The Effect of Variable Thermal Conductivity and the Inclined Magnetic Field on MHD Plane Poiseuille Flow Past Non-Uniform Plate Temperature

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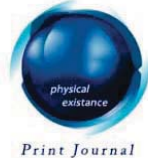*Keywords:* poiseuille-flow, thermal conductivity, inclined uniform magnetic field.

GJSFR-F Classification : MSC 2010: 65H20. 34D10

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I. Introduction

In MHD plane Poiseuille flow, the flow velocity is driven by non- zero pressure gradient and the plates are kept at a standstill. The axis of $\overline{x}$, for the sake of convenience is taken in the middle of the flow field.

The study of heat transfer by thermal conduction is a great importance in fluid dynamics. The temperature difference in fluid, in the span of time is reduced by heat flowing from higher temperature to those of lower temperature in regions. Fourier laws govern the heat transfer by conduction. Cooling procedure can be controlled effectively by theory of variable conduction for which the high quality product may be produced. Small Prandtle number of liquid metals is as used as coolants because of its higher thermal conductivity. The study of magneto hydrodynamic (MHD) Poiseuille flow between two parallel Plates has been on recent years of important research topic due to its numerous applications in solar technology, MHD power generators, MHD pumps, aerodynamics heating, electrostatic precipitation, purification of oil and fluid sprays and

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droplets, etc. MHD plane Poiseuille flow with high conducting fluid is also considered to have importance in transpiration cooling. Several engines can be protected from the influence of hot gases, applying high conducting fluid for its character coolants and are effective in heat transfer between the fluid and boundary with much application to exhaust nozzles, combustion chamber walls, and cooling of rockets and jet. He (2000, 2009), Bizarre, and Ghazvini (2008) perceived the solution of non-linear coupled equations by homotopy perturbation technique.

The subject of the above applications, different researchers, and scholars have made a series of investigations.

Alfven (1942) considered the existence of electromagnetic hydrodynamic waves. Nahme (1940) examined the temperature dependant viscosity in Couette flow. Hausenblas (1950) analysed the viscosity and temperature relation keeping both the walls at the same temperature in plane Poiseuille flow. Bansal and Jain (1975) discussed the same problem when both the walls are different temperature. Shercliff (1956), Cowling (1957), Schlichting (1960), Sinha et al. (1965), and Palm et al. (1972) studied on the steady free convection in porous matrix and extended their work in an isotropic porosity with heat exchange effect. Arunchalm and Rajappa (1978) discussed the force convection in liquid metal with variable conductivity and capacity. Drake (1965) measured the flow in a channel with periodic pressure gradient. Raptis et al. (1982) considered MHD free convective flow past parallel plates with porous medium, Ram et al. (1984) studied Hall Effect and heat with mass transfer through porous matrix. Singh (1992) analyzed MHD fluid flow between two parallel plates and extended his work in (2000) with the study of unsteady flow of fluid under the influence of inclined magnetic field through channels with changing pressure gradient exponentially. Al-Hadhrami et al. (2003) considered fluid flow through horizontal channels and resulted velocity in terms of Reynolds numbers using the porous matrix. Ganesh et al. (2007) discussed the MHD unsteady stokes flow problem between two parallel plates. They studied fluid being withdrawn through both the walls at the same rate. Mayonge at al. (2013) discussed the flow problem between Poiseuille flow channels if one plate of channel is porous under the influence of the inclined magnetic field. Kiema et al. (2015) analyzed the steady MHD Poiseuille flow between two infinite parallel porous plates under the inclined transverse magnetic field applying the finite difference method.

The proposed study on the effect of variable thermal conductivity and the inclined uniform magnetic field on steady MHD plane Poiseuille flow are through non-uniform plate temperature and constant suction.

a) Mathematical Formulation and its Solution

Consider the viscous incompressible electrically conducting plane Poiseuille fluid flow bounded by two parallel plates separated by a distance 2h. Taking x-axis along the centre line of the parallel plates and the $y^{\emptyset}$-axis is perpendicular to the plates i.e. $y^{\emptyset} = \pm$h. A uniform transverse magnetic field β_0 is applied normal to the wall. Both of the plates are kept stationary and maintained at constant dissimilar temperatures $\theta^{\emptyset}{}_0$ & $\theta^{\emptyset}{}_1$. It is assumed that the magnetic Reynold number is very small, so that the induce electric field caused by induce magnetic field is assumed negligible. The poiseuille flow is driven by the constant pressure gradient. The flow in the region is unidirectional, steady laminar and fully developed so all the physical variables except pressure depend on $y^{\emptyset}$ only. The suction velocity V= -V$_0$ is at one porous plate $y^{\emptyset} = $ -h so $\frac{\partial V_0}{\partial y^{\emptyset}} = 0$

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b) Governing Equations
Equation of momentum:

$$-V_0 \frac{\partial u^{\emptyset}}{\partial y^{\emptyset}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u^{\emptyset}}{\partial y^{2\emptyset}} - \frac{\sigma\beta^2{}_0 u^{\emptyset}}{\rho} \qquad \ldots\ldots\ldots\ldots(1)$$

Equation of energy with thermal conductivity is:

$$-\rho C_p V_0 \frac{\partial\theta^{\emptyset}}{\partial y^{\emptyset}} = \frac{\partial}{\partial y^{\emptyset}}\left(K^{\emptyset}\frac{\partial\theta^{\emptyset}}{\partial y^{\emptyset}}\right) + \mu\left(\frac{\partial u^{\emptyset}}{\partial y^{\emptyset}}\right)^2 \qquad \ldots\ldots\ldots\ldots(2)$$

Corresponding boundary conditions are

$$y^{\emptyset} = h \qquad u^{\emptyset} = 0 \qquad \theta^{\emptyset} = \theta_1$$

And

$$y^{\emptyset} = -h \qquad u^{\emptyset} = 0 \qquad \theta^{\emptyset} = \theta_0 \qquad \ldots\ldots\ldots\ldots(3)$$

Following Arunachalam and Rajappa (1978), the thermal conductivity is assumed to vary linearly with temperature and it is of the form:

$$K^{\emptyset} = K\,(1 + \epsilon\theta)$$

$$V = -V_0 \text{ Suction constant velocity} \qquad \ldots\ldots\ldots\ldots(4)$$

Introducing dimensionless quantities:

$$y = \frac{y^{\emptyset}}{h}, \; Re = \frac{+V_0 h}{\upsilon}, \quad P = \frac{-h^2}{2\mu U_m}\frac{dp}{dx}$$

$$\theta = \frac{\theta^{\emptyset} - \theta^{\emptyset}{}_0}{\theta^{\emptyset}{}_1 - \theta^{\emptyset}{}_0}, \qquad \bar{M}^2 = \frac{\sigma\beta_0{}^2 h^2}{\upsilon}$$

$$Pr = \frac{\rho C p\upsilon}{K}, \; Ec = \frac{U_m{}^2}{Cp(\theta^{\emptyset}{}_1 - \theta^{\emptyset}{}_0)} \qquad \ldots\ldots\ldots\ldots(5)$$

Fluid motion is maintained owing to the constant pressure gradient. It is sufficiently assumed that the maximum velocity $\left(U_m = -\frac{h^2}{2\mu}\frac{dp}{dx}\right)$ contained in the middle of the channel in the plane Poiseuille flow with constant fluid properties (Schilichting). Equation (1) and (2) ease into non-dimensional momentum and energy equations using equation (5) (the dimensionless parameters).
Non-dimensional equation of momentum:

$$\frac{d^2 u}{dy^2} + Re\frac{du}{dy} - \bar{M}^2 u = -P$$

Or

$$\frac{d^2 u}{dy^2} + Re\frac{du}{dy} - \bar{M}^2 Sin^2\alpha u = -P \qquad \ldots\ldots\ldots\ldots(6)$$

Where α is the angle between v and β_0 which means that the two fields able to be assessed at any one angle α for $0 \leq \alpha \leq \pi$

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Equation (6) became

$$\frac{d^2u}{dy^2} + Re\frac{du}{dy} - M^2u = -P \qquad \text{.......(7)}$$

Non-dimensional equation of energy:

$$(1 + \varepsilon\theta)\frac{d^2\theta}{dy^2} + RePr\frac{d\theta}{dy} + \epsilon\left(\frac{d\theta}{dy}\right)^2 = EcPr\left(\frac{du}{dy}\right)^2 \qquad \text{.........(8)}$$

Normalize boundary conditions are

$$y = -1, u = 0, \theta = 0$$

$$y = 1, u = 0, \theta = 1 \qquad \text{------ (9)}$$

where σ is the coefficient of electrical conductivity, Cp is the specific heat, β_0 is the applied magnetic field, υ is the kinematic viscosity, μ is the viscosity of the fluid and $-V_0$ is the suction velocity, 2h is the distance between plates, K is thermal conductivity, ε is variable conductivity of fluid, u is dynamic velocity, ρ is density, θ is the temperature of fluid at any point, α is the inclined angle between fluid velocity and applied magnetic field, Cp is specific heat, θ_1, θ_0 are the temperature of the upper and the lower plate temperature where $\theta_1 > \theta_0$. $\bar{M}$ the Hartmann number, M $= \bar{M}$ sin α the Hartmann number with inclined angle α, Pr the Prandtl number, Ec the Ekert number, Re the Reynold number. P the normalize constant pressure gradient, U_m the maximum velocity of fluid.

II. Solutions

The boundary value problem described by the equations, non-coupled (7) and coupled (8) through (9) which provide analytical solutions. Firstly, the solution of the momentum ordinary differential equation (7) is obtained which used to solve the equation (8) by homotopy perturbation technique. Solution of equation (7) using boundary conditions (9) is obtained as given below:

$$u(y) = C_1e^{a_1y} + C_2e^{a_2y} + \frac{P}{M^2} \qquad \text{.........(10)}$$

Where integration constants C_1 & C_2 are computed with boundary conditions (9) and are obtained as:

$$C_1 = \frac{-\left[C_2e^{a_2} + \frac{P}{M^2}\right]}{e^{a_1}}$$

And

$$C_2 = \frac{P}{M^2}\left[\frac{e^{a_1} - e^{-a_1}}{e^{(a_2-a_1)} - e^{(a_1-a_2)}}\right]$$

Again

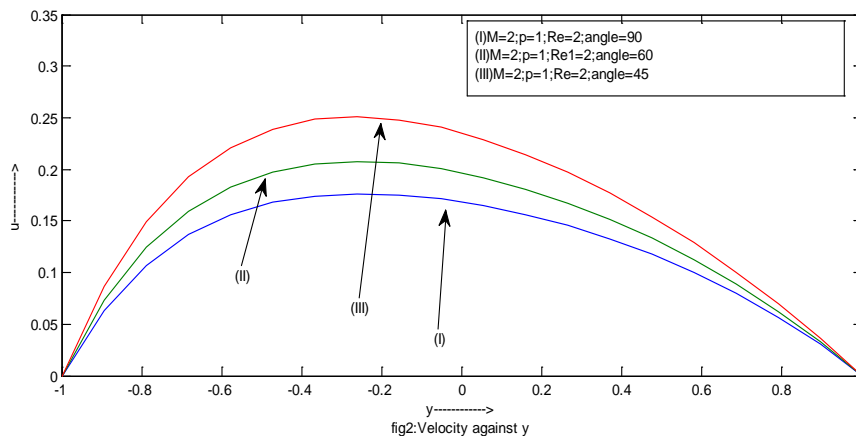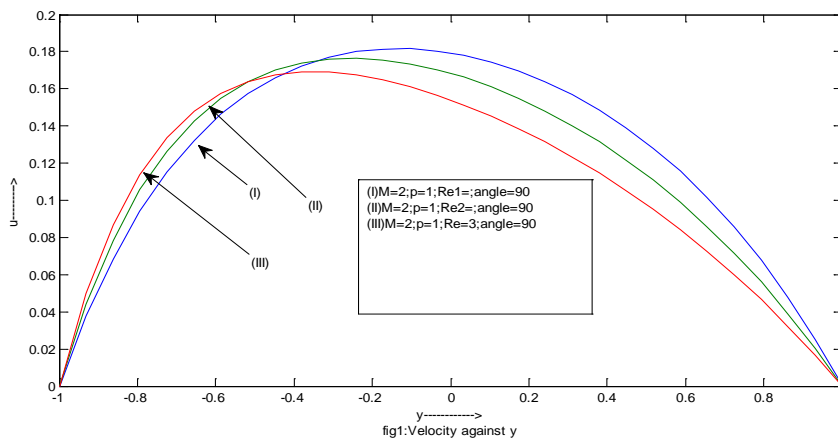$$a_1 = \frac{-Re + \sqrt{Re^2 + 4M^2}}{2}$$

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24. Biazar, j. and Ghazvini, H., (2008), Homotopy perturbation method for solving hyperbolic partial differential equations, An International Journal Coumpters and Mathematics with applications, 56,453-458.

$$a_2 = \frac{-Re - \sqrt{Re^2 + 4M^2}}{2}$$

And M = $\bar{M}$ Sin α

On using dynamic velocity of fluid u, given by equation (10) in the coupled energy equation (8) and is solved by homotopy perturbation technique.
Construct homotopy for energy equation, [Ref. 24, 25, 26]:

$$H = L (\theta) - L (\theta_i) + P [L (\theta_i) + N (\theta) - F (r)] = 0 \qquad \text{-------- (11)}$$

Where L (θ) and N (θ) are the Linear and Non-linear term of θ and L (θ_i) is the initial term of linearity.

$$\text{Let } \theta (y) = \theta_{00} + p\, \theta_{01} + \qquad \text{.............. (12)}$$

We get the solution of energy equation (8) for the temperature θ in view of boundary conditions (9) is obtained as follows:

$$\theta(y) = \frac{1}{2}\left(1 + Sin\frac{\pi}{2}y\right) + \gamma_2 + \gamma_1 e^{-PrRey} - \beta_2 Sin\frac{\pi}{2}y - \beta_3 Cos\frac{\pi}{2}y$$

$$+\beta_4 Cos\pi y - \beta_5 Sin\pi y + \beta_6 e^{2a_1 y} + \beta_7 e^{2a_2,y} + \beta_8 e^{(a_1+a_2)y} \qquad \text{........... (13)}$$

Where γ_2, γ_1, β_2, β_3, β_4, β_5, β_6, β_7 and β_8 are constants and their values are not given here for the sake of brevity.

III. Result & Discussion

The study on the effect of variable thermal conductivity and the inclined uniform magnetic field on steady MHD plane Poiseuille flow through non-uniform plate temperature and constant suction have been discussed numerically and are performed for the velocity and temperature profile. The consequences are displayed graphically in figure (1) to (5) for pertinent parameters such as Hartmann number M, Reynold number Re, thermal conductivity Parndtl parameter (Pr) and variable thermal conductivity (ε).

Figure 1 shows the effect of different inclination angle of magnetic parameter M on the velocity profile against y. It is evident that an increase in inclination angle (α), reducing the velocity of the flow field, maximum retardation of the velocity of flow occurs at the inclination angle α = 90°, signifies the increase of maximum resistive type force (Lorenz force) which tends to parallel opposite direction of flow field has a tendency to slow down the motion of fluid flow.

Figure 2: illustrates the velocity profile for different values of the suction parameter (Re) when taking all other parameters constant. The fluid velocity accelerates near the lower plate and decelerates near the upper plate [Das and Jana (2013)]. Suction parameter (Re) affects the main velocity of the flow field, which accelerate at the hand of suction (Lower plate) because suction parameter sucks the obstacle dust particles thus decreases the boundary layer thickness.

Figure 3: It is encountered to conclude that the temperature profile for different values of Reynold number (Re), increasing of suction parameter (Re) reduces the temperature at all the points of fluid flow. It is attributed to the fact that suction parameter absorbs the heat.

Figure 4: It is observed that decreasing of Parndtl (Pr) number increases the thermal conductivity and therefore, heat is able to diffuse away from plate than the higher value of Pr. Hence, in case of small Prandtl number, the thermal boundary layer is thicker, and the temperature profile increasing.

Figure 5: The effect of variable thermal conductivity parameter is shown in figure 5 for the high conducting fluid. It is observed from this depict that the increasing value of ε results in increasing the magnitude of temperature causing thermal boundary layer thickening reaches at the certain point of the fluid, but after reaching a certain point, the figure shows the effect of thermal layer thinning thus heat may be transferred from the fluid to plate

IV. Conclusion

(1) Increasing Re (Reynold number) decreases the dynamic velocity near to the suction plate, but the reversal effect shows at another plate coincides with the results of [Das and Jana 2013].

(2) Maximum retardation of velocity occurs at the inclination angle 90 degree that are between velocity and magnetic field.

(3) The effect of Prandtl number is to decrease the thermal boundary layer thickness.

(4) The thermal variable thermal conductivity also has an impact in enhancing the temperature at the certain point of temperature profile, then decay to the adjacent to the heating plate for high conducting fluid.

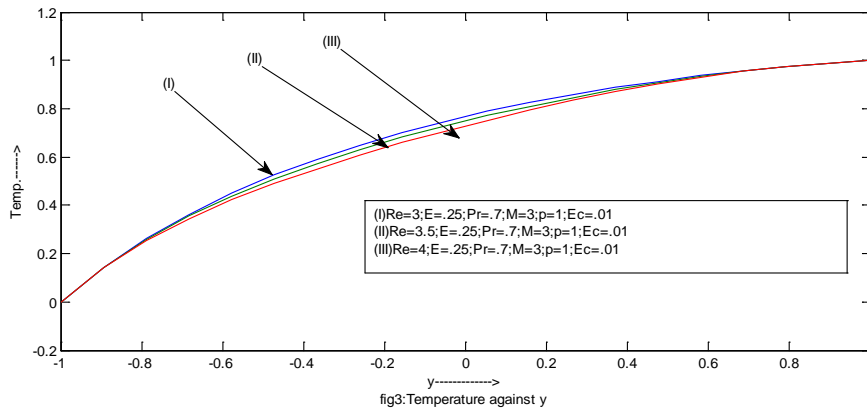(5) Fluid temperature decreases with the increases in Reynold number

(I)M=2;p=1;Re1=;angle=90
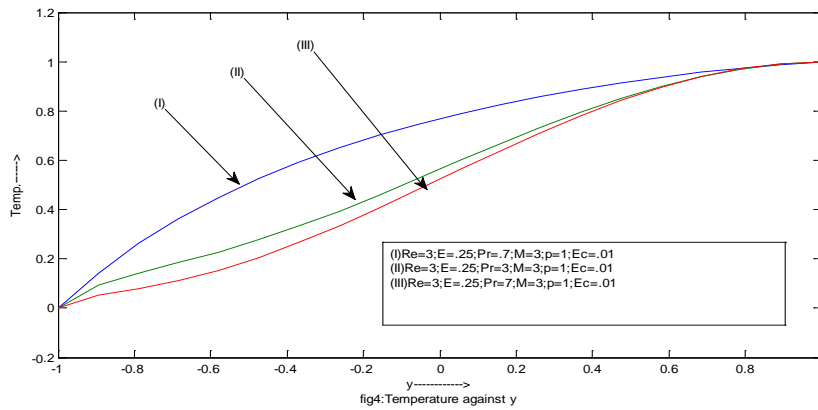(II)M=2;p=1;Re2=;angle=90
(III)M=2;p=1;Re=3;angle=90

fig1:Velocity against y

(I)M=2;p=1;Re=2;angle=90
(II)M=2;p=1;Re1=2;angle=60
(III)M=2;p=1;Re=2;angle=45

fig2:Velocity against y

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fig3:Temperature against y

(I)Re=3;E=.25;Pr=.7;M=3;p=1;Ec=.01
(II)Re=3.5;E=.25;Pr=.7;M=3;p=1;Ec=.01
(III)Re=4;E=.25;Pr=.7;M=3;p=1;Ec=.01

fig4:Temperature against y

(I)Re=3;E=.25;Pr=.7;M=3;p=1;Ec=.01
(II)Re=3;E=.25;Pr=3;M=3;p=1;Ec=.01
(III)Re=3;E=.25;Pr=7;M=3;p=1;Ec=.01

fig5: Temperature against y

(i)Re=3;E=00;Pr=.7;M=3;p=1;Ec=.01
(II)Re=3;E=.25;Pr=.7;M=3;p=1;Ec=.01
(III)Re=3;E=.5;Pr=.7;M=3;p=1;Ec=.01

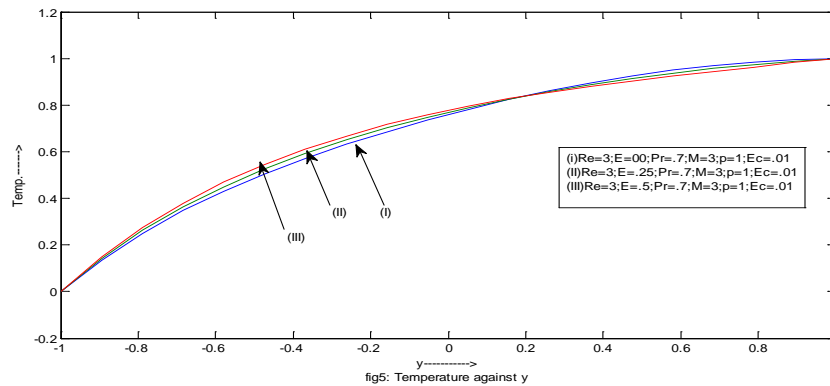## References Références Referencias

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