# The effect of the white dwarf magnetic field on dwarf nova outbursts 

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#### Abstract

Summary. If the white dwarf in a cataclysmic variable has a strong magnetic field, the inner region of the accretion disc is disrupted. We calculate the effect of this on the form of dwarf nova outbursts, in models for disc instabilities and in models for a mass-transfer instability. We find that disruption of the inner disc region leads to shorter outbursts, and, for disc instabilities starting near the inner disc edge, to much longer intervals between outbursts. For moderate mass-transfer rates, the disc instability is suppressed altogether. Our models indicate that recently observed variability in cataclysmic variables with a magnetized white dwarf may be associated with dwarf nova type outbursts.


## 1 Introduction

Dwarf novae are cataclysmic variables that show outbursts, during which the visual brightness increases temporarily by $2-5 \mathrm{mag}$. The duration of the outbursts varies from days to weeks. The intermediate polars, also called DQ Her systems after the prototype are cataclysmic variables in which the white dwarf has a fairly strong magnetic field. The exact range of magnetic-field strengths in these systems is uncertain, but the field strength at the white dwarf surface is presumably in excess of $10^{6} \mathrm{G}$. On the whole, it appears that these two classes of cataclysmic variables are mutually exclusive, i.e. the white dwarfs in dwarf novae do not show evidence for a strong magnetic field, and DQ Her systems do not undergo dwarf nova outbursts. However, there are DQ Her systems that do show outbursts. One of these is the dwarf nova GK Per. EXOSAT observations showed that the X-ray flux of this system is strongly modulated, with a period of 351 s . This modulation is thought to arise because the white dwarf magnetic field channels the accretion onto the magnetic poles, whose aspect changes with the white dwarf rotation (Watson, King \& Osborne 1985; Norton, Watson \& King 1987). Outbursts have also been observed in the intermediate polars TV Col and EX Hya. EX Hya was originally classified as a U Gem system, on the basis of its dwarf-nova-like behaviour (e.g. Bateson 1978). In TV Col, variations on the time-scale of hours, both in the
ultraviolet (Szkody \& Mateo 1984) and in the optical (Schwarz et al. 1988), were interpreted as very small dwarf nova outbursts.

A strong magnetic field of the white dwarf disrupts the flow of the matter in the accretion disc, in the region close to the white dwarf. In this paper we make a first investigation of how this disruption may affect the form and frequency of dwarf nova outbursts. We compare model outbursts for an accretion disc which extends all the way to the white dwarf, with those for systems in which the inner disc is disrupted. We make the comparison for two current models for dwarf nova outbursts. In the first model the dwarf nova outburst is due to increased mass transfer from the mass-donor star (e.g. Bath 1975; Bath \& Pringle 1981). In the second model, the outburst is due to an instability within the accretion disc (e.g. Meyer \& Meyer-Hofmeister 1981; Smak 1984). We describe our calculations in Section 2, and interpret and explain the results in Section 3. In Section 4 we make the comparison with observations, and give a discussion of our assumptions.

## 2 Numerical calculations

The diffusion equation for a time-dependent disc can be written
$\frac{\partial \Sigma}{\partial t}=\frac{1}{R} \frac{\partial}{\partial R}\left[\frac{1}{R \Omega} \frac{\partial}{\partial R}\left(3 v \Sigma R^{2} \Omega\right)\right]+\frac{1}{R} \frac{\partial}{\partial R}\left(\frac{\Sigma}{\pi \Omega} \frac{G M_{\mathrm{wd}}}{R} \frac{2}{q^{2}} f\right)$,
where $\Sigma$ is the vertically integrated density of the disc, $R$ the distance to the white dwarf centre, $\Omega$ the Keplerian angular velocity $\left(\Omega=\sqrt{G M_{\mathrm{wd}} / R^{3}}\right), M_{\mathrm{wd}}$ the mass of the white dwarf, $q$ the mass ratio of the binary $q=M_{\mathrm{wd}} / M_{\mathrm{c}}$, and $v$ the kinematic viscosity coefficient. $f$ is a dimensionless function giving the loss of angular momentum caused by the tidal force exerted on the disc by the mass-donor star: $f=q^{2} / 2$ means that all Keplerian angular momentum is lost in one single orbit. Equation (1) expresses the change of the local density as the result of a divergence in the mass flow, which is in turn the consequence of viscosity in the disc and of the tidal force exerted by the companion star.

If the white dwarf has a strong magnetic field, the accreting matter close to the white dwarf is forced to flow along the magnetic field lines, and the inner disc radius is some distance away from the white dwarf surface. We will make the drastic simplification that the only effect of the magnetic field is to move the inner disc radius $R_{\mathrm{in}}$ from the white dwarf surface to a fixed larger radius. For a non-magnetic white dwarf we have $R_{\mathrm{in}}=R_{\mathrm{wd}}$, for the magnetic white dwarf we use $R_{\mathrm{in}}=12.5 R_{\mathrm{wd}}$. This corresponds to the magnetospheric radius of a white dwarf with a moderately strong $\left(\sim 10^{6} \mathrm{G}\right)$ magnetic field. For a lower magnetic field the effects may be expected to be smaller than the effects that we find in our calculations. We will neglect the change in the radius of the magnetosphere with variation of the mass-transfer rate in the disc. For the inner boundary condition, we use the one proposed by Shakura \& Sunyaev (1973), which has viscosity, and hence temperature, equal to zero at $R=R_{\mathrm{in}}$. We neglect the detailed structure of the boundary region between disc and magnetosphere, and its dependence on the mass-transport rate through the disc. This simple description is analogous to the neglect of the boundary layer between white dwarf and disc in the usual description of accretion discs around non-magnetic white dwarfs.

The code that we use to calculate the disc instabilities is the one described in Pringle, Verbunt \& Wade (1986). It solves the diffusion equation for the mass flow in the accretion disc, using the approximate relations between surface density and viscosity given by Meyer \& MeyerHofmeister (1981). In the model, the viscosity is assumed to be proportional to the pressure, with a proportionality constant $\alpha$. The disc instability arises because the relation between surface density and viscosity is multivalued; a global, as opposed to local, instability arises only if
the value of $\alpha$ is more than about three times higher in outburst than in quiescence (Smak 1984). Addition of mass to the disc, and the boundary conditions, are also handled, as described in Pringle et al. (1986). In particular, the angular momentum of the matter added to the disc, $\dot{J}_{\dot{M}}$, is expressed in terms of a characteristic radius, $R_{\mathrm{h}}$, as $\dot{J}_{\dot{M}}=$ $\sqrt{G M_{\mathrm{wd}} R_{\mathrm{h}}}$. The code ignores the tidal force within the accretion disc; instead a maximum allowed disc radius $R_{\text {max }}$ is prescribed.

To calculate the transfer instabilities, we modify the relation between surface density and viscosity, to make it single-valued and so stable for any given mass-transfer rate, as described in Pringle et al. (1986). This allows a considerable simplification of the code, in which the disc is always in thermal equilibrium locally. For the calculations of the mass-transfer instability we use a simplified, and hence faster, version of the code (see Priedhorsky \& Verbunt 1988). In this code the tidal force is described with
$f=\left(\frac{R}{R_{\text {tid }}}\right)^{5} \simeq \frac{1}{0.17}\left(\frac{R}{a}\right)^{5}$
(see Smak 1984). $R_{\text {tid }}$ can be adjusted to obtain a disc radius in accordance with observation. We use $R_{\text {tid }} \simeq 0.7$ a (equation 2 corrects a printing error in Priedhorsky \& Verbunt 1988, repeated in Livio \& Verbunt 1988). Table 1 lists the disc parameters used in our calculations.

Table 1. Disc parameters.

|  | 'TV Col' | 'GK Per' |
| :--- | :--- | :--- |
|  |  |  |
| $M_{w d}$ | $0.8 M_{\odot}$ | $0.8 M_{\odot}$ |
| $R_{w d}$ | $5 \times 10^{8} \mathrm{~cm}$ | $5 \times 10^{8} \mathrm{~cm}$ |
| $R_{i n}$ | $R_{w d}$ or $12.5 R_{w d}$ | $12.5 R_{w d}$ or $87.5 R_{w d}$ |
| $R_{\text {max }}$ | $5.19 \times 10^{10} \mathrm{~cm}$ | $2.31 \times 10^{11} \mathrm{~cm}$ |
| $R_{h}$ | $1.18 \times 10^{10} \mathrm{~cm}$ | $6.64 \times 10^{10} \mathrm{~cm}$ |
| $R_{t i d}$ | $1.72 \times 10^{11} \mathrm{~cm}$ | $3.31 \times 10^{11} \mathrm{~cm}$ |
| $d$ | 100 pc |  |

Table 2. Model parameters.

|  | Mass-transfer instability |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\alpha_{1}$ | $\alpha_{2}$ | $\dot{M}_{q}$ | $\dot{M}_{o}$ | $\Delta t_{o}$ |
|  |  |  | $(\mathrm{~g} / \mathrm{s})$ | $(\mathrm{g} / \mathrm{s})$ | (days) |
| 1 | 1.0 | 1.0 | $1.0 \times 10^{16}$ | $1.0 \times 10^{18}$ | 0.25 |

Disc instability

|  | $\alpha_{1}$ | $\alpha_{2}$ | $\dot{M}$ | outburst | quiescence |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $(\mathrm{g} / \mathrm{s})$ | (days) | (days) |
| 2a | 0.1 | 1.0 | $1.6 \times 10^{16}$ | 3.5 | 60 |
| 2b | 0.1 | 1.0 | $1.6 \times 10^{16}$ | 2.0 | 800 |
| 3a | 0.1 | 1.0 | $2.0 \times 10^{17}$ | 3.5 | 5 |
| 3b | 0.1 | 1.0 | $2.0 \times 10^{17}$ | 1.5 | 5 |

In the mass-transfer instability calculation, we start from an equilibrium disc at the quiescent mass-transfer rate $\dot{M}_{\mathrm{q}}$, and then enhance the mass transfer to an outburst mass-transfer rate $\dot{M}_{0}$ during a time interval $\Delta t_{0}$. The values used for model 1 are listed in Table 2, the results of the calculations are shown in Fig. 1. In Fig. 1 the changes in $\dot{M}$ are reflected in the luminosity of the hot spot, which is calculated as in Pringle et al. (1986). The total disc luminosity $L_{\mathrm{d}}$ reacts with some delay to the mass-transfer burst.

We write the luminosity at the inner disc radius as a sum of two components, namely the luminosity of the boundary layer $L_{\mathrm{bl}}$ and the luminosity released by the matter falling from the


Figure 1. Results for the mass-transfer instability model calculations. On the left-hand side are the graphs for a white dwarf without a magnetic field, on the right-hand side those for a white dwarf surrounded by a magnetosphere of 12.5 white dwarf radii. From top to bottom are shown: (a) the light curves of the disc luminosity $L_{\mathrm{d}}$ (solid line), the boundary layer luminosity $L_{\mathrm{bl}}$ (dotted line), and the hot spot luminosity $L_{\mathrm{hs}}$ (dashed line); (b) the visual magnitude $V$ of the disc through the outburst; (c) temperature profiles of the disc during the decline from maximum (radius in units of $R_{\max }$ ), and ( d ) the corresponding disc spectra during decline from maximum (the flux is in units of $\mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2} \mathrm{~Hz}^{-1}$ ).
inner disc radius to the white dwarf surface, $L_{\mathrm{ff}}$;
$L_{\mathrm{in}}=\frac{G M_{\mathrm{wd}} \dot{M}_{\mathrm{in}}}{2 R_{\mathrm{in}}}+G M_{\mathrm{wd}} \dot{M}_{\mathrm{in}}\left(\frac{1}{R_{\mathrm{wd}}}-\frac{1}{R_{\mathrm{in}}}\right) \equiv L_{\mathrm{bl}}+L_{\mathrm{ff}}$,
where $\dot{M}_{\text {in }}$ is the mass-transfer rate at the inner disc radius. For a non-magnetic white dwarf, $L_{\mathrm{ff}}$ is zero. In Fig. 1, we show $L_{\mathrm{b}}$ as a function of time; $L_{\mathrm{in}}$ is a multiple of $L_{\mathrm{b}}$, with a fixed constant (see equation 3). For the magnetic white dwarf, $L_{\mathrm{b}}$ is larger than the disc luminosity $L_{\mathrm{d}}$ throughout the outburst.

The main effects of the large $R_{\text {in }}$ are the drastic shortening of the outburst, in particular in the visual magnitude; the removal from the disc of the high-temperature regions, and the lowering of the ultraviolet fluxes. The spectra shown in Fig. 1 (and in Figs 2 and 3) are those calculated for the disc only, i.e. contributions by the boundary layer and the hot spot, the white dwarf and the mass donor, are not taken into account. (For a description of the procedures followed in calculating the disc spectra, see Pringle et al. 1986.) The distance assumed in the calculations of the spectra and the visual magnitudes is always 100 pc .

Model 2: Disc instability starting inside


Figure 2. Results for model 2, the disc instability that starts at the inside fo the disc. Otherwise as in Fig. 1, except that the temperature profiles and the disc spectrum are of the rise to maximum.

Model 3: Disc instability starting outside


Figure 3. Results for model 3, the disc instability that starts at the outside of the disc. Otherwise as in Fig. 2.

The results for two disc-instability calculations are shown in Figs 2 and 3, for an instability starting at the inner disc edge, and at the outer disc edge, respectively. The parameters used in these models are listed in Table 2. The main effects of a large $R_{\mathrm{in}}$ are the same as those found in the mass-transfer calculation: shortening of the outburst; removal of the high-temperature regions, and lower ultraviolet fluxes. In Table 2 we also list the interval between outbursts, under the heading 'quiescence'.

For instabilities starting at the inner disc, the length of the interval between large outbursts is about 60 d . For the parameters of model $2(\mathrm{a})$, a small outburst that only affects a few zones close to the white dwarf occurs in the middle between every two larger outbursts. For the larger inner disc radius, we find only large outbursts, and the interval between the outbursts has increased dramatically. We have run a model with a slightly larger viscosity in quiescence, $\alpha_{1}=0.3$, but with other parameters as in model $2(\mathrm{a}$ and b$)$. For $R_{\mathrm{in}}=R_{\mathrm{wd}}$, we find large outbursts only, which last 2.5 d , and are separated by quiescent intervals of about 5 d . For $R_{\mathrm{in}}=12.5 R_{\mathrm{wd}}$, the disc is stable.

For the outbursts that start at the outer edge of the disc, the interval between outbursts is hardly affected by the change in inner disc radius.

## 3 Interpretation

In a stationary accretion disc, the effective temperature, $T_{\text {eff }}$, can be written as a function of radius, $R$ (Shakura \& Sunyaev 1973):
$\sigma T_{\text {eff }}^{4}=\frac{3 G M_{\mathrm{wd}} \dot{M}}{8 \pi R^{3}}\left(1-\sqrt{\frac{R_{\text {in }}}{R}}\right)$.
The luminosity of the disc is found by integrating over the disc surface
$L_{\mathrm{d}}=\int_{R_{\text {in }}}^{R_{\text {out }}} 4 \pi R \sigma T_{\text {eff }}^{4} d R$,
which for a stationary disc gives
$L_{\mathrm{d}, \mathrm{stat}}=\frac{G M_{\mathrm{wd}} \dot{M}}{2 R_{\text {in }}}\left[1-\frac{R_{\text {in }}}{R_{\text {out }}}\left(3-2 \sqrt{\frac{R_{\text {in }}}{R_{\text {out }}}}\right)\right]$,
where $R_{\text {out }}$ is the outer disc radius. If $R_{\mathrm{in}}=R_{\mathrm{wd}}$ the disc luminosity is close to the luminosity of the boundary layer $L_{\mathrm{b}}$, whenever the disc approaches the stationary solution. For larger inner radius, however, $L_{\mathrm{b}}$ can be substantially larger than $L_{\mathrm{d}}$. This effect is seen in all models 1,2 and 3.

For the mass-transfer instability, and for the disc instability that starts at the outer disc edge, the rise to outburst maximum consists of two parts: first, the temperature in the outer disc region rises, and then the high-temperature region gradually extends inwards. As a result, the optical flux, with originates mostly from the outer disc regions at this stage, reaches its maximum before the ultraviolet flux, which originates in the inner disc regions. The rise to optical maximum is not much affected by the change in inner disc radius. However, the rise to the ultraviolet and bolometric maximum is shortened by an increase of $R_{\mathrm{in}}$, due to the shortening of the time that it takes the high-temperature region to reach the inner disc edge. The ultraviolet and bolometric maxima occur close to the optical maximum in this case.

An additional reason for the shorter outburst in the magnetic case, is the smaller size of the disc, in quiescence and throughout the outburst, as compared to the non-magnetic case (see Fig. 1). To explain this smaller size we consider the flow of angular momentum to and from the accretion disc. Angular momentum is added to the disc with the mass stream from the donor star, it is taken from the disc by the mass flowing through the boundary layer, and by tidal forces. In equilibrium, the disc does not change its angular momentum, hence
$\dot{M} \sqrt{G M_{\mathrm{wd}} R_{\mathrm{h}}}-\dot{M} \sqrt{G M_{\mathrm{wd}} R_{\mathrm{in}}}=\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \Sigma G M_{\mathrm{wd}} \frac{2}{q^{2}} f d R$.
For a large value of $R_{\text {in }}$, the tidal force needed to remove angular momentum from the disc in equilibrium, is smaller, and hence the disc can be smaller. (Note that the tidal force is very small at small $R$, so that the change in the lower integral boundary hardly affects the right-hand side of equation 7.)

For the disc instability that starts at the inside, the rise to outburst maximum consists of an initial rise of the temperature in the innermost disc region, followed by an extension of the high-temperature region outwards. In model $2(\mathrm{a})$, with $R_{\mathrm{in}}=R_{\mathrm{wd}}$, the temperature of the innermost zones keeps rising during the second phase, as $\dot{M}_{\text {in }}$ continues to increase. In model $2(\mathrm{~b})$, with $R_{\mathrm{in}}=12.5 R_{\mathrm{wd}}$, the temperature of the innermost zone reaches its maximum value long before the outburst has spread over the full disc. As a result, the rise to outburst maximum is very short. This effect is a consequence in part of the lower temperature of the maximum (see equation 4), in part of the high values of $\dot{M}_{\text {in }}$ before the onset of the outburst, and related to the long interval between outbursts, which will be discussed below.

For non-magnetic white dwarfs, the decline from outburst maximum is very similar for all types of outburst models, as discussed in Pringle et al. (1986): the hot disc region gradually converges on the white dwarf region. In the final stages of the decline, the disc luminosity is dominated by the disc region close to the white dwarf. These latter stages are suppressed when a magnetosphere around the white dwarf disrupts the inner disc region. As a result, the decline of the outburst is shorter. This effect is particularly pronounced in the mass-transfer instability.

In all outburst models, the removal of the inner disc regions leads to smaller ultraviolet and bolometric disc luminosities (see equation 4.6). The maximum of the optical flux, depending more on the outer regions, is less affected. In the mass-transfer instability model and in the disc instability that start in the outer disc region, the visual disc maximum is about 1 mag weaker for the magnetized white dwarf. For the disc instability starting in the inner disc region, the visual maxima are at almost the same magnitude.

In our calculations for the mass-transfer instability (model 1) the distance between outbursts is a free parameter. In this model, it is the physics of the surface layers of the mass donor which determine when an outburst will start, and the distance between outbursts is not expected to vary with $R_{\text {in }}$. In the disc-instability model with outbursts starting at the outer disc edge, the distance between outbursts depends on the physical situation in the outermost disc regions, and our finding that the quiescent interval between outbursts is little affected by the change in $R_{\text {in }}$, is in agreement with expectations.

Perhaps the most surprising result of our calculations is found for the disc instabilities starting at the inner disc edge. Increasing the value of $R_{\text {in }}$ here leads to a dramatic increase in the length of the interval between outbursts, or even to complete suppression of outbursts. The explanation for this lies in the nature of the disc instability, which is caused by the transition of the disc matter between a mostly neutral and a mostly ionized state. This transition occurs at approximately the same temperature, somewhat less than 6000 K , everywhere in the disc. This can be seen in more detail in the S-curves, as given by, for example, Smak (1984). The highest temperatures in the accretion disc tend to occur close to the white dwarf. To reach a temperature $\simeq 6000 \mathrm{~K}$, demands a much higher $M$ at $R=12.5 R_{\mathrm{wd}}$ than at $R \simeq R_{\mathrm{wd}}$ (see equation 4). The mass-transfer rate through the disc is determined by the viscosity, and is high only when the surface density in the disc is high. Thus, to set off the instability at large $R_{\mathrm{in}}$, a high density must be built up throughout the accretion disc, to produce the required high $\dot{M}_{\text {in }}$. In Fig. 2, it is seen that just before the outburst, $L_{\mathrm{b} \mid}$ and a fortiori $\dot{M}_{\mathrm{in}}$ (see equation 3) are much higher for the model with $R_{\mathrm{in}}=12.5 R_{\mathrm{wd}}(\operatorname{model} 2 \mathrm{~b})$ than for $R_{\mathrm{in}}=R_{\mathrm{wd}}($ model 2a). The required build-up of a large density throughout the disc leads to the long quiescent intervals.

If the $\dot{M}_{\text {in }}$ required is larger than the mass-transfer from the companion to the outer edge of the disc, the whole disc can become stable, on the lower branch of the S-curve. This is what happens in our calculation with $\alpha_{1}=0.3$ and other parameters equal to those of model 2 b . (The dependence on $\alpha$ of the temperature at which the disc become unstable is very small; the stabilization between $\alpha=0.1$ and 0.3 is therefore to a large extent accidental.)

## 4 Discussion and comparison with observations

## 4.1 the disc spectrum

The outburst characteristics that may be compared with observations are the outburst spectra, and the outburst lengths. The spectra were calculated for the disc only. Other contributions may be expected from the mass-donor star, the hot spot, the boundary layer, and the white dwarf. The contribution of the mass donor is important only in the optical wavelength region, only during quiescence. The contribution of the hot spot is important in quiescence, and (in the transfer instability) during the burst of mass transfer. The hot-spot contribution can be separated from the disc by its characteristic phase dependence. The white dwarf can affect the ultraviolet flux if it is very hot, as it may be during outburst. If such a white dwarf contribution were important, it would show up in spectra of DQ Her systems at the short wavelength range of the ultraviolet spectra, as observed with IUE. However, the IUE observations do not indicate the presence of a hot component in the ultraviolet spectrum of DQ Her systems (Verbunt 1987). Together with the strong X-ray flux observed from many intermediate polars, this suggests that the bulk of $L_{\text {in }}$ is emitted in the X-ray region. Although it therefore seems that the calculated spectra do represent the expected ultraviolet - optical flux close to outburst maximum, care must be taken in making detailed comparisons, because of evidence in several cataclysmic variables that disc spectra cannot be constructed by adding stellar spectra (e.g. RU Peg, see La Dous et al. 1985; Wade 1988).

The optical spectrum of a DQ Her system can contain a component from the white dwarf or the boundary layer, either directly, or via irradiation of the disc. This may affect the magnitude of the optical and ultraviolet outburst, but not its length, because the duration of the increase in $L_{b l}$ is always shorter than the duration of enhanced disc luminosity $L_{d}$ (see Figs 1-3). Thus our conclusions about the length of the outbursts are not affected by this uncertainty.

### 4.2 THE RADIUS OF THE MAGNETOSPHERE

In the simplest approach, the radius of the magnetosphere is found by equating the magnetic pressure of a dipole field to the ram pressure of matter moving with the free-fall (or in the case of a disc, Keplerian rotation) velocity (Davidson \& Ostriker 1973). The radius $R_{\mathrm{m}}$ of a magnetosphere around an accreting white dwarf can be written
$R_{\mathrm{m}} \simeq 9 \times 10^{9} \mathrm{~cm}\left(\frac{B_{0} R_{\mathrm{wd}}^{3}}{3.7 \times 10^{32} \mathrm{G} \mathrm{cm}^{3}}\right)^{4 / 7}\left(\frac{10^{17} \mathrm{~g} \mathrm{~s}^{-1}}{\dot{M}}\right)^{2 / 7}\left(\frac{0.8 M_{\odot}}{M_{\mathrm{wd}}}\right)^{1 / 7}$,
where $B_{0}$ is the magnetic field strength at the white dwarf surface. Thus $9 \times 10^{9} \mathrm{~cm}$ is the radius of the magnetosphere of a white dwarf of $0.8 M_{\odot}$, with radius $7.2 \times 10^{8} \mathrm{~cm}$, and surface field strength of $10^{6} \mathrm{G}$, if it is accreting at a rate of $10^{17} \mathrm{~g} \mathrm{~s}^{-1}$.

Equation (8) is derived for stationary accretion. During outbursts, $\dot{M}$ can vary over several orders of magnitude. Equation (8) suggests that the radius of the magnetosphere may vary noticeably too. The exact range of variation depends on the density of matter remaining in the disc, and on the details of the interaction between the disc matter and a moving magnetosphere. It is beyond the scope of this paper to go into these details; we shall limit ourselves to a few remarks. The general conclusions of this paper, namely that the magnetosphere shortens the length of the outburst, and suppresses the ultraviolet flux, are valid, independent of the exact size of the magnetosphere. Similarly, our finding that the interval between disc instability outburst starting at the inner disc edge is longer in the presence of a magneto-
sphere, does not depend on the exact size of the magnetosphere, although the magnitude of the change does.

The sudden rise of the mass transfer rate at the inner edge could give rise to transient phemonena, as the magnetosphere shrinks. It would be interesting to see if such phenomena could be traced in radio observations. If the mass donor is less massive than about $0.3 M_{\odot}$, a magnetosphere of $9 \times 10^{9} \mathrm{~cm}$ is smaller than the angular momentum radius $R_{\mathrm{h}}$ (see the fitting formula for $R_{\mathrm{h}} / \alpha$ in Verbunt \& Rappaport 1988). Similarly, for more massive companions $R_{\mathrm{m}}$ can still be larger than $R_{\mathrm{h}}$ if the mass transfer rate is low or the surface magnetic field strength high.

As shown by equation (7), the disc disappears if $R_{\mathrm{h}}=R_{\mathrm{m}}$ for a sufficiently long time, and no persistent disc exists if $R_{\mathrm{h}}<R_{\mathrm{m}}$. Variations in $\dot{M}$ may therefore lead to temporary disappearances of the disc (see King 1986; Lamb \& Melia 1986 and references therein). We note that in systems where the disc is absent, no tidal force between the disc and the donor star is present to restore the angular momentum lost from the orbit with the mass flow (and given by $\dot{M} \sqrt{G M_{\text {wd }} R_{\mathrm{h}}}$ ) to the orbit. This speeds up the evolution of the binary, and may even lead to unstable mass transfer in systems with a relatively massive mass-donor star.

### 4.3 TV Col

In TV Col, the observations indicate that the ultraviolet flux rose more or less simultaneous with the optical flux, in the outburst observed by Szkody and Mateo (1984; see also Verbunt 1987). Within the appreciable uncertainties, this seems also to be the case in all models presented here for magnetic white dwarfs; as explained in Section 3, the absence of the inner disc region removes most of the delay between ultraviolet and optical rises. The length of each of the three outbursts of TV Col reported in the literature is about 6 hr (Szkody \& Mateo 1984; Schwarz et al. 1988). We have not tried to fit these outbursts in detail, but our calculations show that in a small disc around a large magnetosphere, such a short outburst is quite possible.

The question arises of why outbursts of intermediate polars are rare, if not in nature, then at least in the literature? For disc instabilities starting at the inner disc edge, we found that intervals between outbursts can be very long, or that the disc may be stable altogether. Outbursts starting at the inner disc edge are therefore predicted to be rare in intermediate polars. This cannot be the full explanation, however, because we find that outbursts starting at the outer edge are as frequent in intermediate polars as they are in non-magnetic systems. Interestingly, the outbursts in TV Col observed by Schwarz et al. (1988) occurred within 10 d of each other. If these outbursts are disc instabilities, they should therefore start at the outer edge of the accretion disc. The short distance between these outburst suggests that small dwarf-nova-like outbursts may be rather more frequent amongst DQ Her systems than hitherto suspected. The observation of an outburst in a very limited number of IUE observations also indicates that such outbursts are frequent. Mass-transfer bursts and disc instabilities starting in the outer disc region lead to optical maxima about 10 mag less bright in systems with a magnetosphere than in non-magnetic systems. This may help in explaining that many outbursts in DQ Her systems have hitherto gone unobserved.

### 4.4 GK Per

GK Per is an interesting system for students for the theory for dwarf nova outbursts. It differs from other dwarf novae in having a longer orbital period and hence a large Roche lobe around the white dwarf. In our models we use $R_{\text {out }}=3.3 R_{\odot}$. Its outbursts last about $50-100 \mathrm{~d}$ and
occur at intervals of about 400 d . Cannizzo \& Kenyon (1986) found that very small values of the viscosity in the disc are required to explain these outbursts in terms of the disc instability model. In terms of $\alpha$, both the outburst and the quiescent value of $\alpha$ must be smaller by an order of magnitude than the values used in modelling dwarf nova outbursts in systems with shorter orbital periods. Cannizzo \& Kenyon did not take into account the fact that GK Per is an intermediate polar. We have therefore also modelled the GK Per outbursts, using $R_{\text {in }}=9 \times 10^{9} \mathrm{~cm}$. Whereas we find that long outburst intervals do not require low values for $\alpha$, provided the outbursts start at the inner edge, we also find that long-lasting outbursts do require a very low value for $\alpha_{2}$, and hence also for $\alpha_{1}$. The finding of Cannizzo \& Kenyon (1986) therefore remains intact.

We have also calculated mass-transfer instability models for GK Per, for two different values of the magnetospheric radius. For $R_{\mathrm{in}}=9 \times 10^{9} \mathrm{~cm}$, the amount of mass transferred during the outburst to get a noticeable effect must be larger by at least an order of magnitude that the amounts transferred in calculations for outbursts in shorter-period systems. The reason for this is the large mass of the large accretion disc in GK Per, of about $2 \times 10^{23} \mathrm{~g}$ (for the parameters in Table 2, with $R_{\mathrm{in}}=12.5 R_{\mathrm{wd}}$ ); unless the amount of added matter is more than the disc mass, nothing much happens. If we use $R_{\mathrm{in}}=6.3 \times 10^{10} \mathrm{~cm}$, as appropriate for a surface field strength of $3 \times 10^{7} \mathrm{G}$, the disc mass is reduced to about $1.4 \times 10^{22} \mathrm{~g}$, and a much smaller mass-transfer burst will cause a noticeable outburst. However, in this model the temperatures never rise above a few thousand K . If the radius of the magnetosphere in GK Per is small, one has to explain either (in the disc-instability model) that the viscosity is an order-of-magnitude smaller than elsewhere, or (in the mass-transfer instability model) that the amount of transferred matter in the burst is an order-of-magnitude larger than elsewhere.

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