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The Effective Index Method and Its Application to Semiconductor Lasers

JENS BUUS, MEMBER, IEEE

Abstract—By the effective index method a two-dimensional field problem is transformed to a problem for a one-dimensional effective waveguide. This method is applied to semiconductor lasers having a gradual lateral variation in the complex permittivity. For the special case of a parabolic variation, analytical formulas for the required gain in the center and the half width of the intensity distribution are derived. The results are compared with a numerical method and very good agreement is found except in some cases where convergence problems occur for the numerical method. This agreement is taken as evidence for the validity of results obtained using the effective index method for analysis of semiconductor laser structures.

I. INTRODUCTION

THE effective index method was originally proposed for analysis of rectangular dielectric waveguides [1]. By this method a two-dimensional problem is transformed to a problem for a one-dimensional effective waveguide; this latter problem can then be solved analytically or by simpler methods

than required for the two-dimensional case. The method has been shown to be better than other simple methods for rectangular dielectric waveguides.

The analysis of waveguiding in semiconductor lasers is a two-dimensional problem and the effective index method has been used in different forms to simplify the solution of this problem, e.g., [2]–[6].

In this paper some general formulas related to the analysis of waveguiding in laser structures are presented. For the special case of parabolic permittivity variation the general formulas are used to derive analytical expressions for the gain in the center of the active region required to overcome the loss, and for the full width at half intensity of the field distribution in the junction plane.

For the parabolic profile a numerical method was presented in [7], this method is summarized in Section III and it is shown how the method can be somewhat simplified, reducing the required computing time considerably.

Finally, in Section IV the results from the two methods are presented and discussed.

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II. FIELD CALCULATION BY THE "EFFECTIVE INDEX METHOD"

A. Slab Structure

We first consider a simple slab structure with permittivity variation in the x -direction only, as shown in Fig. 1. Using the scalar wave equation we find the propagation constant for the TE modes (y polarization) (e.g., [8])

$$\left(\frac{\beta_z}{k}\right)^2 = \epsilon_{\text{eff}} = b\epsilon_1 + (1-b)\epsilon_2 \quad (1)$$

where the normalized propagation constant b is the solution to the characteristic equation, which for the fundamental mode reads

$$v\sqrt{1-b} = \arctan \sqrt{\frac{b}{1-b}} \quad (2)$$

$$v = kt\sqrt{\epsilon_1 - \epsilon_2}. \quad (3)$$

The imaginary parts of ϵ_1 and ϵ_2 are much smaller than the real parts and hence we consider the imaginary parts as perturbations of the corresponding slab structure with real indexes. This gives for the effective refractive index

$$n_{\text{eff}} = \text{Re} \{ \sqrt{\epsilon_{\text{eff}}} \} = \frac{1}{k} \text{Re} \{ \beta_z \} = \sqrt{b_o n_1^2 + (1-b_o) n_2^2} \quad (4)$$

where b_o is the solution to the characteristic equation (2) with the normalized frequency v replaced by the real number v_o

$$v_o = kt\sqrt{n_1^2 - n_2^2}. \quad (5)$$

We now include the imaginary parts of ϵ_1 and ϵ_2 and get a complex value for b with $b_i = \text{Im} \{ b \} \ll b_o$, i.e.,

$$b = b_o + jb_i = b_o + \Delta v \left. \frac{db}{dv} \right|_{v=v_o} \quad (6)$$

where Δv is the change in v due to the imaginary parts of ϵ_1 and ϵ_2

$$\Delta v = j \frac{v_o}{n_1^2 - n_2^2} \frac{n_1 g + n_2 \alpha}{2k}. \quad (7)$$

This gives a complex value for ϵ_{eff} when (6) is used in (1), and we introduce an effective gain g_{eff} defined by (neglecting $(g_{\text{eff}}/k)^2$)

$$\epsilon_{\text{eff}} = n_{\text{eff}}^2 + j \frac{n_{\text{eff}} g_{\text{eff}}}{k}. \quad (8)$$

Use of (6) and (7) in (1) gives

$$g_{\text{eff}} = 2 \text{Im} \{ \beta_z \} = k \frac{\text{Im} \{ \epsilon_{\text{eff}} \}}{n_{\text{eff}}} = \frac{\Gamma n_1 g - (1-\Gamma) n_2 \alpha}{n_{\text{eff}}} \quad (9)$$

where the weighing factor Γ is given by

$$\Gamma = b_o + \frac{v_o}{2} \left. \frac{db}{dv} \right|_{v=v_o} = b_o + (1-b_o) \frac{v_o \sqrt{b_o}}{1 + v_o \sqrt{b_o}}. \quad (10)$$

Γ is also defined as the fraction of the intensity propagating

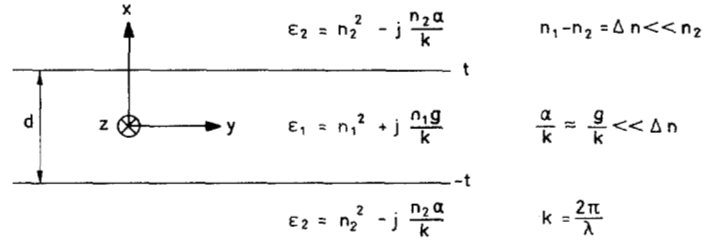


Fig. 1. Definition of slab structure with ϵ denoting permittivity, n refractive indexes, α loss, g gain, and k wavenumber. The terms $(\alpha/k)^2$ and $(g/k)^2$ are neglected.

in the guiding layer; direct calculation gives the same expression as (10).

Equation (9) can alternatively be derived in the following way [9]: the scalar field equation is multiplied by the complex conjugate of the field; the resulting expression is integrated from $-\infty$ to ∞ . This gives a variational expression for β_z^2 , and taking the imaginary parts of ϵ_1 and ϵ_2 as perturbations, the field (to first order) remains real whereas β_z becomes complex, taking the imaginary part of the expression for β_z^2 gives: $E(x)$ being the field distribution

$$\text{Im} \{ \epsilon_{\text{eff}} \} = \text{Im} \left\{ \frac{\beta_z^2}{k^2} \right\} = \frac{\int_{-\infty}^{\infty} \text{Im} \{ \epsilon(x) \} |E(x)|^2 dx}{\int_{-\infty}^{\infty} |E(x)|^2 dx} \quad (11)$$

defining Γ as the fraction of the intensity between $-t$ and t (9) is obtained. Since $n_2 \leq n_{\text{eff}} \leq n_1$ and $(n_1 - n_2) \ll n_2 \lesssim n_1$ (9) gives

$$g_{\text{eff}} \approx \Gamma g - (1-\Gamma) \alpha. \quad (12)$$

B. Slab with Lateral ϵ Variation

We now consider the case where ϵ_1 and ϵ_2 are functions of y given by

$$\left. \begin{aligned} \epsilon_1(y) &= \epsilon_1(0) - f_1(y) \\ \epsilon_1(0) &= n_1^2 + j \frac{n_1 g_o}{k} \\ f_1(0) &= 0 \end{aligned} \right\} \quad (13a)$$

$$\left. \begin{aligned} \epsilon_2(y) &= \epsilon_2(0) - f_2(y) \\ \epsilon_2(0) &= n_2^2 - j \frac{n_2 \alpha}{k} \\ f_2(0) &= 0. \end{aligned} \right\} \quad (13b)$$

We define an effective permittivity for each plane given by a value of y , according to (1)

$$\epsilon_{\text{eff}}(y) = b(y) \epsilon_1(y) + (1-b(y)) \epsilon_2(y). \quad (14)$$

For the laser structures considered we have

$$|f_1(y)|, |f_2(y)| \ll |\epsilon_1(0) - \epsilon_2(0)|; \quad (15)$$

hence, we only use one term in the expressions of b

$$b(y) = b(0) + (v(y) - v(0)) \left. \frac{db}{dv} \right|_{y=0}. \quad (16)$$

Insertion of (16) and (13) in (14) then gives for the variation of ϵ_{eff} in the y direction

$$\begin{aligned}\Delta(y) &= \epsilon_{\text{eff}}(0) - \epsilon_{\text{eff}}(y) \\ &= b(0)f_1(y) + (v(0) - v(y)) \left. \frac{db}{dv} \right|_{y=0} \epsilon_1(0) \\ &\quad + (1 - b(0))f_2(y) - (v(0) - v(y)) \left. \frac{db}{dv} \right|_{y=0} \epsilon_2(0)\end{aligned}\quad (17)$$

$$\begin{aligned}v(y) &= kt \sqrt{(\epsilon_1(0) - f_1(y)) - (\epsilon_2(0) - f_2(y))} \\ &\approx v(0) \left(1 - \frac{1}{2} \frac{f_1(y) - f_2(y)}{\epsilon_1(0) - \epsilon_2(0)} \right).\end{aligned}\quad (18)$$

Using the expression for the derivative of b with respect to v [also used in (10)] (17) and (18) give

$$\Delta(y) = \Gamma f_1(y) + (1 - \Gamma) f_2(y).\quad (19)$$

In (19) Γ is the filling factor for the simple real slab guide, i.e., (10) with v_0 given by (5) and hence independent of y . This formula, of course, is valid only when the expansion (16) is valid, i.e., (15) must be satisfied. If this is not the case the definition of $\epsilon_{\text{eff}}(y)$ given in (14) must be used directly.

The result (19) is also derived in [10], [11] and was first used in [3]. In [12] the y dependence and imaginary part of b is neglected, thus giving an equation similar to (19) but with Γ replaced by b_0 . When the imaginary part of b is included (but the y dependence ignored) one again gets an equation similar to (19), but with b_0 as weighing factor for the real part and Γ for the imaginary part [5], [13]. If the guiding parallel to the active layer is dominated by variations in the thickness of the active layer we get [2]

$$\Delta(y) = (b(0) - b(y)) (\epsilon_1(0) - \epsilon_2(0)).\quad (20)$$

In this case, the imaginary part of b and the y dependence of ϵ_1 and ϵ_2 are unimportant for the guiding.

The effective index method was originally proposed for rectangular structures [1]. The method is shown [14] to be better than another simple method [15] for rectangular dielectric waveguides.

The y variation of the effective index gives a y variation of the field which can be described by a lateral propagation constant β_y ($|\beta_y| \ll |\beta_z|$ if (15) is satisfied). Compared with the slab without variations in the y direction we now get modified values of the propagation constant, the effective index, and the effective gain (mode gain), which we define as

$$\beta'_z = \sqrt{\beta_z^2 - \beta_y^2}\quad (21)$$

$$n'_{\text{eff}} = \frac{1}{k} \text{Re} \{ \beta'_z \}\quad (22)$$

$$G = 2 \text{Im} \{ \beta'_z \}\quad (23)$$

where β_z is the result for the slab guide without y variations ($f_1(y) = f_2(y) = 0$). Expansion of the square root of β'_z in (22) and (23) and use of (4) and (9) gives

$$n'_{\text{eff}} = n_{\text{eff}} - \frac{\text{Re} \{ \beta_y^2 \}}{2n_{\text{eff}}k^2}\quad (24)$$

$$G = g_{\text{eff}} - \frac{\text{Im} \{ \beta_y^2 \}}{n_{\text{eff}}k}.\quad (25)$$

C. The Parabolic Profile

We now apply the results to the case of a parabolic variation of index and gain in the active layer [7]

$$\left. \begin{aligned}f_1(y) &= a^2 y^2 \\ f_2(y) &= 0\end{aligned} \right\}\quad (26)$$

hence from (19)

$$\Delta(y) = \Gamma a^2 y^2 = a_{\text{eff}}^2 y^2.\quad (27)$$

In order to introduce convenient parameters we assume that the gain in the active layer equals 0 at $y = \pm y_0$ (half gain width), i.e.,

$$\epsilon_1(y_0) = n_1^2 - 2n_1 \delta n\quad (28)$$

where $\delta n \ll (n_1 - n_2)$ is the change of the real part of the refractive index going from $y = 0$ to $y = \pm y_0$. Thus

$$a^2 y_0^2 = \frac{n_1 g_0}{k} (R + j)\quad (29)$$

with the ratio R between the variation in the real and imaginary parts of ϵ_1 (note that R is defined differently in [7])

$$R = \delta n \frac{2k}{g_0}.\quad (30)$$

The effective guiding is described by a_{eff}

$$a_{\text{eff}} = \sqrt{\frac{\Gamma n_1 g_0}{k y_0^2} (R + j)}.\quad (31)$$

As is well known, the solution to the field equation with an effective permittivity variation given by (27) is Hermite-Gauss functions and the field variation in the y -direction for the fundamental mode becomes

$$E(y) = e^{-ka_{\text{eff}}(y^2/2)}\quad (32)$$

with the intensity variation

$$I(y) = |E(y)|^2 = e^{-k \text{Re} \{ a_{\text{eff}} \} y^2}\quad (33)$$

and eigenvalue (square of lateral propagation constant)

$$\beta_y^2 = ka_{\text{eff}}.\quad (34)$$

Thus (24) and (25) give

$$n'_{\text{eff}} = n_{\text{eff}} - \frac{\text{Re} \{ a_{\text{eff}} \}}{2n_{\text{eff}}k}\quad (35)$$

$$G = g_{\text{eff}} - \frac{\text{Im} \{ a_{\text{eff}} \}}{n_{\text{eff}}}.\quad (36)$$

In the lasing state the required mode gain G_{req} must balance mirror and internal scattering losses (i.e., losses not included in the net material gain); using (9), (36), and (31) this condition can be written as

$$G_{\text{req}} = \alpha_{\text{mirror}} + \alpha_{\text{scattering}} = \frac{\Gamma n_1 g_o - (1 - \Gamma) n_2 \alpha}{n_{\text{eff}}}$$

$$-\frac{\sqrt{\frac{\Gamma n_1 g_o}{k y_o^2}} \text{Im} \{ \sqrt{R+j} \}}{n_{\text{eff}}} \quad (37)$$

which can be rearranged as

$$g_o - \sqrt{\frac{g_o}{\Gamma n_1 k y_o^2}} \text{Im} \{ \sqrt{R+j} \} = G_{\infty} \quad (38)$$

with G_{∞} being the required gain in the active region for $y_o \rightarrow \infty$

$$G_{\infty} = \frac{G_{\text{req}} + (1 - \Gamma) \alpha \frac{n_2}{n_{\text{eff}}}}{\Gamma \frac{n_1}{n_{\text{eff}}}} \quad (39a)$$

$$\approx \frac{G_{\text{req}} + (1 - \Gamma) \alpha}{\Gamma} \quad (39b)$$

We define a new parameter C as

$$C = \text{Re} \{ \sqrt{R+j} \} = \sqrt{\frac{R + \sqrt{1+R^2}}{2}} = \frac{1}{2 \text{Im} \{ \sqrt{R+j} \}} \quad (40)$$

It should be noted that C is related to the astigmatism factor K , defined in [15]

$$K = \sqrt{1 + \frac{1}{4C^4}} \quad (41)$$

The product of the width of the near and far fields can be shown to be proportional to K , for $R \gg 1$ (index guiding) $K = 1$.

Solving (38) for g_o and the use of (40) gives

$$g_o = \frac{(1 + \sqrt{1 + G_{\infty} \Gamma B C^2})^2}{\Gamma B C^2} \quad (42)$$

$$B = 16 n_1 k y_o^2 \quad (43)$$

Note that this formula is somewhat different from the corresponding formula (35) in [7].

The full width of half intensity w_o is found from

$$I\left(\frac{w_o}{2}\right) = \frac{1}{2} \quad (44)$$

which gives

$$w_o = \frac{4 \sqrt{\ln 2} y_o}{\sqrt[4]{g_o \Gamma B C^2}} \quad (45)$$

$$= \frac{4 \sqrt{\ln 2} y_o}{\sqrt{1 + \sqrt{1 + G_{\infty} \Gamma B C^2}}} \quad (46)$$

The expression (35) can now be written as

$$n'_{\text{eff}} = n_{\text{eff}} - \frac{2 \ln 2}{n_{\text{eff}} (k w_o)^2} \quad (47)$$

Evaluation of (42) in the two limiting cases for small and large values of the thickness $d = 2t$ gives

$$d \rightarrow 0 \Rightarrow w_o \rightarrow \frac{4 \sqrt{\ln 2} y_o}{\sqrt{1 + \sqrt{1 + \frac{n_2}{n_1} (G_{\text{req}} + \alpha) B C^2}}} \quad (48)$$

$$d \rightarrow \infty \Rightarrow w_1 \rightarrow \frac{4 \sqrt{\ln 2} y_o}{\sqrt{1 + \sqrt{1 + G_{\text{req}} B C^2}}} \quad (49)$$

$$\frac{w_o(d \rightarrow \infty)}{w_o(d \rightarrow 0)} \leq \sqrt[4]{\frac{\frac{n_2}{n_1} (G_{\text{req}} + \alpha)}{G_{\text{req}}}} \quad (50)$$

This result shows that w_o is nearly independent of d . For small y_o (46) gives

$$w_o \approx \sqrt{2 \ln 2} (2 y_o) \quad (51)$$

III. FIELD CALCULATION BY PLANEWAVE EXPANSION

Following [7], the field in the active layer of a laser structure with the permittivity given by (26) and (29) is written (retaining a finite number of expansion terms)

$$E(x, y) = \sum_{l=0}^{l_{\text{max}}} A_l \cos(q_l x) H_l(\sqrt{ak} y) \exp\left(-ak \frac{y^2}{2}\right) l_{\text{even}} \quad (52)$$

H_l is the Hermite polynomial of order l . Insertion of (52) in the field equation gives relations between q_l and the propagation constant β'_z

$$k^2 \epsilon_1(0) - \beta_z'^2 = q_l^2 + ak(2l+1) \quad l=0, 2, \dots, l_{\text{max}} \quad (53)$$

Matching (52) to a planewave expansion at $x = \pm t$ (see Fig. 1) results in a set of linear equations for the expansion coefficients A_l appearing in (52)

$$A_l' = \sum_l A_l \Omega_{l,l'} \quad l=0, 2, \dots, l_{\text{max}} \quad (54)$$

The matrix elements are calculated in [7] and can be written as

$$\Omega_{l,l'} = \frac{(-1)^{(l+l')/2} \cos\left(q_l \frac{d}{2}\right)}{q_l \sin\left(q_l' \frac{d}{2}\right)} \sqrt{\beta_z'^2 - \epsilon_2 k^2} \cdot I_{l',l} \quad (55a)$$

$$I_{l',l} = \frac{1}{\sqrt{\pi 2^l l'!}} \int_{-\infty}^{\infty} H_{l'}(z) H_l(z) e^{-z^2} \sqrt{1 + Az^2} dz \quad (55b)$$

$$A = \frac{ak}{\beta_z'^2 - \epsilon_2 k^2} \quad (55c)$$

In order to have solutions for (54) we must have

$$\text{Det}(\bar{\Omega} - \bar{E}) = 0 \quad (56)$$

This condition determines the propagation constant β'_z and the required center gain g_o is determined by varying g_o [and hence the value of a given by (29)] until the condition $2 \text{Im} \{ \beta'_z \} = G_{\text{req}}$ is satisfied, with G_{req} being the mode gain required to overcome the loss.

Since a double iteration is needed in order to calculate β'_z and g_o , the computing time becomes excessive if (55b) is evaluated by numerical integration. The Hermite-Gauss part

of the integrand in (55b) obtains its maximum value for $z = z_{\max} \approx \sqrt{l_{\max}}$ and decreases very rapidly for $z > z_{\max}$. If we have $|A|z'^2 < 1$ with $z' \approx \sqrt{l_{\max}} + 1$ a power series expansion of the square root in (55b) will be valid in the interval of z values where the integrand is nonvanishing, for $l_{\max} = 10$ the condition is satisfied for all parameter values considered, this will not be the case for extremely small values of the thickness. Thus, retaining m terms

$$\sqrt{1 + Az^2} = 1 + \sum_{i=1}^m b_i A^i z^{2i}. \quad (57)$$

Expansion of the powers in Hermite polynomials ($c_{i,n}$ given in [17])

$$z^{2i} = \sum_{n=0}^i c_{i,n} H_{2n}(z) \quad (58)$$

gives

$$\sqrt{1 + Az^2} = 1 + \sum_{i=1}^m \left\{ b_i A^i \sum_{n=0}^i (c_{i,n} H_{2n}(z)) \right\}. \quad (59)$$

Equation (55b) now gives

$$I_{l',l} = \delta_{l',l} + \frac{1}{\sqrt{\pi} 2^{l'} l'!} \sum_{i=1}^m \left\{ b_i A^i \left(\sum_{n=0}^i c_{i,n} I_{2n,l',l} \right) \right\} \quad (60)$$

where [18]

$$\begin{aligned} I_{2n,l',l} &= \int_{-\infty}^{\infty} H_{2n}(z) H_{l'}(z) H_l(z) e^{-z^2} dz \\ &= \frac{\sqrt{\pi} 2^s (2n)! l'! l!}{(s-2n)! (s-l')! (s-l)!}, \quad s = \frac{2n+l'+l}{2}. \end{aligned} \quad (61)$$

$I_{l',l}$ differ from 0 only if $|l' - l| \leq 2n \leq (l' + l)$. Insertion of (61) in (60) gives

$$I_{l',l} = \delta_{l',l} + \sum_{i=1}^m A^i d_{i,l',l}. \quad (62)$$

The coefficients $d_{i,l',l}$ need only be calculated once

$$d_{i,l',l} = \frac{l!}{2^{l'}} b_i \sum_{n=0}^i c_{i,n} \frac{2^s (2n)!}{(s-2n)! (s-l')! (s-l)!}. \quad (63)$$

This very fast method for calculation of the matrix elements has been compared with results obtained by numerical integration, and fine agreement was found. The computing time is now determined by the time required to solve (56) rather than 55b).

IV. RESULTS

The center gain g_o and full width at half intensity w_o are first calculated by the effective index method according to (42) and (46). Some results are shown in Figs. 2 and 3.

The planewave expansion method is applied to the following cases: $n_2 = 3.4, 3.5$; $R = -3, -2, -1, 0, 1$; $d = 0.1, 0.15, 0.2, 0.4 \mu\text{m}$; $y_o = 2, 4, 6, 8, 10 \mu\text{m}$, using $m = 4, 5$ in (57) and $l_{\max} \leq 10$ in (52).

In contrast to [7], very good agreement with the effective

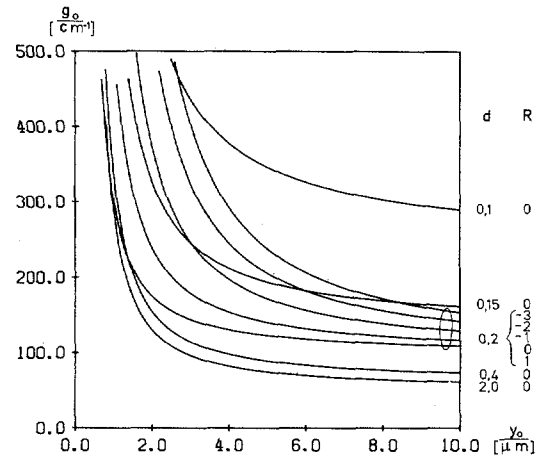


Fig. 2. Center gain g_o as function of the gain width y_o for various values of the active layer thickness d and the guiding parameter R . Other parameter values: $\lambda = 0.9 \mu\text{m}$, $G_{\text{req}} = 50 \text{ cm}^{-1}$, $\alpha = 20 \text{ cm}^{-1}$, $n_1 = 3.6$, and $n_2 = 3.4$.

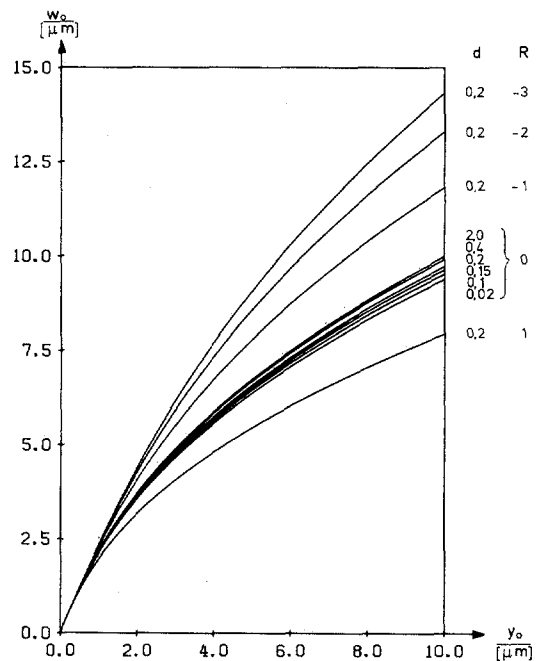


Fig. 3. Full width at half intensity w_o calculated under the same conditions as g_o in Fig. 1.

index method is found; a few examples are given in Table I.

Use of the simple (39b) instead of (39a) in the effective index calculation changes the values for g_o in Table I by 4–5 cm^{-1} and w_o by 0.05–0.06 μm , thus degrading the agreement with the planewave method significantly.

The normalized intensity distributions corresponding to three of the cases considered in Table I are shown in Fig. 4.

For $R = -1$ the result converges rapidly with an increasing number of expansion functions [l_{\max} in (52)], in the case of $R = -2$ a shoulder develops for $l_{\max} \geq 4$ and for $R = -3$ a large peak appears for $l_{\max} \geq 6$. This behavior is attributed to the complex parameter a in (52), for increasing negative values of R the real part of a decreases and the imaginary part increases. It is to be noted, however, that a correct value of the width of half intensity is found also for large negative R , but a larger value for l_{\max} is required to give a reliable result. Even for $R = -3$ a reasonable value for the center gain g_o is found for low values (4 or 6) of l_{\max} .

TABLE I

g_0 in cm^{-1}	$R = -3$	-2	-1	0	1
eff index	206.54	182.21	156.49	131.99	118.14
plane wave $l_{\text{max}} = 10$	206.55	182.22	156.50	132.01	118.16
8	206.55	182.22	156.50	132.01	118.16
6	206.55	182.22	156.50	132.01	118.18
4	206.55	182.22	156.51	132.02	118.18
2	206.69	182.33	156.58	132.05	118.20
0	212.13	186.38	159.25	133.52	119.04

w_0 in μm	$R = -3$	-2	-1	0	1
eff index	10.309	9.6861	8.7423	7.3184	6.0361
plane wave $l_{\text{max}} = 10$	10.381	9.7288	8.7606	7.3220	6.0353
8	10.480	9.7505	8.7620	7.3217	6.0352
6	10.217	9.7096	8.7677	7.3250	6.0368
4	10.201	9.5191	8.6664	7.2944	6.0208
2	17.711	12.6050	9.4898	7.5235	6.1516
0	8.946	8.4135	7.6034	6.3744	5.2627

Center gain g_0 and full width at half intensity calculated for $\lambda = 0.9 \mu\text{m}$, $G_{\text{req}} = 50 \text{ cm}^{-1}$, $\alpha = 20 \text{ cm}^{-1}$, $n_1 = 3.6$, $n_2 = 3.4$, $d = 0.2 \mu\text{m}$, $y_0 = 6 \mu\text{m}$.

Inspection of the expansion coefficients shows a behavior similar to the results in Table I of [7], i.e., an increase of l_{max} only gives very small changes of the coefficients A_l . As seen in Fig. 4(c) the consequences of higher values of l_{max} are a better convergence for low y -values and a shift of the peak to larger y -values.

From the calculations it is found that the boundary between convergence [as in Fig. 4(a)] and development of a peak [as in Fig. 4(c)] can be described by the following empirical relation:

$$\Gamma C \approx 0.2. \quad (64)$$

Below the curve given by (64) the planewave method may still be used to calculate center gain and intensity width, but it cannot give correct intensity distributions far from the center. In extreme cases of large negative R values and small values for the thickness d , reliable results cannot be obtained even for $l_{\text{max}} = 10$. The validity of the expansion in (57) has been checked and it was found that the influence of a finite number of terms (i.e., $m = 5$) was negligible except in a few extreme cases where the results were wrong anyway.

The bad agreement between the two methods reported in [7] is probably mainly due to the use of a formula for the gain g_0 somewhat different from (42).

As noted in [19] the far field in the $x = 0$ plane is easily computed. We have considered the three examples from [19] which can be described by $\Delta n = 0.1$, $d = 0.1 \mu\text{m}$, $R = -0.35$, -0.70 , and -1.05 . As these values lie below the $\Gamma C = 0.2$ curve shown in Fig. 5, it is not surprising that the same convergence problems as seen for the near field occurs. The shape

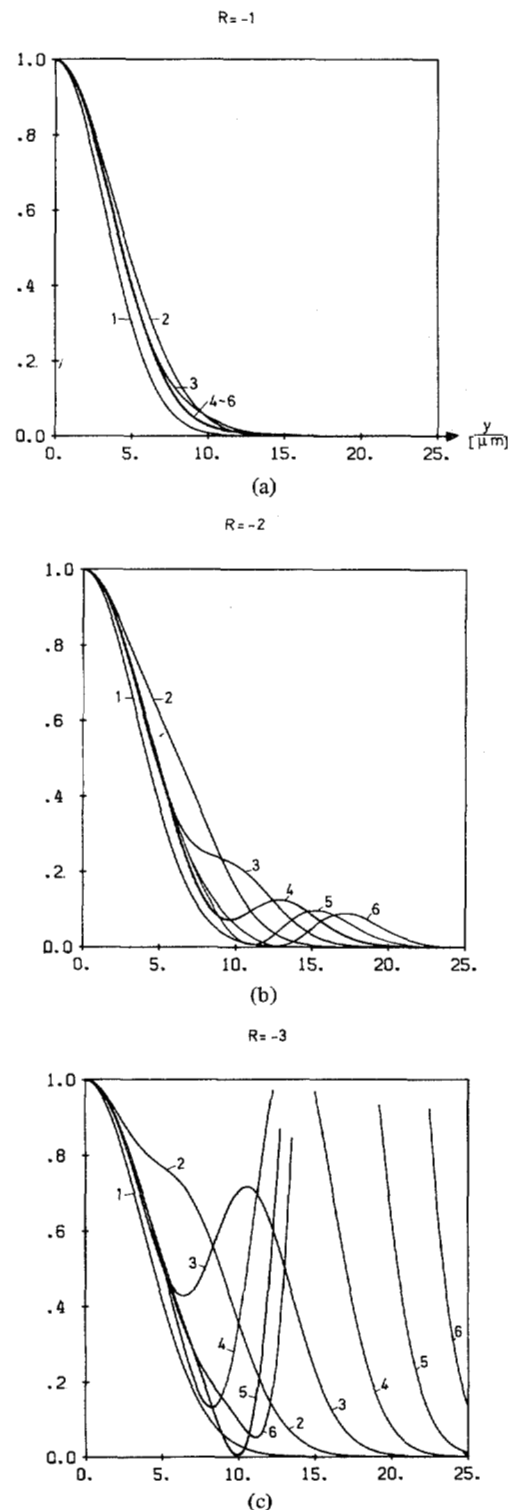


Fig. 4. Intensity distributions for three of the cases considered in Table I: (a) $R = -1$, (b) $R = -2$, and (c) $R = -3$. In all three cases the intensities are calculated for $l_{\text{max}} = 0, 2, 4, 6, 8$, and 10 .

of the far fields depends on the number of expansion terms in (52), and the shoulders reported in [19] are numerical. For an infinite parabolic profile in the active layer the effective index method always gives Gaussian far fields. For other profiles such as $(\cosh^2)^{-1}$ [20], [21], Lorentzian [21], and truncated parabolic [21], more peculiar far fields appear.

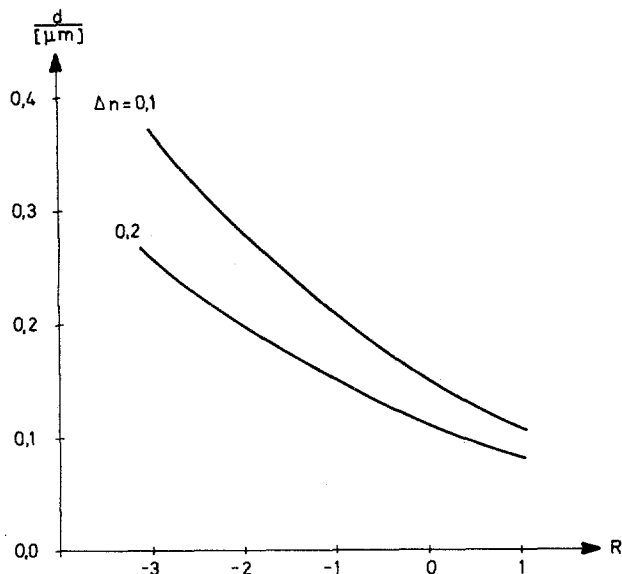


Fig. 5. The relation $\Gamma C = 0.2$, where Γ is defined in (10) and C in (40), plotted as function of the parameter R for $n_1 = 3.6$ and two values of $\Delta n = n_1 - n_2$.

V. CONCLUSION

Application of the effective index method to the special case of a laser structure with a parabolic profile in the active layer results in simple formulas for center gain and half intensity width. These are compared with results obtained by a numerical planewave expansion method, and very good agreement is found for all cases where the numerical method gives reliable results. This agreement does not mean that the parabolic profile is a good description of the guiding in the laser, only that for this particular case the effective index method is both efficient and precise. The planewave expansion is only of practical interest for profiles where the field in the active layer and its Fourier transform can be expressed analytically. The good agreement for the parabolic case, however, gives good evidence for the validity of the effective index method also for other profiles.

The small difference in the expression used for the required gain in case of a broad active region (39a) and the usual expression (39b) is only of theoretical interest and will not affect practical laser design.

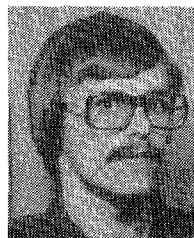
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