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The effect of structure formation on the expansion of the universe

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Abstract.
Observations of the expansion rate of the universe at late times disagree by a factor of 1.5–2 with the prediction of homogeneous and isotropic models based on ordinary matter and gravity. We discuss how the departure from linearly perturbed homogeneity and isotropy due to structure formation could explain this discrepancy. We evaluate the expansion rate in a dust universe which contains non-linear structures with a statistically homogeneous and isotropic distribution. The expansion rate is found to increase relative to the exactly homogeneous and isotropic case by a factor of 1.1–1.3 at some tens of billion of years. The timescale follows from the cold dark matter transfer function and the amplitude of primordial perturbations without additional free parameters.

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Three assumptions and a factor of two. Cosmological observations show that the early universe is well described by a model which contains only ordinary matter (i.e. baryons, leptons, photons, and dark matter), evolves according to ordinary general relativity and is exactly homogeneous and isotropic (up to linear perturbations). However, such a model underpredicts the cosmological distances measured in the late universe by a factor of about 2.

In a homogeneous and isotropic model, the distance scale is determined in terms of the expansion rate and spatial curvature, and the disrepancy can be summarised by saying that the observed Hubble parameter is a factor of 2 larger than expected given the matter density (i.e. $3H^2 \approx 4 \times 8\pi G_{\rm N}\rho_{\rm m}$ instead of $3H^2 \approx 8\pi G_{\rm N}\rho_{\rm m}$), or a factor of 1.5 larger given the age of the universe (i.e. $Ht \approx 1$ instead of $Ht = 2/3$). More precisely, the Hubble parameter has fallen more slowly than predicted, corresponding to acceleration.

Explaining the factor of 2 requires abandoning at least one of the three assumptions of standard matter, standard gravity and perfect homogeneity and isotropy. Keeping to homogeneity and isotropy, it is possible to account for the distance observations by adding a factor of 3 to the energy density in the form of exotic matter with negative pressure or introducing repulsive gravity in the same measure. Such models have two shortcomings.

First, it is difficult to understand why the contributions of ordinary matter and the repulsive component are roughly equal today, at around 10 billion years. This *coincidence problem* is somewhat philosophical in nature: it does not contradict any known physical law or observation.

In contrast, the second problem of homogeneous and isotropic models is unambiguous: the universe is not perfectly homogeneous and isotropic (or even perturbatively near homogeneity and isotropy). There are non-linear structures which are not described by perturbations around a smooth background, with a distribution that is statistically homogeneous and isotropic above a scale of about 100 Mpc [1].

A universe which is homogeneous and isotropic only statistically does not in general expand like an exactly homogeneous and isotropic universe, even on average. This feature of general relativity was discussed under the name *fitting problem* by George Ellis in 1983 [2]. However, at the time the observational situation was not clear enough for factors of order one to be important. Now that cosmological observations have become more precise and a discrepancy has arisen, the complication due to non-linear structures can no longer be neglected. Also, the fact that structure formation is the most prominent change in the universe at late times suggests that it might solve the coincidence problem [3].

The Buchert equations. The effect of inhomogeneity and/or anisotropy on the average evolution is called backreaction [4]. Backreaction is quantified by the Buchert equations, which describe the average evolution of a rotationless dust space and provide a partial answer to the fitting problem [5]:

$$3\frac{\ddot{a}}{a} = -4\pi G_{\rm N}\langle\rho\rangle + \mathcal{Q} \tag{1}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_{\rm N}\langle\rho\rangle - \frac{1}{2}\langle{}^{(3)}R\rangle - \frac{1}{2}\mathcal{Q} \tag{2}$$

$$\partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0 \ , \tag{3}$$

where dot is a derivative with respect to proper time t, $\langle\rho\rangle$ is the average energy density, $\langle{}^{(3)}R\rangle$ is the average spatial curvature, $\frac{1}{3}\langle\theta\rangle = \dot{a}/a \equiv H$ is the average Hubble parameter (θ is the local volume expansion rate), and the backreaction variable $\mathcal{Q}$ contains the effect of inhomogeneity and anisotropy:

$$\mathcal{Q} \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle \ , \tag{4}$$

where $\langle\sigma^2\rangle$ is the average shear scalar. The averages are taken on the hypersurface of constant proper time; for details and discussion, see [6, 7].

If the variance of the expansion rate in (4) is large enough compared to the shear and the energy density, the average expansion rate accelerates, as (1) shows. The physical reason is simply that the fraction of space in faster expanding regions grows, so the average expansion rate can rise.

Structure formation is by definition a non-perturbative problem, and evaluating the expansion rate in a model with realistic structures is more involved than introducing a new source term in homogeneous and isotropic models. It is not feasible to obtain an exact metric. However, in a statistically homogeneous and isotropic universe, only statistical information is needed for calculating the average expansion rate. The Buchert equations reduce the effect of structures to the function $\mathcal{Q}$, which depends only on global statistics.

We may draw an analogy with a classical system of particles. For a couple of particles, or for small perturbations about a smooth background, it may be reasonable to look for an exact solution. However, with many particles and sizeable local fluctuations, the system can only be treated statistically. Statistical treatment is also sufficient for evaluating the interesting properties of such systems, at least when the coherence length of fluctuations is much smaller than the scales of interest. In cosmology, practically all observations are made over scales larger than the homogeneity scale.

The peak model. We will evaluate the average expansion rate in a simple model. We consider a homogeneous and isotropic, spatially flat, matter-dominated background with an initial spectrum of linear Gaussian perturbations. We follow the evolution of the perturbations into the non-linear regime statistically.

We model structures as isolated spherical peaks of the initial density field smoothed on scale R, as in the peak model of structure formation [8]. The number density of peaks is determined as a function of $\delta/\sigma_0(t,R)$, the linear density contrast divided by the rms density contrast smoothed on scale R. The smoothing scale R is fixed by taking the rms density contrast to be unity at all times, $\sigma_0(t,R)=1$. The scale R thus measures the size of typical structures forming at time t, and its growth corresponds to the progress of structure formation to larger scales.

Since the individual regions are spherical and isolated, they evolve (in the Newtonian limit) like independent homogeneous and isotropic universes (see e.g. [9]). Overdense structures slow down and collapse, underdense voids expand less slowly and become emptier.

The evolution of the number density is determined by the power spectrum of linear perturbations. We take a scale invariant primordial spectrum with the observed amplitude, and consider two different approximations of the cold dark matter transfer function. In addition to the well-known BBKS function [8], we use the simple form introduced by Bonvin and Durrer [10]. The different results give some indication of the degree of uncertainty in the results due our simplified modelling assumptions.

The average Hubble rate at time t is given by

$$H(t) = \int_{-\infty}^{\infty} \mathrm{d}\delta \, v_\delta(t) H_\delta(t) \ , \tag{5}$$

where $v_\delta(t)$ is the fraction of volume in regions with linear density contrast δ, and $H_\delta(t)$ is the Hubble rate of such regions. The volume fraction is composed of two parts, $v_\delta(t) = s_\delta f(\delta,t)/(\int_{-\infty}^{\infty}\mathrm{d}\delta\, s_\delta f(\delta,t))$, where s_δ is the volume of a region with linear density contrast δ relative to an unperturbed region, and $f(\delta,t)$ is the number density of such regions. See [7] for details.

Given the initial power spectrum and the transfer function, the evolution is completely fixed, there are no free parameters to adjust. The result for H multiplied by t is shown in figure 1. In the early universe, the expansion is near the homogeneous and isotropic case, with $Ht \approx 2/3$. At late times Ht rises, saturating to a final value somewhat less than unity as voids come to dominate the volume of the universe. However, the rise is not rapid enough to correspond to acceleration.

The timescale $\approx 10^{10}$ years comes from a combination of the matter-radiation equality time $t_{\mathrm{eq}} \approx 10^5$ years encoded in the transfer function and the primordial perturbation amplitude $\approx 10^{-5}$. Because the initial amplitude is small, it takes long for structures to become important.

Outlook. We have demonstrated with a simple model how non-linear structures lead to an increase in the expansion rate from the homogeneous and isotropic value. It is remarkable that the era when the expansion rate increases comes out roughly correctly without free parameters, showing how structure formation may solve the coincidence problem. Given the level of approximation, the fact that the model does not show acceleration is not particularly worrisome. In a universe which is not perfectly homogeneous and isotropic, there is no fundamental difference between acceleration and deceleration; it is merely a question of how rapidly the faster expanding regions come to dominate. Acceleration has been explicitly demonstrated in a dust model with two spherical regions [6].

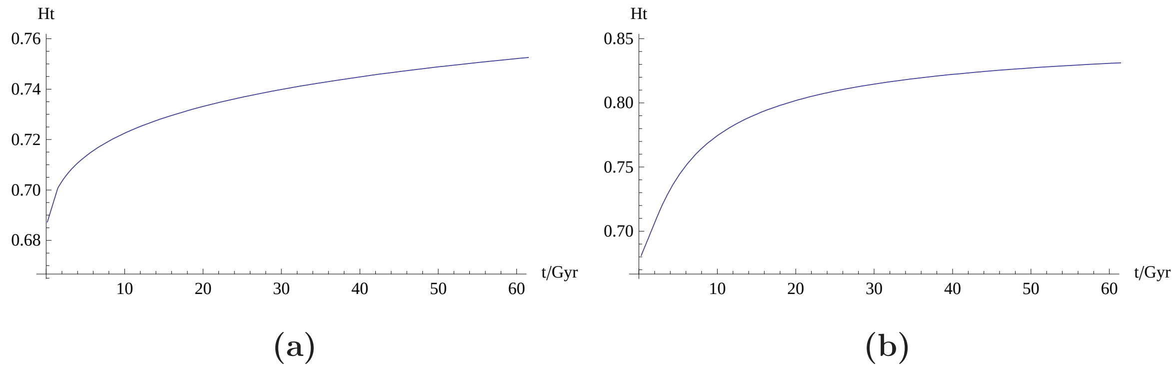

Figure 1. The expansion rate Ht as a function of time for (a) the BBKS transfer function and (b) the Bonvin and Durrer transfer function.

It may be instructive to compare the current situation to the early years of Newtonian gravity. Newtonian theory explained the local gravity observations on Earth, and the two-body solution was very successful in describing the orbits of the planets. However, when the two-body solution was applied to the nearby Earth-Moon system, the result for the lunar perigee was wrong by a factor of 2. It was proposed that the inverse square law of gravity be modified at small distances (of the order of the Earth-Moon separation) to correct the discrepancy [11]‡. However, the solution turned out to lie in the non-linear aspects of Newtonian gravity: the influence of the Sun cannot be neglected. Even after a correction of the right order of magnitude was demonstrated, it took decades before the non-linear three-body calculation was fully worked out.

Similarly, general relativity has explained the local observations in the solar system, and the application of the homogeneous and isotropic solution to the early universe has been very successful. However, the prediction for the universe nearer to us in time is wrong by a factor of 2. Whether non-linear structures can explain this discrepancy is not yet known. However, their effect has to be quantified before it is possible to draw definite conclusions about other explanations.

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‡ In his book *The Structure of Scientific Revolutions*, Thomas Kuhn uses this as an example of an inadmissible solution to a scientific puzzle [12].

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