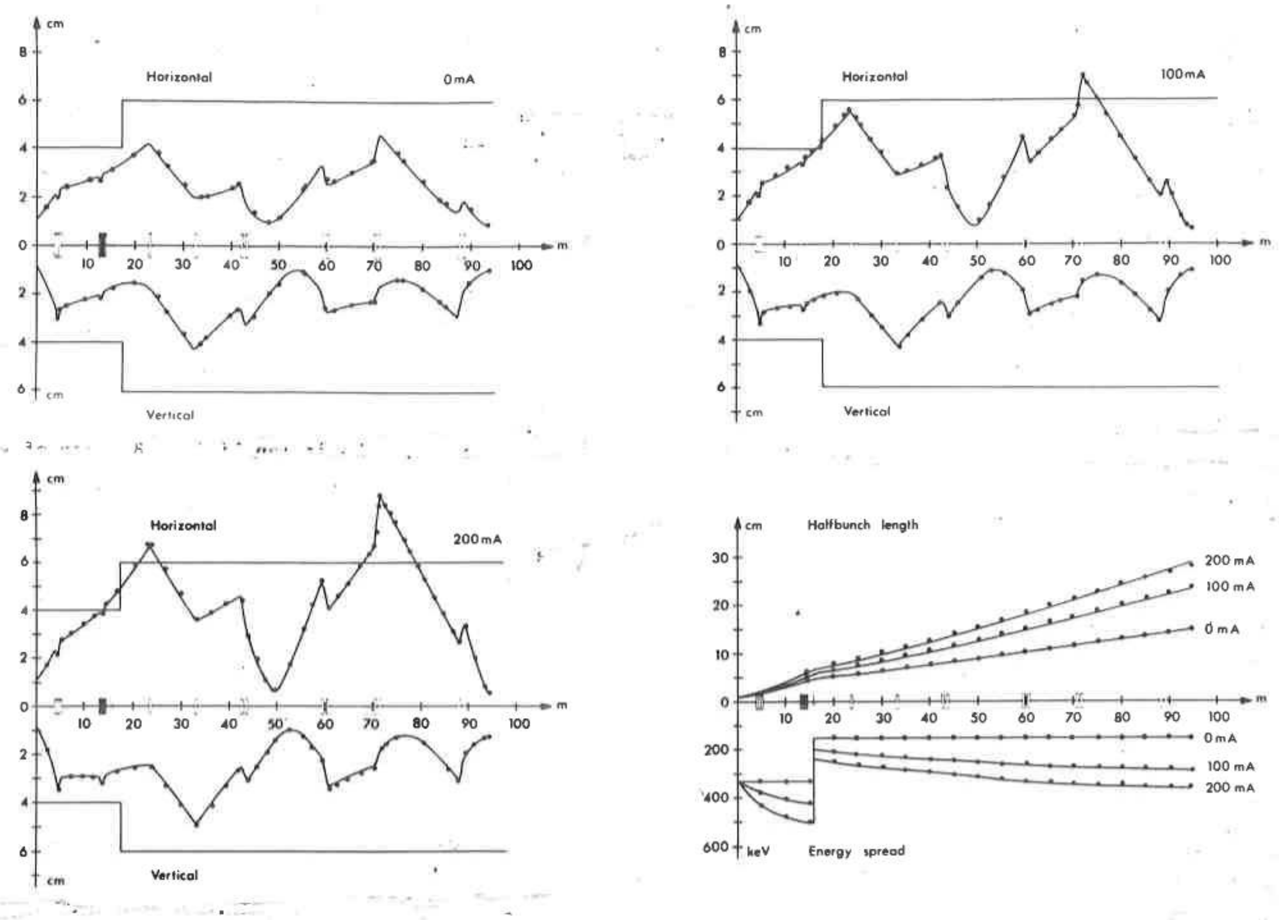
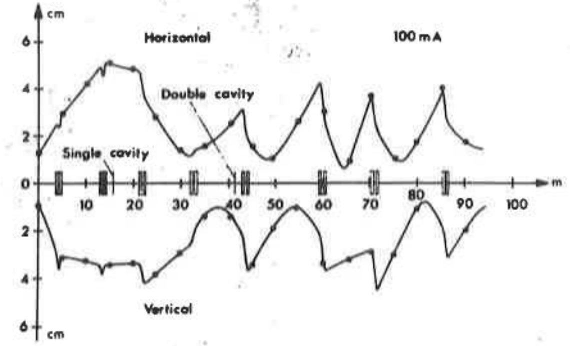
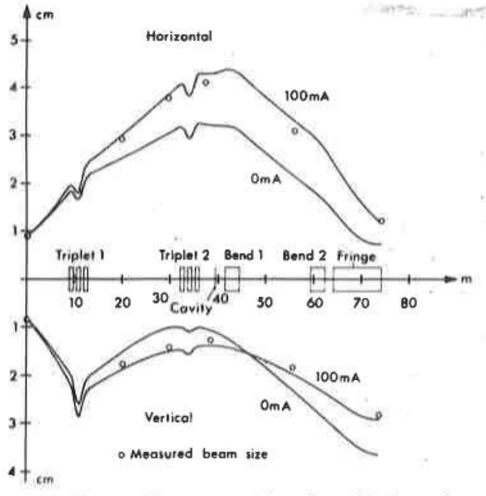


<u>Fig. 1</u> Envelopes (2 standard deviations) for 0, 100 and 200 mA calculated with LINEAR are shown for a transfer line that is matched for zero current. The results from the non-linear program BEAM are also shown (dots) for comparison. The useful aperture is indicated. Quadrupole magnets ▯, RF cavities | .



<u>Fig. 2</u> Final design of PSB injection line showing 100 mA envelopes obtained from LINEAR and BEAM.



<u>Fig. 3</u> Comparison of measured and calculated envelopes for existing 50 MeV Linac-PS injection line.

<u>Distribution (open)</u>

Abstract sent to MPS and SI Scientific Staff